Nucleon sigma terms at $m_\pi = 222$ MeV with a variational analysis from lattice QCD

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DESY Zeuthen Particle Physics Theory

In collaboration with:

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(U. Regensburg)

FOR 5269

DESY.

The 43rd International Symposium on Lattice Field Theory (Lattice 2026)

University of Maryland, College Park, 26/06/2026 - 01/08/2026

Motivation

- Relevant quantity for Standard Model and Beyond Standard Model Physics

- Current tension between pheno and most lattice determinations of $\sigma_{\pi N}$

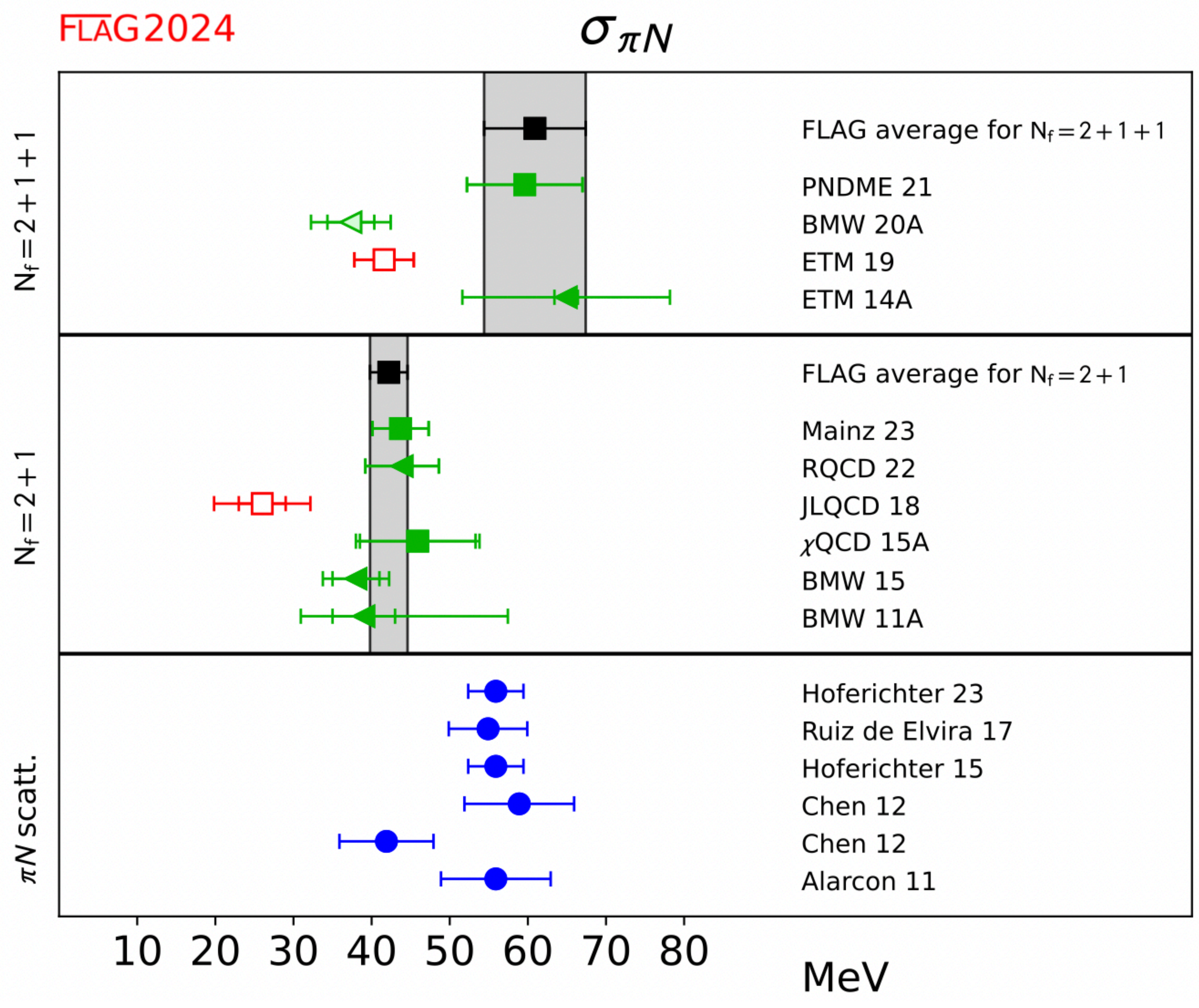
- Excited-state contamination (ESC) must be properly accounted for precise determinations

R. Gupta et al. [*Phys.Rev.Lett.* 127, 242002 (2021)]

L. Barca, G. Bali, S. Collins [*Phys.Rev.D* 111 (2025) 3, L031505]

L. Barca [*Phys.Rev.D* 112 (2025) 9, L091503]

[G. Bali Fri 11:00]



[FLAG Review 2024]

Sigma terms and direct method

$$\sigma_{\pi N} = \sigma_{uN} + \sigma_{dN} \qquad \sigma_{qN} = m_q \langle N | S_q | N \rangle \qquad S_q = q I \bar{q}$$

$$\langle \mathrm{O}_N(t)\ S_q(\tau)\ \bar{\mathrm{O}}_N(0) \rangle = \sum_{n',n} \langle 0 | \mathrm{O}_N | n' \rangle \langle n' | S_q | n \rangle \langle n | \bar{\mathrm{O}}_N | 0 \rangle \frac{e^{-E'_n(t-\tau)} e^{-E_n \tau}}{4 E'_n E_n V_{n'} V_n}$$ (after VEV-subtraction)

$$= |Z_N|^2 \langle N | S_q | N \rangle\ e^{-E_N t}\ +\ \ldots\ (\mathrm{ESC})$$

At large t, $\langle N | \bar{q} q | N \rangle$ can be reliably determined as ESC is exponentially suppressed.

Unfortunately, StN hinders the analysis at large t, τ. Need to account for ESC at moderate t.

Some contributions from multi-particle states are not suppressed by the spatial volume.

$$R_q(t,\tau) = \frac{\langle \mathrm{O}_N(t)\ S_q(\tau)\ \bar{\mathrm{O}}_N(0) \rangle}{\langle \mathrm{O}_N(t)\ \bar{\mathrm{O}}_N(0) \rangle} = \underset{(g_S^q)}{\langle N | \bar{q} q | N \rangle}\ +\ \ldots\ (\mathrm{ESC})$$

Current-enhanced operators/states in $\langle \mathrm{O}_H(\vec{p}\,', t)\; J(\vec{q}, \tau)\; \bar{\mathrm{O}}_H(\vec{p}, 0)\rangle$

L. Barca [*Phys.Rev.D* 112 (2025) 9, L091503]

Solve GEVP: $C(\vec{p}, t)V(\vec{p}, t; t_0) = C(\vec{p}, t_0)\; \Lambda(\vec{p}, t; t_0)\; V(\vec{p}, t; t_0)$ $\qquad \mathbb{B}_H = \left\{ \mathrm{O}_H, \mathrm{O}_{HM}, \mathrm{O}_{HMM}, \mathrm{O}_{HM_1}, \mathrm{O}_{HM_2}, \ldots \right\}$

GEVP-improved operators: $\Omega_H = \sum_{\mathrm{O}_i \in \mathbb{B}_H} v_i^H \mathrm{O}_i$ $\qquad V(\vec{p}, t; t_0) = \left(\vec{v}^H, \ldots\right)$

GEVP-improved 3-pts: $\langle \Omega_H(\vec{p}\,', t)\; J(\vec{q}, \tau)\; \Omega_H^\dagger(\vec{p}, 0)\rangle \;=\; \sum_{\mathrm{O}_i, \mathrm{O}_j \in \mathbb{B}_H} v_i^H(\vec{p}\,', \tilde{t}_2; t_0) \langle \mathrm{O}_i(\vec{p}\,', t)\; J(\vec{q}, \tau)\; \mathrm{O}_j^\dagger(\vec{p}, 0)\rangle v_j^H(\vec{p}, \tilde{t}_1; t_0)$

Terms like $\langle \mathrm{O}_{HM}(\vec{p}\,', t)\; J(\vec{q}, \tau)\; \mathrm{O}_H^\dagger(\vec{p}, 0)\rangle$ are much larger at short-intermediate distances than the others

if they contain (non-vanishing) quark-line disconnected diagrams, i.e., when (Direct diagrams in spectroscopy calculations)

$$\langle \mathrm{O}_H(\vec{p}\,'_H, t)\; \mathrm{O}_H^\dagger(\vec{p}, 0)\rangle, \;\; \langle \mathrm{O}_M(\vec{p}\,'_M, t)\; J(\vec{q}, \tau)\rangle \;\neq 0$$

(*HM* are then “current-enhanced”)

They grow with the volume because of translational invariance (backup).

Current-enhanced states: another viewpoint

CERN Theory Institute 2026
Thankful for discussions with
J. Green, M. Hansen, H. Meyer

$\vec{p} = \vec{p}\,' = \vec{q} = \vec{0}$

Infinite volume

$$\langle \mathrm{O}_N(t)\ J(\tau)\ \bar{\mathrm{O}}_N(0)\rangle = |Z_N|^2 \langle N|J|N\rangle\ e^{-m_N t}$$

$$+\ Z_{N'M}\ Z_N\ \langle N'M|J|N\rangle\ e^{-E_{N'M}(t-\tau)}e^{-E_N\tau}\ +\ \dots$$

Factorisation (*spectator-disconnected* + *connected*):

$$\langle N'M|J|N\rangle = \langle N'|N\rangle\langle M|J|0\rangle + \mathscr{M}_{conn.}(N'M \leftarrow N)$$

$$2E_N(2\pi)^3\delta(p_{N'} - p)\langle M|J|0\rangle + \mathscr{M}_{conn.}(N'M \leftarrow N)$$

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Infinite volume

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$$+\ Z_{N'M}\ Z_N\ \langle N'M|J|N\rangle\ e^{-E_{N'M}(t-\tau)} e^{-E_N \tau}\ +\ \dots$$

Finite volume ($V = T \times L^3$)

$$\langle \mathrm{O}_N(t)\ J(\tau)\ \bar{\mathrm{O}}_N(0)\rangle_L = |Z_N(L)|^2 \langle N|\mathcal{J}|N\rangle_L\ e^{-m_N t}$$

$$+\ Z_{N'M}(L)\ Z_N(L)\ \langle N'M|J|n\rangle_L\ e^{-E_{N'M}(t-\tau)} e^{-E_N \tau}\ +\ \dots$$

$$\boxed{(2\pi)^3 \delta(\vec{p}_{N'} - \vec{p})\ \rightarrow\ L^3 \delta_{\vec{p}_{N'},\vec{p}}}$$

Factorisation (*sp* … *n-interacting + interact.*):

$$\langle N'M|J|N\rangle = \langle N'|N\rangle\langle M|J|0\rangle + \mathcal{M}_{conn.}(N'M \leftarrow N)$$

$$2E_N(2\pi)^3\delta(p_{N'} - p)\langle M|J|0\rangle + \mathcal{M}_{conn.}(N'M \leftarrow N)$$

$$\langle N'M|J|N\rangle_L =$$

$$(2E_N)^2 L^3 \delta_{p_{N'},p}\langle M|J|0\rangle + \mathcal{M}^V_{int.}(N'M \leftarrow N)$$

States $\langle N'M|$ with the same QN and momentum as $\langle N|$ and with $\langle M|J|0\rangle \neq 0$ yield large contributions at short-intermediate distances compared to other excited states.

Current-enhanced states for $\sigma_{\pi N}$

$$\langle \mathrm{O}_N(t)\ q\bar{q}(\tau)\ \bar{\mathrm{O}}_N(0)\rangle_L = |Z_N|^2 \langle N|q\bar{q}|N\rangle_L\ e^{-m_N t} + \ Z_{N'M}\ Z_N\ \langle N'M|q\bar{q}|N\rangle_L\ e^{-E_{N'M}(t-\tau)}e^{-E_N\tau}\ +\ \ldots$$

$$\langle N'M|\bar{q}q|N\rangle_L = \ (2m_N)^2L^3\delta_{p_{N'},0}\langle M|\bar{q}q|0\rangle + \mathscr{M}^V_{int.}(N'M \leftarrow N)$$

$$\langle M|\bar{q}q|0\rangle \neq 0$$

$\langle M|$ must have σ-like QN: Large ESC from $N\sigma\big/N(\pi\pi)_{L=0}$

Current-enhanced states for $\sigma_{\pi N}$

$$\langle \mathrm{O}_N(t)\; q\bar{q}(\tau)\; \bar{\mathrm{O}}_N(0)\rangle_L = |Z_N|^2 \langle N|\, q\bar{q}\,|N\rangle_L\; e^{-m_N t} + \; Z_{N'M}\; Z_N\; \langle N'M|\, q\bar{q}\,|N\rangle_L\; e^{-E_{N'M}(t-\tau)} e^{-E_N \tau} \; + \; \ldots$$

$$\langle N'M|\,\bar{q}q\,|N\rangle_L = \; (2m_N)^2 L^3 \delta_{p_{N'},0} \langle M|\,\bar{q}q\,|0\rangle + \mathscr{M}^{V}_{int.}(N'M \leftarrow N)$$

$$\langle M|\,\bar{q}q\,|0\rangle \neq 0$$

$\langle M|$ must have σ-like QN: Large ESC from $N\sigma \big/ N(\pi\pi)_{L=0}$

Using same argument in other channels:

- $N\pi$ in some $\langle \mathrm{O}_N(\vec{0},t)\; A_\mu(\vec{q},\tau)\; \mathrm{O}_N(-\vec{q},0)\rangle$ as predicted by ChPT in [O. Bär, *PRD* 101 (2020) 3, 034515, …]
  [A. Jüttner Mon 9:15]
- $B^*_\mu \pi$ in $\langle \mathrm{O}_B(\vec{0},t)\; V_\mu(\vec{q},\tau)\; \mathrm{O}_\pi(-\vec{q},0)\rangle$ as also predicted by HMChPT in [O. Bär, et al. *Eur.Phys.J.C* 83 (2023) 8, 757]
- B^*K in $\langle \mathrm{O}_{B_s}(\vec{0},t)\; V_\mu(\vec{q},\tau)\; \mathrm{O}_K(-\vec{q},0)\rangle$, $K^*\pi \big/ (K\pi)\pi$ in $\langle \mathrm{O}_K(\vec{0},t)\; V_\mu(\vec{q},\tau)\; \mathrm{O}_\pi(-\vec{q},0)\rangle$, …
- $N\pi$ contribution is small in $\langle \mathrm{O}_N(\vec{0},t)\; V_\mu(\vec{q},\tau)\; \mathrm{O}_N(-\vec{q},0)\rangle$ [O. Bär, *PRD* 103 (2021) 11, 114514] $\Big(\langle \pi|\,V_\mu\,|0\rangle = 0\Big)$
- …

Nucleon-sigma terms with a variational analysis at $m_\pi = 429\ \text{MeV}$

$a = 0.098\ \text{fm}$ $L = 24a$

L. Barca, G. Bali, S. Collins
[*Phys.Rev.D* 111 (2025) 3, L031505]

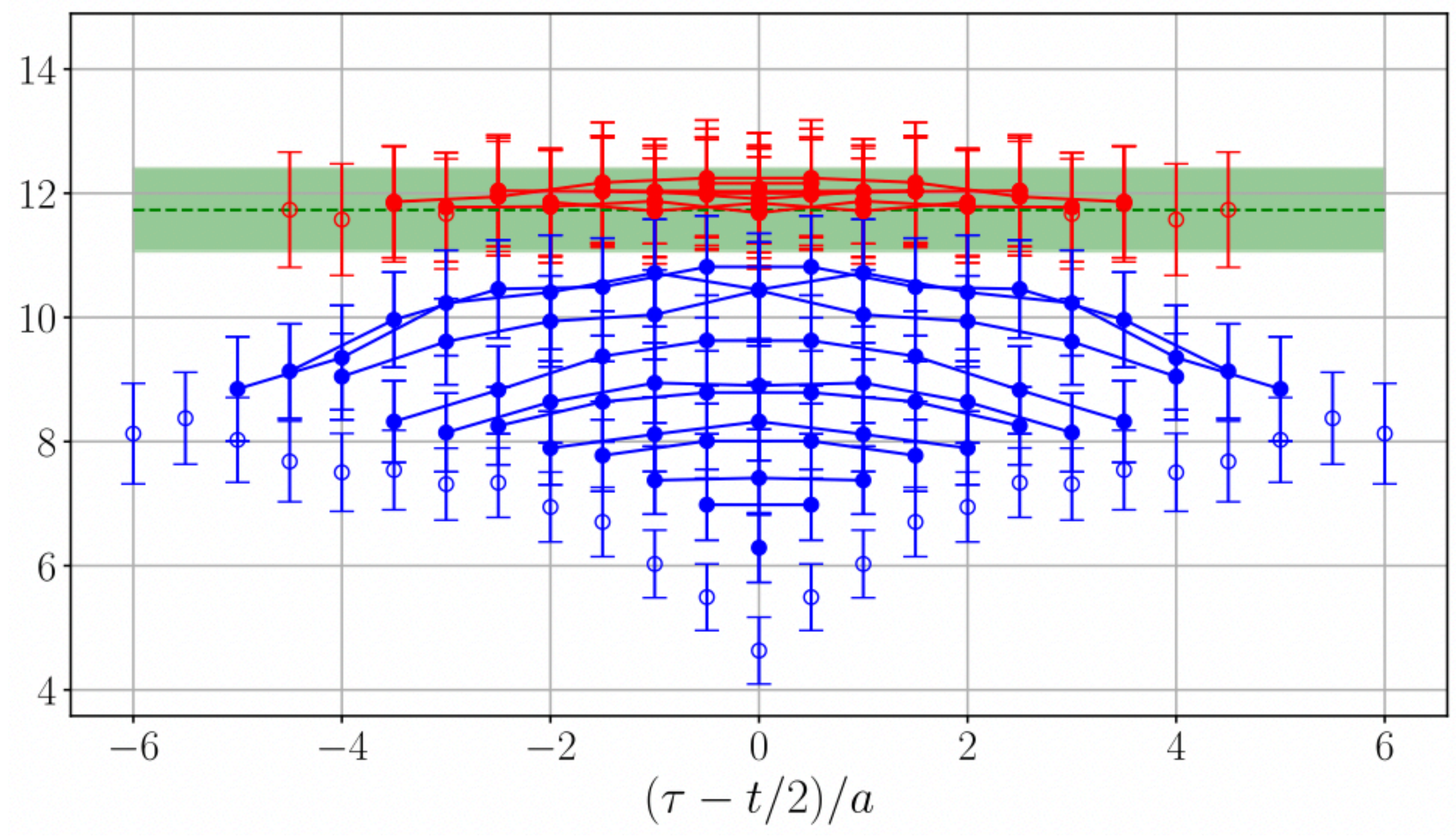
Variational analysis with N and $N\sigma$ interpolators $\sigma \sim \bar{u}u + \bar{d}d$

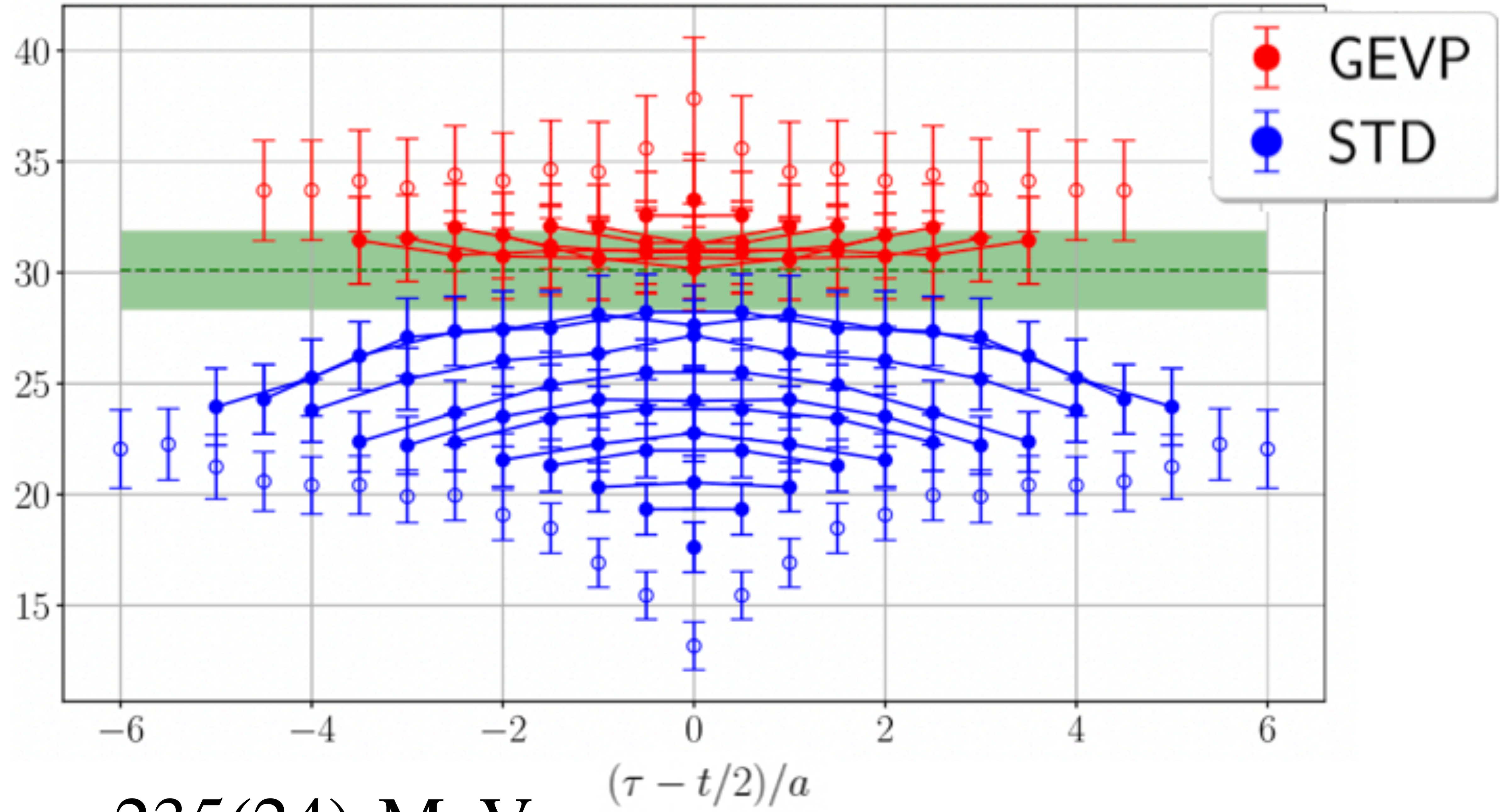
$$R_{q\bar{q}}^{(\text{GEVP})}(t,\tau) = \frac{\langle \text{O}_N^{(\text{GEVP})}(t)\ (q\bar{q})(\tau)\ \bar{\text{O}}_N^{(\text{GEVP})}(0)\rangle}{\langle \text{O}_N^{(\text{GEVP})}(t)\ \bar{\text{O}}_N^{(\text{GEVP})}(0)\rangle} = \langle N|q\bar{q}|N\rangle\ +\ \ldots\ (\text{ESC})$$

σ stable at $\approx 550\ \text{MeV} < \text{E}_{\pi\pi}^{(0)}$

$\boxed{\text{Substantial reduction of ESC}}$

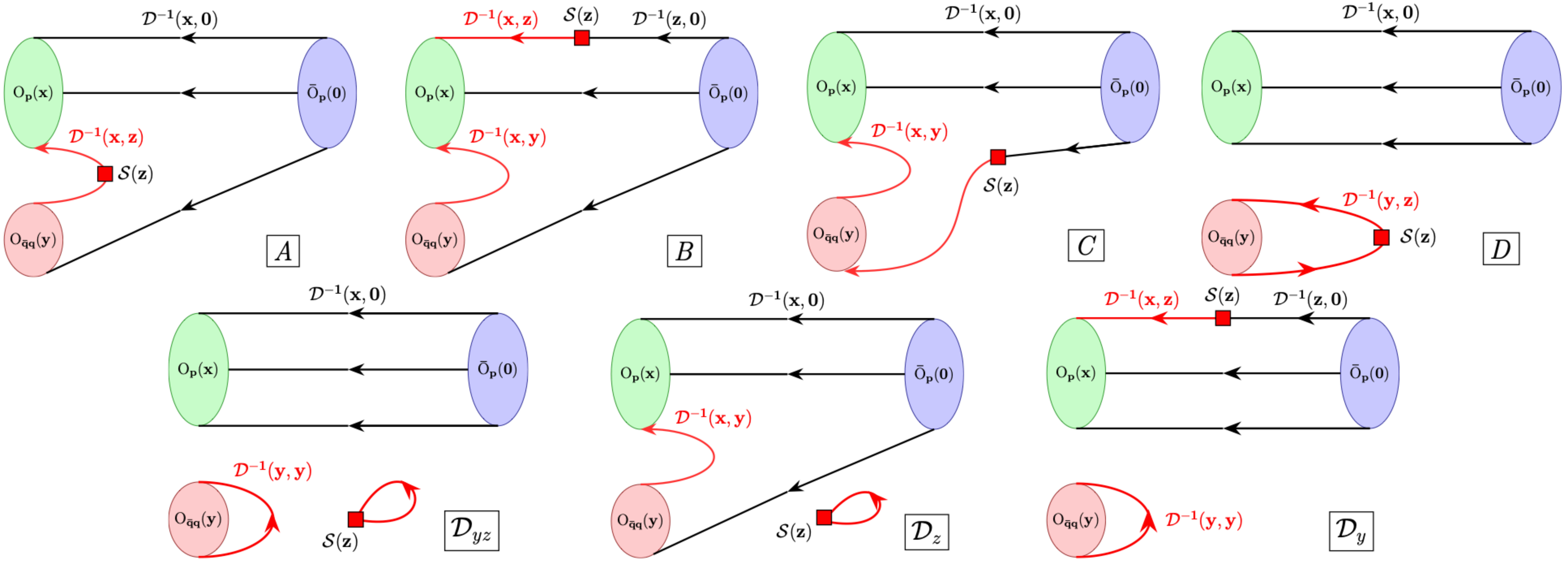
$R_s^{(\text{GEVP})}(t,\tau)$ $R_{u+d}^{(\text{GEVP})}(t,\tau)$





$\sigma_{sN} = 42(16)\ \text{MeV}$ $\sigma_{\pi N} = 235(24)\ \text{MeV}$

Diagrams

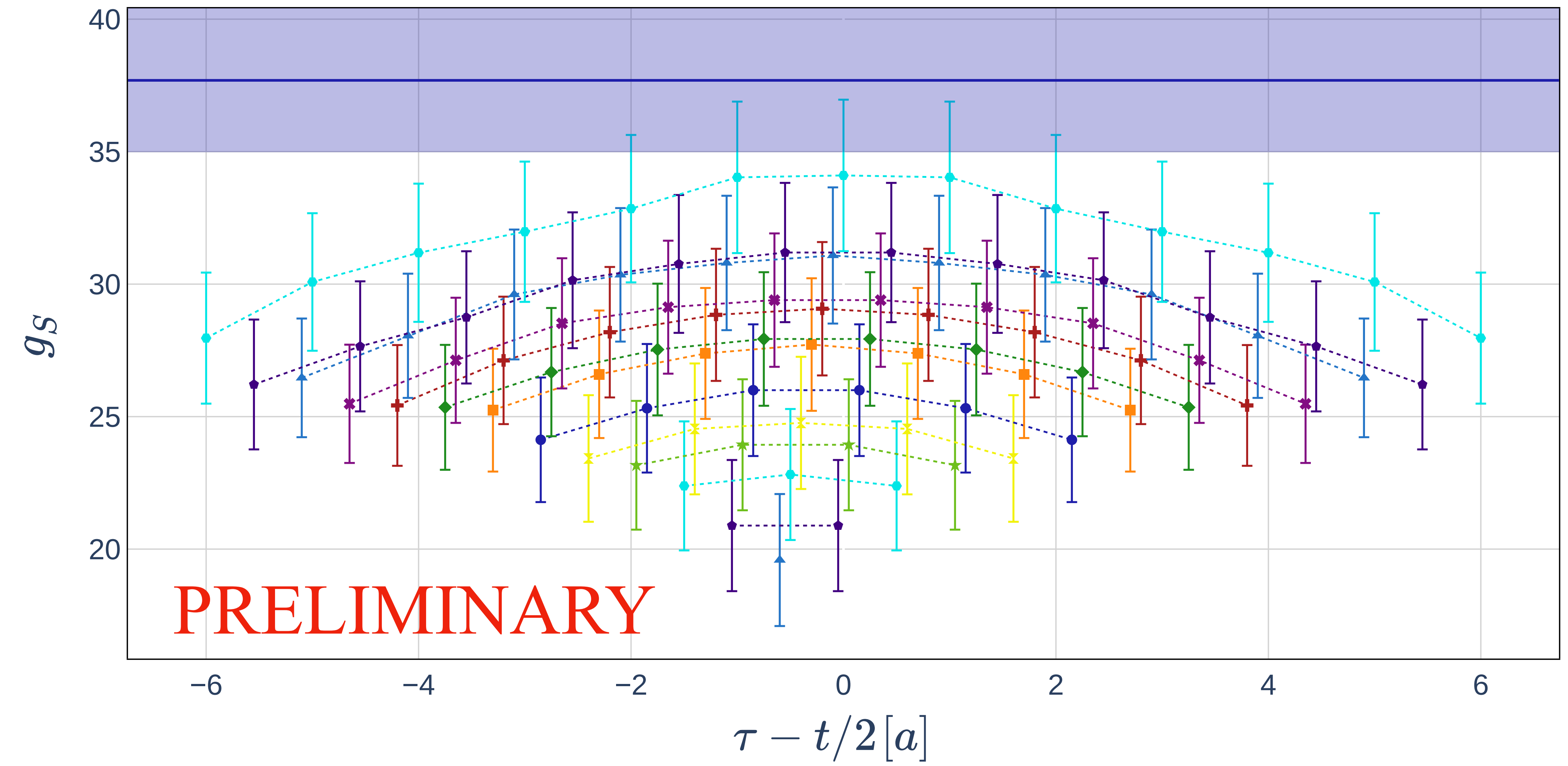


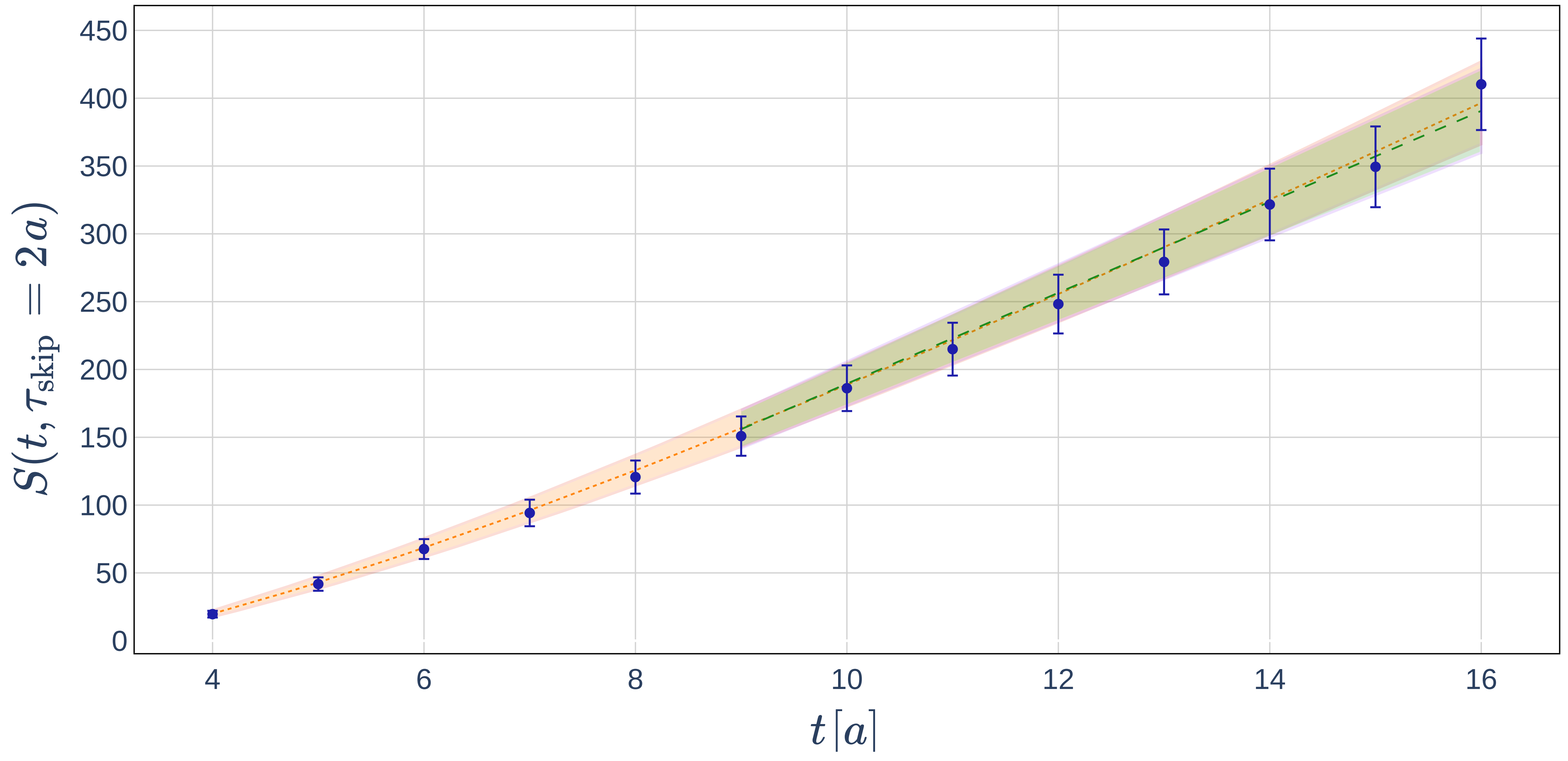
$\mathscr{D}_{yz}$ and D $= \mathcal{O}(100) \times$ other diagrams $(m_\pi = 429\ \text{MeV},\ \text{L} = 24a)$

Nucleon sigma terms at $m_\pi = 222\ \mathrm{MeV}$ with standard correlators

G. Bali, **LB**, S. Collins, M. Rodekamp

$a = 0.0865\ \mathrm{fm}$ $\quad L = 48a$





Summation method: $$S(t, \tau_{\rm skip} = 2a) = \sum_{\tau=2a}^{t-2a} R_{q\bar{q}}(t, \tau)$$

Fit models

<u>Linear fit</u> $\quad S(t) = c_0 + g_s t \quad g_s^{\rm fit} = 33.5(2.7)$

<u>Linear + exp. fit</u> $\quad S(t) = c_0 + g_s t + c_{10} e^{-\Delta E t}$

No prior	$g_s^{\rm fit} = 36(42)$	😟
$\Delta E = E_{N\pi} - m_N$	$g_s^{\rm fit} = 38.3(2.8)$	$\chi^2/\mathrm{dof} = 0.939$
$\Delta E = E_{N\pi\pi} - m_N$	$g_s^{\rm fit} = 37.4(2.7)$	$\chi^2/\mathrm{dof} = 0.933$
ΔE from nucleon 2-pts	$g_s^{\rm fit} = 32.4(2.5)$	$\chi^2/\mathrm{dof} = 1.05$

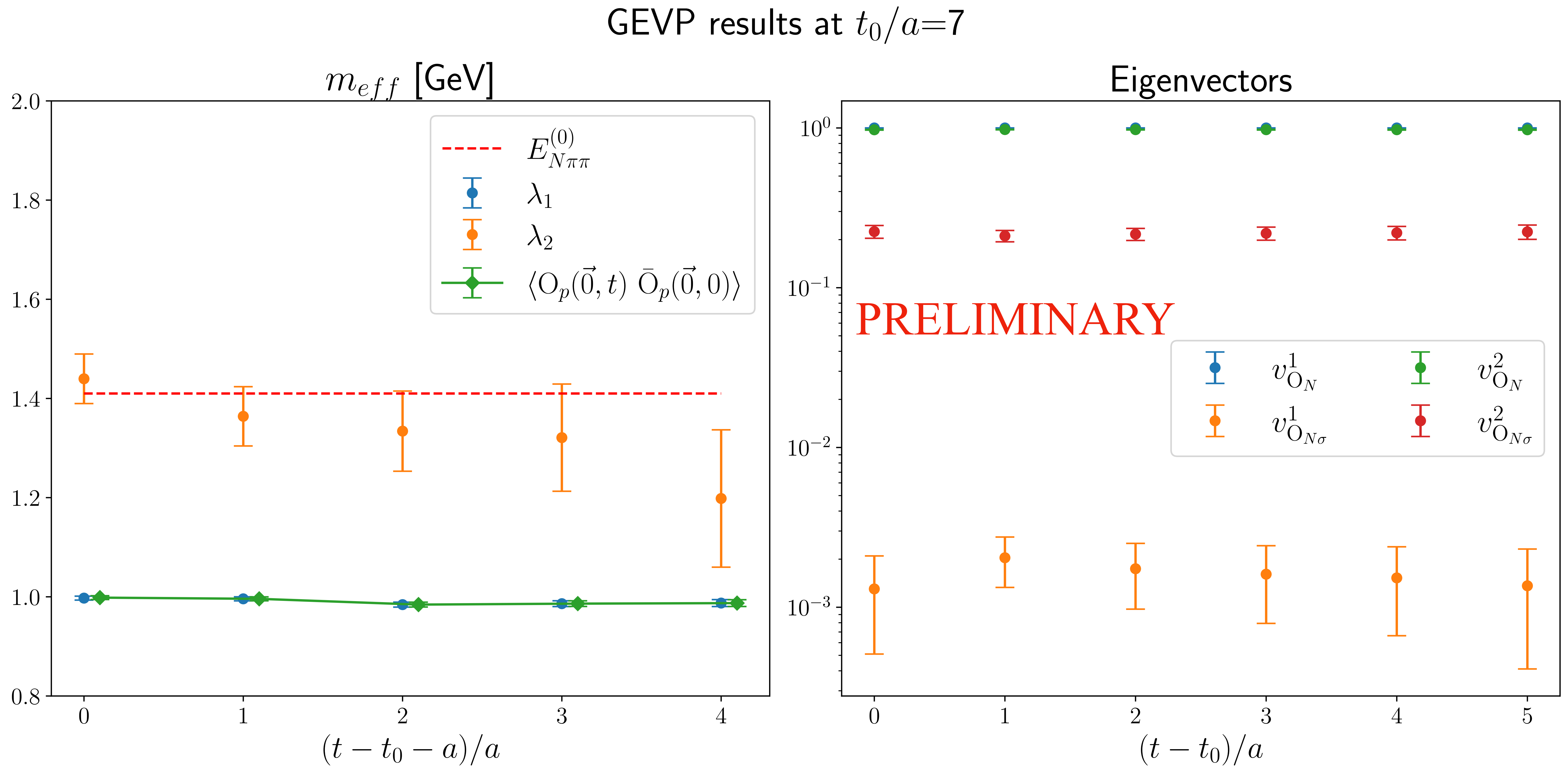
Variational analysis at $m_\pi = 222\ \mathrm{MeV}$

$\langle\sigma(t)\ \sigma(t)\rangle$ decays w/ $m_{\mathrm{eff}} \approx 420\ \mathrm{MeV}$
$\approx E^{(0)}_{\pi\pi}$

G. Bali, **LB**, S. Collins, M. Rodekamp

Basis: N and $N\sigma$ interpolators

σ becomes a broad resonance
[*Phys.Rev.Lett.* 118 (2017) 2, 022002 (HadSpec)] ($m_\pi = 236\ \mathrm{MeV}$)



GEVP-improved operators

$$\mathrm{O}_N^{(\mathrm{GEVP})}(t) = \sum_{i=N,N\sigma} v_i^N \mathrm{O}_i(t)$$

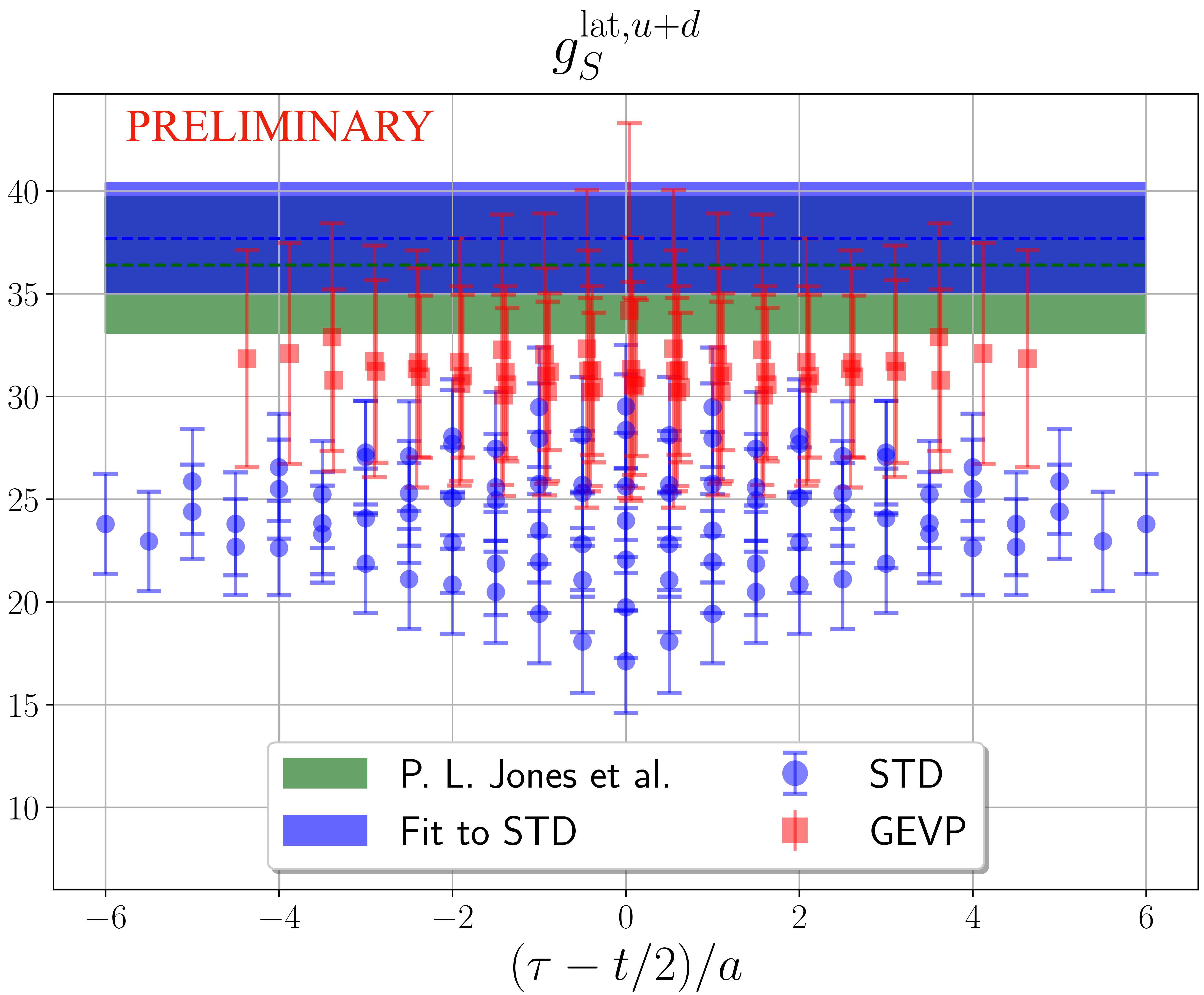
GEVP-improved correlators

$$C_{2\mathrm{pt}}^{(\mathrm{GEVP})}(t) = \langle \mathrm{O}_N^{(\mathrm{GEVP})}(t)\ \bar{\mathrm{O}}_N^{(\mathrm{GEVP})}(0)\rangle$$

$$C_{3\mathrm{pt}}^{(\mathrm{GEVP})}(t,\tau) = \langle \mathrm{O}_N^{(\mathrm{GEVP})}(t)\ S_q(\tau)\ \bar{\mathrm{O}}_N^{(\mathrm{GEVP})}(0)\rangle$$

Note: $v^1_{\mathrm{O}_{N\sigma}} \approx (2/1000) = \mathcal{O}\left(1/\sqrt{L^3}\right)$

Nucleon-sigma terms with a variational analysis at $m_\pi = 222$ MeV



$\mathbb{B}_N = \{\mathrm{O}_N, \mathrm{O}_{N\sigma}\}$ very effective in removing the largest excited state contamination in this channel.

Errors of GEVP ratios are larger because of noise in the eigenvectors.

<u>Comparison</u>

GEVP ratios agree within the errors with other fits.

Fits to summed GEVP ratios yield $g_s^{u+d} = 31.2(4.0)$

<u>In progress</u>

$\mathbb{B}_N = \left\{\mathrm{O}_N, \mathrm{O}_{N\sigma_0}, \mathrm{O}_{N\sigma_8}\right\} \longrightarrow g_s^{\mathrm{lat},u+d},\ g_s^{\mathrm{lat},s}$

Conclusions

[G. Bali, **L. Barca**, S. Collins, M. Rodekamp]

- Standard ratios for $\sigma_{\pi N}$ and summed ratios suffer from large ESC.

- Discussed the mechanism of current-enhanced states in lattice QCD 3-pts. Broad implications in lattice QCD.

- GEVP analysis for g_S^{u+d} ($\sigma_{\pi N}$) and g_S^s (σ_{sN}) with O_N and $O_{N\sigma}$ interpolating operators at $m_\pi = 429$ MeV, 222 MeV is very effective in removing the dominant ESC.

CLQCD has also obtained very good results with this idea.

<u>Physics interpretation</u>

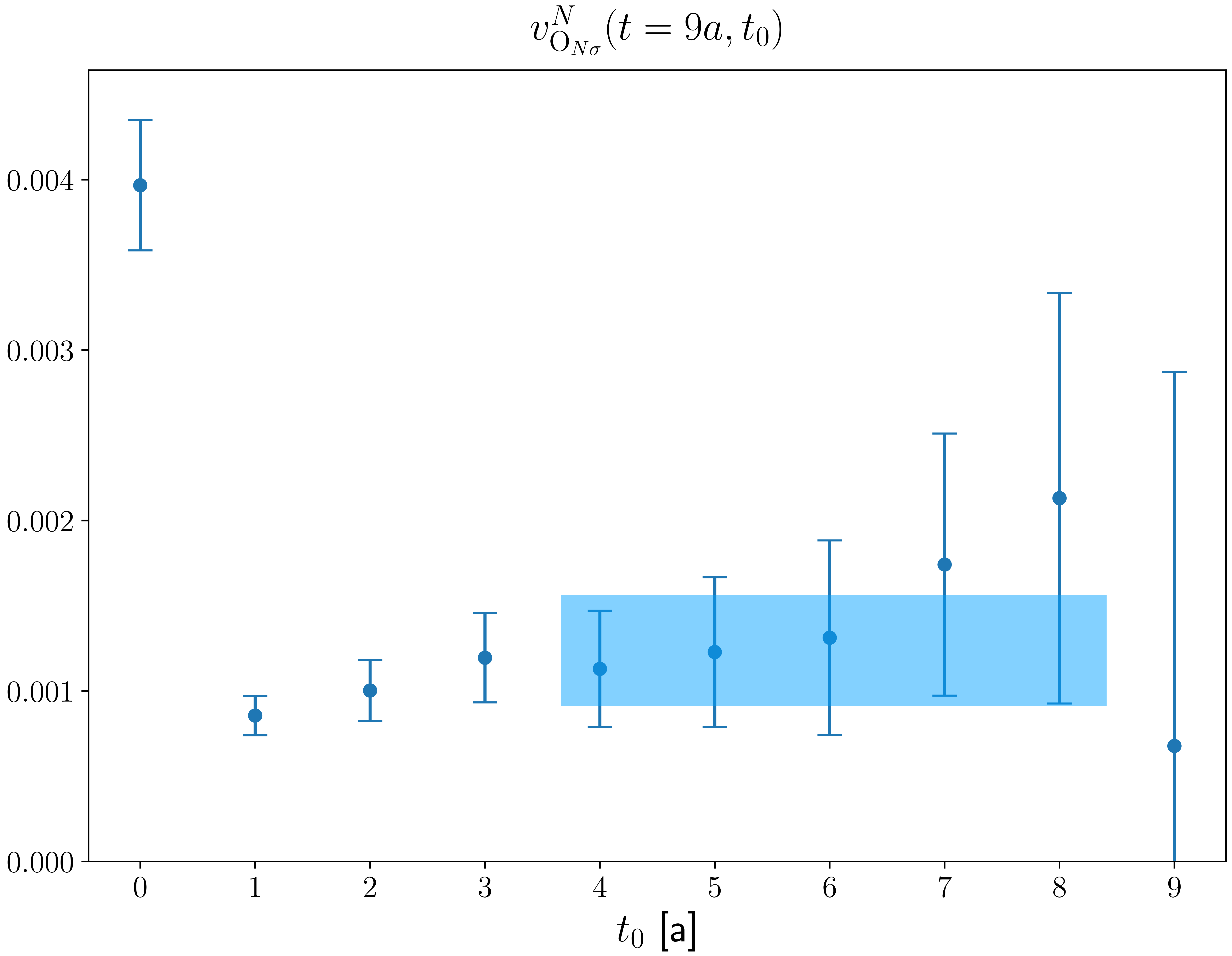
At $m_\pi = 429$ MeV dominant ESC is $N\sigma$, while at $m_\pi = 222$ MeV is $N\pi\pi$

<u>Next</u> Try to reduce statistical errors and carry out variational analysis at or near physical pion masses.

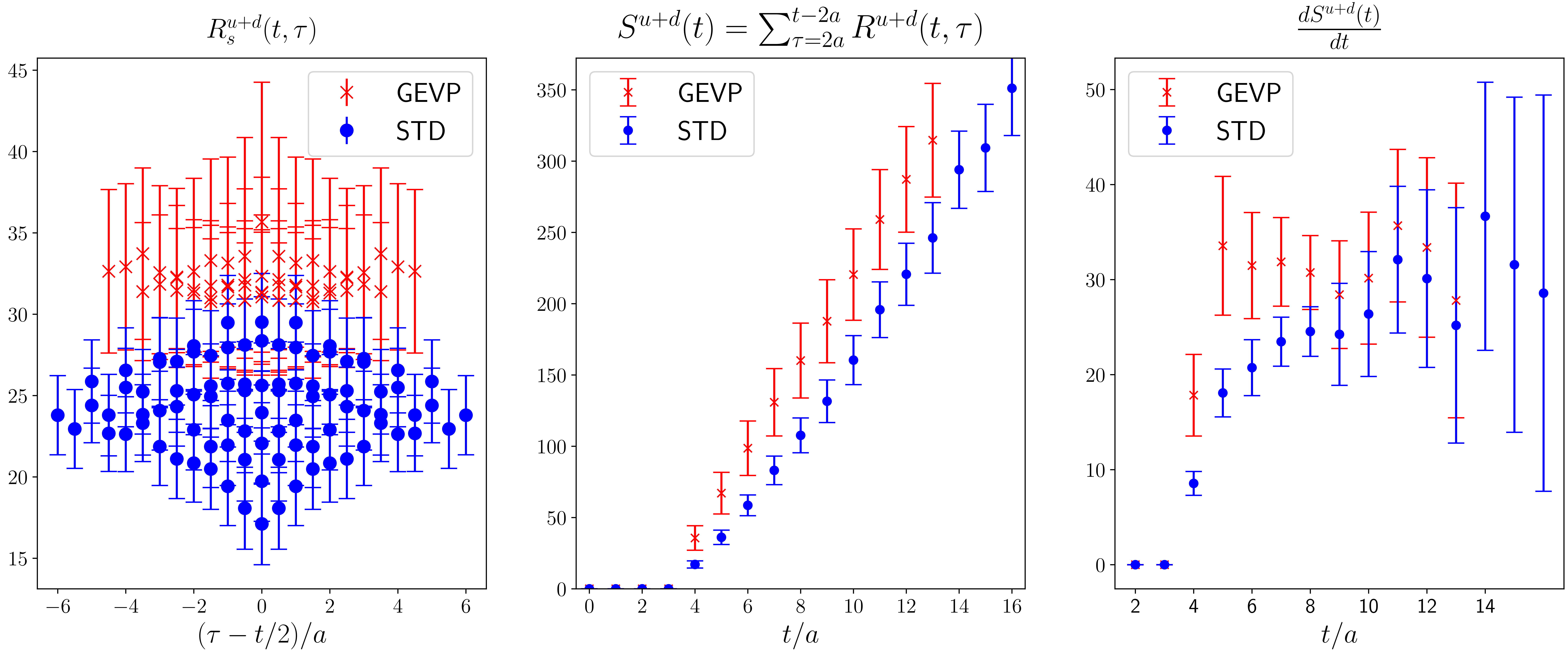
<u>Ambitious Goal</u> Construct comprehensive variational basis for nucleon (include all B, BM, BMM operators)

Exponentially reduced systematics, statistical errors and allows us to extract TME

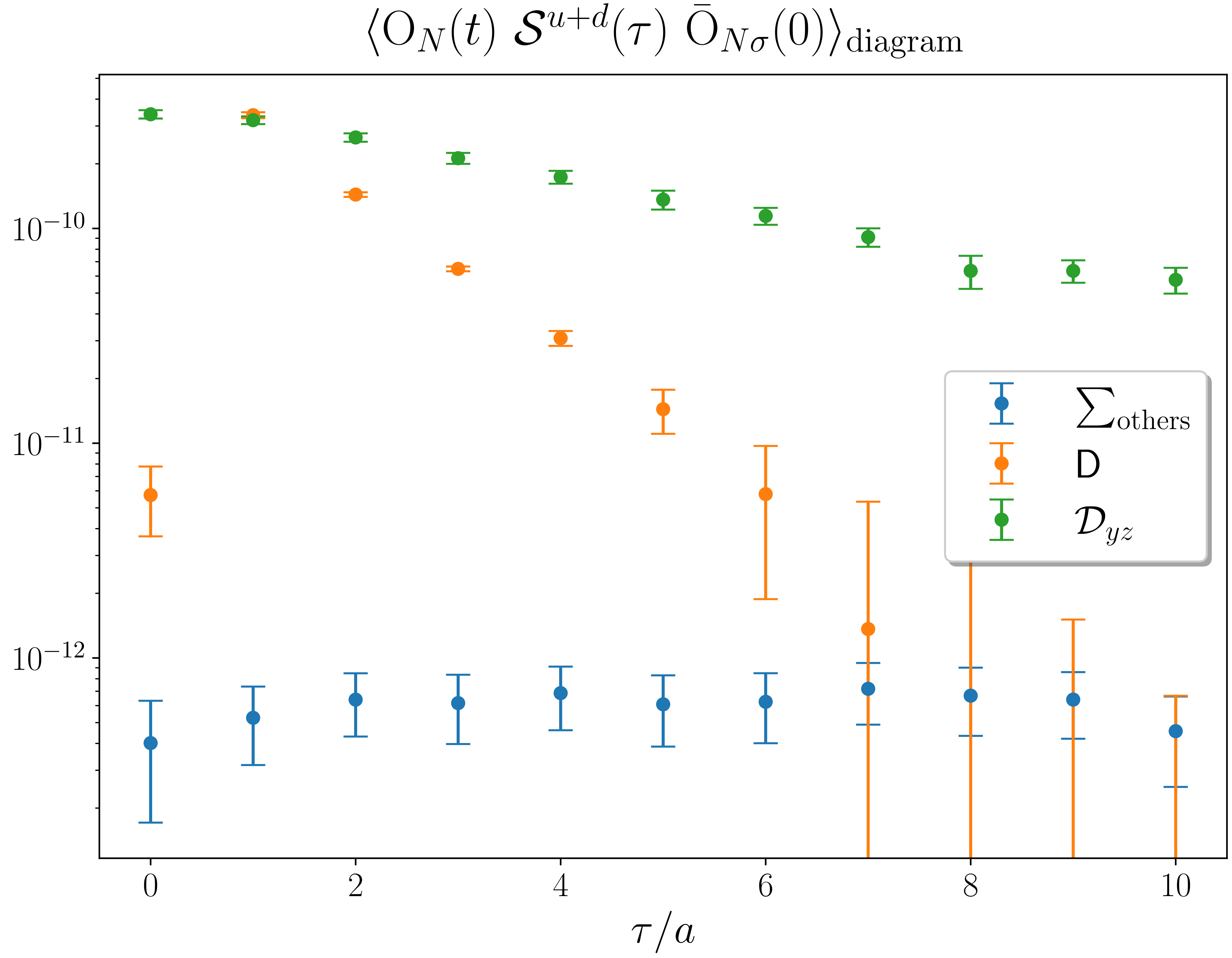
In collaboration also with Y. Fu, and with J. Bulava, F. Romero-López, … (BaSc)



$$\bar{v}^N_{\mathrm{O}_{N\sigma}} = \frac{1}{4} \sum_{t_0=5a}^{8a} v^N_{\mathrm{O}_{N\sigma}}(t = 9a, t_0)$$


$R_s^{u+d}(t,\tau)$
GEVP
STD
$(\tau - t/2)/a$
$S^{u+d}(t) = \sum_{\tau=2a}^{t-2a} R^{u+d}(t,\tau)$
GEVP
STD
t/a
$\frac{dS^{u+d}(t)}{dt}$
GEVP
STD
t/a

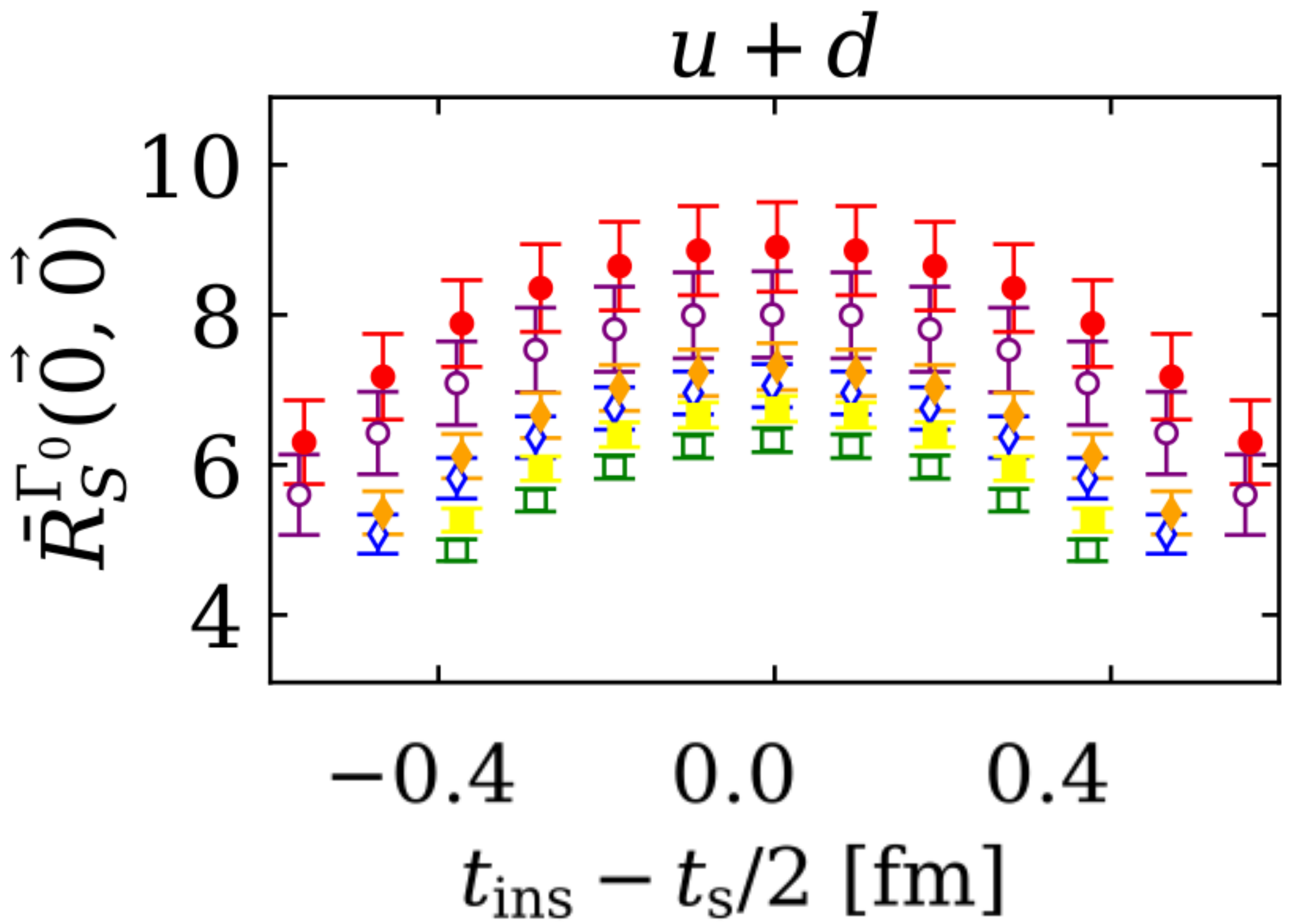
m_π = 222 MeV, a = 0.0865 fm, L = 48a



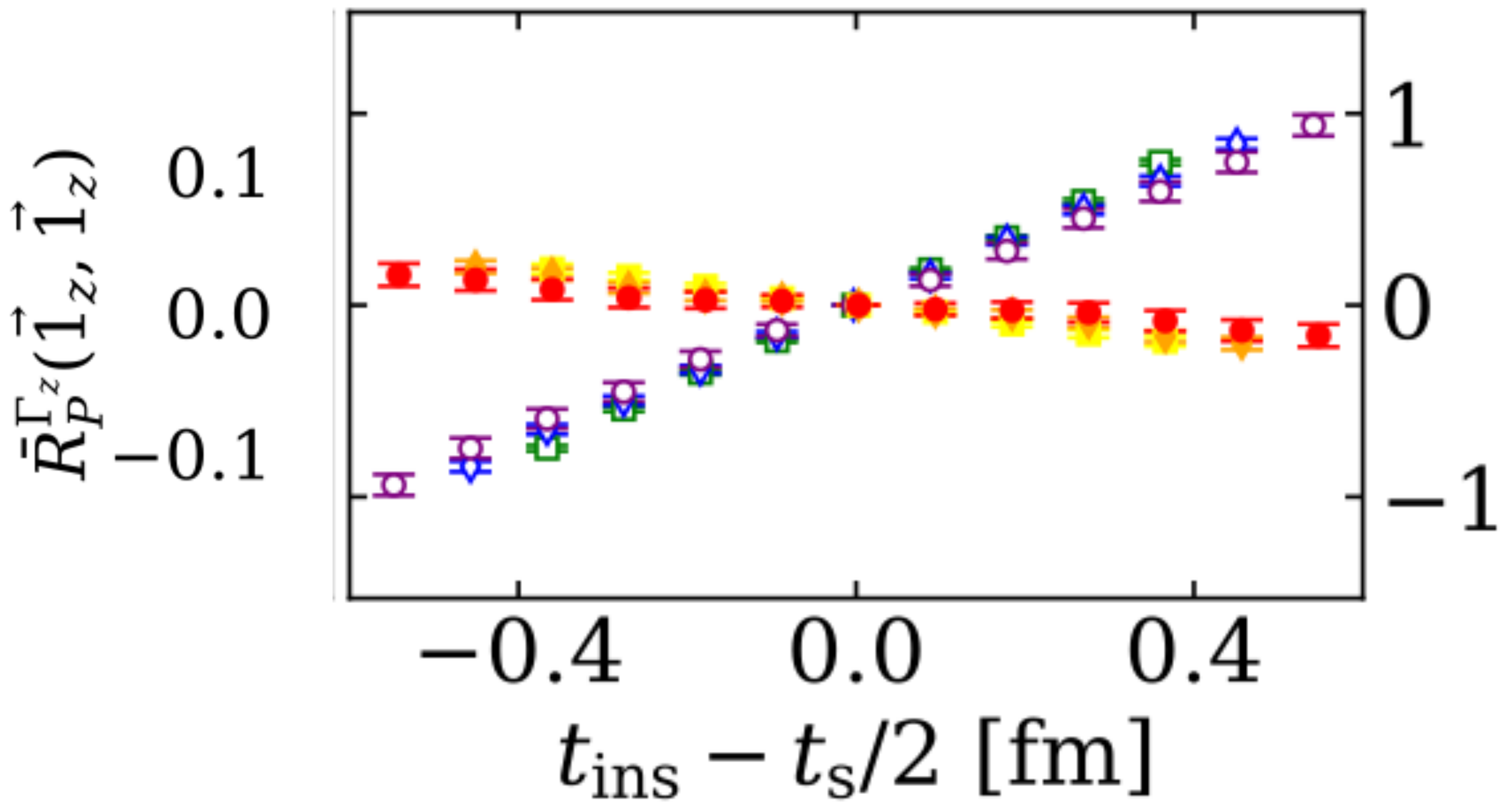
[Y. Li, et al. (ETMC), *Phys.Rev.D* 110 (2024) 9, 094514]

Variational analysis with O_N, $O_{N\pi}$

Ensembles	Flavors (N_f)	$N_L^3 \times N_T$	a [fm]	L [fm]	m_π [MeV]	$m_\pi L$	m_N [MeV]
cA211.53.24	$2+1+1$	$24^3 \times 48$	0.0908	2.27	346	3.99	1193(18)
cA2.09.48	2	$48^3 \times 96$	0.0938	4.50	131	2.98	931(3)



No effect in the isoscalar scalar channel



Tremendous effect in the isovector pseudoscalar channel

Accurate nucleon isovector tensor and scalar charge at the physical point

J.H. Wang, Z.C. Hu et al. (CLQCD) [*arXiv:2511.02326*]

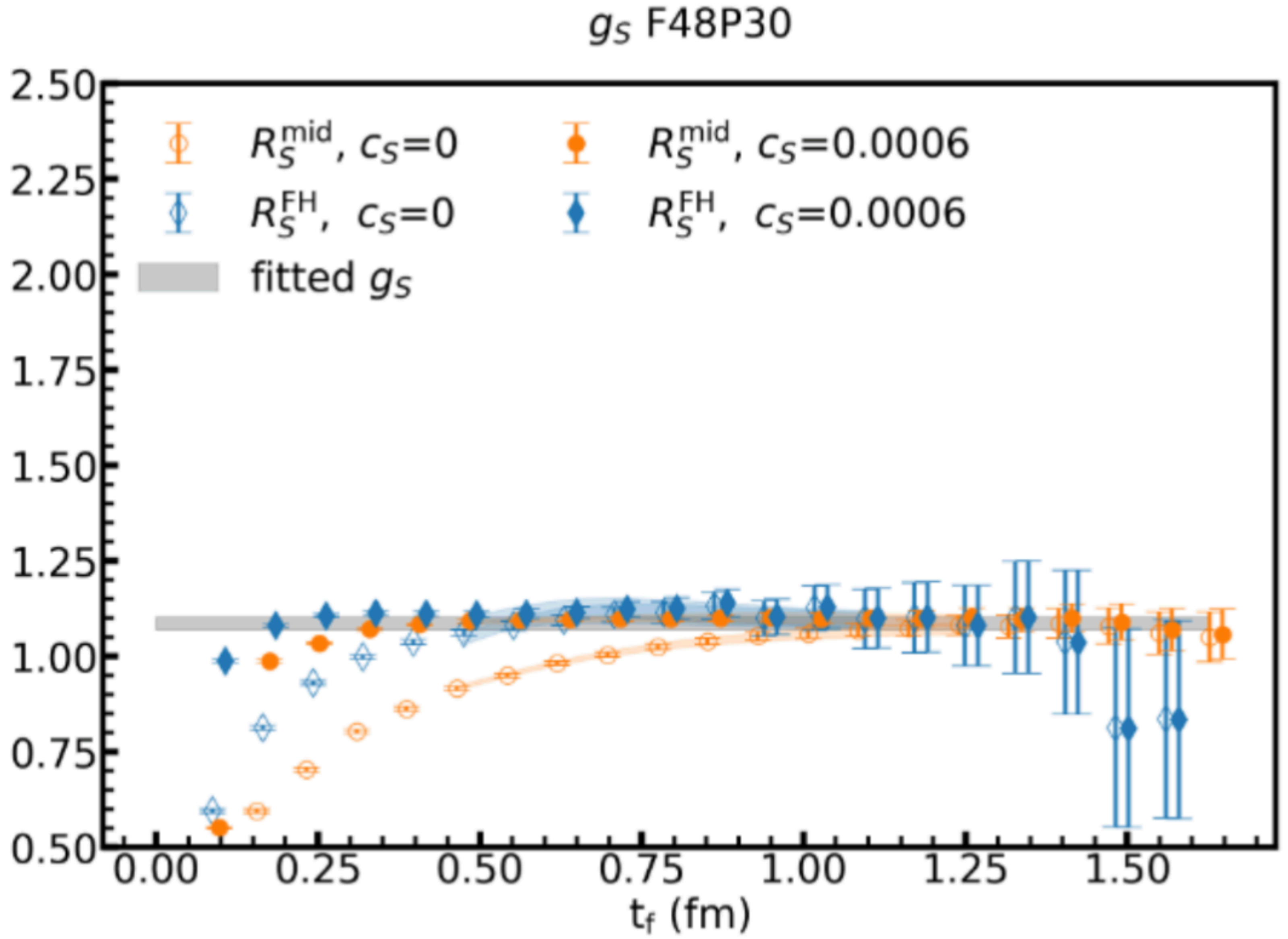
Using the blending method [18] and the idea of current-involved interpolation field [11, 19] proposed recently, our study delivers a new calculation of the nucleon isovector charges g_S and g_T with better control over systematic errors—notably from ESC, chiral extrapolation and FVE—compared to all existing lattice determinations.

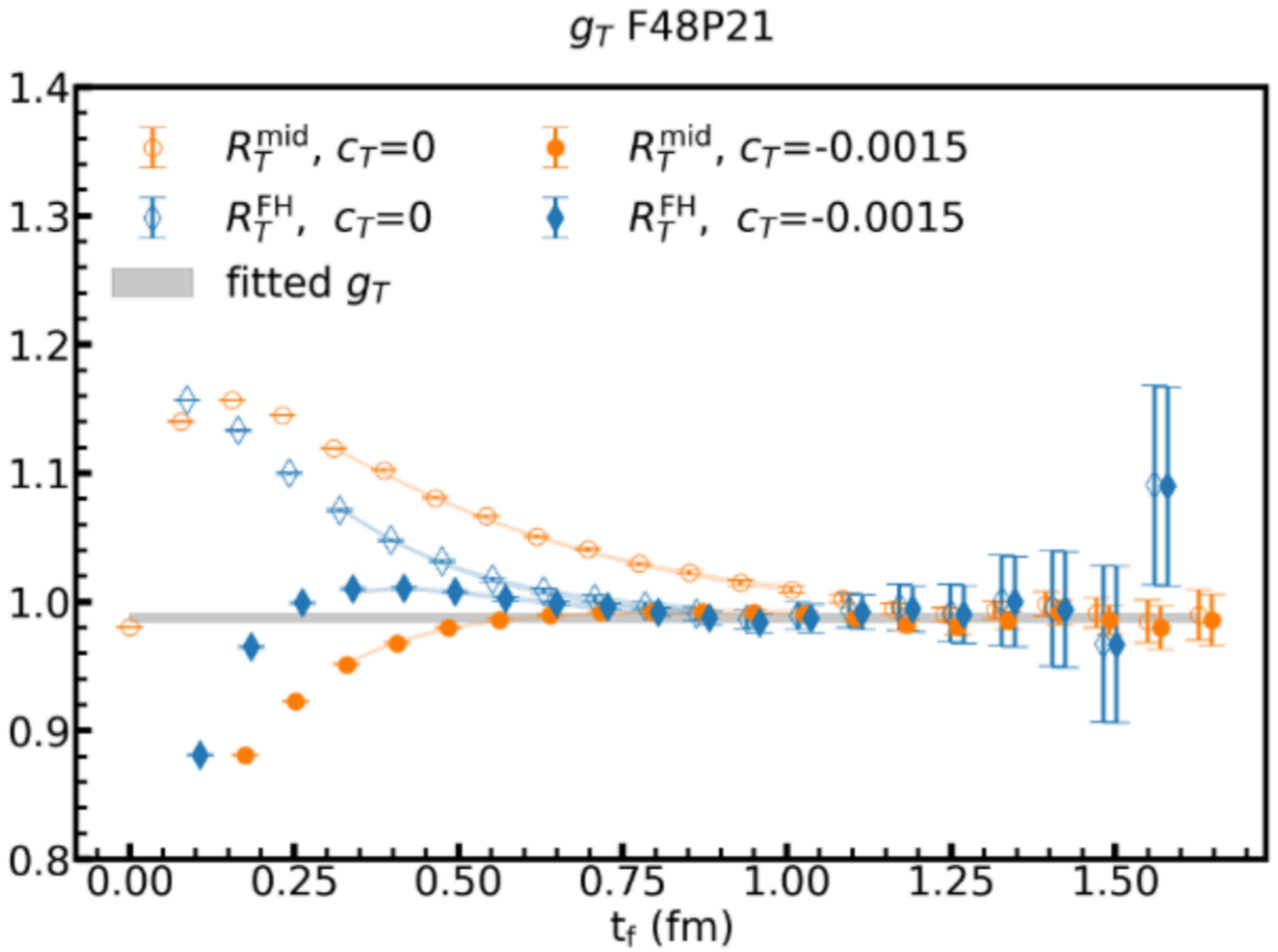
Used my **idea of current-enhanced operators** to remove the leading contamination

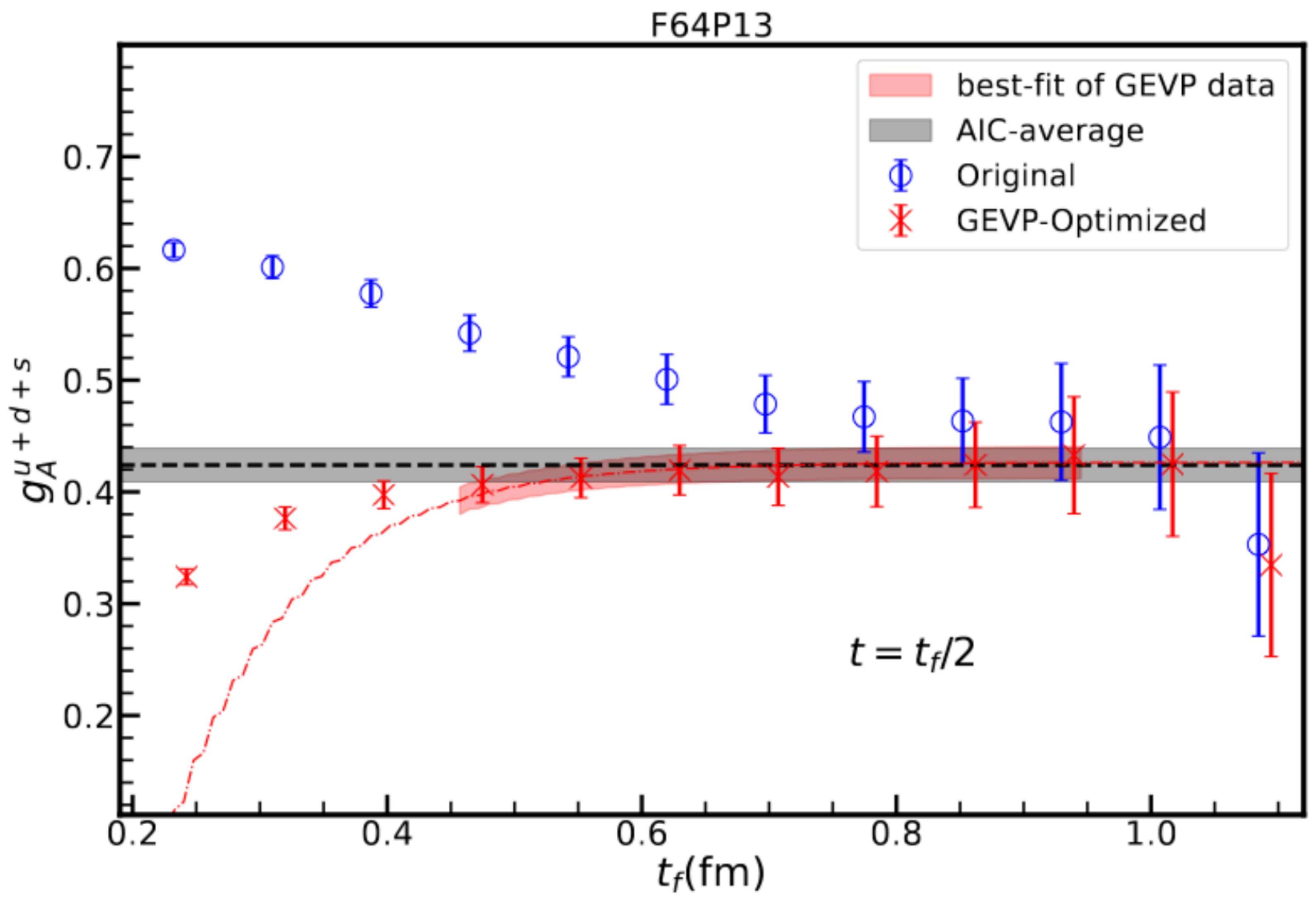
$$\langle \mathrm{O}_N(\vec{0},t)\ \mathcal{J}(\vec{q},t_{\mathrm{sep}})\ \mathrm{O}_N^\dagger(-\vec{q},0)\rangle\ -\ c_{\mathcal{J}}\langle \mathrm{O}_N(-\vec{q},t)\ \mathrm{O}_M(\vec{q},t)\ \mathcal{J}(\vec{q},t_{\mathrm{sep}})\ \mathrm{O}_N^\dagger(-\vec{q},0)\rangle$$

The coefficients $c_{\mathcal{J}}$ are fitted. Using this simple strategy, they obtain the most precise lattice determinations for isovector g_S and g_T ! (Also via GEVP)

works also for g_A^{u+d+s} !





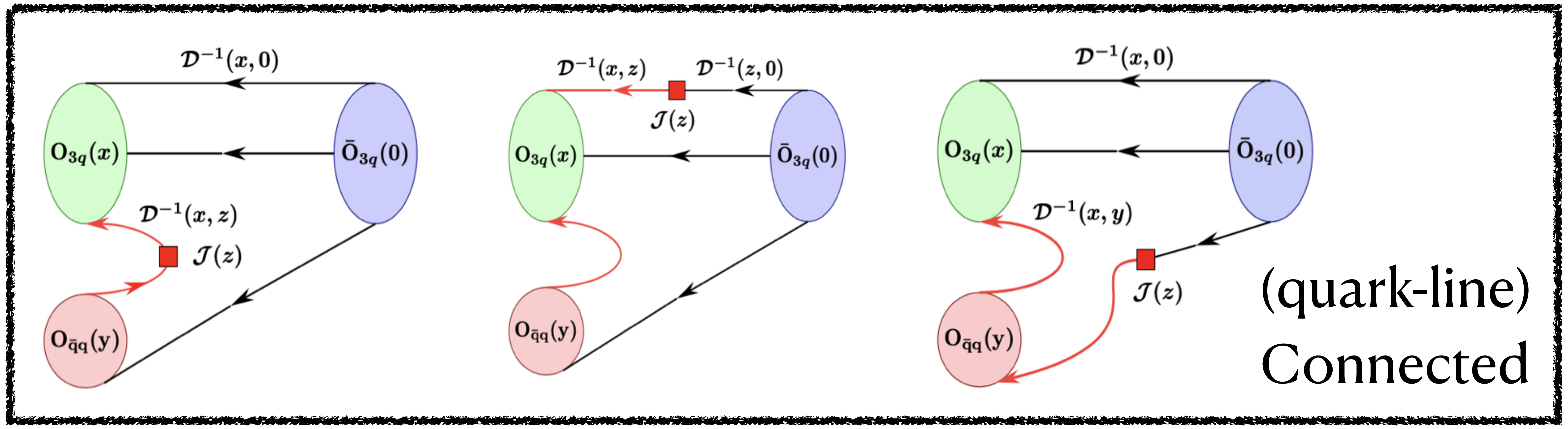


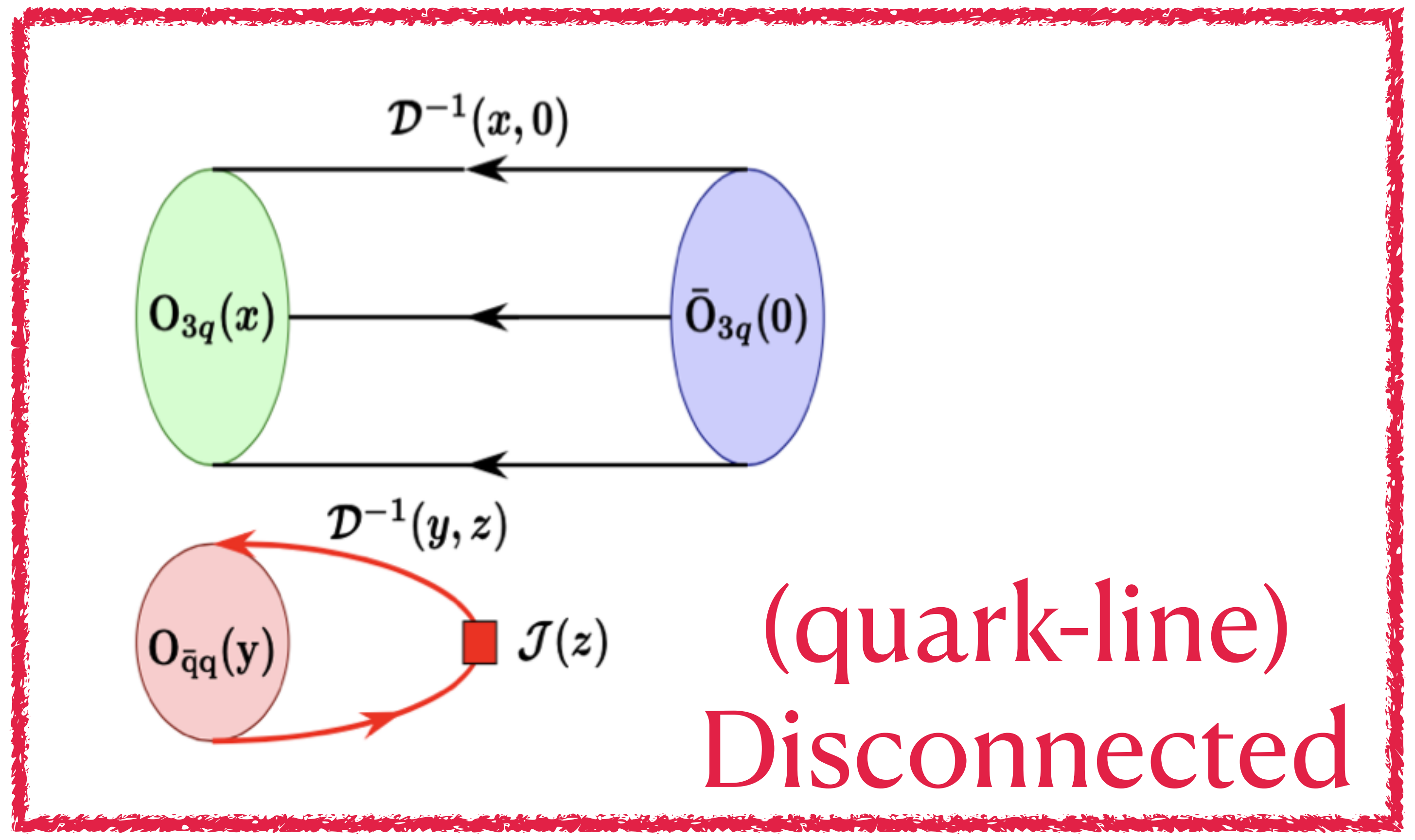
Empty symbols: standard

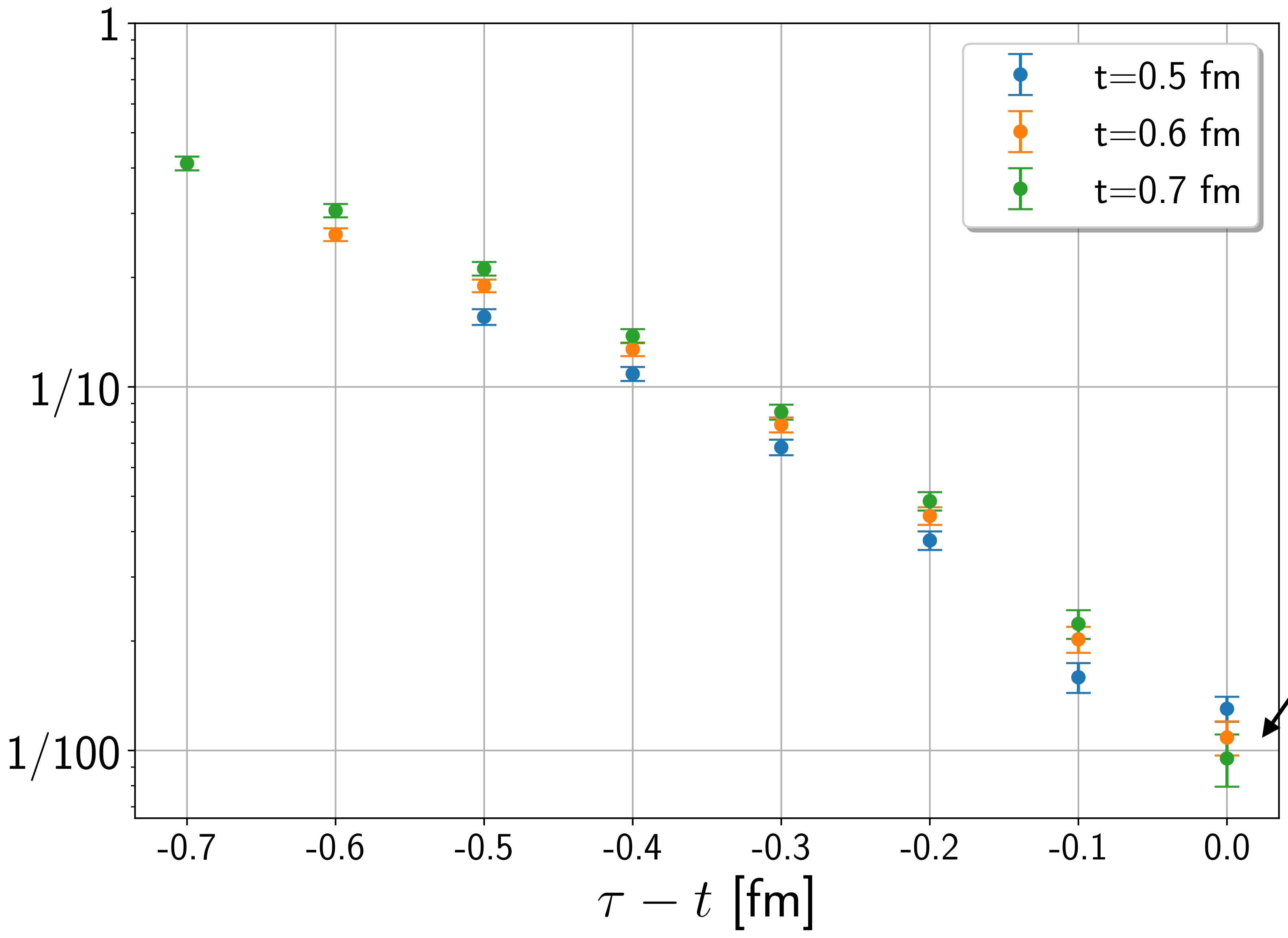
Filled symbols: using $\mathrm{O}_N + c_T \mathrm{O}_N \mathrm{O}_T$ (Current-involved)

Yi-Bo, et al. Private comm.

Lattice diagrams in $\langle \mathrm{O}_{N\pi}(\vec{p}\,', t)\ \mathcal{J}(\vec{q}, \tau)\ \mathrm{O}_N^\dagger(\vec{p}, 0)\rangle$







$$\frac{\langle \mathrm{O}_{N\pi}(\vec{p}\,', t)\ \mathscr{P}(\vec{q}, \tau)\ \mathrm{O}_N^\dagger(\vec{p}, 0)\rangle_{\text{connected}}}{\langle \mathrm{O}_{N\pi}(\vec{p}\,', t)\ \mathscr{P}(\vec{q}, \tau)\ \mathrm{O}_N^\dagger(\vec{p}, 0)\rangle_{\text{disconnected}}} \approx 1/100 \qquad (\tau = t)$$

$m_\pi = 429$ MeV

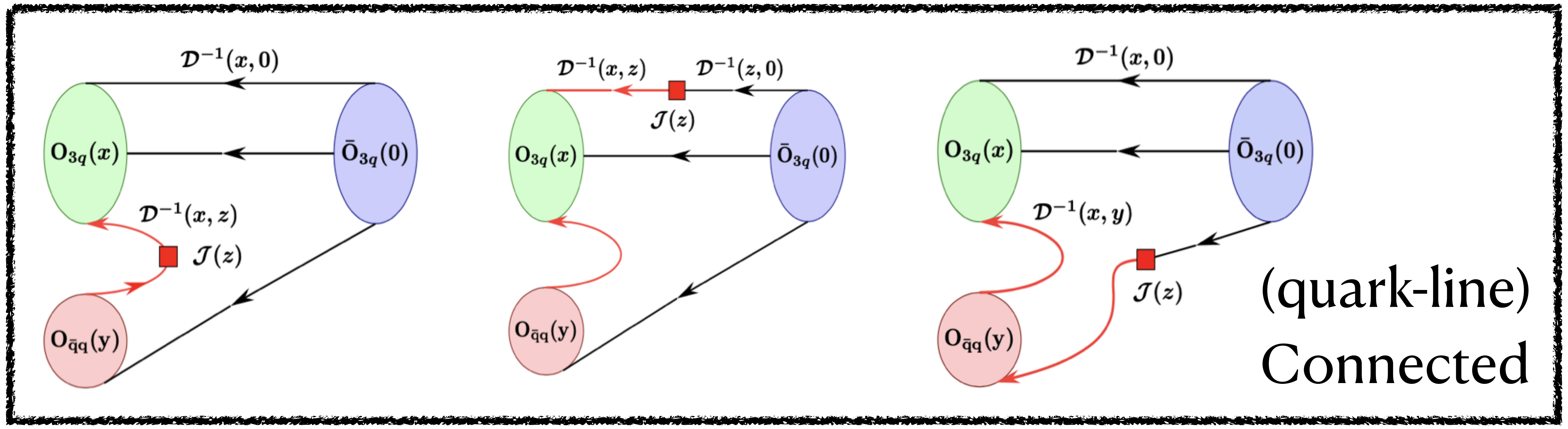
$$\langle \mathrm{O}_{N\pi}(\vec{p}\,', t)\ \mathscr{P}(\vec{q}, \tau)\ \mathrm{O}_N^\dagger(\vec{p}, 0)\rangle_{\text{disconnected}} \approx$$

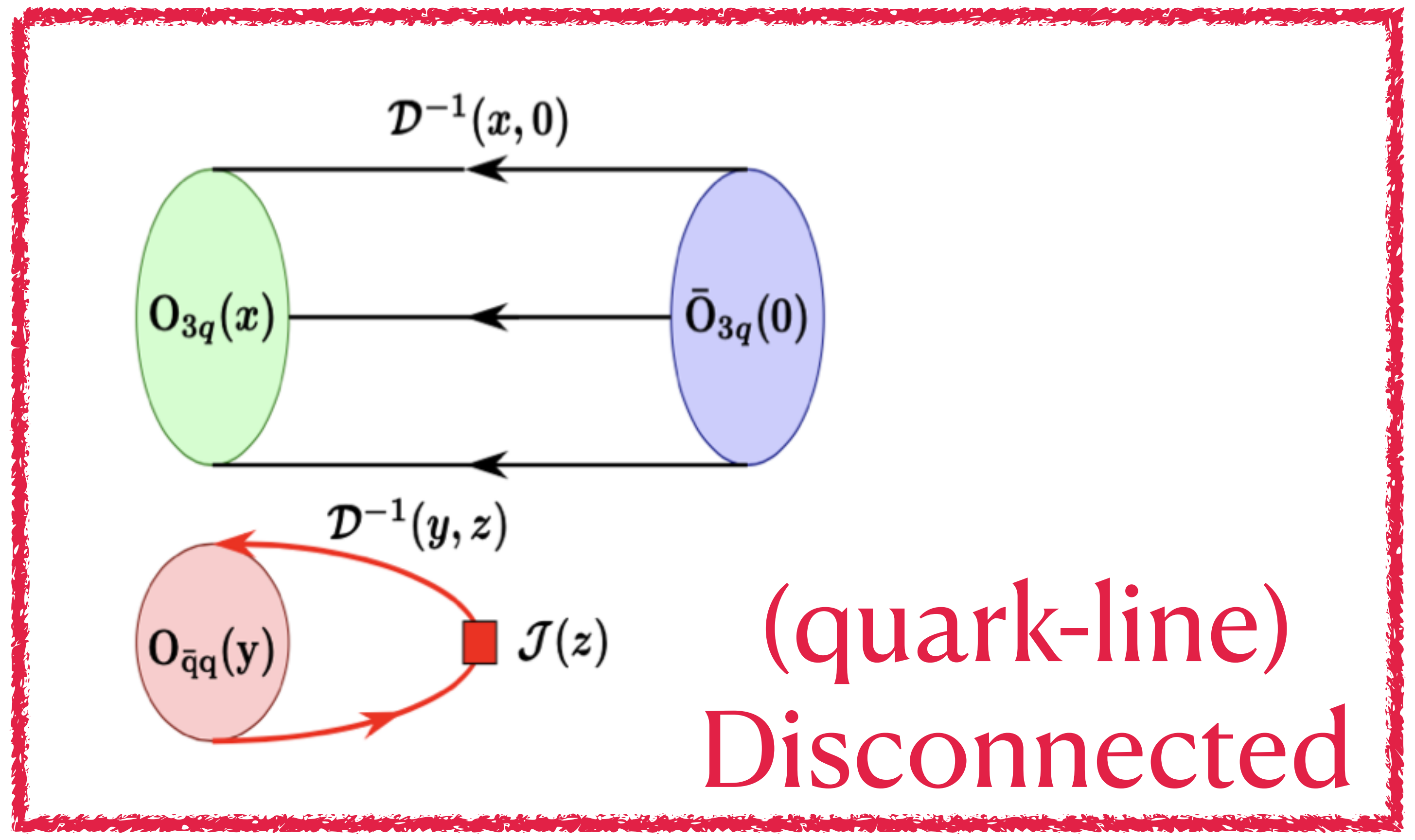
$$\approx \langle \mathrm{O}_N(\vec{p}\,'_N, t)\ \mathrm{O}_N^\dagger(\vec{p}, 0)\rangle\ \langle \mathrm{O}_\pi(\vec{p}\,'_\pi, t)\ \mathscr{P}(\vec{q}, \tau)\rangle$$

non-interacting

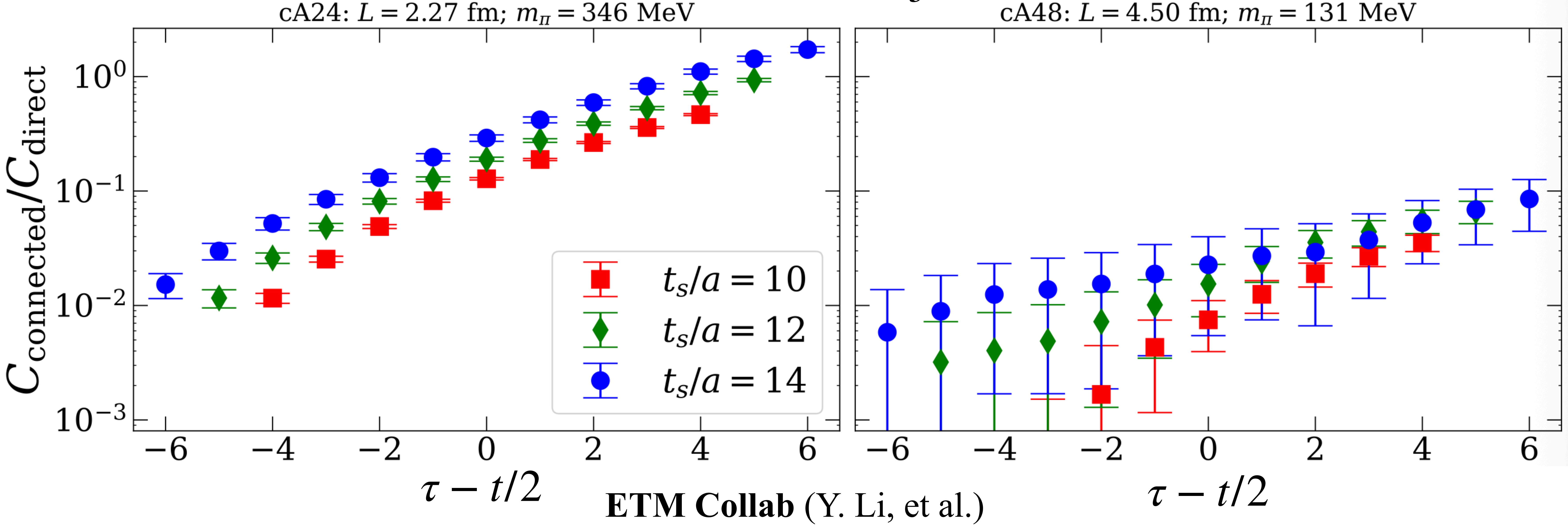
Volume-enhanced

Volume study of diagrams in $\langle O_N(\vec{p}', t)\ \mathcal{J}(\vec{q}, \tau)\ O^\dagger_{N\pi}(\vec{p}, 0)\rangle$





$\mathcal{J} = \bar{\psi}\ \gamma_5\ \psi$



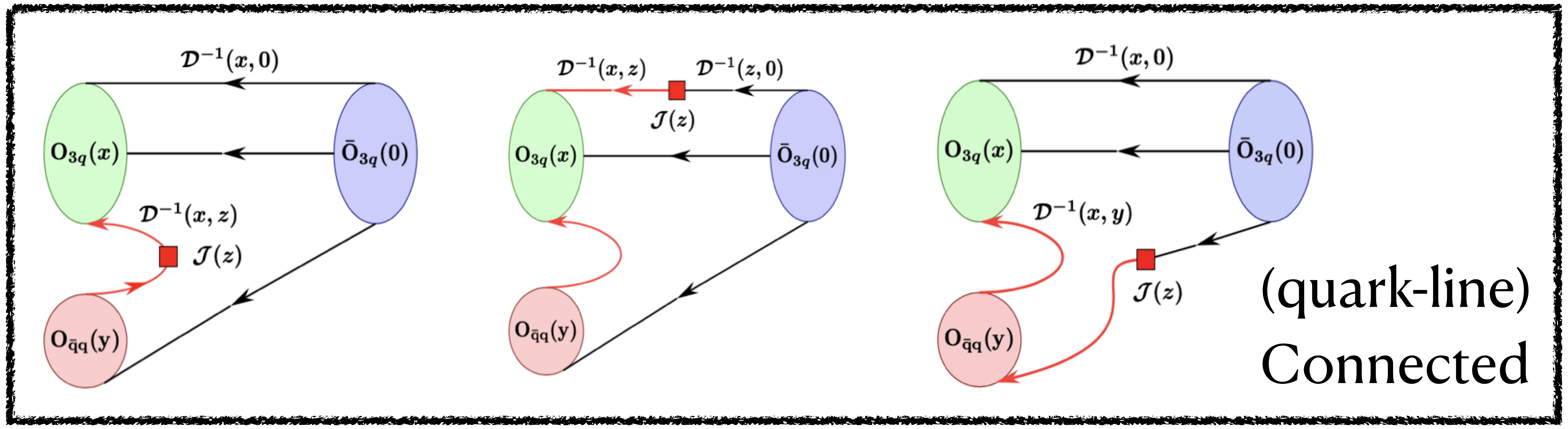
Disconnected diagrams increases with $\sim(L/a)^3$

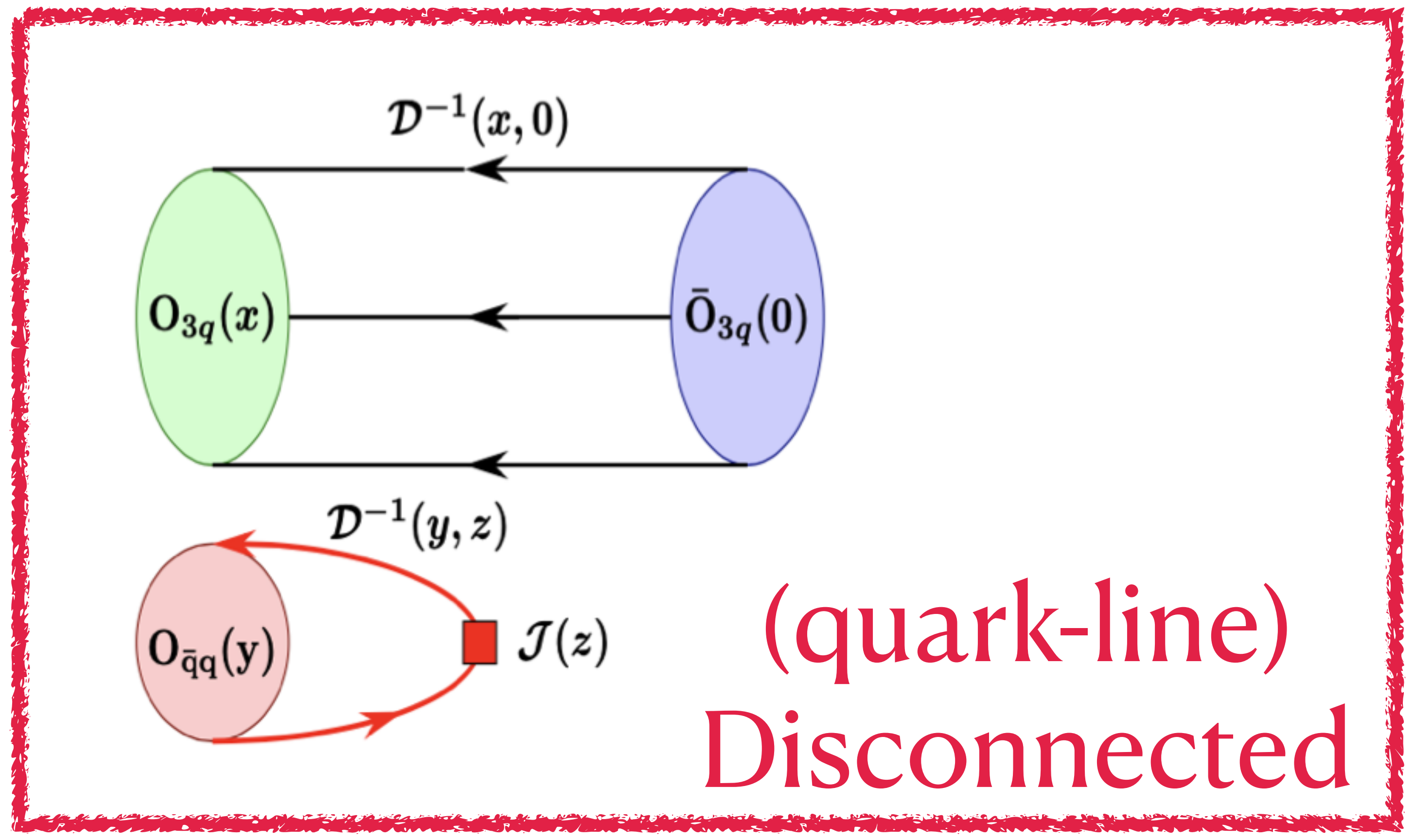
Only O_{NM} which yield disconnected diagrams in 3-pts are volume enhanced.

ETM Collab (Y. Li, et al.)

Private comm. [*Phys.Rev.D* 110 (2024) 9, 094514]

Volume enhancement of quark-line disconnected diagrams.





$$\langle \mathrm{O}_{N\pi}(\vec{p}\,',t)\ \mathscr{P}(\vec{q},\tau)\ \mathrm{O}_N^\dagger(\vec{p},0)\rangle_{\text{disconnected}} \approx \langle\langle \mathrm{O}_N(\vec{p}\,'_N,t)\ \mathrm{O}_N^\dagger(\vec{p},0)\rangle\ \langle \mathrm{O}_\pi(\vec{p}\,'_\pi,t)\ \mathscr{P}(\vec{q},\tau)\rangle\ \left(+\ \mathcal{O}(\alpha_S^2/N_c)\right)$$

(2-gluon exchange +
+ baryon large N [E.Witten])

$$\langle\langle \mathrm{O}_N(\vec{p}\,'_N,t)\ \mathrm{O}_N^\dagger(\vec{p},0)\rangle\ \langle \mathrm{O}_\pi(\vec{p}\,'_\pi,t)\ \mathscr{P}(\vec{q},\tau)\rangle = \sum_{\vec{x}\,'_\pi}\sum_{\vec{z}} e^{-i\vec{p}\,'_\pi\cdot\vec{x}\,'_\pi}e^{i\vec{q}\cdot\vec{z}}\ \langle\langle \mathrm{O}_N(\vec{p}\,'_N,t)\ \mathrm{O}_N^\dagger(\vec{p},0)\rangle\ \langle \mathrm{O}_\pi(\vec{x}\,'_\pi,t)\ \mathscr{P}(\vec{z},\tau)\rangle$$

$$(\vec{z} = \vec{x}\,'_\pi + \vec{r},\ \ \vec{p}\,'_\pi = \vec{q}) \qquad = \sum_{\vec{x}\,'_\pi}\sum_{\vec{r}} e^{-i\vec{p}\,'_\pi\cdot\vec{x}\,'_\pi}e^{i\vec{q}\cdot\vec{z}}\ \langle \mathrm{O}_N(\vec{p}\,'_N,t)\ \mathrm{O}_N^\dagger(\vec{p},0)\rangle\ \langle \mathrm{O}_\pi(\vec{x}\,'_\pi,t)\ \mathscr{P}(\vec{x}\,'_\pi-\vec{r},\tau)\rangle$$

$$(\text{Translational invariance}) \qquad = (L/a)^3\sum_{\vec{r}} e^{-i\vec{p}\,'_\pi\cdot\vec{x}\,'_\pi}e^{i\vec{q}\cdot\vec{z}}\ \langle\langle \mathrm{O}_N(\vec{p}\,'_N,t)\ \mathrm{O}_N^\dagger(\vec{p},0)\rangle\ \langle \mathrm{O}_\pi(\vec{0},t)\ \mathscr{P}(\vec{r},\tau)\rangle\ .$$