Measuring Mass Splittings in Baryon SU(2) Multiplets

SIMONE BACCHIO, CONSTANTIA ALEXANDROU, ANTONIO EVANGELISTA, CHRISTOS IONA, CHRISTIAN SCHNEIDER

UNIVERSTIY OF CYPRUS

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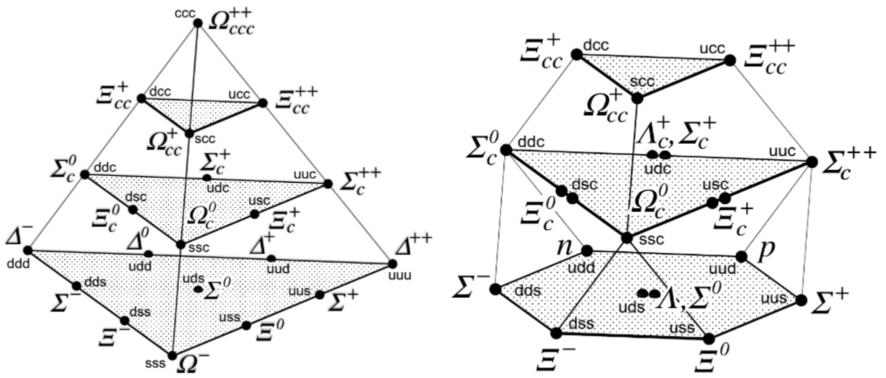

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- Apply the RM123 framework to baryon mass splittings. G. M. de Divitiis et al. Phys. Rev. D 87.11 (2013), p. 114505
- We measure all 40 baryons of $J = \frac{1}{2}$ and $J = \frac{3}{2}$
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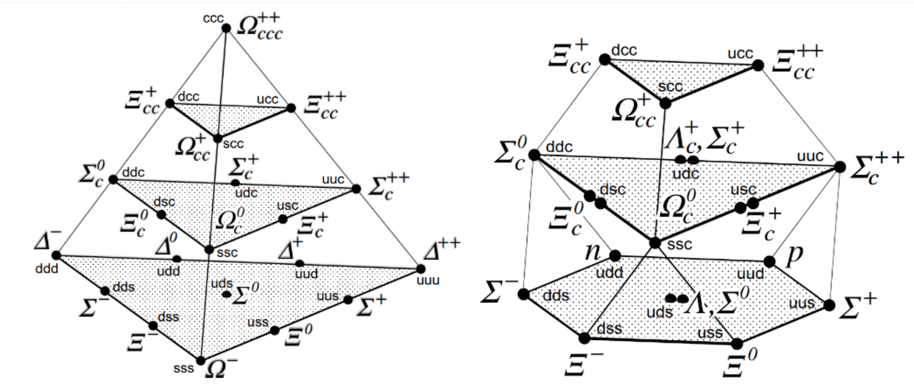

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Goal: Implement a first-principles framework for computing baryon mass splittings

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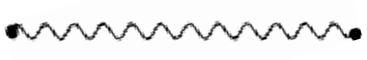
Taking into account isospin breaking effects: $m_u \neq m_d$ &  $\propto e^2$

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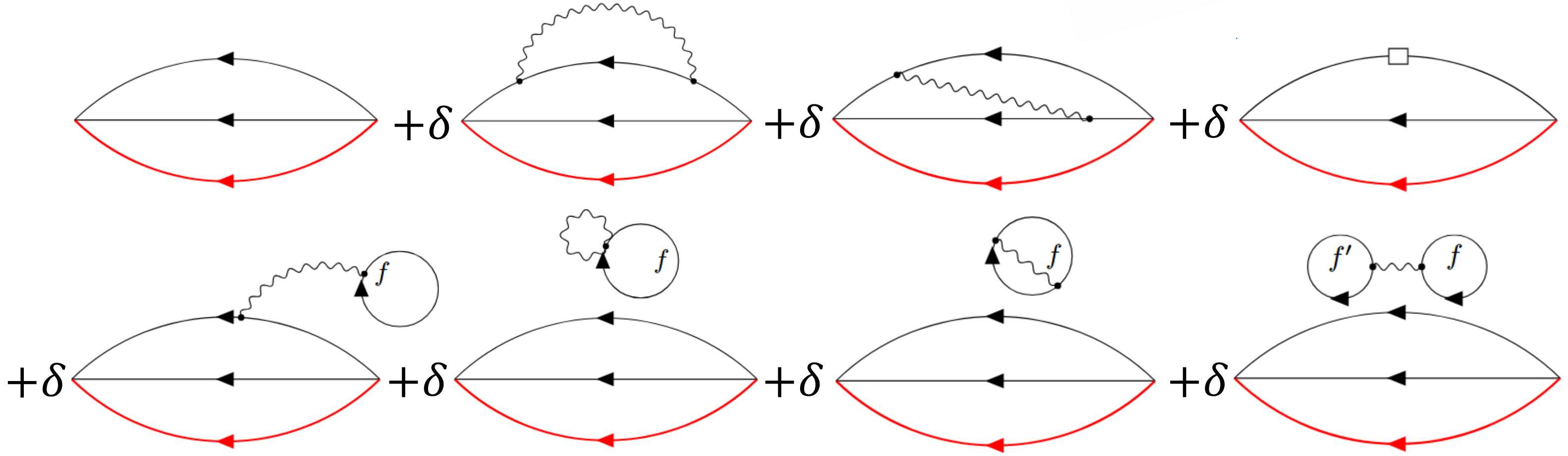

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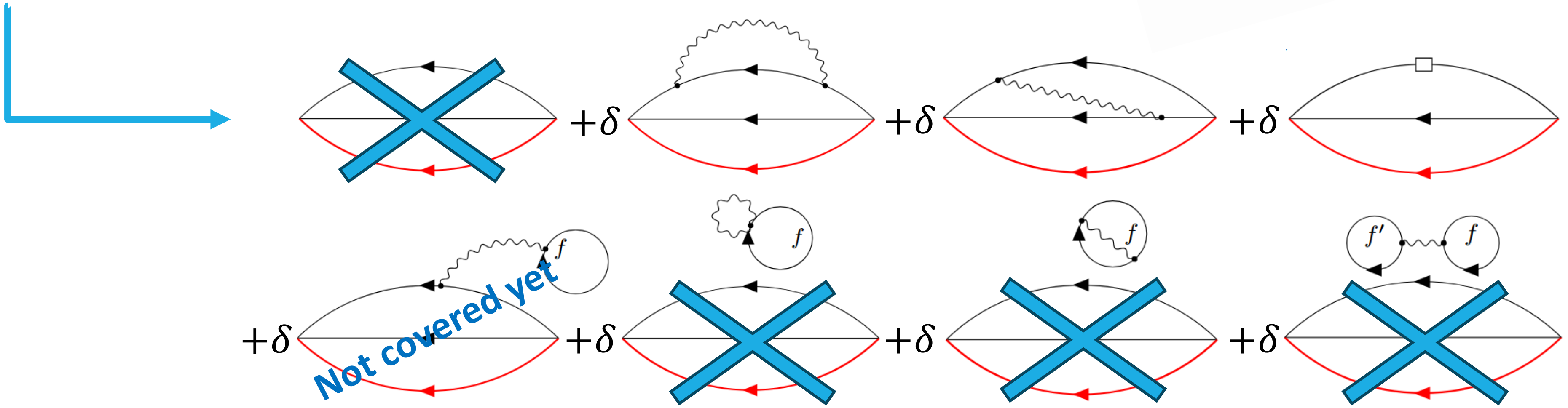

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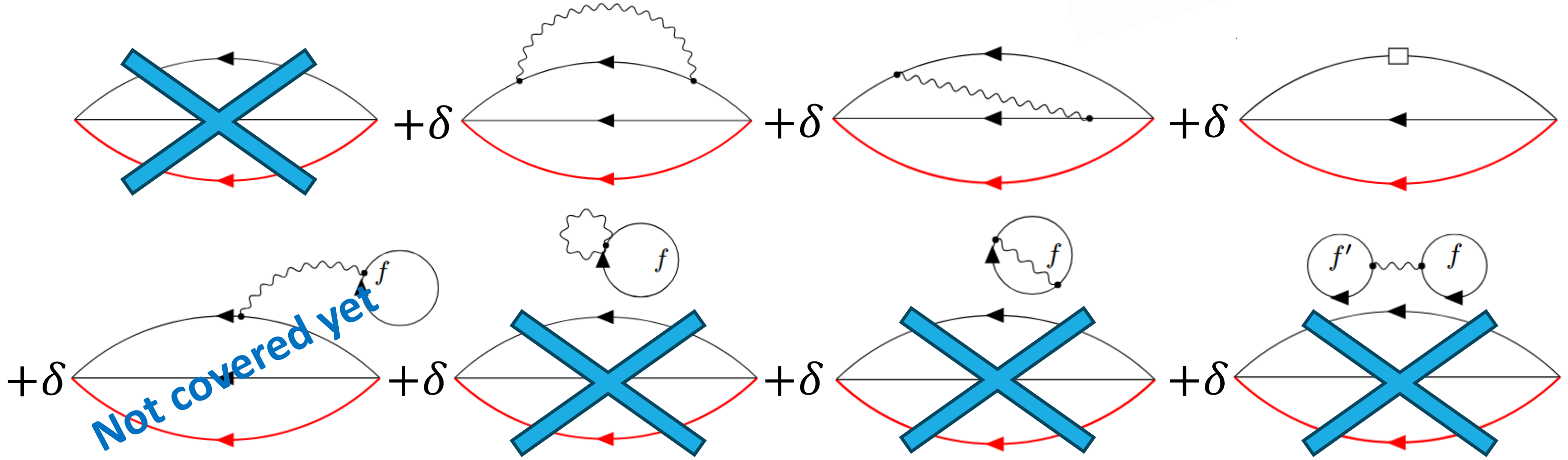

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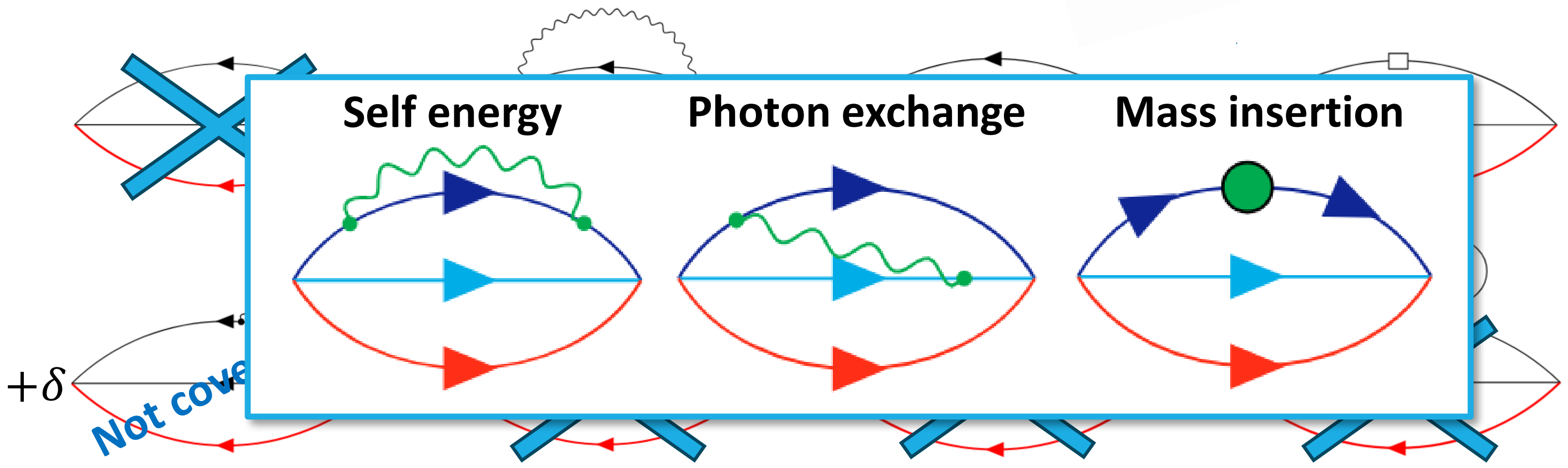

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Perturbative expansion of the path integral

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Expand around the isospin-symmetric theory

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_0 + \sum_i \left(\theta_i - \theta_i^0\right) \frac{\partial \langle \mathcal{O} \rangle}{\partial \theta_i}\bigg|_{\vec{\theta}=\vec{\theta}^0} + \mathcal{O}\left(\left(\vec{\theta} - \vec{\theta}^0\right)^2\right)$$

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Ensemble	$\left(\frac{L}{a}\right)^3 \times \left(\frac{T}{a}\right)$	a [fm]	m_π [MeV]	$m_\pi L$
cB211.072.64 (B64)	$64^3 \times 128$	0.07948(11)	140.2(2)	3.62
cD211.054.96 (D96)	$96^3 \times 192$	0.056850(90)	140.8(2)	3.90

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A. Evangelista et al. [ETMC], PoS LATTICE2024 (2025) 260.

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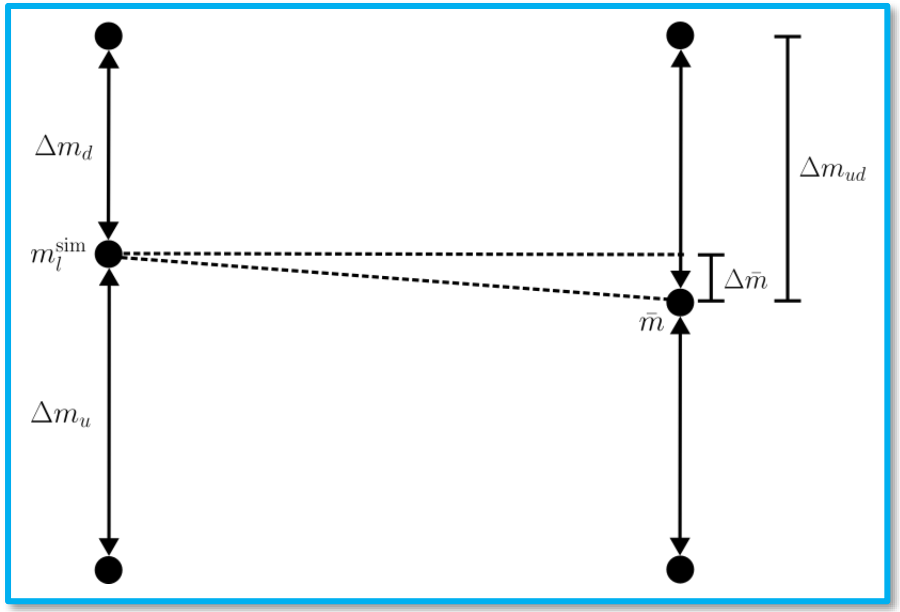

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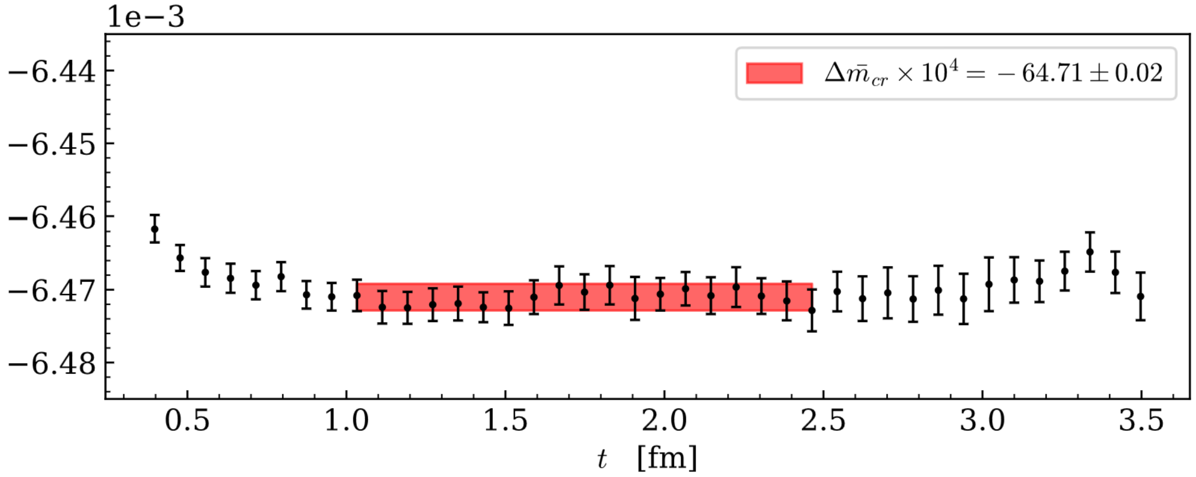

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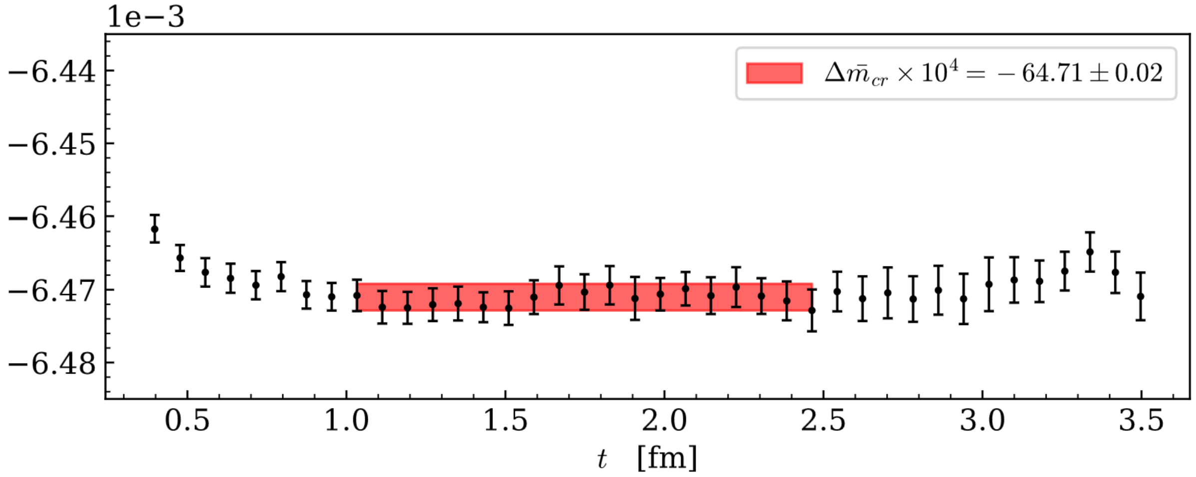

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Ensemble	$a\,\Delta\bar{m}_{\rm cr} \times 10^4$	$a\,\Delta\bar{m} \times 10^4$	$a\,\Delta m_{ud} \times 10^4$
B64	-64.71 ± 0.02	-0.196 ± 0.001	2.56 ± 0.02
D96	-64.0745 ± 0.0047	-0.0757 ± 0.0007	1.8542 ± 0.0076

Numerical realization of isospin-breaking corrections

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Implementation strategy

Apply a QED phase to the selected propagator:

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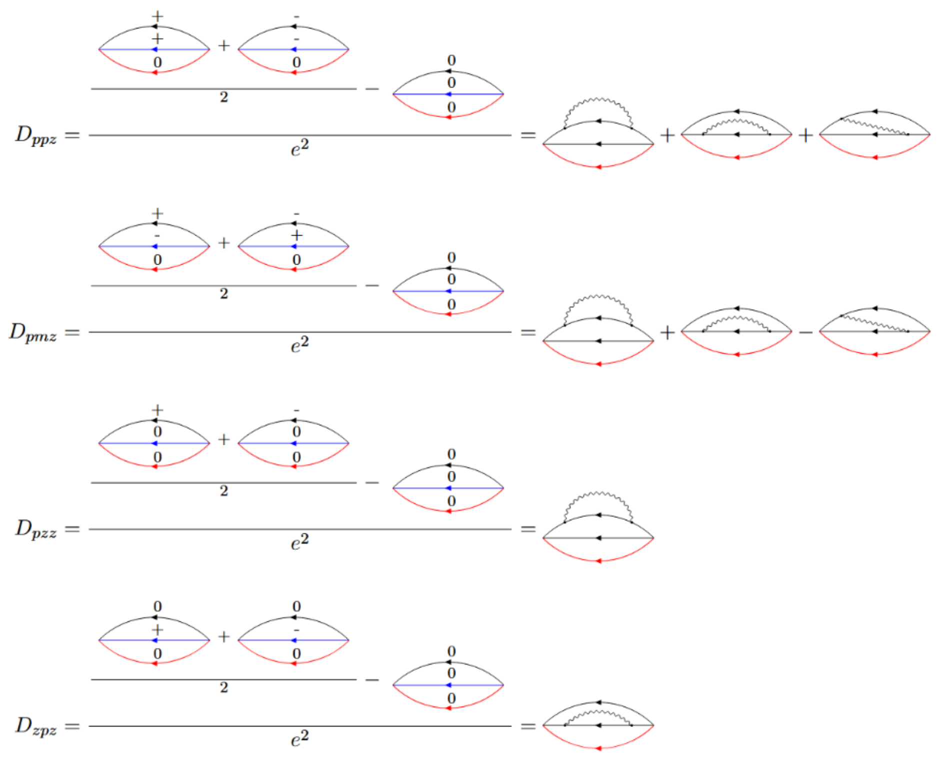
Example for three distinct quark flavors



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$$\frac{\Delta C'(t)}{C(t)} = \underbrace{\frac{\Delta A}{A}}_{\text{overlap}} - \underbrace{t\,\Delta M}_{\text{slope}}$$

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$$\frac{\Delta C'(t)}{C(t)} = \underbrace{\frac{\Delta A}{A}}_{\text{overlap}} - \underbrace{t\,\Delta M}_{\text{slope}}$$

$$\boxed{\Delta M = -\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\Delta C'(t)}{C(t)}\right]}$$

→ Mass shift extracted from time derivative

Determining mass corrections

For a first-order corrected correlator

$$C'(t) = (A + \Delta A)\, e^{-(M+\Delta M)t}$$

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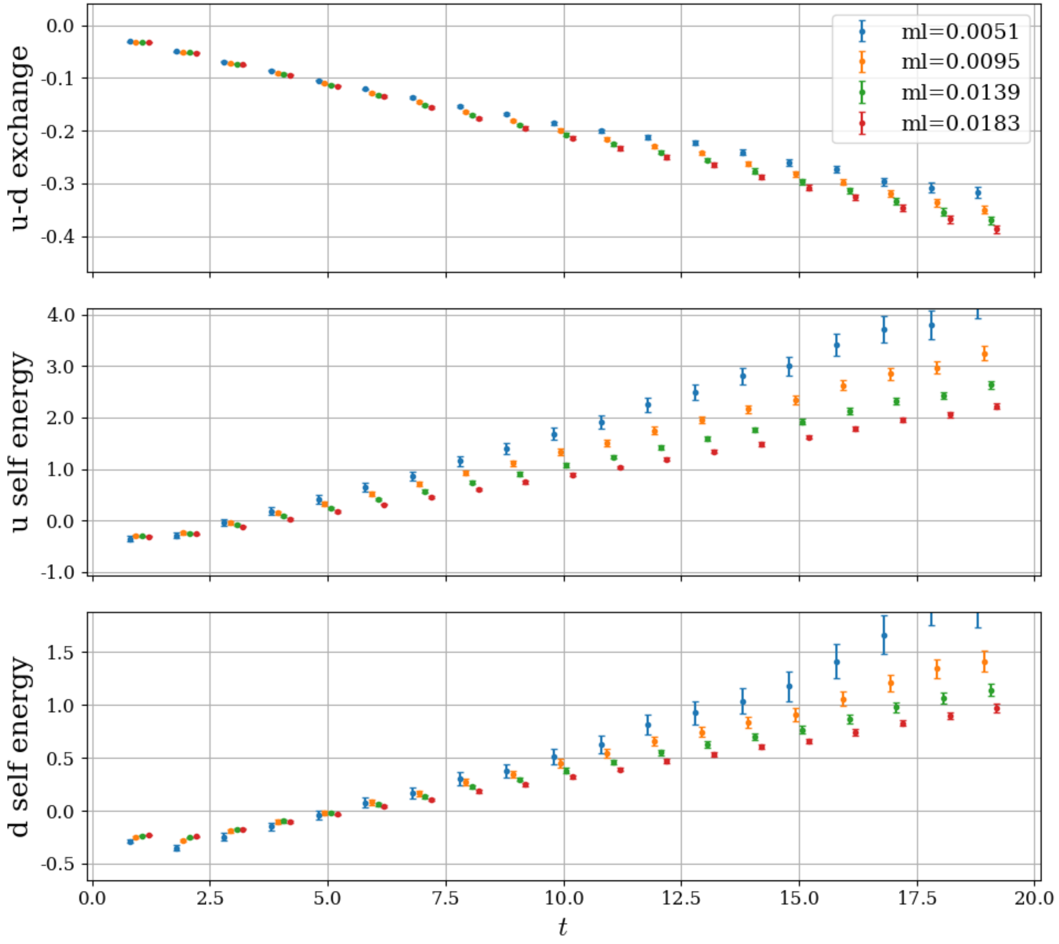
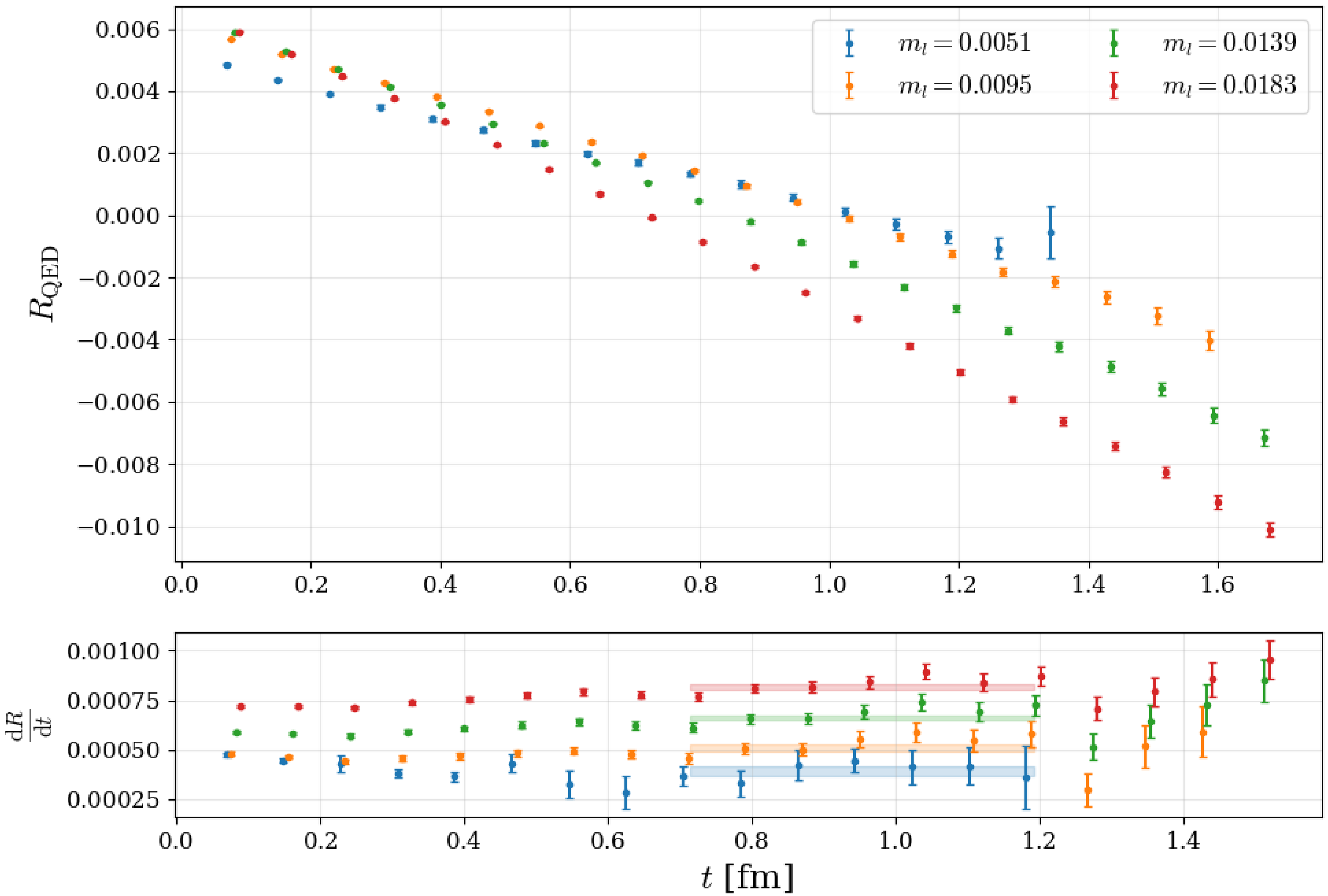


Slope extraction



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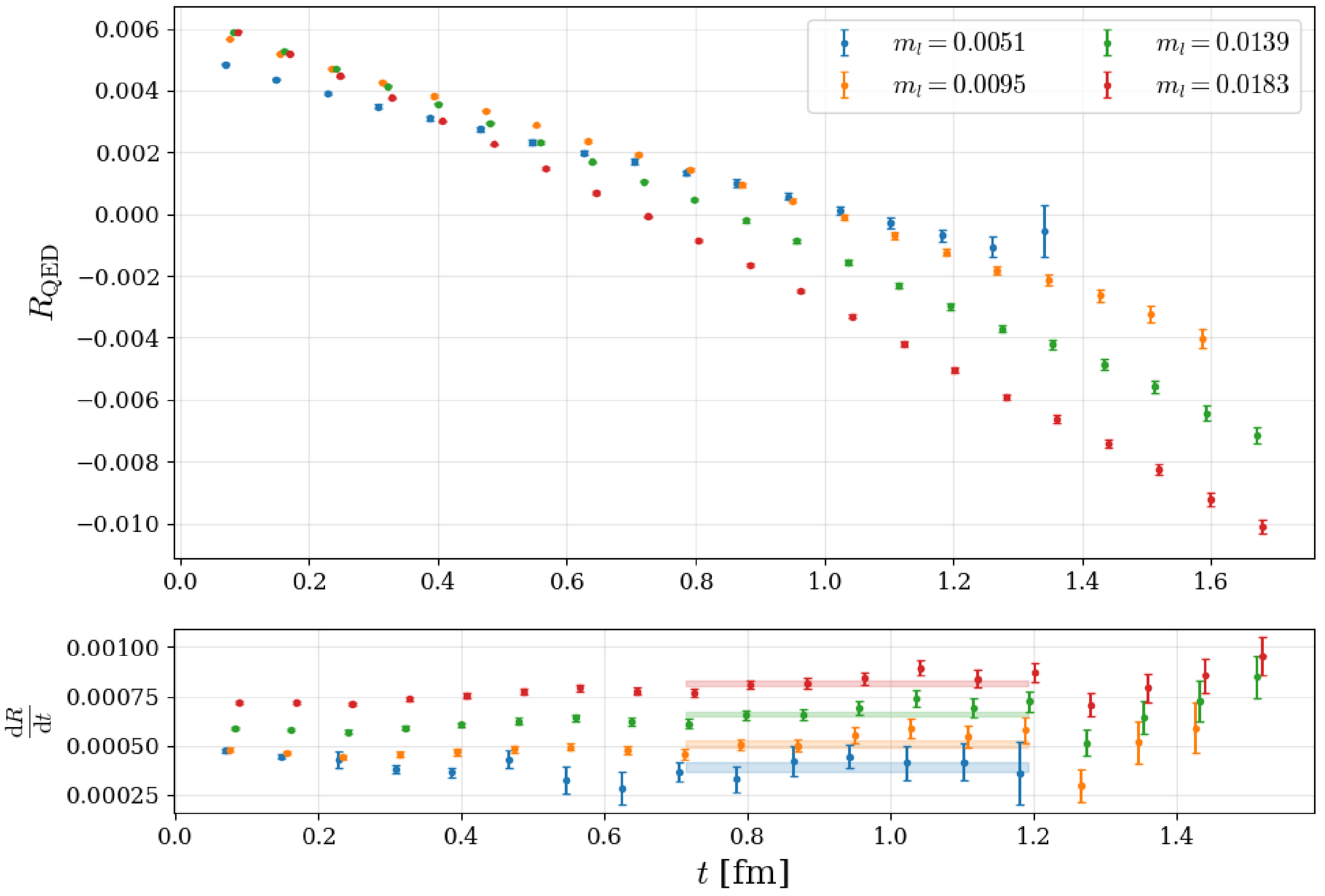
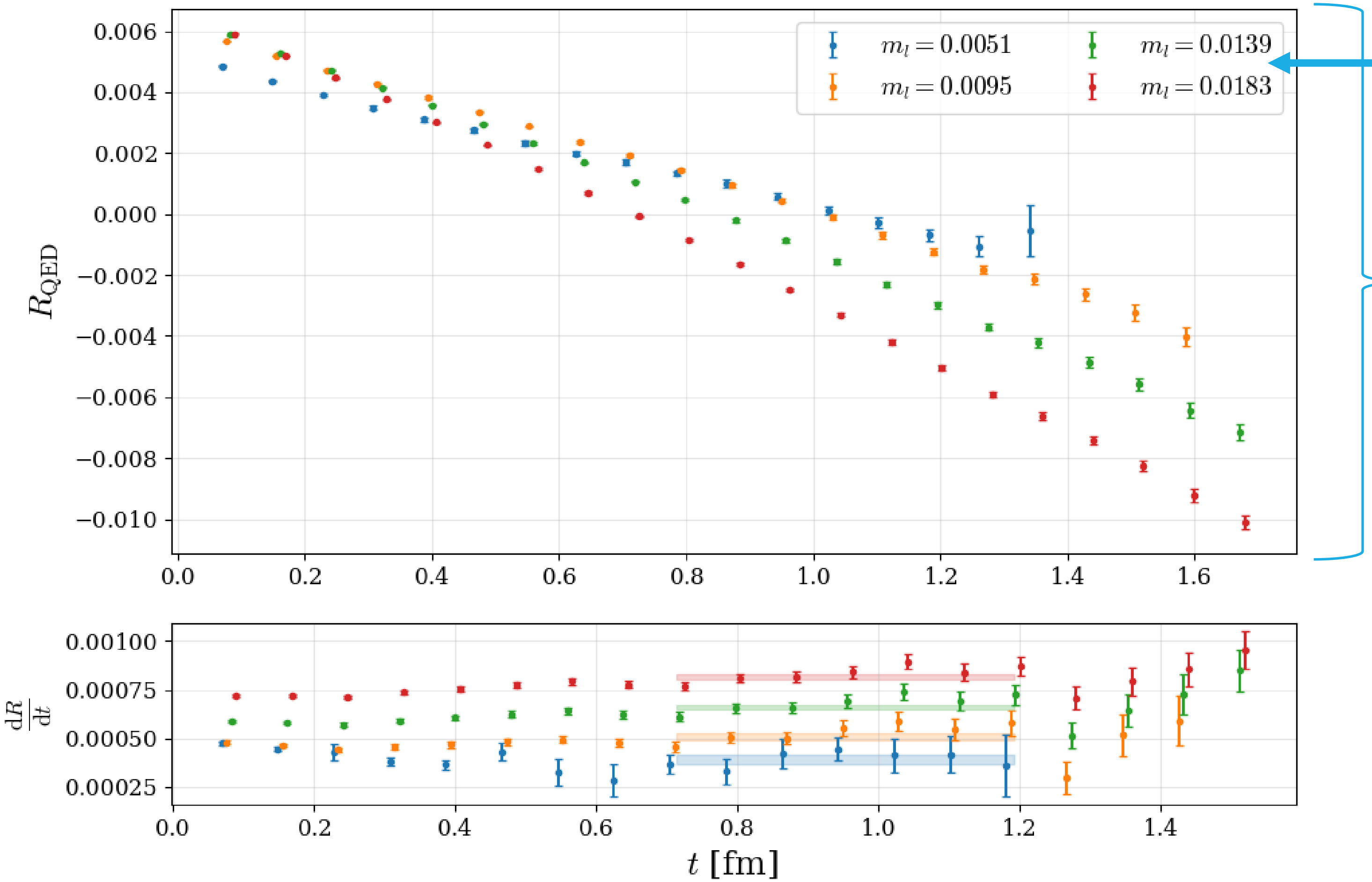


Combined EM + CM = QED correction (highly correlated)

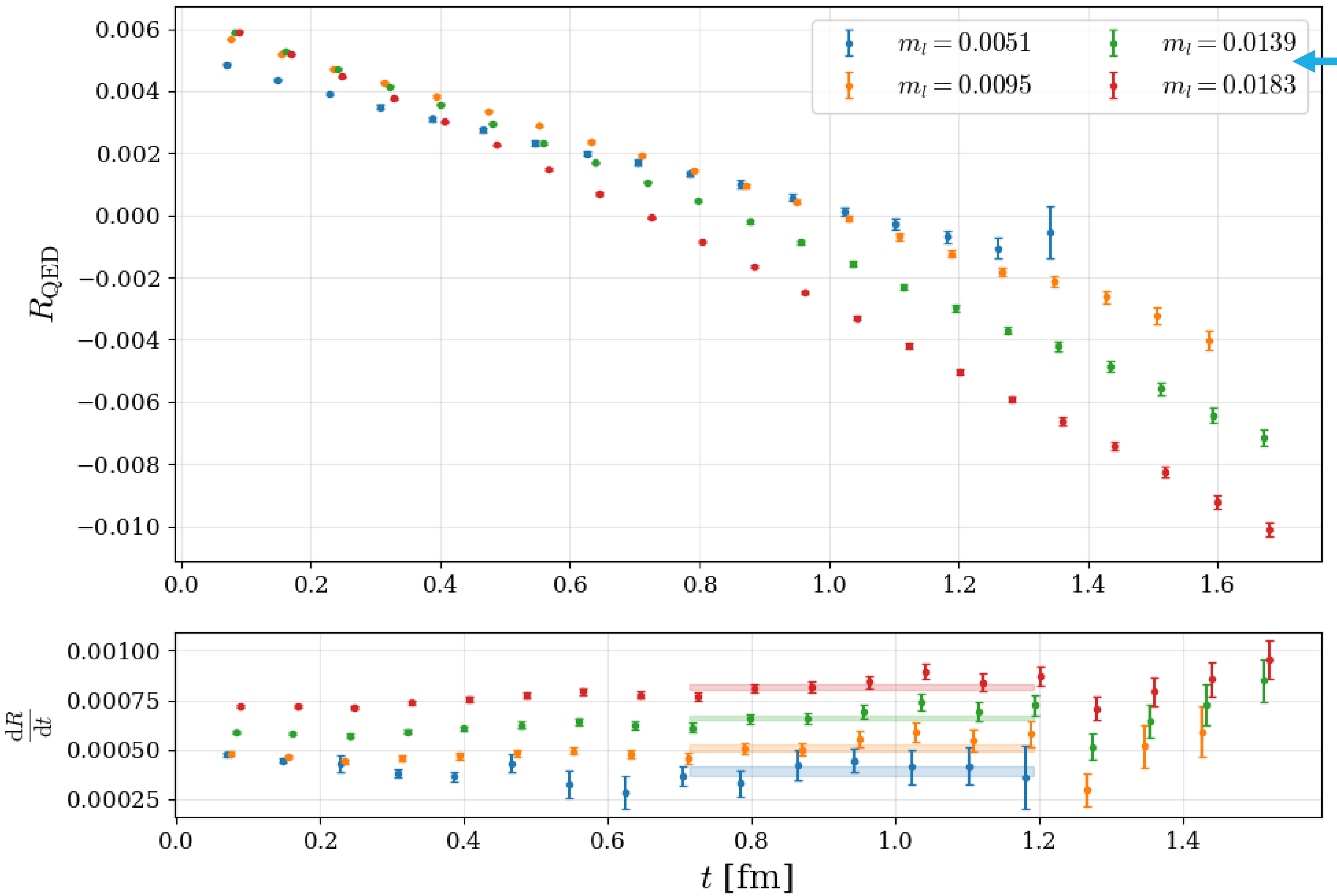
Slope extraction



Multiple valence light quark masses

Combined EM + CM = QED correction (highly correlated)

Slope extraction

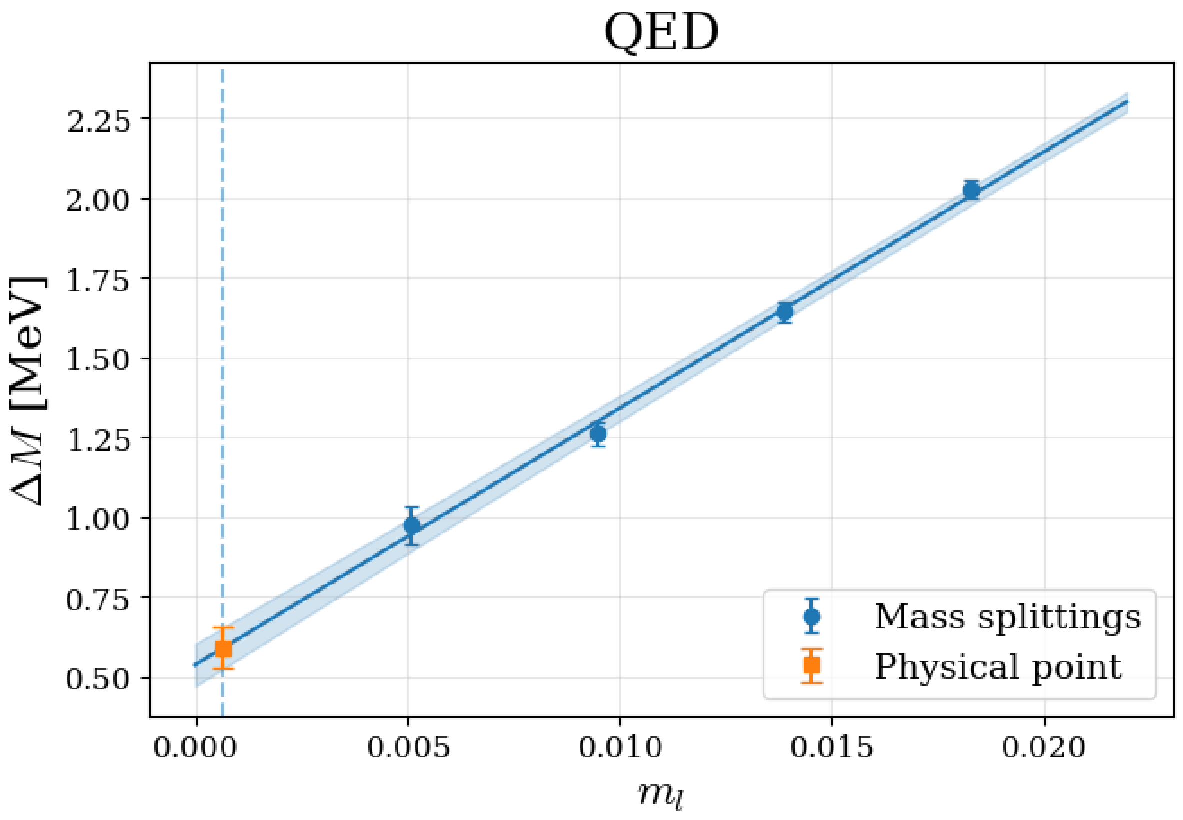


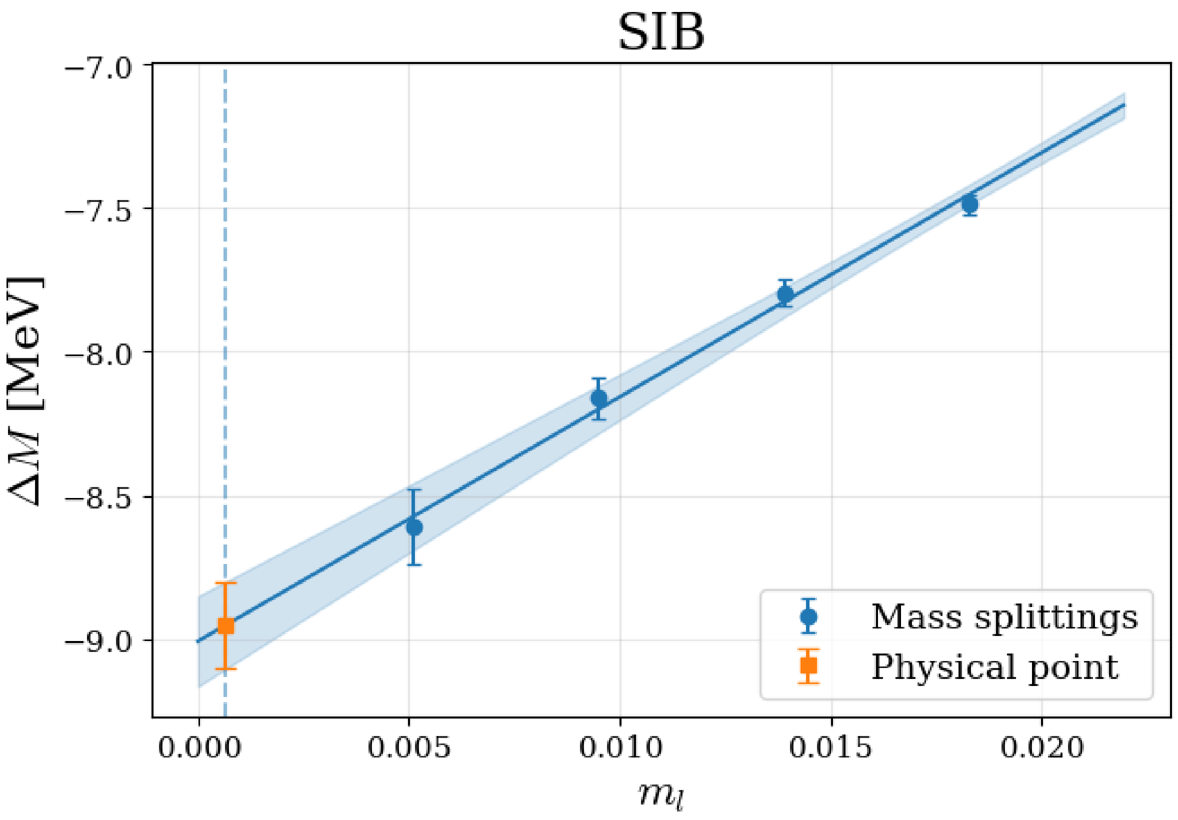
Multiple valence light quark masses

Combined EM + CM = QED correction (highly correlated)

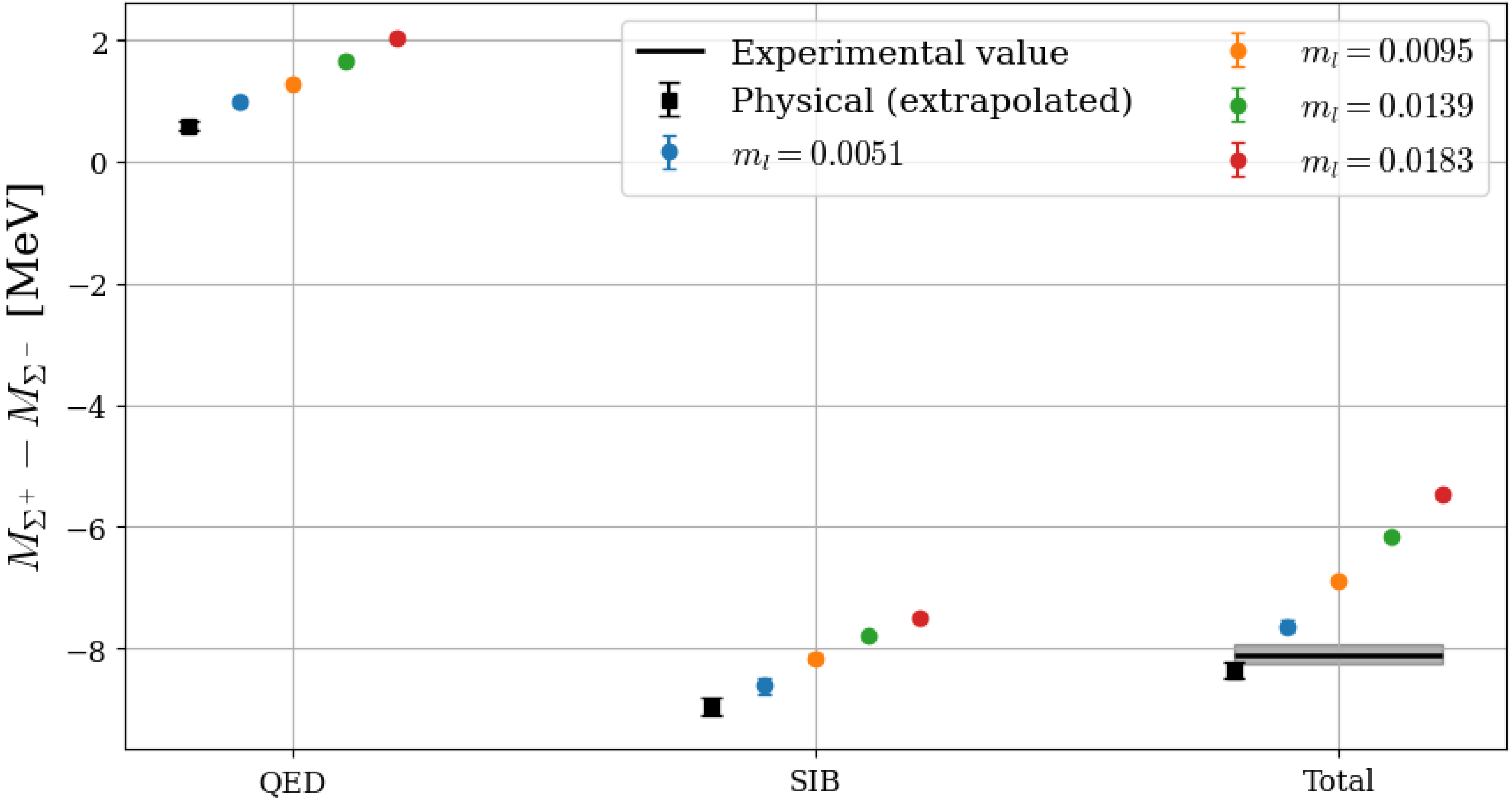
Time derivative of the ratio

Physical-point extrapolation





Example splitting result



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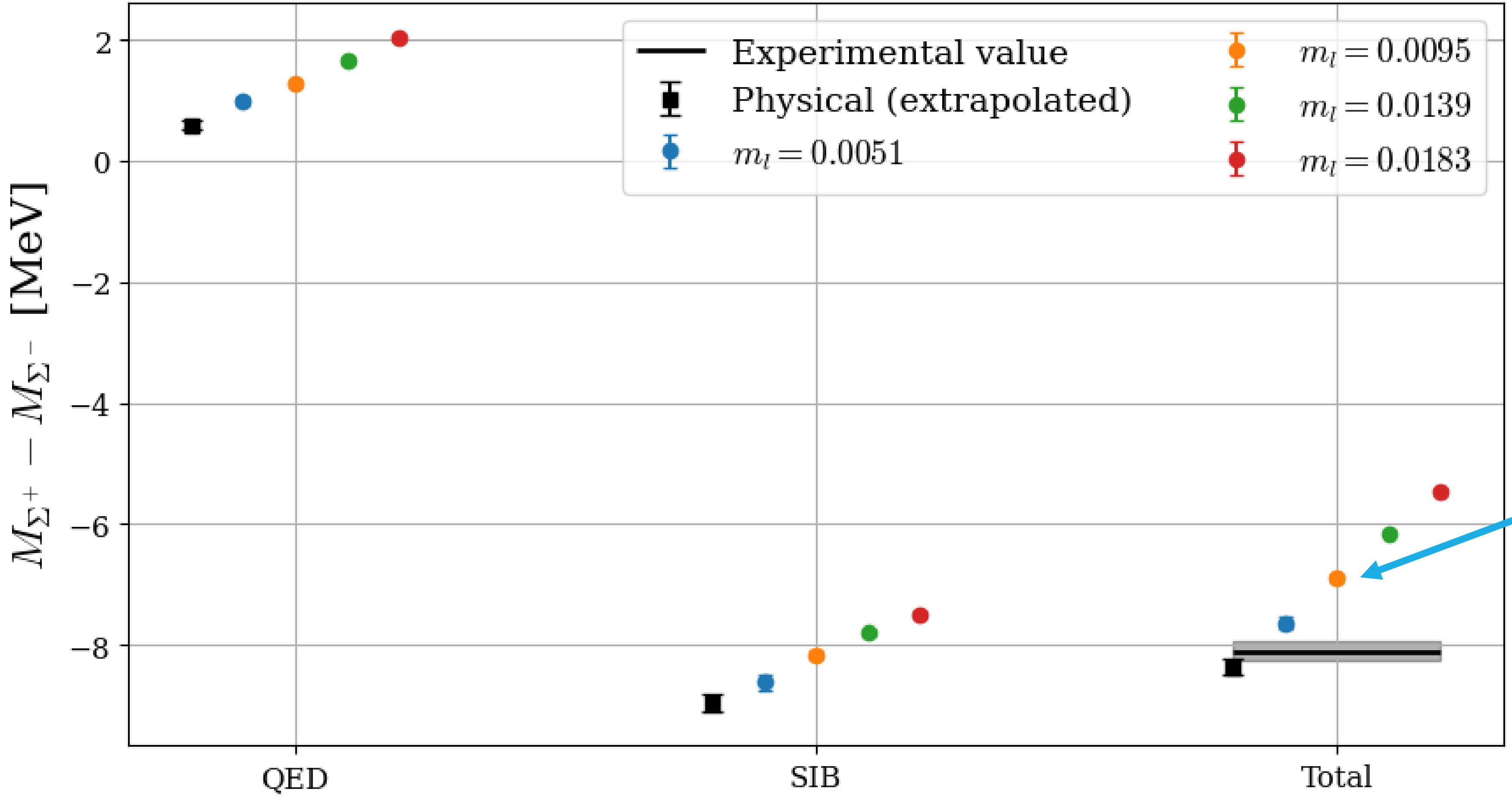
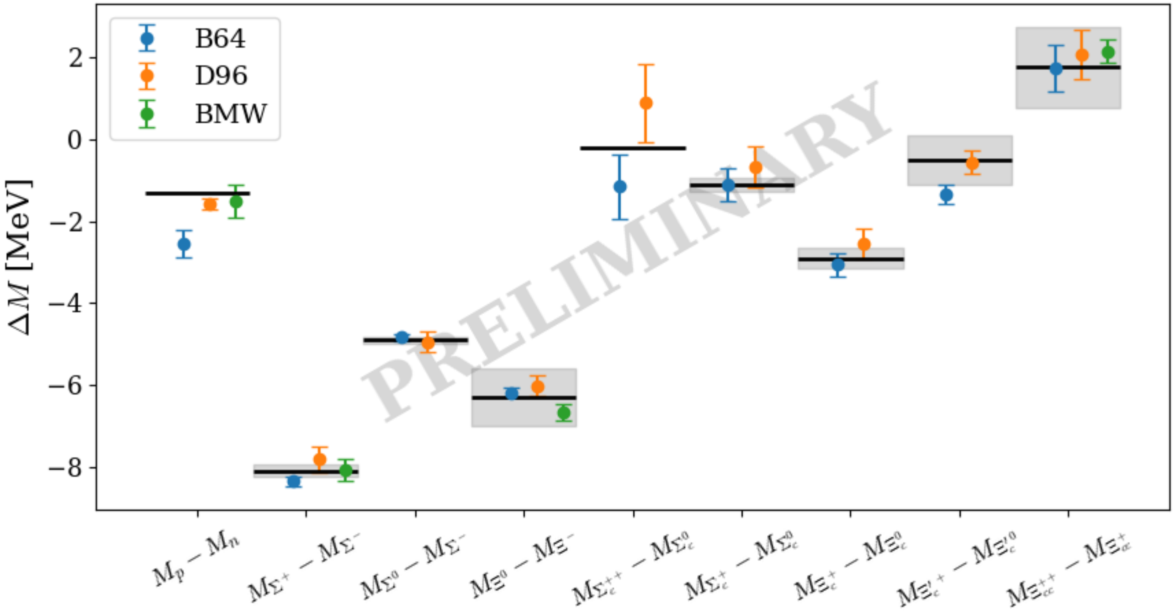


$$\mathrm{FV} = \left(q_{B_1}^2 - q_{B_2}^2\right)\alpha\frac{\kappa}{2L}$$

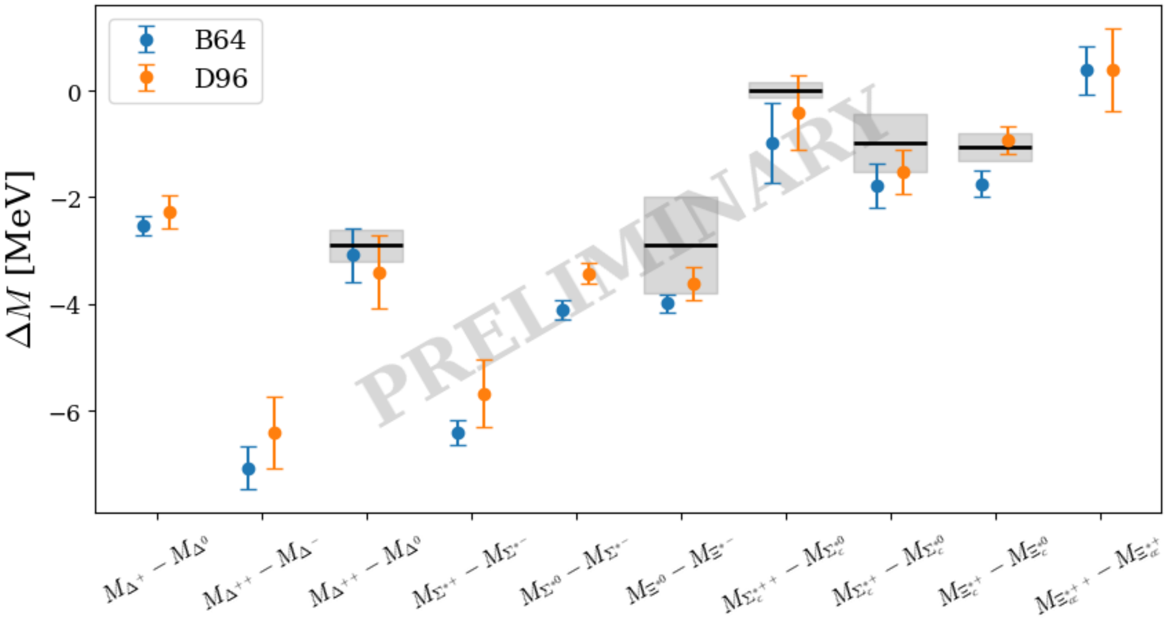
Finite-volume correction applied!

Preliminary mass splittings for the SU(2) multiplets

J=1/2



J=3/2



The framework reproduces the qualitative pattern of the experimental splittings!

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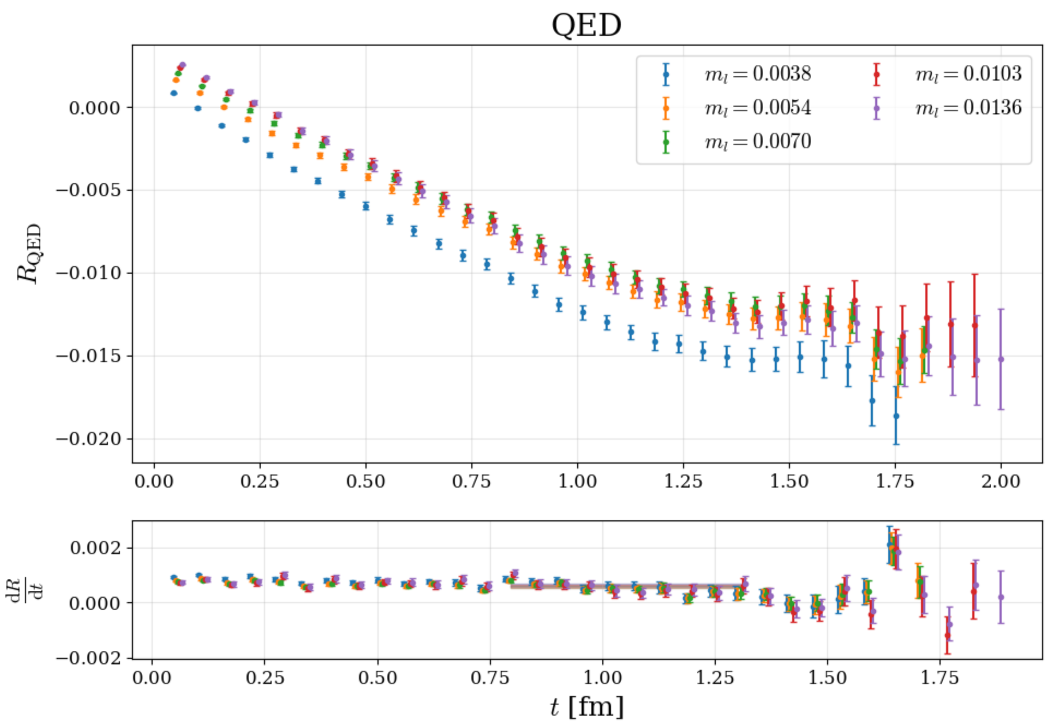
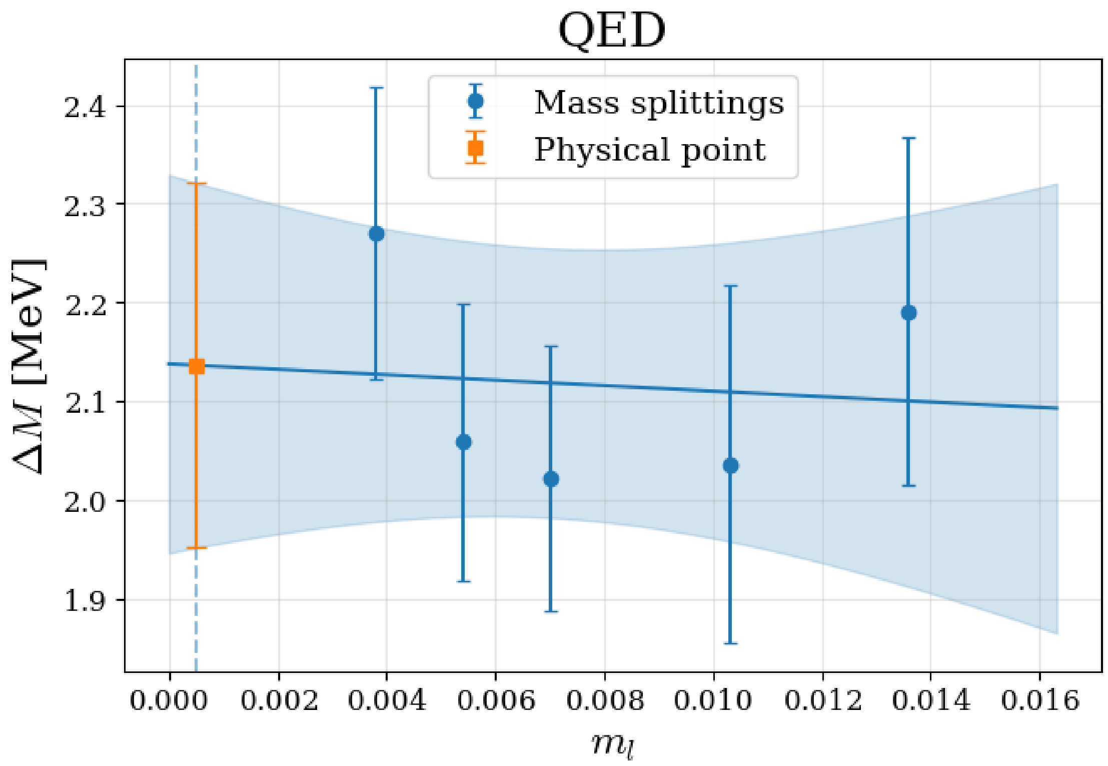
- Future work: sea-valence contribution

Thank you for your attention!

Example: Analysis with less favorable physical point extrapolation





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