

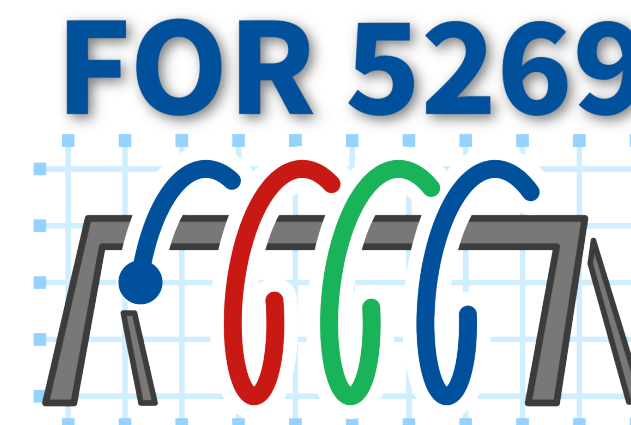
Nucleon sigma terms at $m_\pi = 222$ MeV with a variational analysis from lattice QCD

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DESY Zeuthen Particle Physics Theory

In collaboration with:

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Motivation

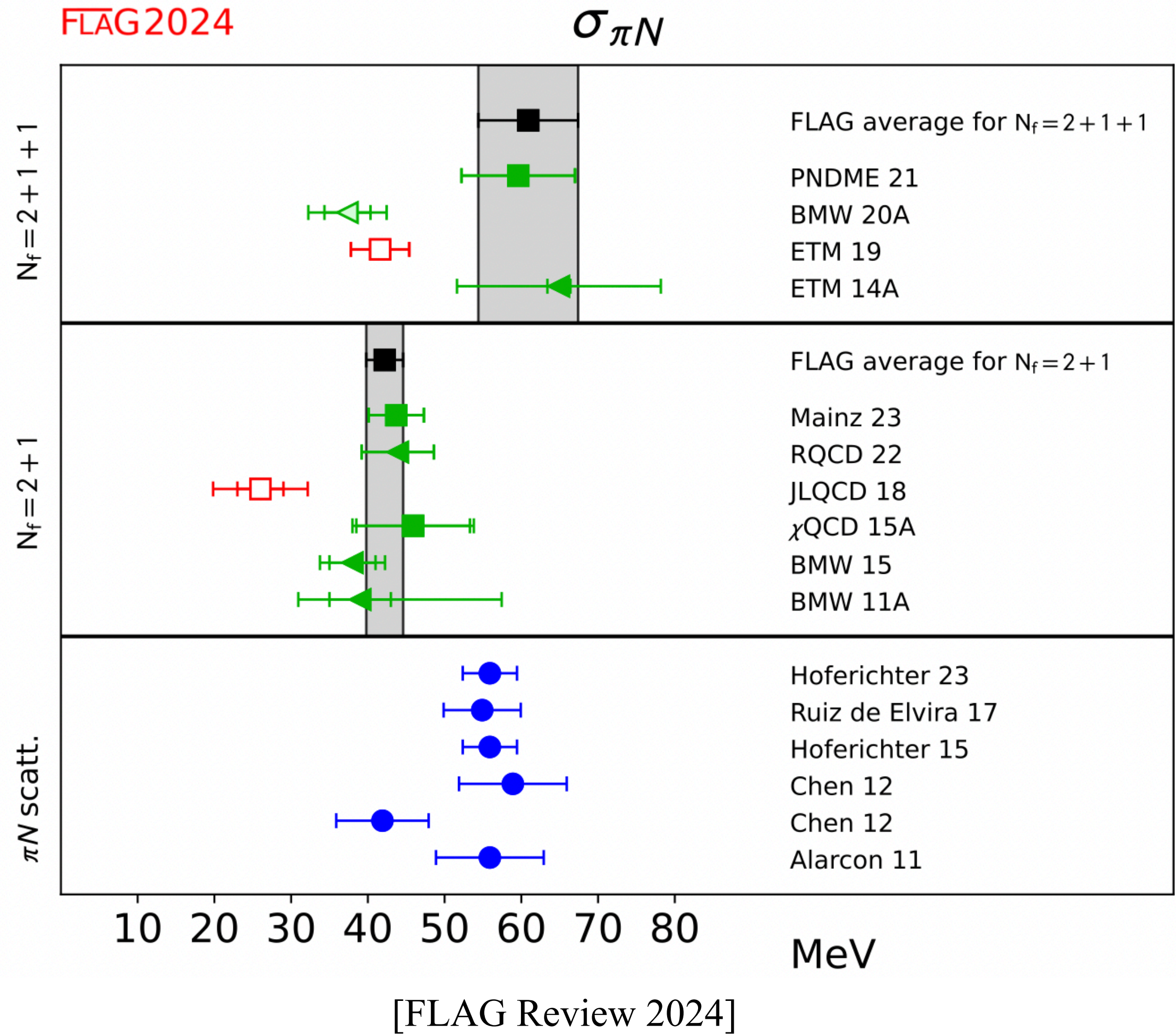
- Relevant quantity for Standard Model and Beyond Standard Model Physics
- Current tension between pheno and most lattice determinations of $\sigma_{\pi N}$
- Excited-state contamination (ESC) must be properly accounted for precise determinations

R. Gupta et al.
[Phys.Rev.Lett. 127, 242002 (2021)]

L. Barca, G. Bali, S. Collins
[Phys.Rev.D 111 (2025) 3, L031505]

L. Barca *[Phys.Rev.D 112 (2025) 9, L091503]*

[G. Bali Fri 11:00]



Sigma terms and direct method

$$\sigma_{\pi N} = \sigma_{uN} + \sigma_{dN} \qquad \sigma_{qN} = m_q \langle N | S_q | N \rangle \qquad S_q = qI\bar{q}$$

$$\begin{aligned} \langle O_N(t) S_q(\tau) \bar{O}_N(0) \rangle &= \sum_{n',n} \langle 0 | O_N | n' \rangle \langle n' | S_q | n \rangle \langle n | \bar{O}_N | 0 \rangle \frac{e^{-E'_n(t-\tau)} e^{-E_n\tau}}{4E'_n E_n V_{n'} V_n} \quad (\text{after VEV-subtraction}) \\ &= |Z_N|^2 \langle N | S_q | N \rangle e^{-E_N t} + \dots \text{ (ESC)} \end{aligned}$$

At large t , $\langle N | \bar{q}q | N \rangle$ can be reliably determined as ESC is exponentially suppressed.

Unfortunately, StN hinders the analysis at large t, τ . Need to account for ESC at moderate t .

Some contributions from multi-particle states are not suppressed by the spatial volume.

$$R_q(t, \tau) = \frac{\langle O_N(t) S_q(\tau) \bar{O}_N(0) \rangle}{\langle O_N(t) \bar{O}_N(0) \rangle} = \underbrace{\langle N | \bar{q}q | N \rangle}_{(g_S^q)} + \dots \text{ (ESC)}$$

Current-enhanced operators/states in $\langle O_H(\vec{p}', t) J(\vec{q}, \tau) \bar{O}_H(\vec{p}, 0) \rangle$

L. Barca [*Phys.Rev.D* 112 (2025) 9, L091503]

Solve GEVP: $C(\vec{p}, t) V(\vec{p}, t; t_0) = C(\vec{p}, t_0) \Lambda(\vec{p}, t; t_0) V(\vec{p}, t; t_0)$ $\mathbb{B}_H = \left\{ O_H, O_{HM}, O_{HMM}, O_{HM_1}, O_{HM_2}, \dots \right\}$

GEVP-improved operators: $\Omega_H = \sum_{O_i \in \mathbb{B}_H} v_i^H O_i$ $V(\vec{p}, t; t_0) = (\vec{v}^H, \dots)$

GEVP-improved 3-pts: $\langle \Omega_H(\vec{p}', t) J(\vec{q}, \tau) \Omega_H^\dagger(\vec{p}, 0) \rangle = \sum_{O_i, O_j \in \mathbb{B}_H} v_i^H(\vec{p}', \tilde{t}_2; t_0) \langle O_i(\vec{p}', t) J(\vec{q}, \tau) O_j^\dagger(\vec{p}, 0) \rangle v_j^H(\vec{p}, \tilde{t}_1; t_0)$

Terms like $\langle O_{HM}(\vec{p}', t) J(\vec{q}, \tau) O_H^\dagger(\vec{p}, 0) \rangle$ are much larger at short-intermediate distances than the others if they contain (non-vanishing) quark-line disconnected diagrams, i.e., when (Direct diagrams in spectroscopy calculations)

$$\langle O_H(\vec{p}'_H, t) O_H^\dagger(\vec{p}, 0) \rangle, \quad \langle O_M(\vec{p}'_M, t) J(\vec{q}, \tau) \rangle \neq 0 \quad (HM \text{ are then “current-enhanced”})$$

They grow with the volume because of translational invariance (backup).

Current-enhanced states: another viewpoint

$$\vec{p} = \vec{p}' = \vec{q} = \vec{0} \quad \underline{\text{Infinite volume}}$$

$$\begin{aligned} \langle O_N(t) J(\tau) \bar{O}_N(0) \rangle &= |Z_N|^2 \langle N | J | N \rangle e^{-m_N t} \\ &+ Z_{N'M} Z_N \langle N'M | J | N \rangle e^{-E_{N'M}(t-\tau)} e^{-E_N \tau} + \dots \end{aligned}$$

Factorisation (*spectator-disconnected* + *connected*):

$$\langle N'M | J | N \rangle = \langle N' | N \rangle \langle M | J | 0 \rangle + \mathcal{M}_{conn.}(N'M \leftarrow N)$$

$$2E_N(2\pi)^3 \delta(p_{N'} - p) \langle M | J | 0 \rangle + \mathcal{M}_{conn.}(N'M \leftarrow N)$$

Current-enhanced states: another viewpoint

$\vec{p} = \vec{p}' = \vec{q} = \vec{0}$ Infinite volume

$$\langle O_N(t) J(\tau) \bar{O}_N(0) \rangle = |Z_N|^2 \langle N | J | N \rangle e^{-m_N t} \\ + Z_{N'M} Z_N \langle N'M | J | N \rangle e^{-E_{N'M}(t-\tau)} e^{-E_N \tau} + \dots$$

Finite volume ($V = T \times L^3$)

$$\langle O_N(t) J(\tau) \bar{O}_N(0) \rangle_L = |Z_N(L)|^2 \langle N | \mathcal{J} | N \rangle_L e^{-m_N t} \\ + Z_{N'M}(L) Z_N(L) \langle N'M | J | n \rangle_L e^{-E_{N'M}(t-\tau)} e^{-E_N \tau} + \dots$$

Factorisation (sp) $(2\pi)^3 \delta(\vec{p}_{N'} - \vec{p}) \rightarrow \textcolor{red}{L^3} \delta_{\vec{p}_{N'}, \vec{p}} \text{ (non-interacting + interact.):}$

$$\langle N'M | J | N \rangle = \langle N' | N \rangle \langle M | J | 0 \rangle + \mathcal{M}_{conn.}(N'M \leftarrow N)$$

$$2E_N (2\pi)^3 \delta(p_{N'} - p) \langle M | J | 0 \rangle + \mathcal{M}_{conn.}(N'M \leftarrow N)$$

$$\langle N'M | J | N \rangle_L =$$

$$(2E_N)^2 \textcolor{red}{L^3} \delta_{p_{N'}, p} \langle M | J | 0 \rangle + \mathcal{M}_{int.}^V(N'M \leftarrow N)$$

States $\langle N'M |$ with the same QN and momentum as $\langle N |$ and with $\langle M | J | 0 \rangle \neq 0$ yield large contributions at short-intermediate distances compared to other excited states.

Current-enhanced states for $\sigma_{\pi N}$

$$\langle O_N(t) q\bar{q}(\tau) \bar{O}_N(0) \rangle_L = |Z_N|^2 \langle N | q\bar{q} | N \rangle_L e^{-m_N t} + Z_{N'M} Z_N \langle N'M | q\bar{q} | N \rangle_L e^{-E_{N'M}(t-\tau)} e^{-E_N \tau} + \dots$$

$$\langle N'M | \bar{q}q | N \rangle_L = (2m_N)^2 L^3 \delta_{p_{N'},0} \langle M | \bar{q}q | 0 \rangle + \mathcal{M}_{int.}^V(N'M \leftarrow N)$$

$$\langle M | \bar{q}q | 0 \rangle \neq 0$$

$\langle M |$ must have σ -like QN: Large ESC from $N\sigma / N(\pi\pi)_{L=0}$

Current-enhanced states for $\sigma_{\pi N}$

$$\langle O_N(t) q\bar{q}(\tau) \bar{O}_N(0) \rangle_L = |Z_N|^2 \langle N | q\bar{q} | N \rangle_L e^{-m_N t} + Z_{N'M} Z_N \langle N'M | q\bar{q} | N \rangle_L e^{-E_{N'M}(t-\tau)} e^{-E_N \tau} + \dots$$

$$\langle N'M | \bar{q}q | N \rangle_L = (2m_N)^2 L^3 \delta_{p_{N'},0} \langle M | \bar{q}q | 0 \rangle + \mathcal{M}_{int.}^V(N'M \leftarrow N)$$

$$\langle M | \bar{q}q | 0 \rangle \neq 0$$

$\langle M |$ must have σ -like QN: Large ESC from $N\sigma / N(\pi\pi)_{L=0}$

Using same argument in other channels:

- $N\pi$ in some $\langle O_N(\vec{0}, t) A_\mu(\vec{q}, \tau) O_N(-\vec{q}, 0) \rangle$ as predicted by ChPT in [O. Bär, *PRD* 101 (2020) 3, 034515, ...] [A. Jüttner Mon 9:15]
- $B_\mu^* \pi$ in $\langle O_B(\vec{0}, t) V_\mu(\vec{q}, \tau) O_\pi(-\vec{q}, 0) \rangle$ as also predicted by HMChPT in [O. Bär, et al. *Eur.Phys.J.C* 83 (2023) 8, 757]
- $B^* K$ in $\langle O_{B_s}(\vec{0}, t) V_\mu(\vec{q}, \tau) O_K(-\vec{q}, 0) \rangle$, $K^* \pi / (K\pi)\pi$ in $\langle O_K(\vec{0}, t) V_\mu(\vec{q}, \tau) O_\pi(-\vec{q}, 0) \rangle$, ...
- $N\pi$ contribution is small in $\langle O_N(\vec{0}, t) V_\mu(\vec{q}, \tau) O_N(-\vec{q}, 0) \rangle$ [O. Bär, *PRD* 103 (2021) 11, 114514] $\left(\langle \pi | V_\mu | 0 \rangle = 0 \right)$
- ...

Nucleon-sigma terms with a variational analysis at $m_\pi = 429$ MeV

$a = 0.098$ fm $L = 24a$

L. Barca, G. Bali, S. Collins
[Phys.Rev.D 111 (2025) 3, L031505]

Variational analysis with $N(0)$ and $N(0)\sigma(0)$ interpolators

$\sigma \sim \bar{u}u + \bar{d}d$

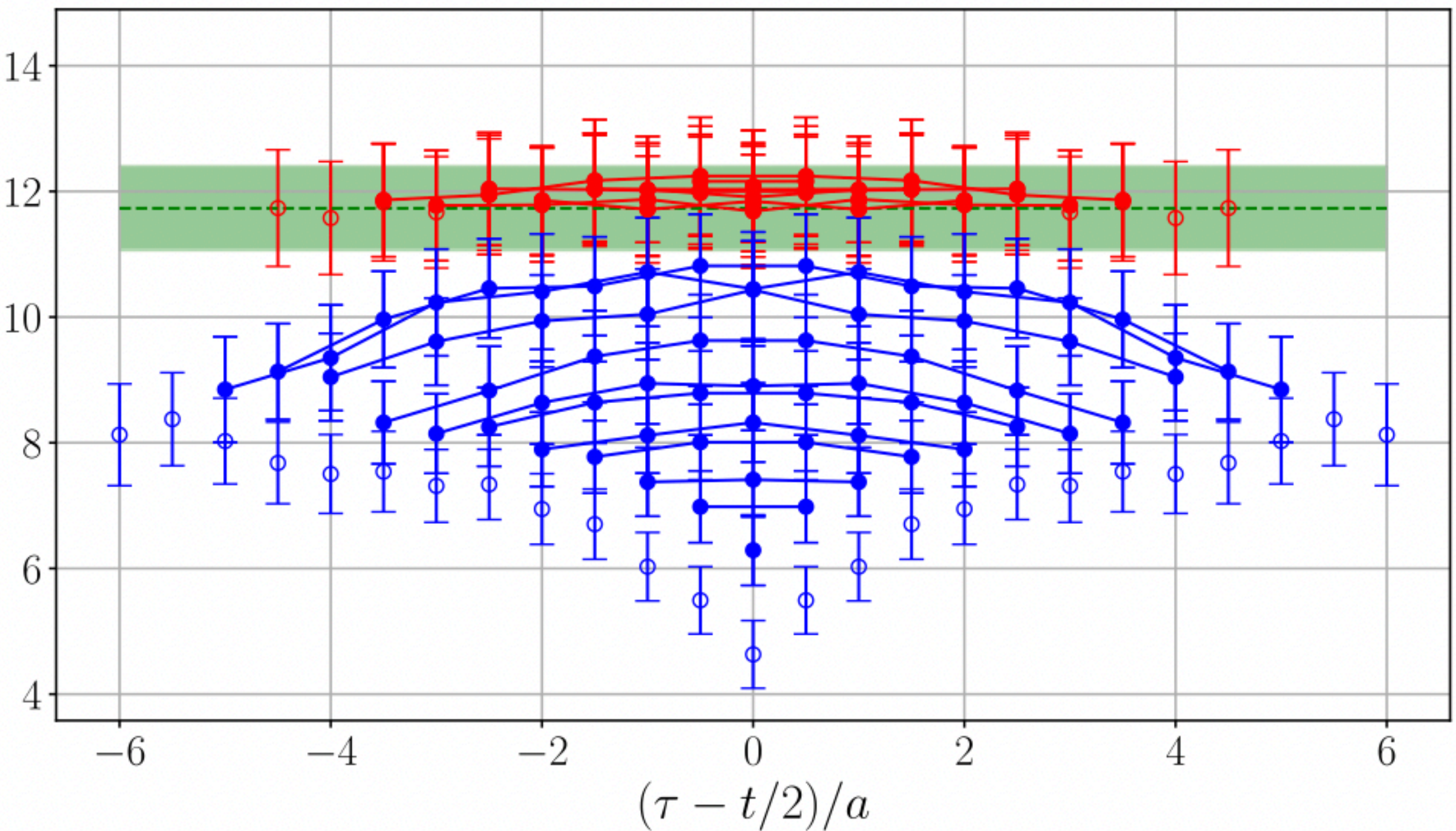
$$R_{q\bar{q}}^{(\text{GEVP})}(t, \tau) = \frac{\langle O_N^{(\text{GEVP})}(t) (q\bar{q})(\tau) \bar{O}_N^{(\text{GEVP})}(0) \rangle}{\langle O_N^{(\text{GEVP})}(t) \bar{O}_N^{(\text{GEVP})}(0) \rangle} = \langle N | q\bar{q} | N \rangle + \dots (\text{ESC})$$

σ stable at ≈ 550 MeV $< E_{\pi\pi}^{(0)}$

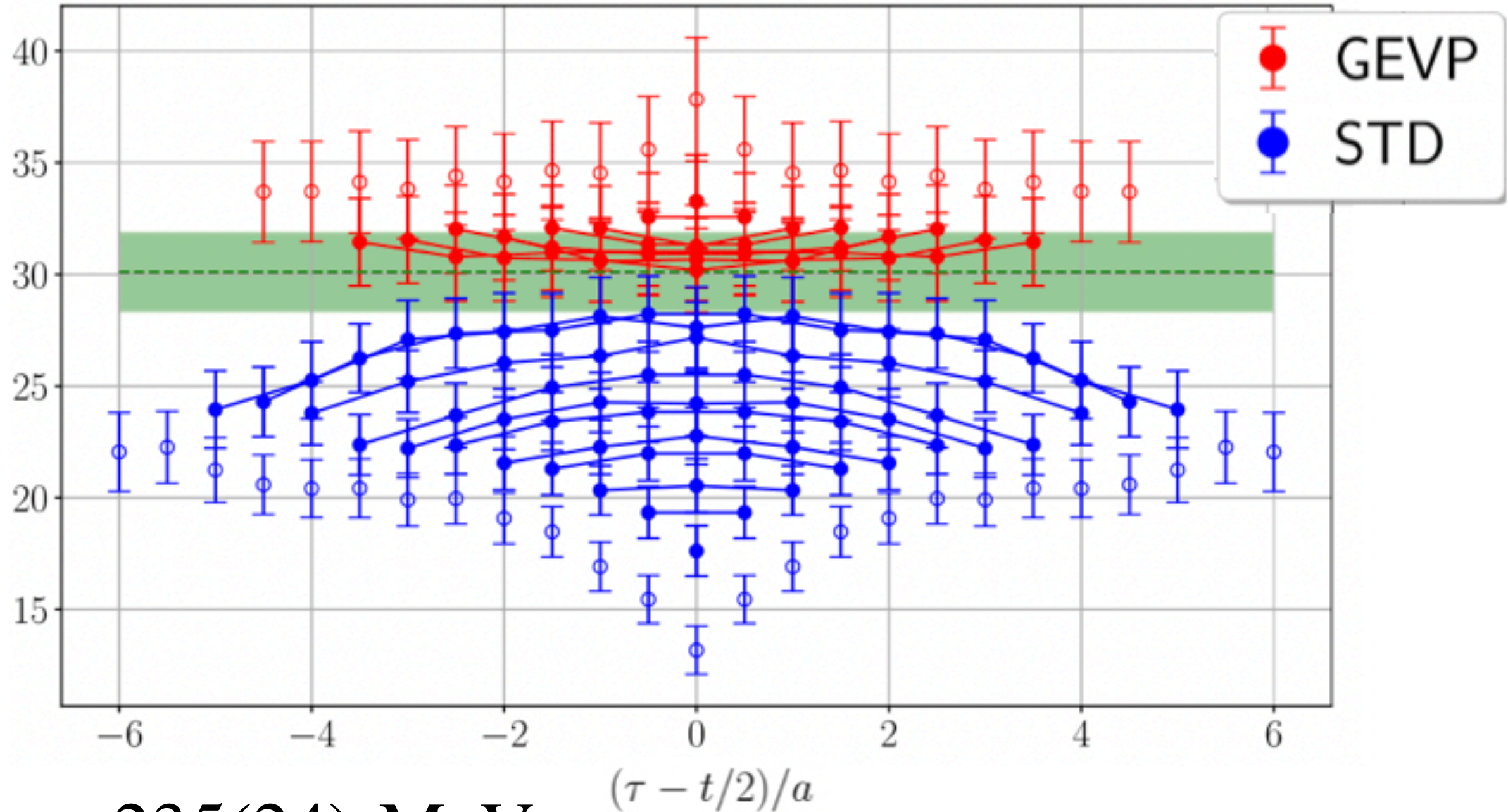
$R_s^{(\text{GEVP})}(t, \tau)$

Substantial reduction of ESC

$R_{u+d}^{(\text{GEVP})}(t, \tau)$

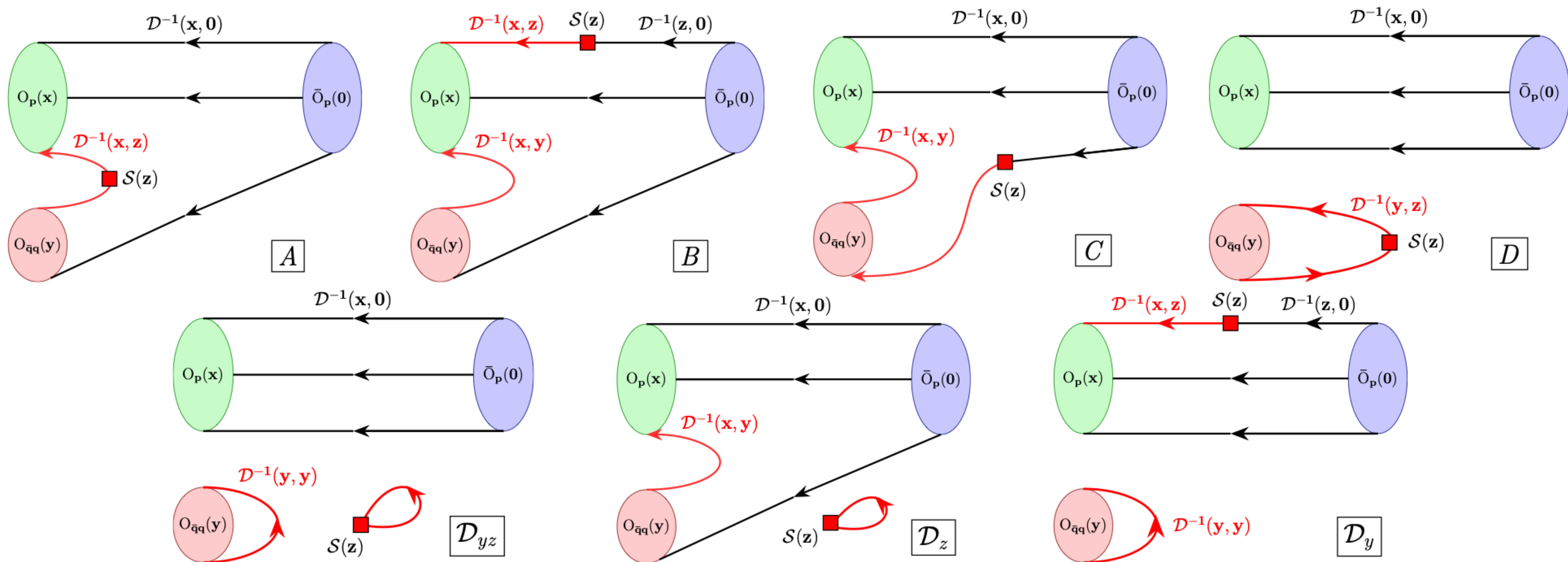


$\sigma_{sN} = 42(16)$ MeV



$\sigma_{\pi N} = 235(24)$ MeV
8/13

Diagrams

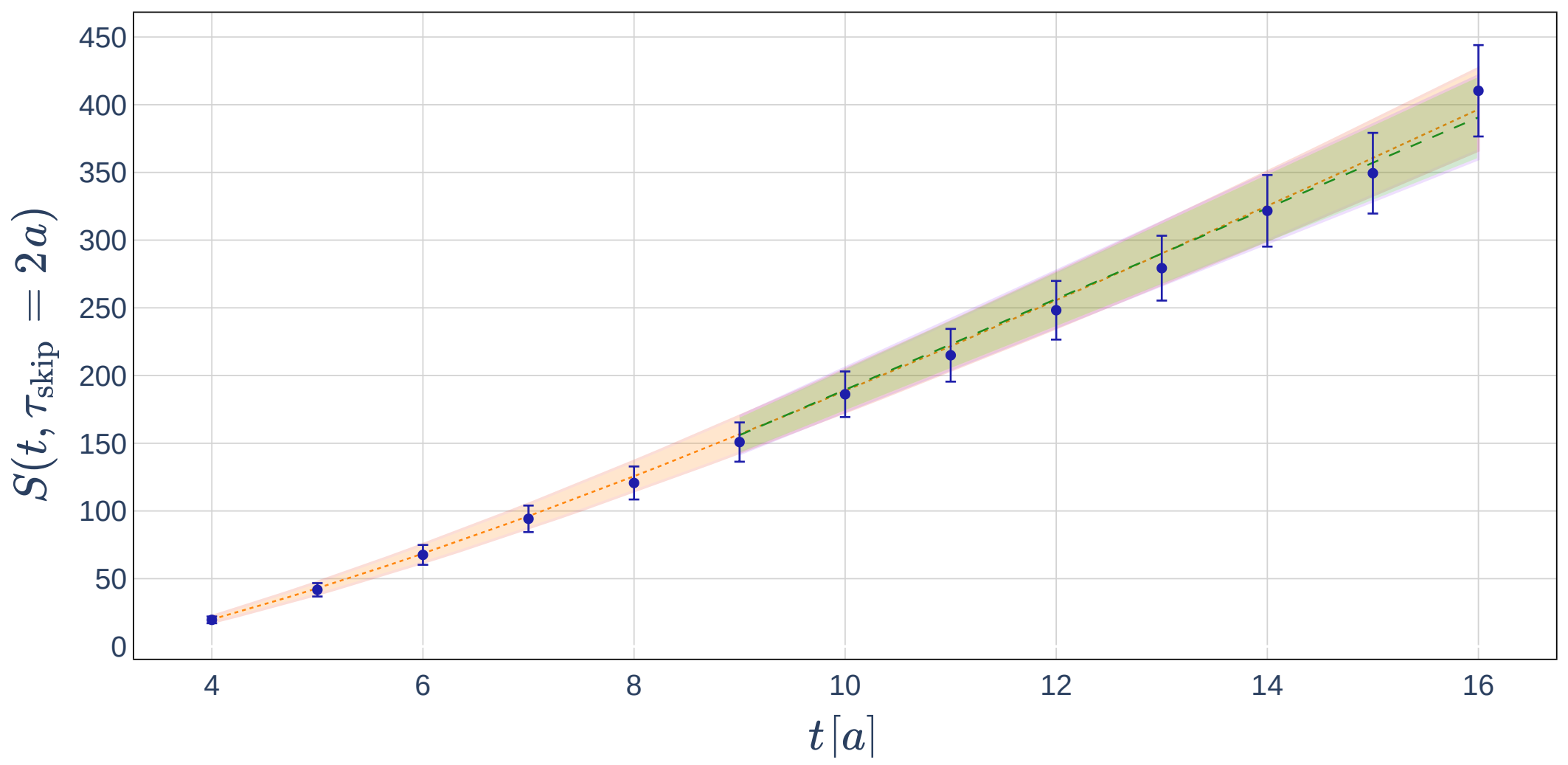
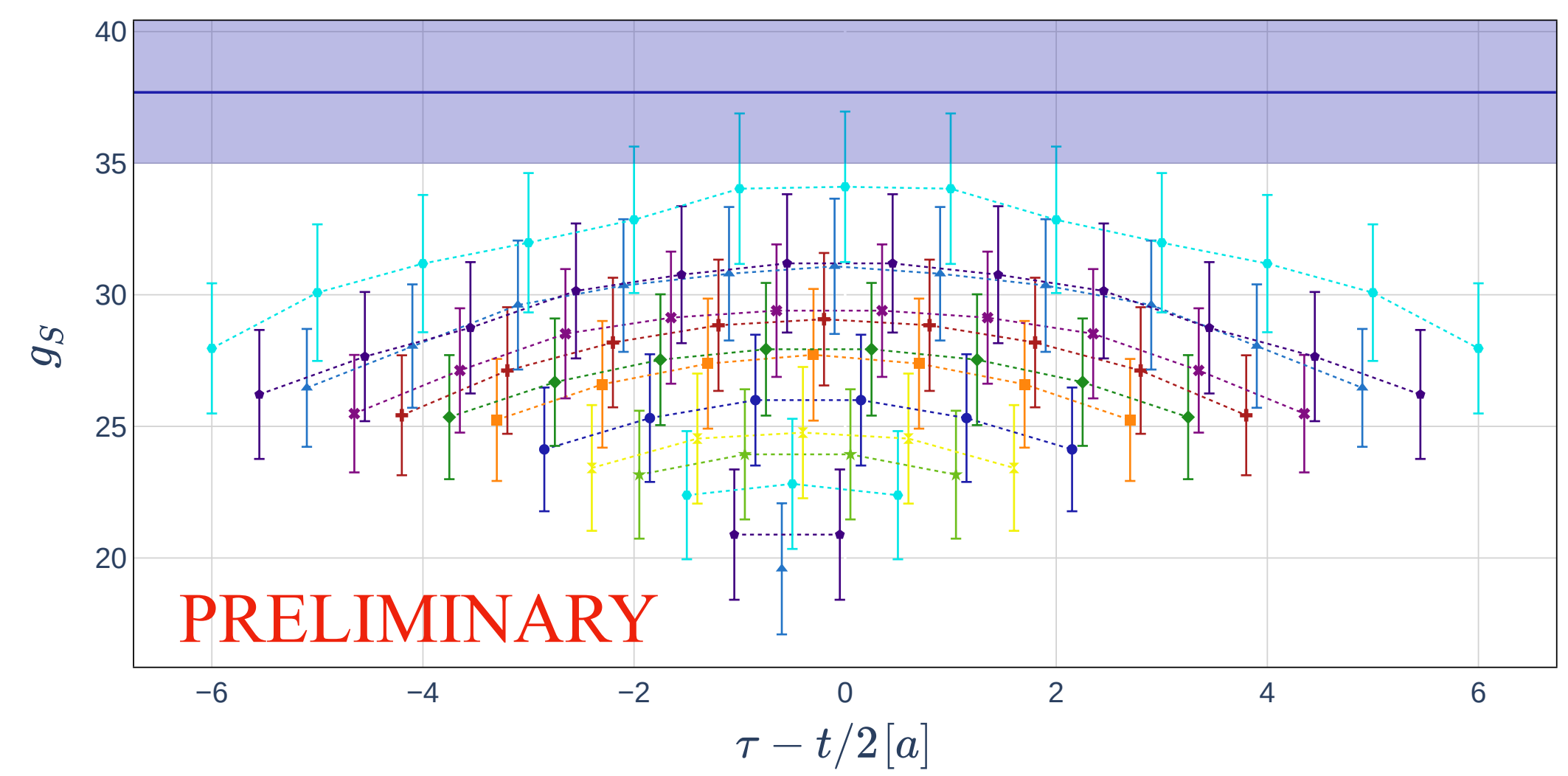


$\mathcal{D}_{yz}$ and $D = \mathcal{O}(100) \times$ other diagrams $(m_\pi = 429 \text{ MeV}, L = 24a)$

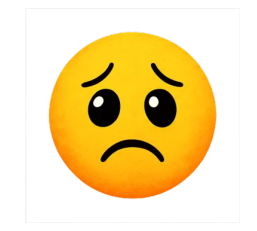
Nucleon sigma terms at $m_\pi = 222$ MeV with standard correlators

G. Bali, **LB**, S. Collins, M. Rodekamp $a = 0.0865$ fm $L = 48a$

Summation method:
$$S(t, \tau_{\text{skip}} = 2a) = \sum_{\tau=2a}^{t-2a} R_{q\bar{q}}(t, \tau)$$



Fit models

<u>Linear fit</u>	$S(t) = c_0 + g_s t$	$g_s^{\text{fit}} = 33.5(2.7)$
<u>Linear + exp. fit</u>	$S(t) = c_0 + g_s t + c_{10} e^{-\Delta E t}$	
No prior	$g_s^{\text{fit}} = 36(42)$	
$\Delta E = E_{N\pi} - m_N$	$g_s^{\text{fit}} = 38.3(2.8)$	$\chi^2/\text{dof} = 0.939$
$\Delta E = E_{N\pi\pi} - m_N$	$g_s^{\text{fit}} = 37.4(2.7)$	$\chi^2/\text{dof} = 0.933$
ΔE from nucleon 2-pts	$g_s^{\text{fit}} = 32.4(2.5)$	$\chi^2/\text{dof} = 1.05$

Variational analysis at $m_\pi = 222$ MeV

$\langle \sigma(t) \sigma(t) \rangle$ decays w/ $m_{\text{eff}} \approx 420$ MeV
 $\approx E_{\pi\pi}^{(0)}$

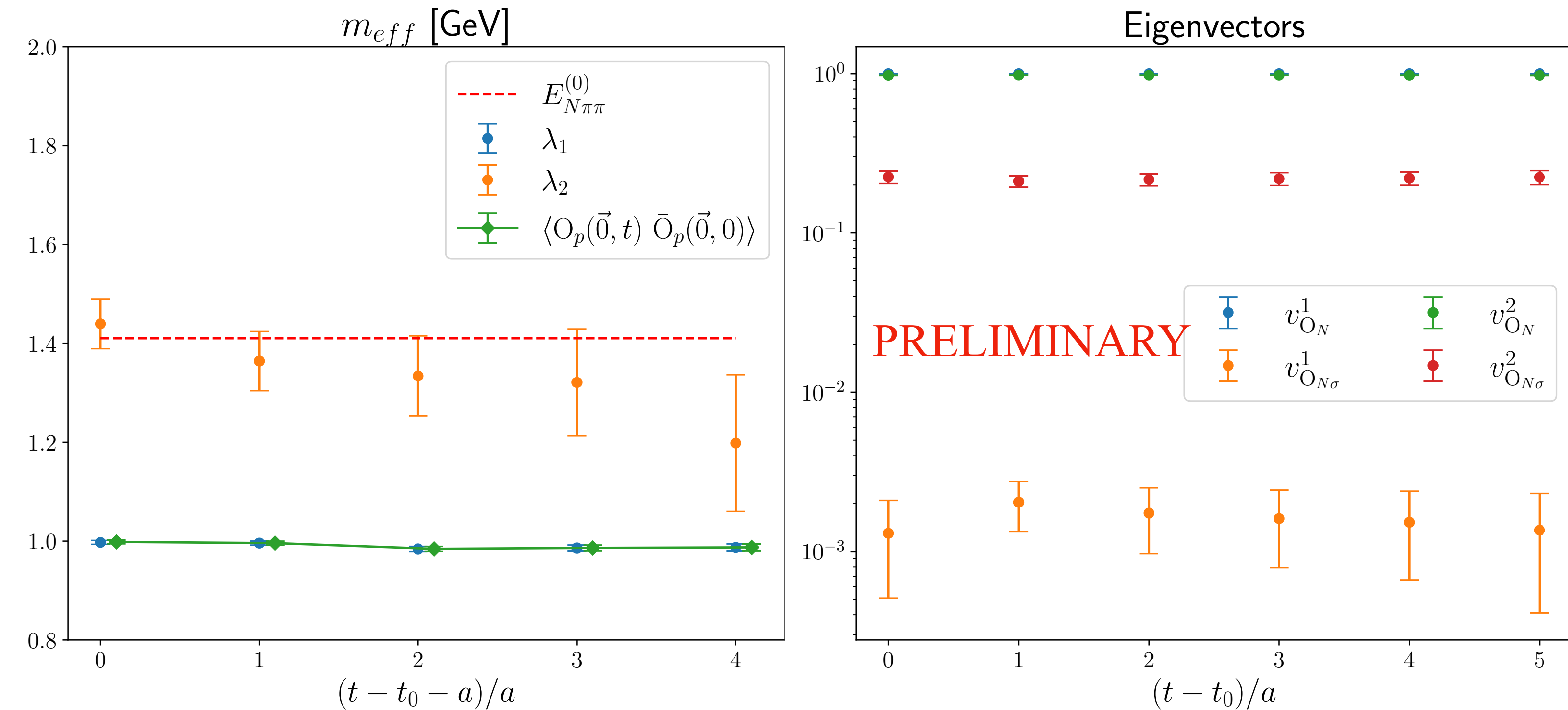
G. Bali, **LB**, S. Collins, M. Rodekamp

Basis: N and $N\sigma$ interpolators

σ becomes a broad resonance

[*Phys.Rev.Lett.* 118 (2017) 2, 022002 (HadSpec)] ($m_\pi = 236$ MeV)

GEVP results at $t_0/a=7$



GEVP-improved operators

$$O_N^{(\text{GEVP})}(t) = \sum_{i=N, N\sigma} v_i^N O_i(t)$$

GEVP-improved correlators

$$C_{2\text{pt}}^{(\text{GEVP})}(t) = \langle O_N^{(\text{GEVP})}(t) \bar{O}_N^{(\text{GEVP})}(0) \rangle$$

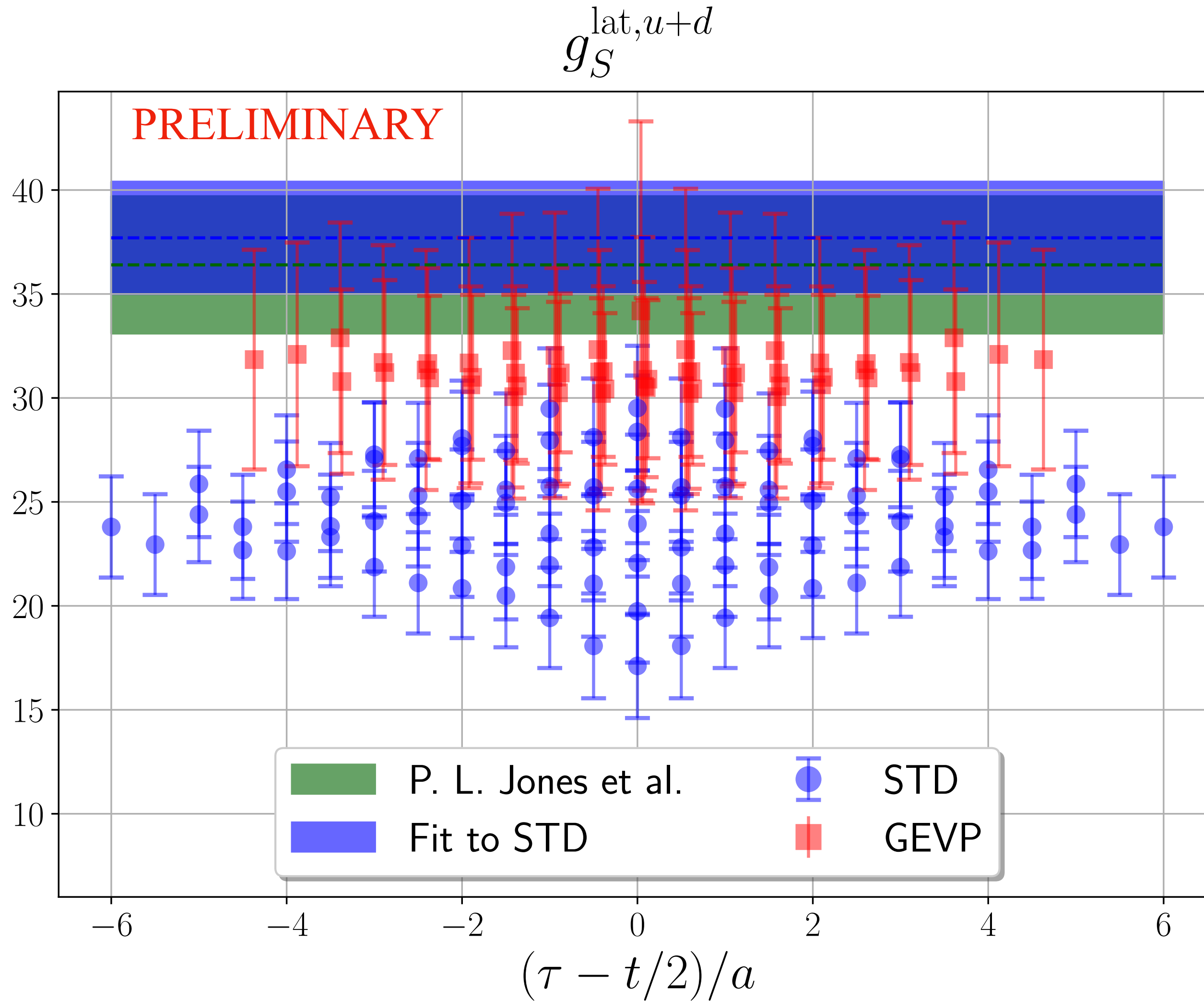
$$C_{3\text{pt}}^{(\text{GEVP})}(t, \tau) = \langle O_N^{(\text{GEVP})}(t) S_q(\tau) \bar{O}_N^{(\text{GEVP})}(0) \rangle$$

Including $N(p)\pi(-p)$ interpolators would be even better for a precise determination of $v_{N\sigma}^1$

Note: $v_{O_{N\sigma}}^1 \approx (2/1000) = \mathcal{O}(1/\sqrt{L^3})$

Therefore, difficult to get from fit with 2-pts

Nucleon-sigma terms with a variational analysis at $m_\pi = 222$ MeV



$\mathbb{B}_N = \{O_N, O_{N\sigma}\}$ very effective in removing the largest excited state contamination in this channel.

Errors of GEVP ratios are larger because of noise in the eigenvectors.

Comparison

GEVP ratios agree within the errors with other fits.

Fits to summed GEVP ratios yield $g_S^{u+d} = 31.2(4.0)$

In progress

$$\mathbb{B}_N = \{O_N, O_{N\sigma_0}, O_{N\sigma_8}\} \longrightarrow g_S^{\text{lat}, u+d}, g_S^{\text{lat}, s}$$

Conclusions

[G. Bali, L. Barca, S. Collins, M. Rodekamp]

- Standard ratios for $\sigma_{\pi N}$ and summed ratios suffer from large ESC.
- Discussed the mechanism of current-enhanced states in lattice QCD 3-pts. Broad implications in lattice QCD.
- GEVP analysis for $g_S^{u+d}(\sigma_{\pi N})$ and $g_S^s(\sigma_{sN})$ with O_N and $O_{N\sigma}$ interpolating operators at $m_\pi = 429$ MeV, 222 MeV is very effective in removing the dominant ESC.

CLQCD has also obtained very good results with this idea.

Physics interpretation

At $m_\pi = 429$ MeV dominant ESC is $N(0)\sigma(0)$, while at $m_\pi = 222$ MeV is $N(0)\pi(0)\pi(0)$ consistent with R. Gupta et al. [Phys.Rev.Lett. 127, 242002 (2021)]

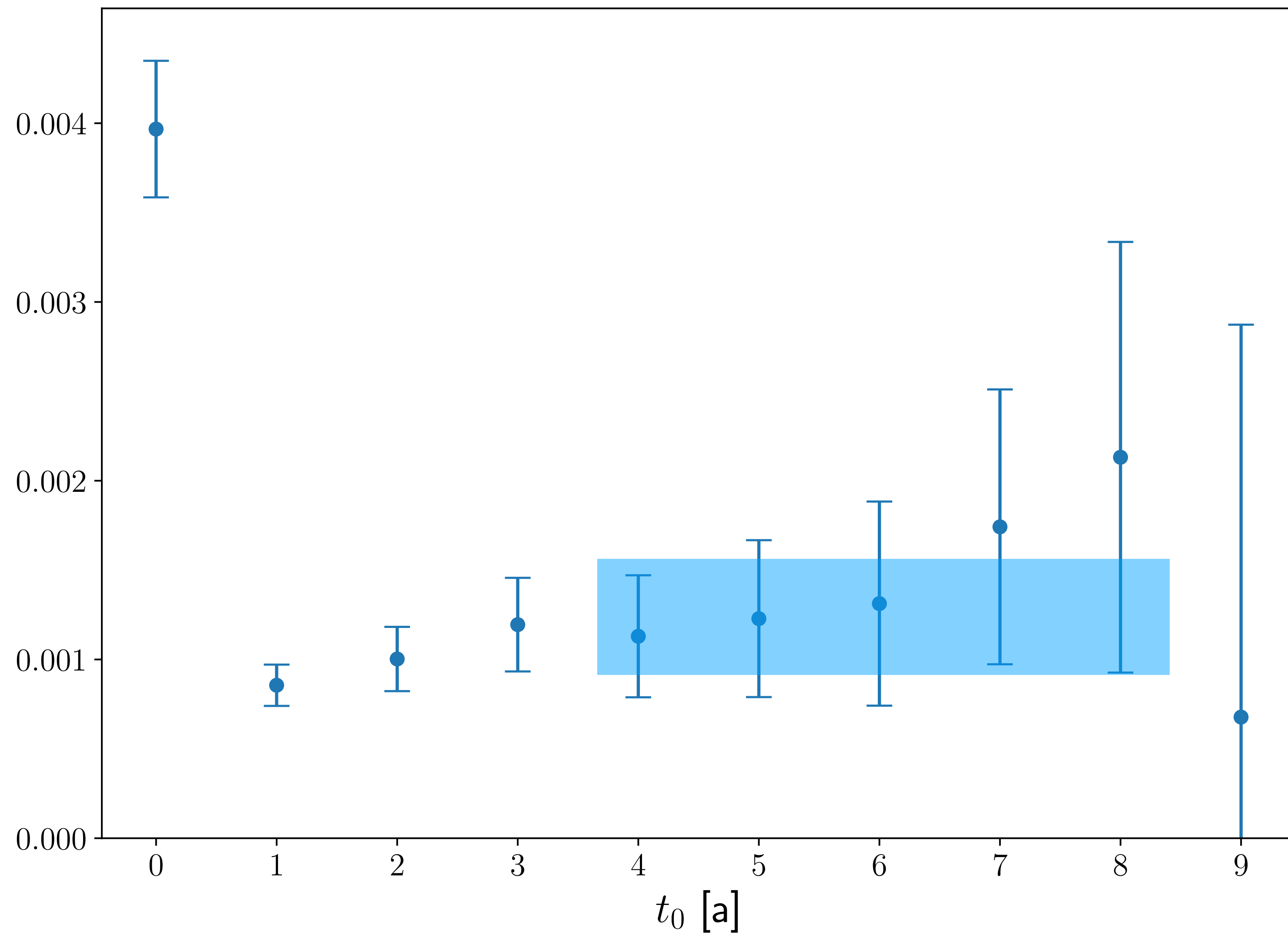
Next Try to reduce statistical errors and carry out variational analysis at or near physical pion masses.

Ambitious Goal Construct comprehensive variational basis for nucleon (include all B, BM, BMM operators)

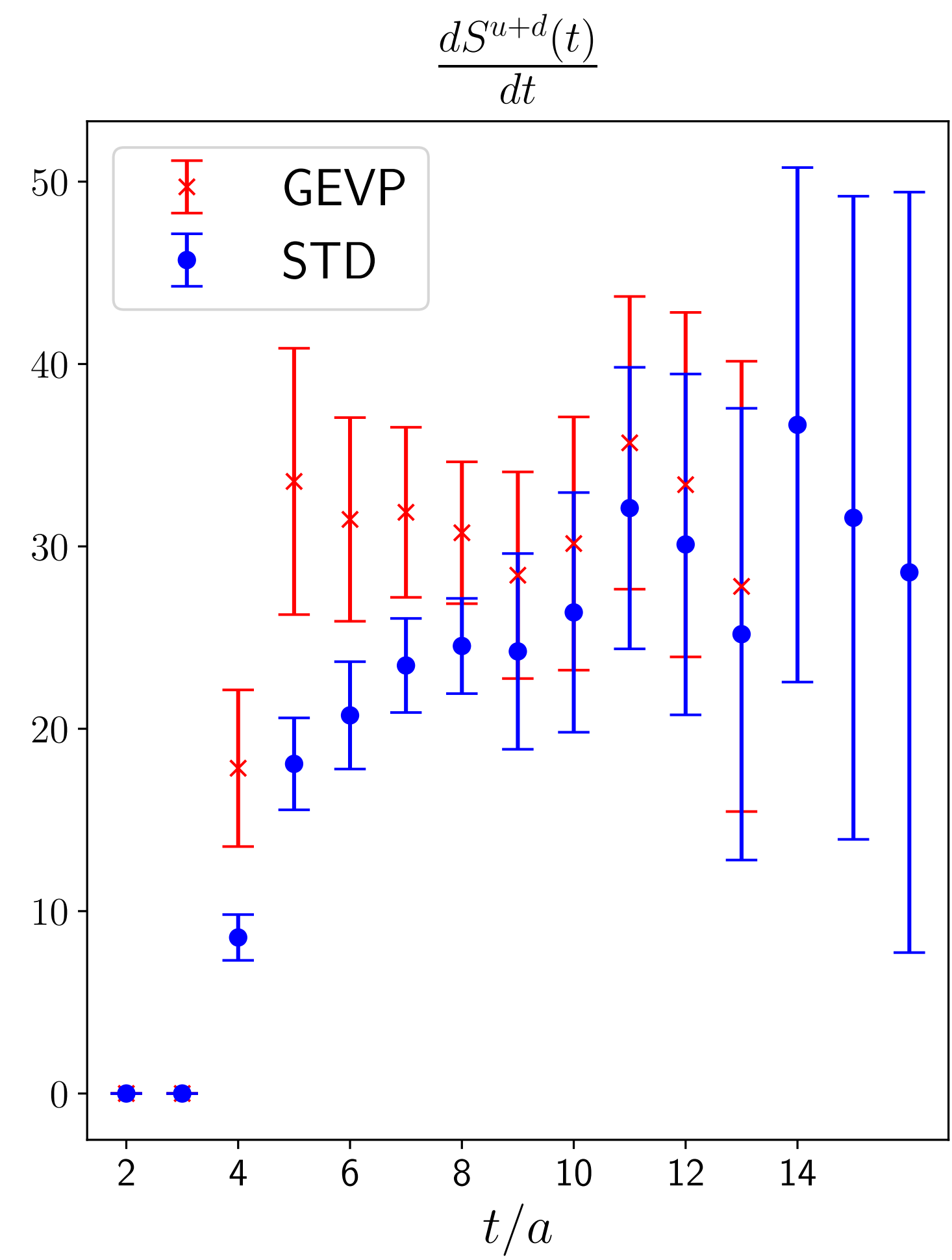
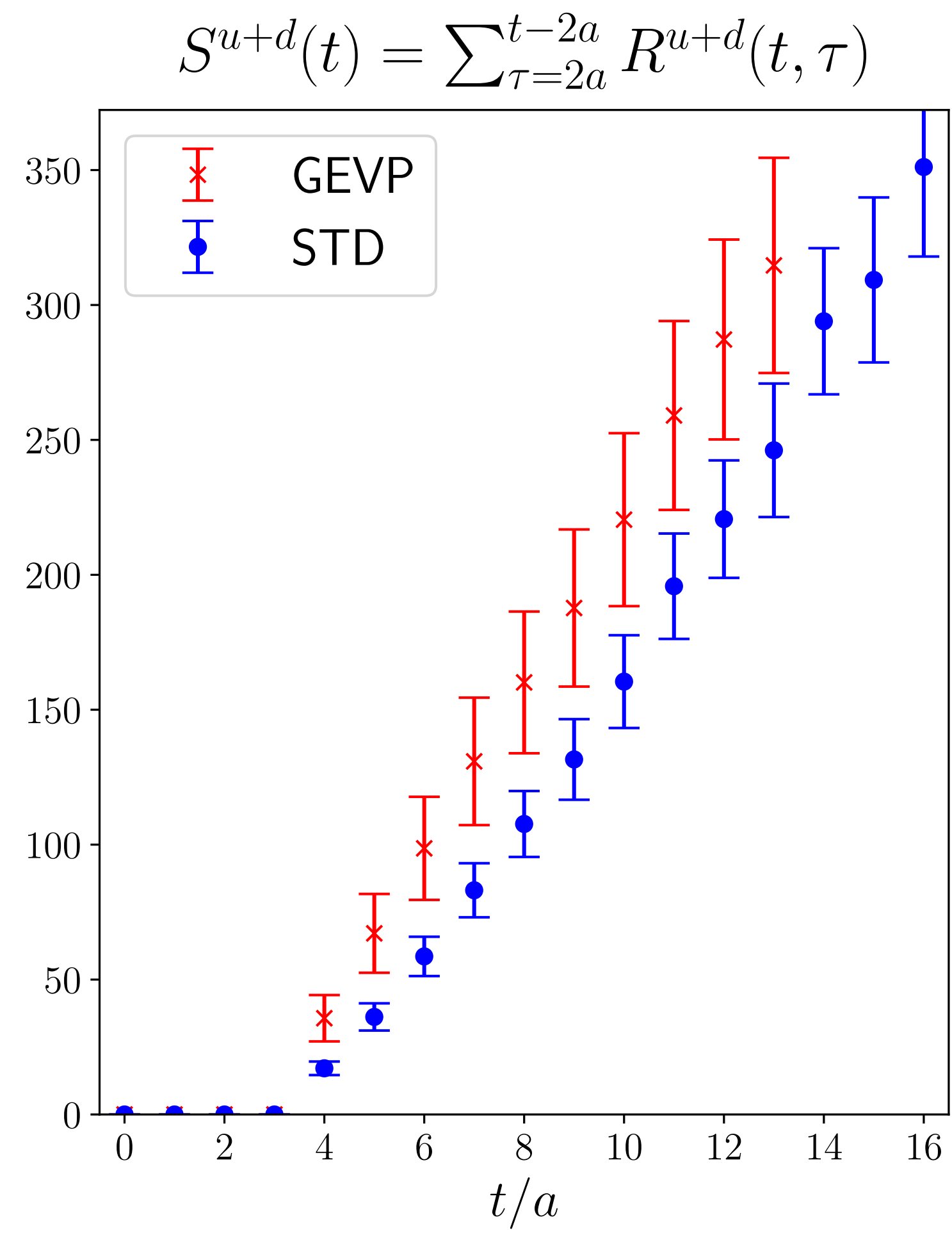
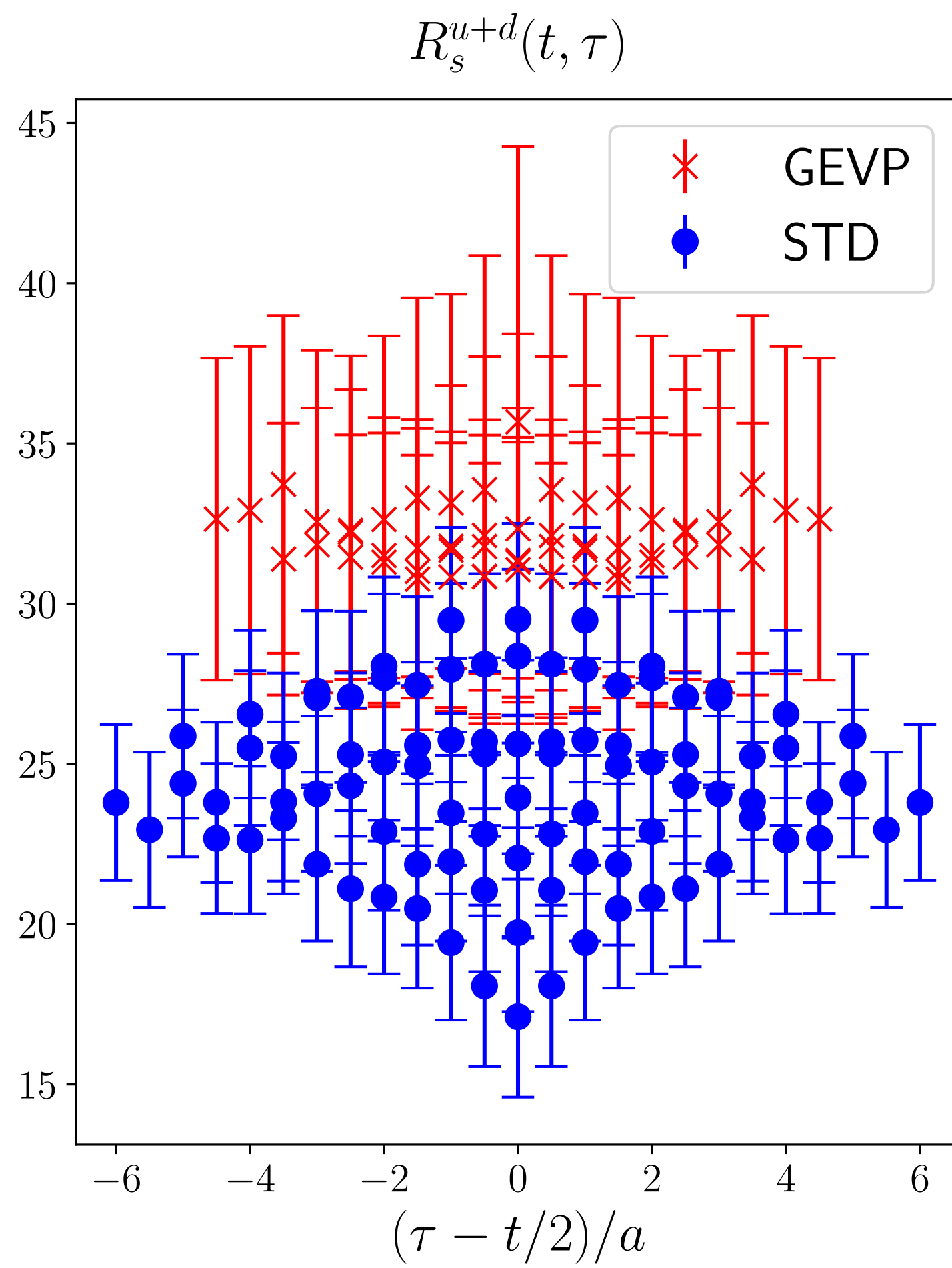
Exponentially reduced ESC, statistical errors and allows us to extract TME

In collaboration also with Y. Fu, and with J. Bulava, F. Romero-López, ... (BaSc)

$$v_{O_{N\sigma}}^N(t = 9a, t_0)$$

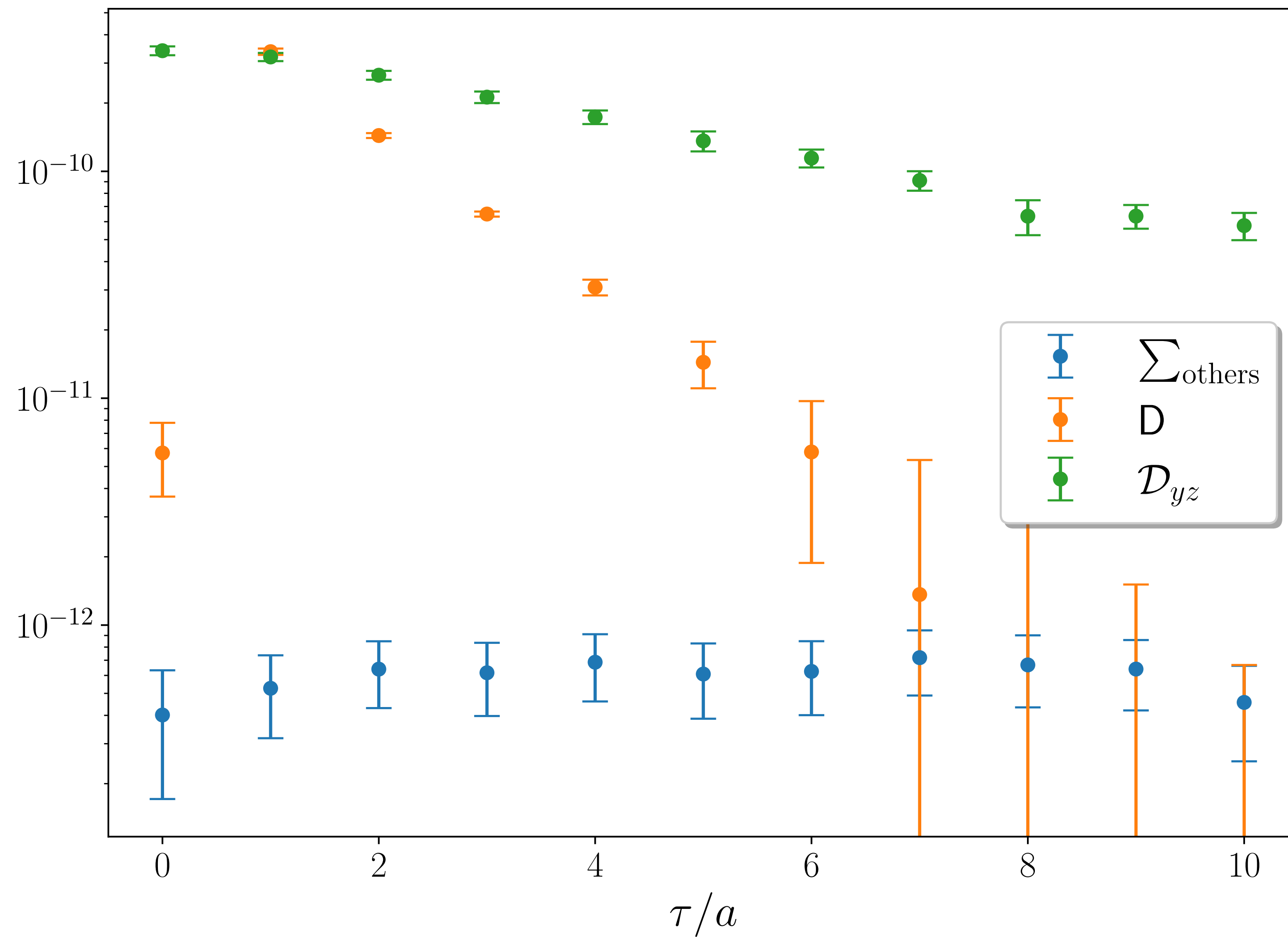


$$\bar{v}_{O_{N\sigma}}^N = \frac{1}{4} \sum_{t_0=5a}^{8a} v_{O_{N\sigma}}^N(t = 9a, t_0)$$



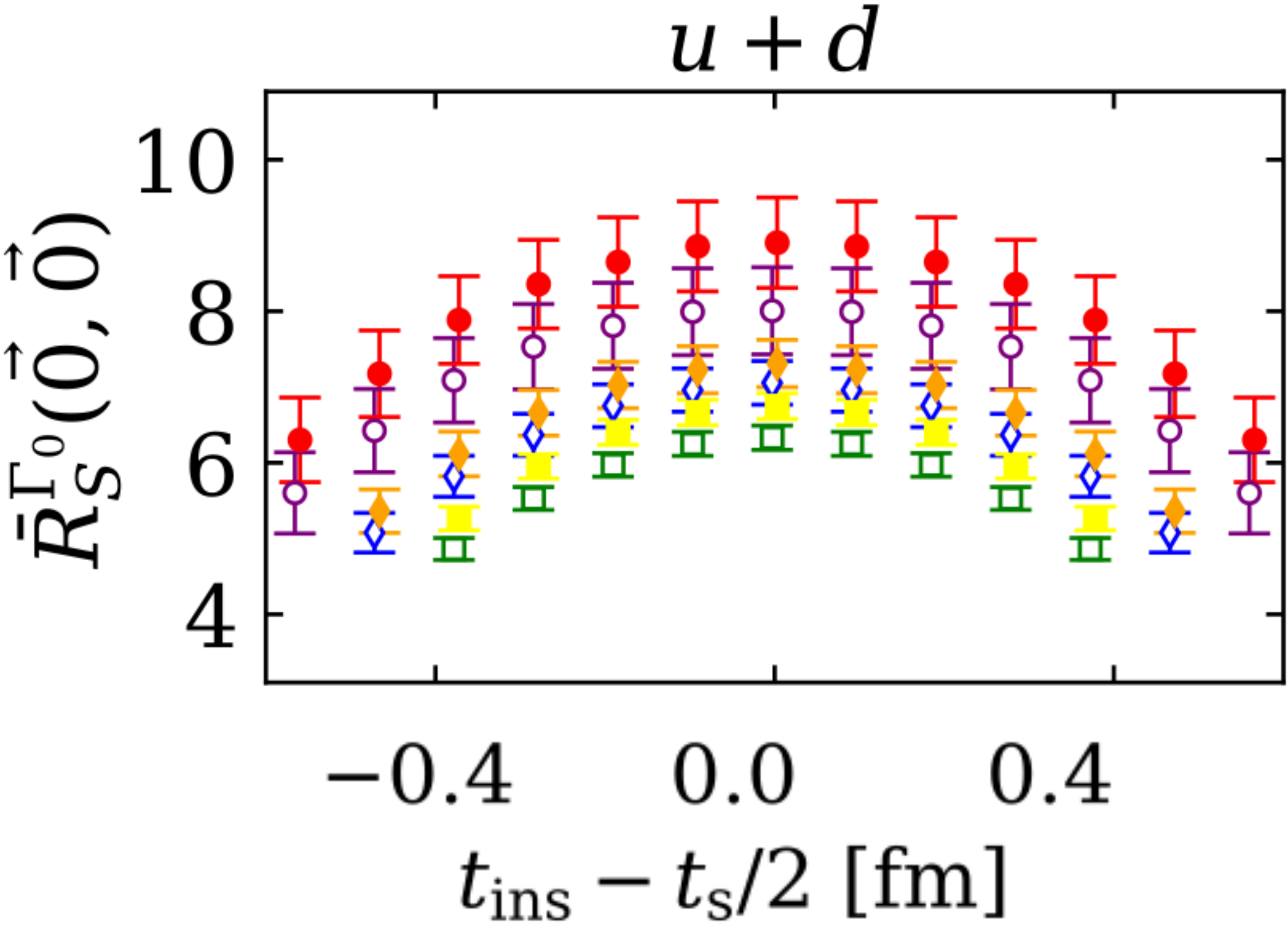
$$m_\pi = 222 \text{ MeV}, a = 0.0865 \text{ fm}, L = 48a$$

$$\langle O_N(t) \mathcal{S}^{u+d}(\tau) \bar{O}_{N\sigma}(0) \rangle_{\text{diagram}}$$

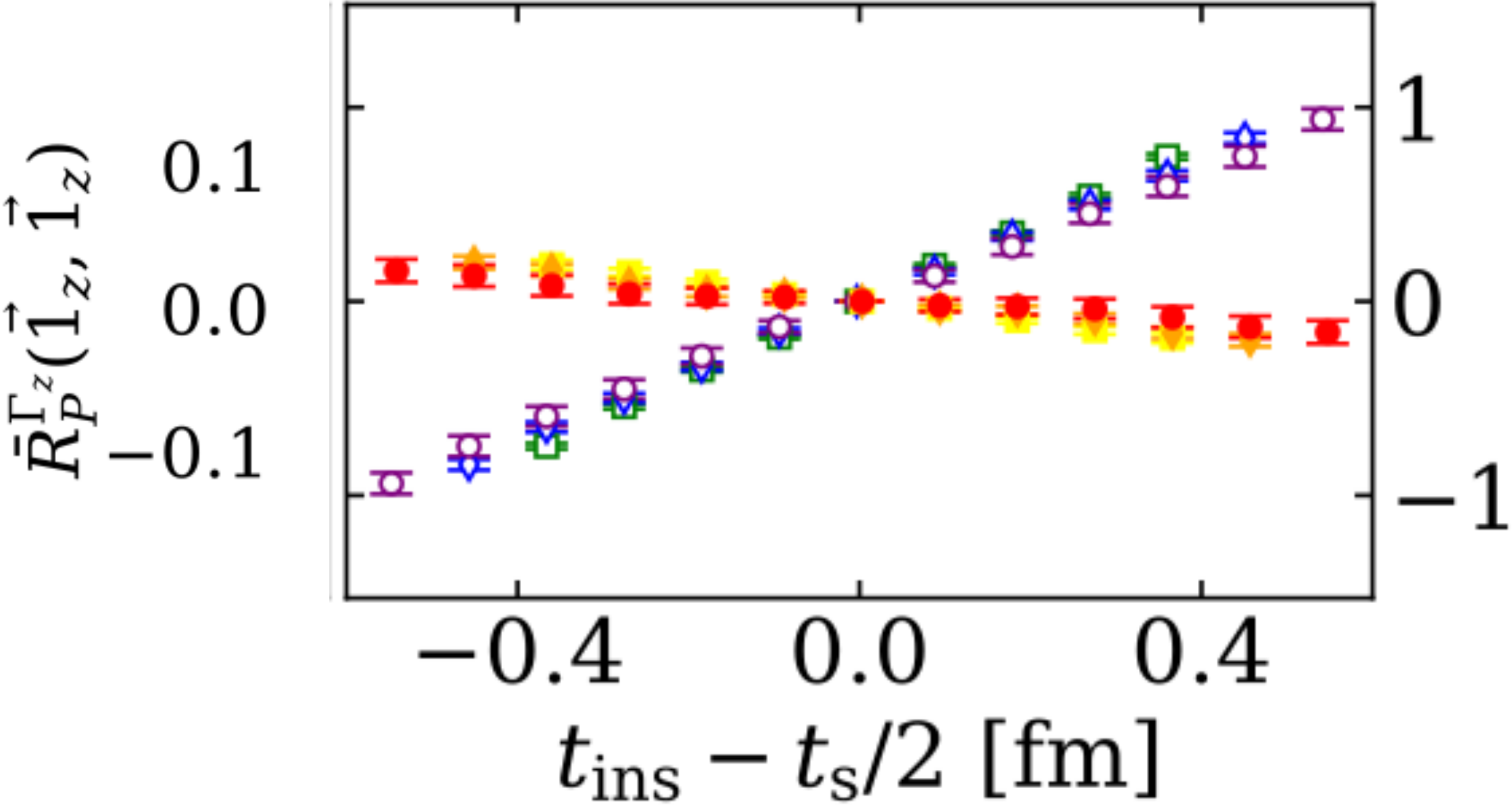


Variational analysis with $\mathcal{O}_N$, $\mathcal{O}_{N\pi}$

Ensembles	Flavors (N_f)	$N_L^3 \times N_T$	a [fm]	L [fm]	m_π [MeV]	$m_\pi L$	m_N [MeV]
cA211.53.24	2 + 1 + 1	$24^3 \times 48$	0.0908	2.27	346	3.99	1193(18)
cA2.09.48	2	$48^3 \times 96$	0.0938	4.50	131	2.98	931(3)



No effect in the isoscalar scalar channel



Tremendous effect in the isovector
pseudoscalar channel

Accurate nucleon isovector tensor and scalar charge at the physical point

J.H. Wang, Z.C. Hu et al. (CLQCD)
[arXiv:2511.02326]

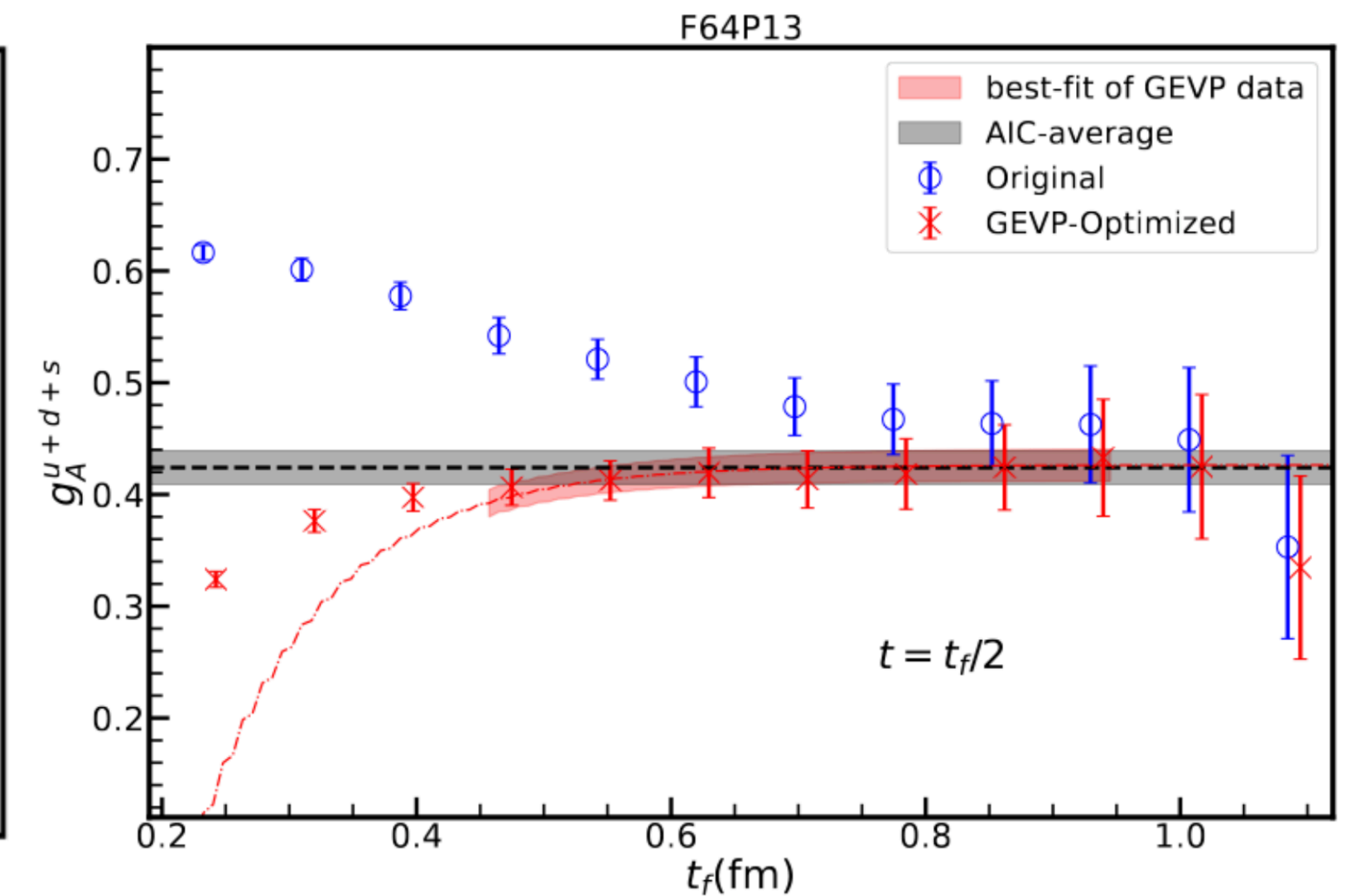
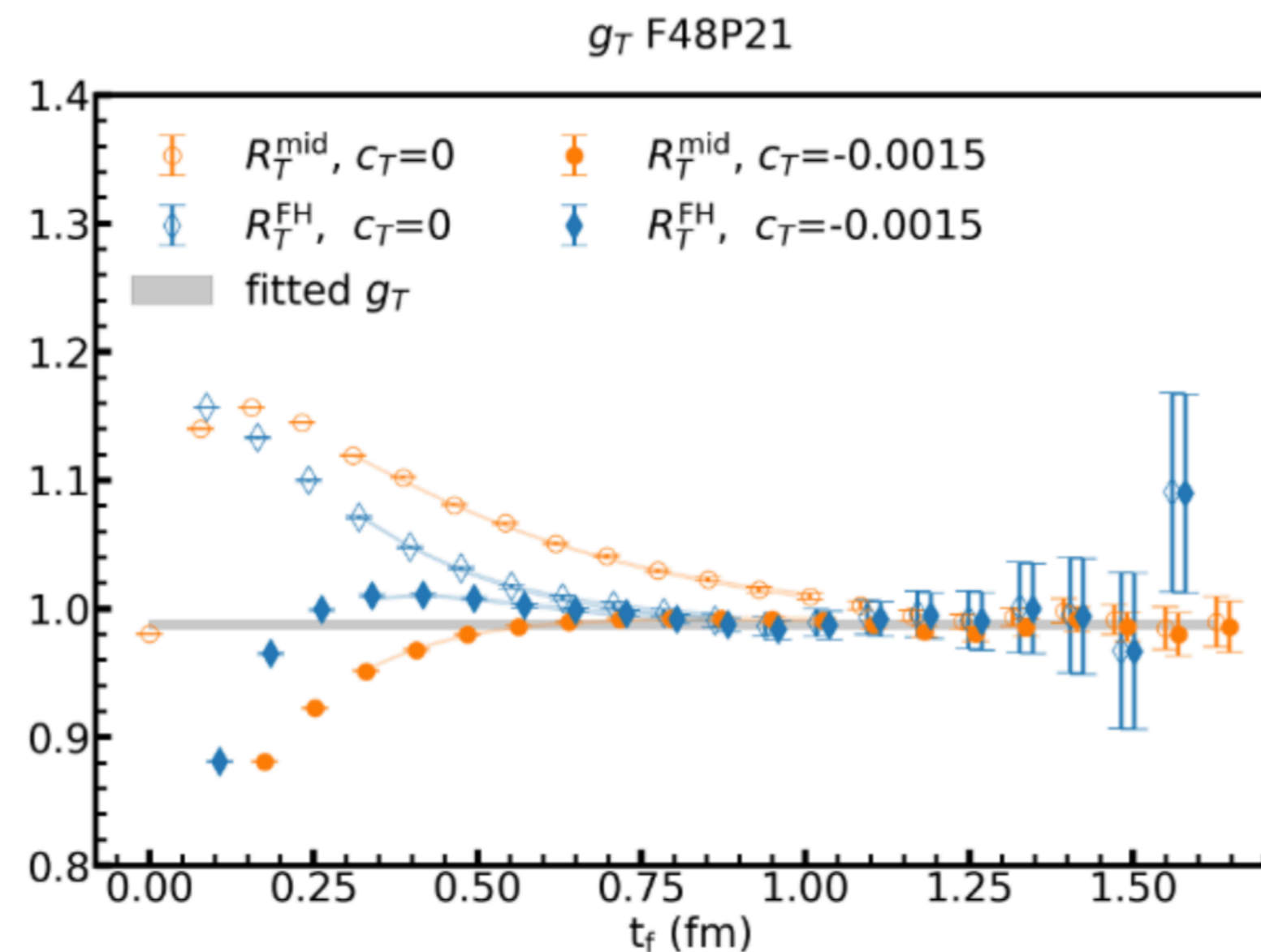
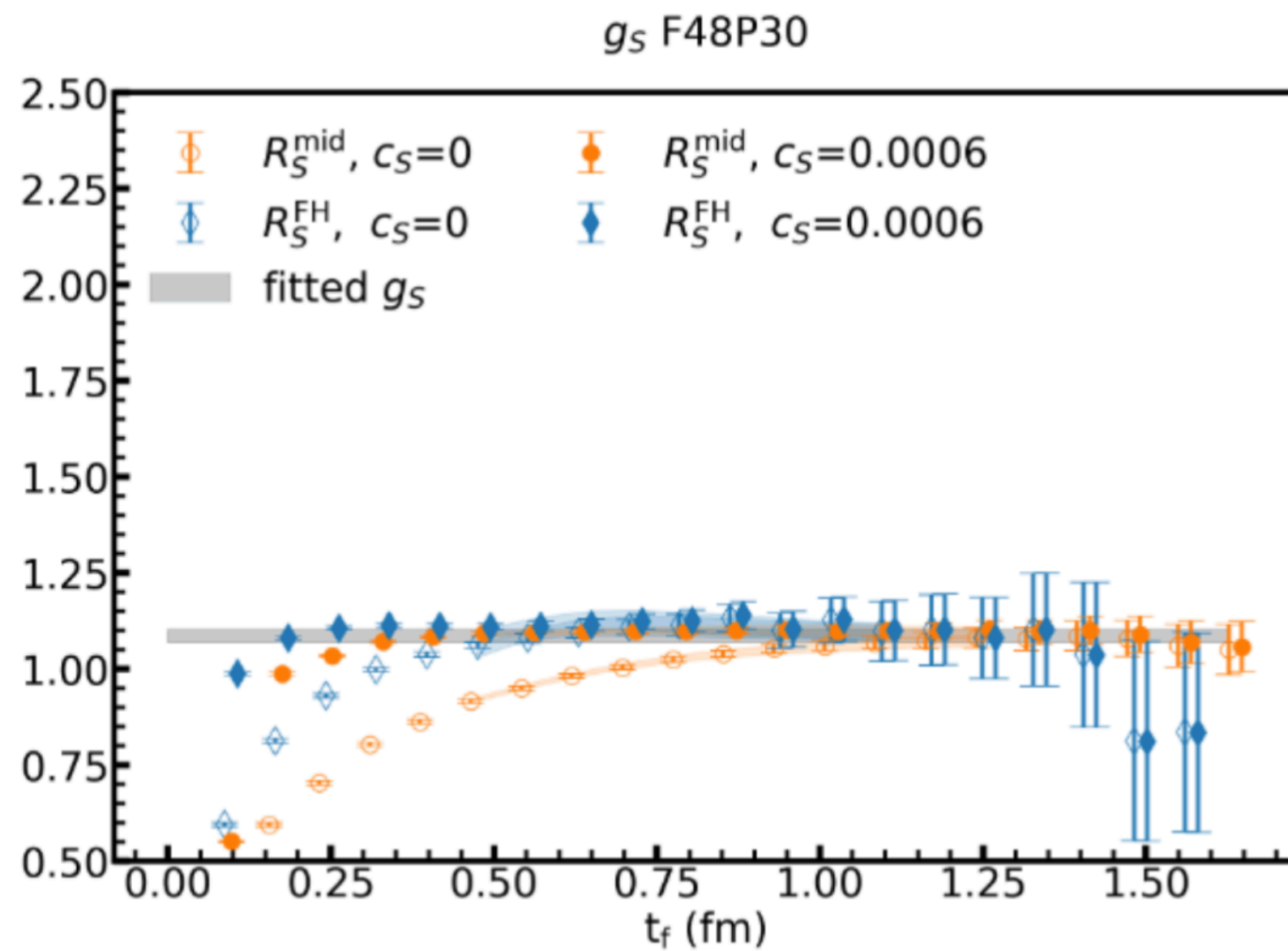
Used my **idea of current-enhanced operators** to remove the leading contamination

$$\langle O_N(\vec{0}, t) \mathcal{J}(\vec{q}, t_{\text{sep}}) O_N^\dagger(-\vec{q}, 0) \rangle - c_{\mathcal{J}} \langle O_N(-\vec{q}, t) O_M(\vec{q}, t) \mathcal{J}(\vec{q}, t_{\text{sep}}) O_N^\dagger(-\vec{q}, 0) \rangle$$

The coefficients $c_{\mathcal{J}}$ are fitted. Using this simple strategy, they obtain the most precise lattice determinations for isovector g_S and g_T ! (Also via GEVP)

Using the blending method [18] and the idea of current-involved interpolation field [11, 19] proposed recently, our study delivers a new calculation of the nucleon isovector charges g_S and g_T with better control over systematic errors—notably from ESC, chiral extrapolation and FVE—compared to all existing lattice determinations.

works also for g_A^{u+d+s} !

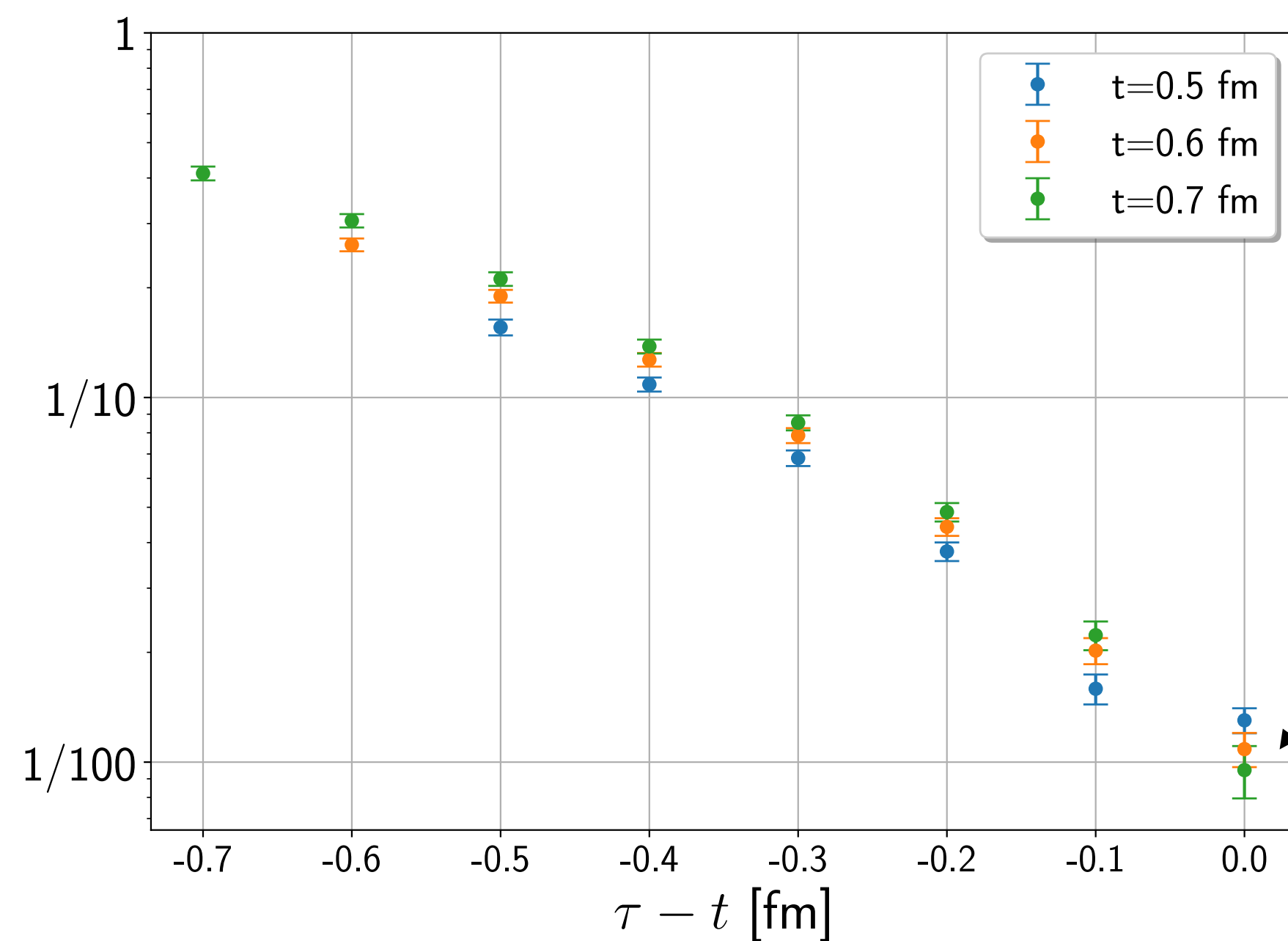
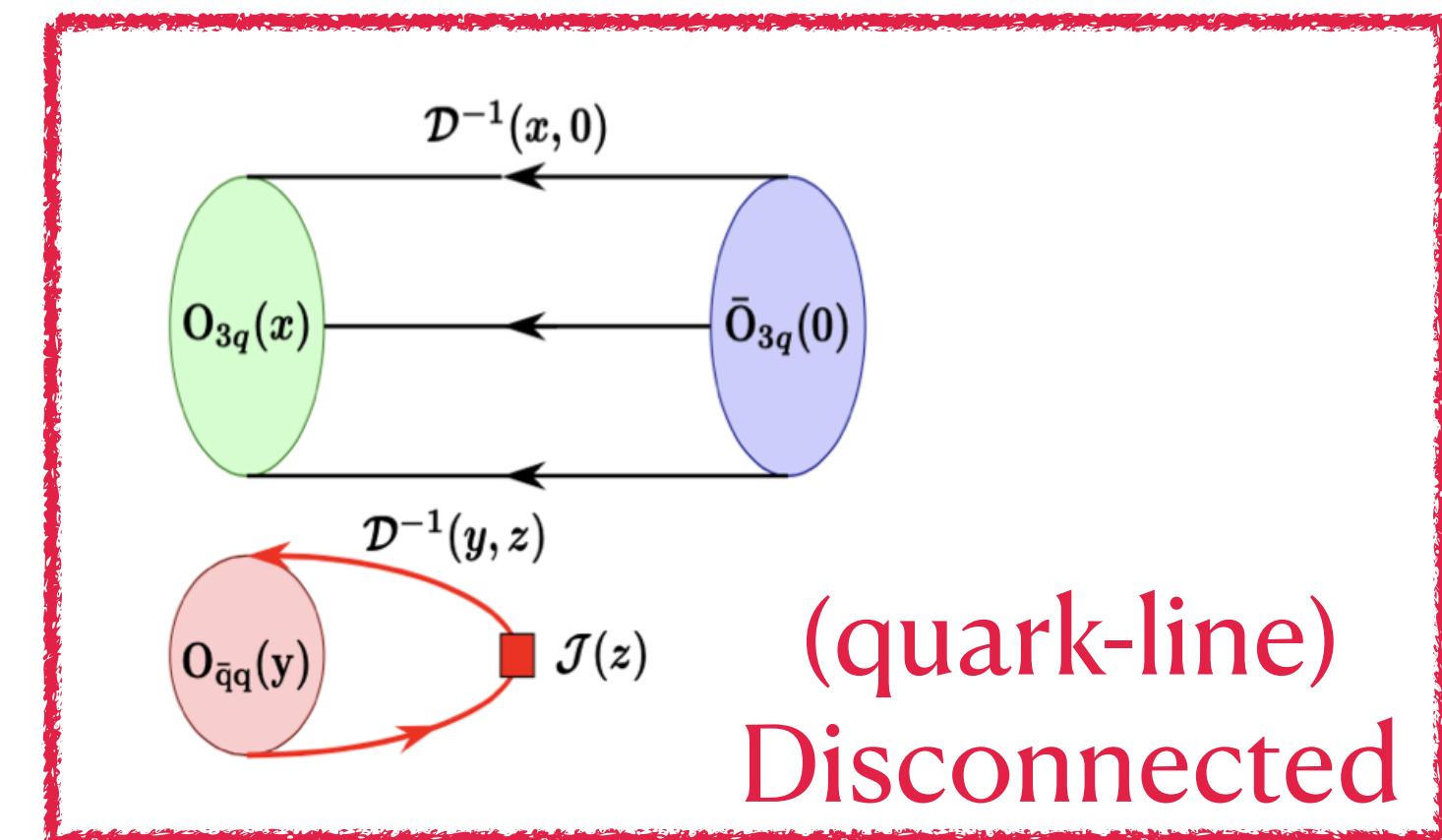
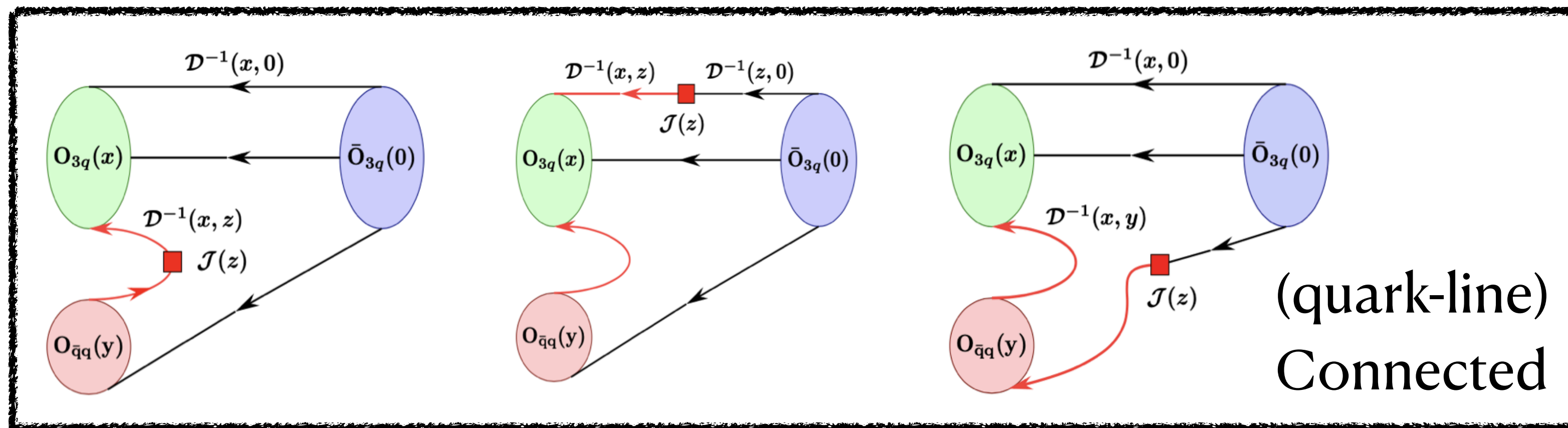


Empty symbols: standard

Filled symbols: using $O_N + c_T O_N O_T$ (Current-involved)

Yi-Bo, et al. Private comm.

Lattice diagrams in $\langle O_{N\pi}(\vec{p}', t) \mathcal{J}(\vec{q}, \tau) O_N^\dagger(\vec{p}, 0) \rangle$

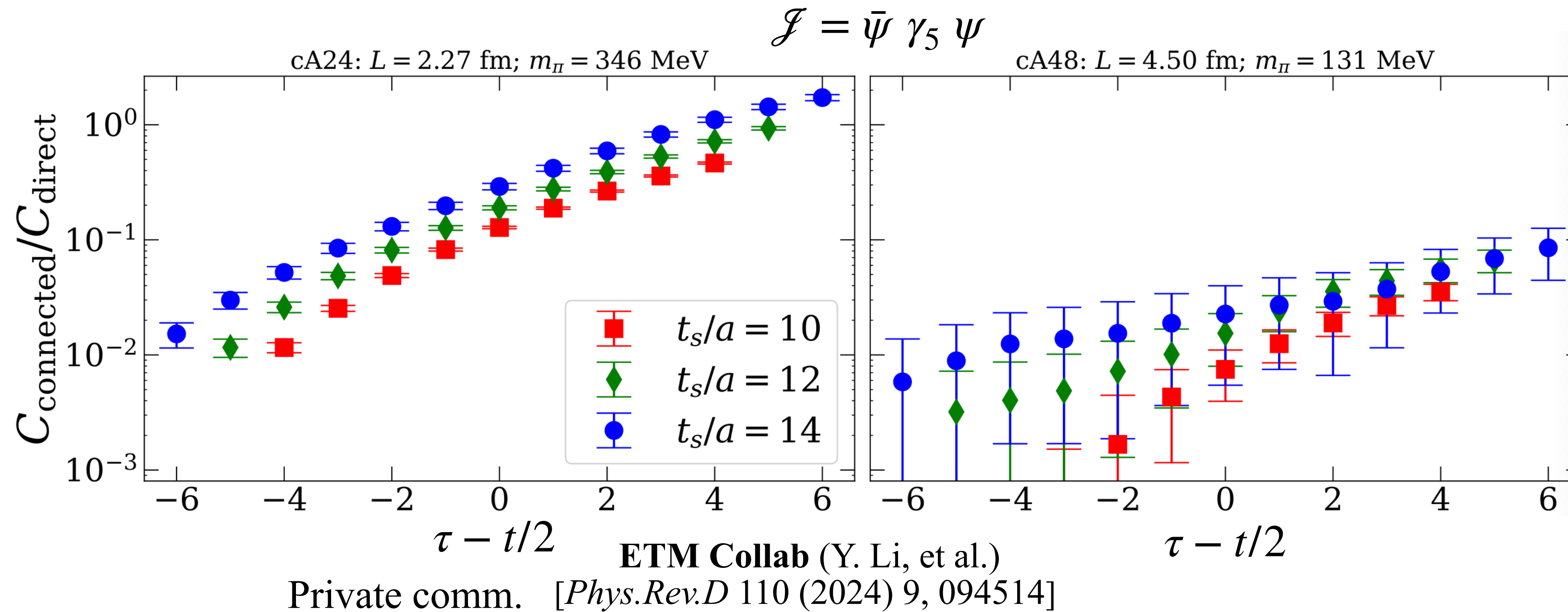
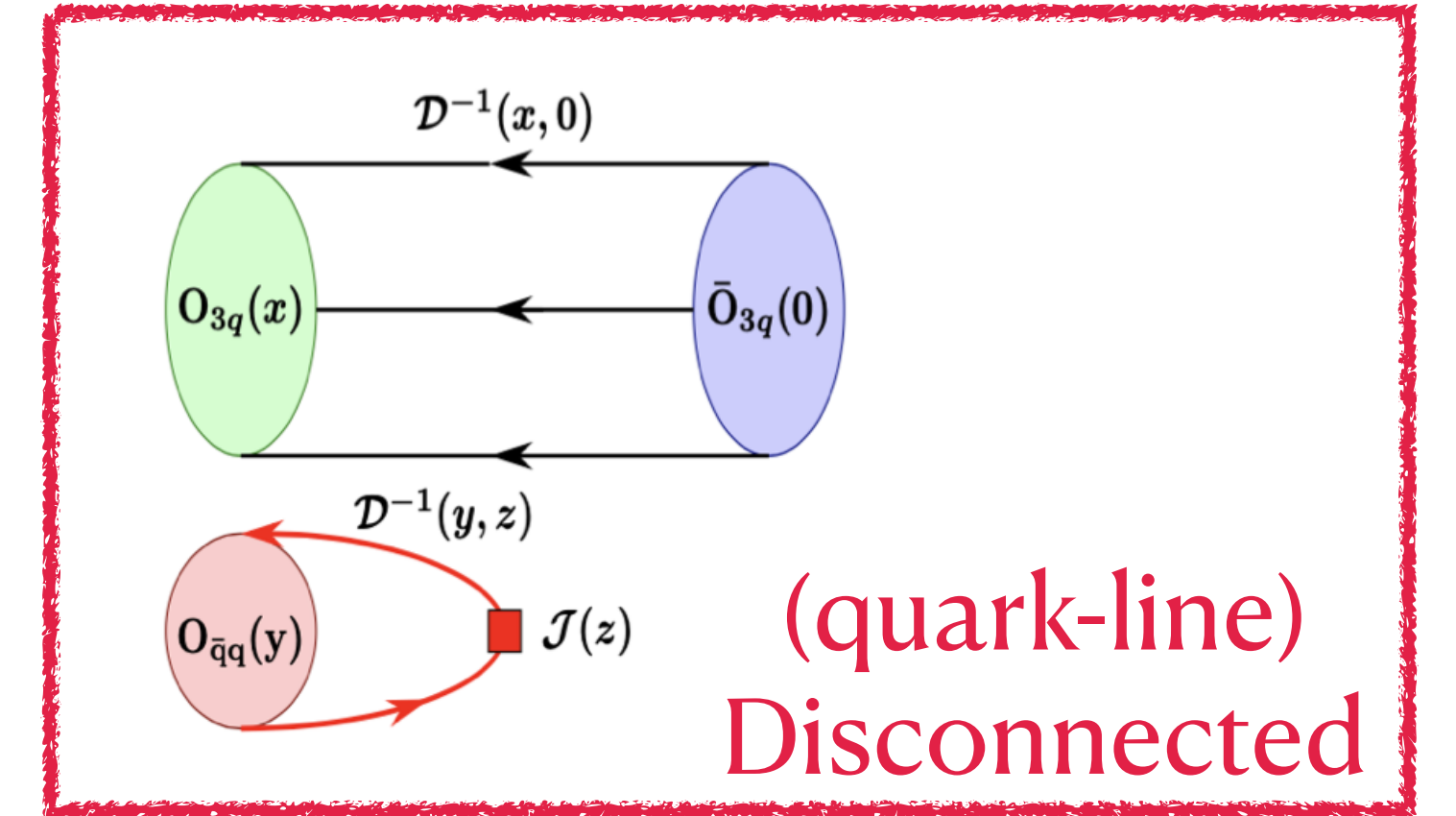
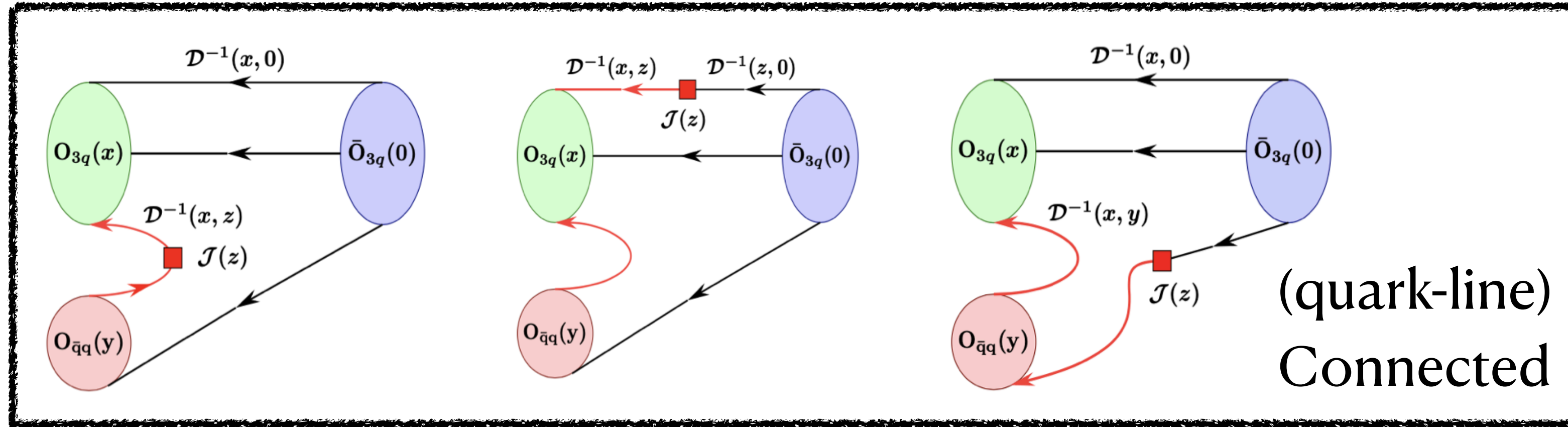


$$\frac{\langle O_{N\pi}(\vec{p}', t) \mathcal{P}(\vec{q}, \tau) O_N^\dagger(\vec{p}, 0) \rangle_{\text{connected}}}{\langle O_{N\pi}(\vec{p}', t) \mathcal{P}(\vec{q}, \tau) O_N^\dagger(\vec{p}, 0) \rangle_{\text{disconnected}}} \approx 1/100 \quad (\tau = t)$$

$m_\pi = 429 \text{ MeV}$

$$\begin{aligned} \langle O_{N\pi}(\vec{p}', t) \mathcal{P}(\vec{q}, \tau) O_N^\dagger(\vec{p}, 0) \rangle_{\text{disconnected}} &\approx \\ &\approx \langle O_N(\vec{p}', t) O_N^\dagger(\vec{p}, 0) \rangle \langle O_\pi(\vec{p}', t) \mathcal{P}(\vec{q}, \tau) \rangle \\ &\text{non-interacting} \\ &\text{Volume-enhanced} \end{aligned}$$

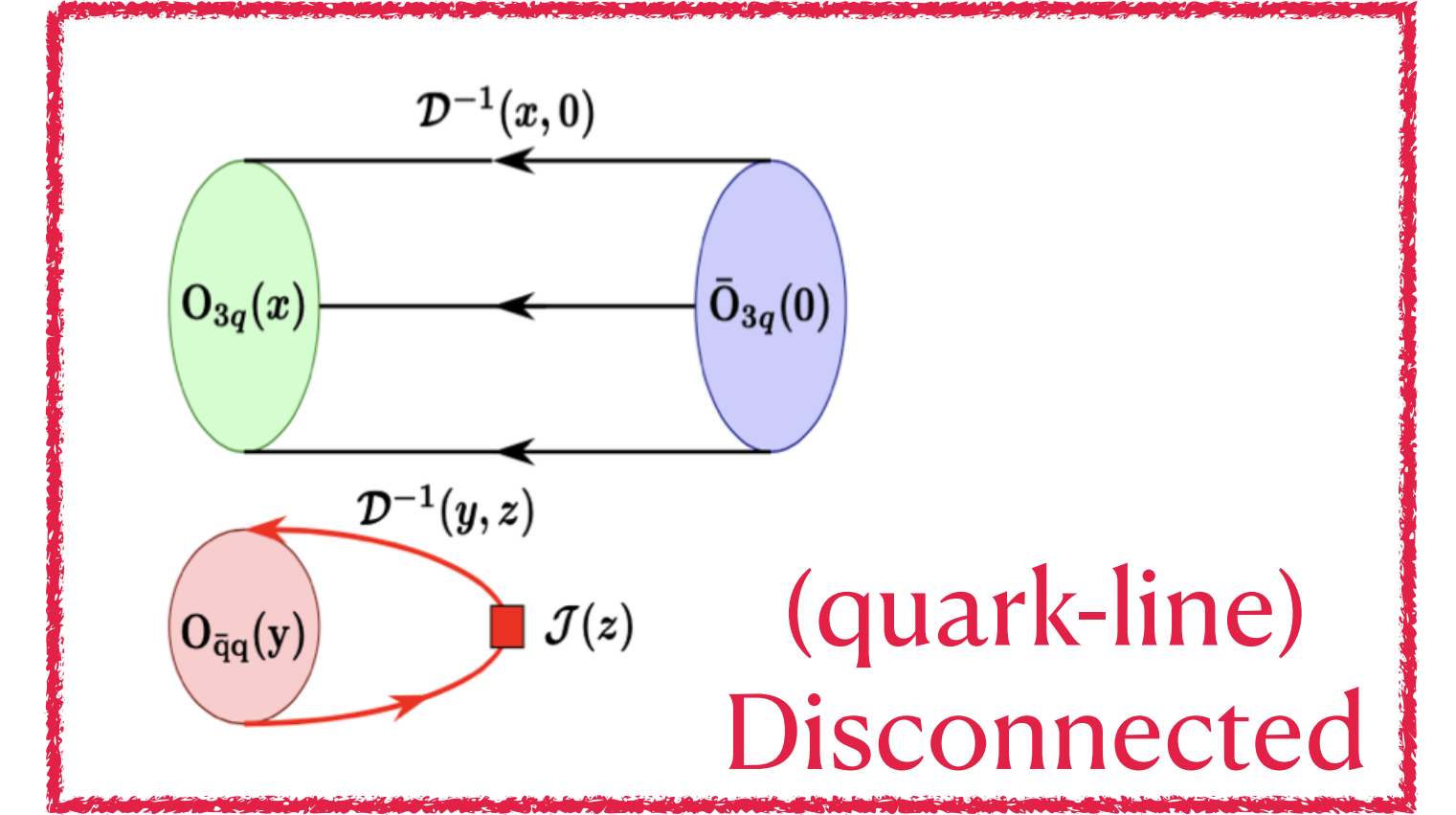
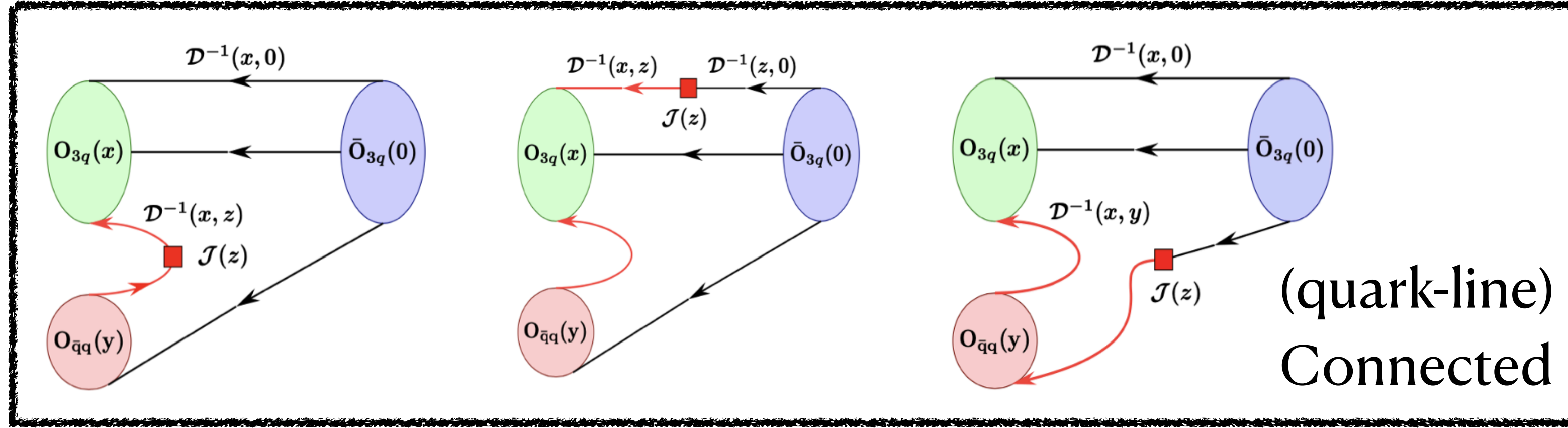
Volume study of diagrams in $\langle O_N(\vec{p}', t) \mathcal{J}(\vec{q}, \tau) O_{N\pi}^\dagger(\vec{p}, 0) \rangle$



Disconnected diagrams
increases with $\sim (L/a)^3$

Only O_{NM} which yield
disconnected diagrams in
3-pts are volume enhanced.

Volume enhancement of quark-line disconnected diagrams.



$$\langle O_{N\pi}(\vec{p}', t) \mathcal{P}(\vec{q}, \tau) O_N^\dagger(\vec{p}, 0) \rangle_{\text{disconnected}} \approx \langle \langle O_N(\vec{p}'_N, t) O_N^\dagger(\vec{p}, 0) \rangle \langle O_\pi(\vec{p}'_\pi, t) \mathcal{P}(\vec{q}, \tau) \rangle \left(+ \mathcal{O}(\alpha_S^2/N_c) \right) \quad \begin{array}{l} \text{(2-gluon exchange +} \\ \text{+ baryon large N [E.Witten])} \end{array}$$

$$\langle \langle O_N(\vec{p}'_N, t) O_N^\dagger(\vec{p}, 0) \rangle \langle O_\pi(\vec{p}'_\pi, t) \mathcal{P}(\vec{q}, \tau) \rangle = \sum_{\vec{x}'_\pi} \sum_{\vec{z}} e^{-i\vec{p}'_\pi \cdot \vec{x}'_\pi} e^{i\vec{q} \cdot \vec{z}} \langle \langle O_N(\vec{p}'_N, t) O_N^\dagger(\vec{p}, 0) \rangle \langle O_\pi(\vec{x}'_\pi, t) \mathcal{P}(\vec{z}, \tau) \rangle$$

$$(\vec{z} = \vec{x}'_\pi + \vec{r}, \quad \vec{p}'_\pi = \vec{q}) = \sum_{\vec{x}'_\pi} \sum_{\vec{r}} e^{-i\vec{p}'_\pi \cdot \vec{x}'_\pi} e^{i\vec{q} \cdot \vec{z}} \langle O_N(\vec{p}'_N, t) O_N^\dagger(\vec{p}, 0) \rangle \langle O_\pi(\vec{x}'_\pi, t) \mathcal{P}(\vec{x}'_\pi - \vec{r}, \tau) \rangle$$

$$\text{(Translational invariance)} = (L/a)^3 \sum_{\vec{r}} e^{-i\vec{p}'_\pi \cdot \vec{x}'_\pi} e^{i\vec{q} \cdot \vec{z}} \langle \langle O_N(\vec{p}'_N, t) O_N^\dagger(\vec{p}, 0) \rangle \langle O_\pi(\vec{0}, t) \mathcal{P}(\vec{r}, \tau) \rangle .$$