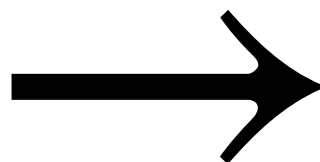


$B \rightarrow \pi \ell \nu$ using HISQ

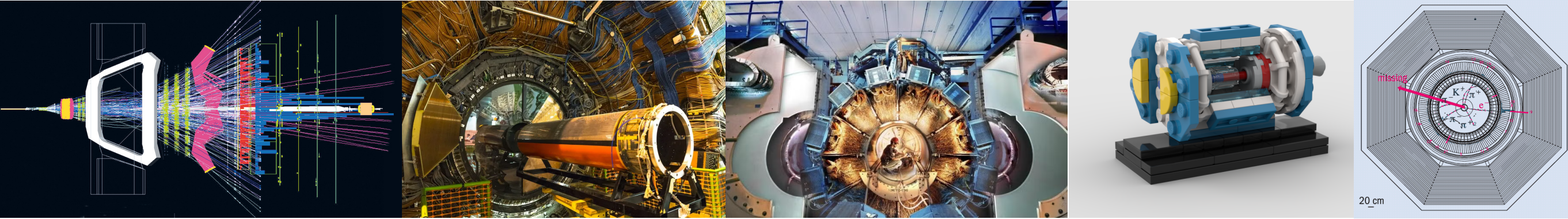


Lattice2026
July 27 - July 31

Nicholas Cassar

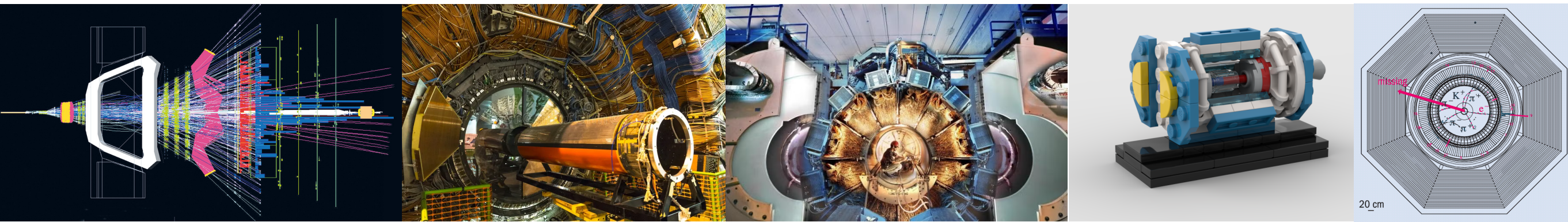
nick.cassar@colostate.edu

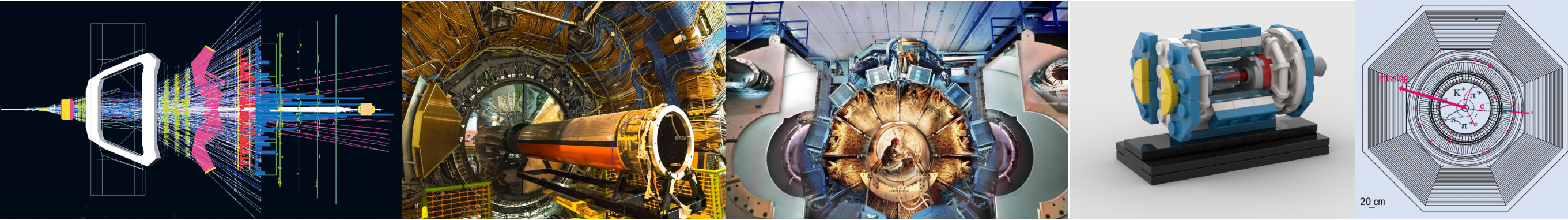




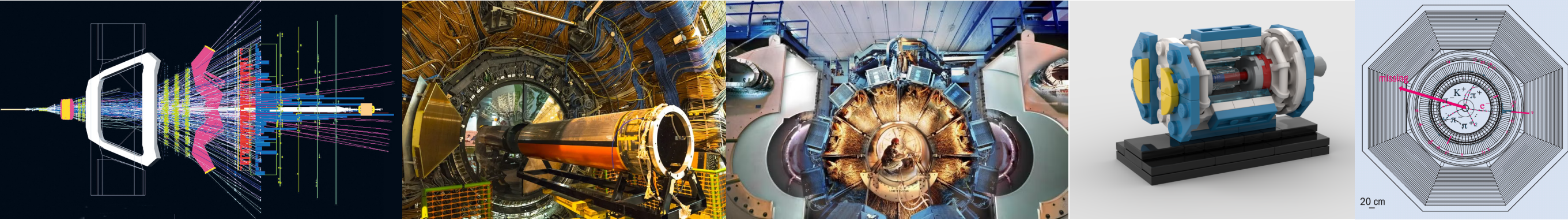
Special thanks to our collaborators:

Nicholas Cassar, Akhil Chauhan, Carleton DeTar, Aida El-Khadra, Elvira Gámiz, Steven Gottlieb, William Jay, Andreas S. Kronfeld, Jack Laiho, Andrew Lytle, Abishek Samlodia, Alejandro Vaquero



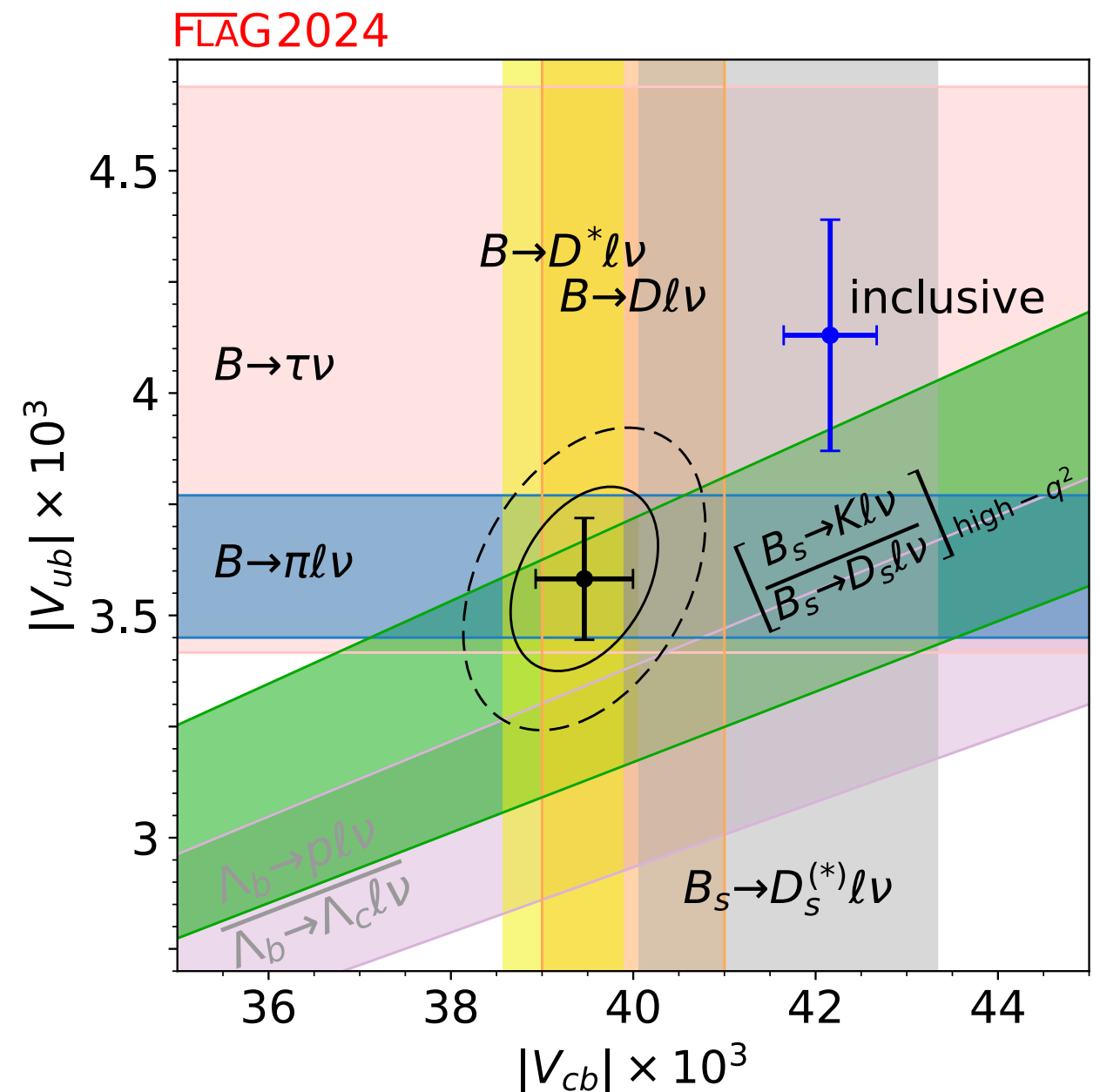


- $|V_{ub} / V_{cb}|$ determines vertex of unitarity triangle
- Unitarity is a prediction of the SM - violations could hint at new physics



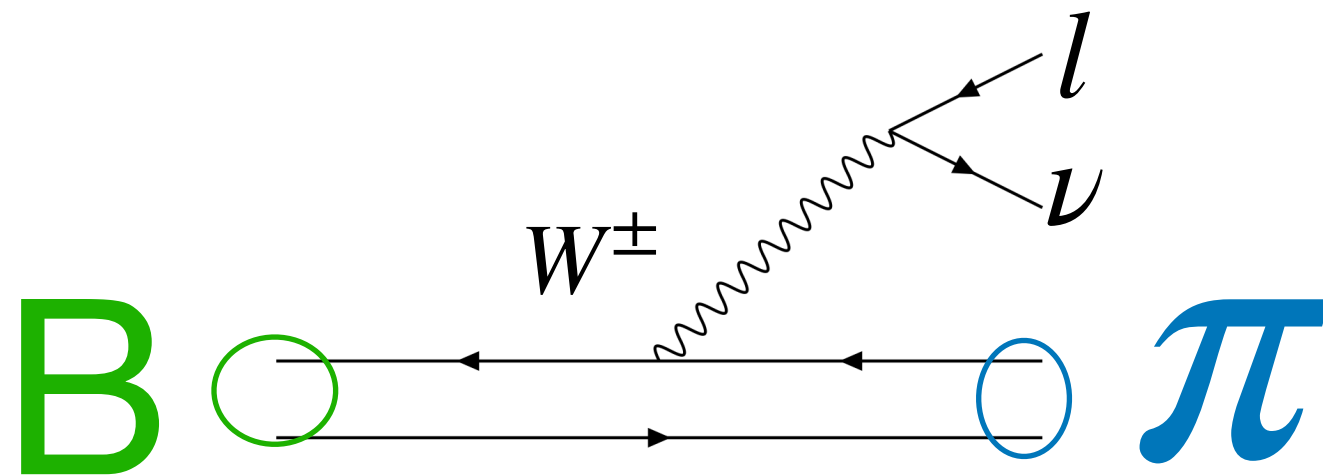
The Heavy Flavor Situation

- Inclusive/Exclusive Tension still around 2σ (for a few decades)
- Requires both theory input and experimental data...
Difficult!



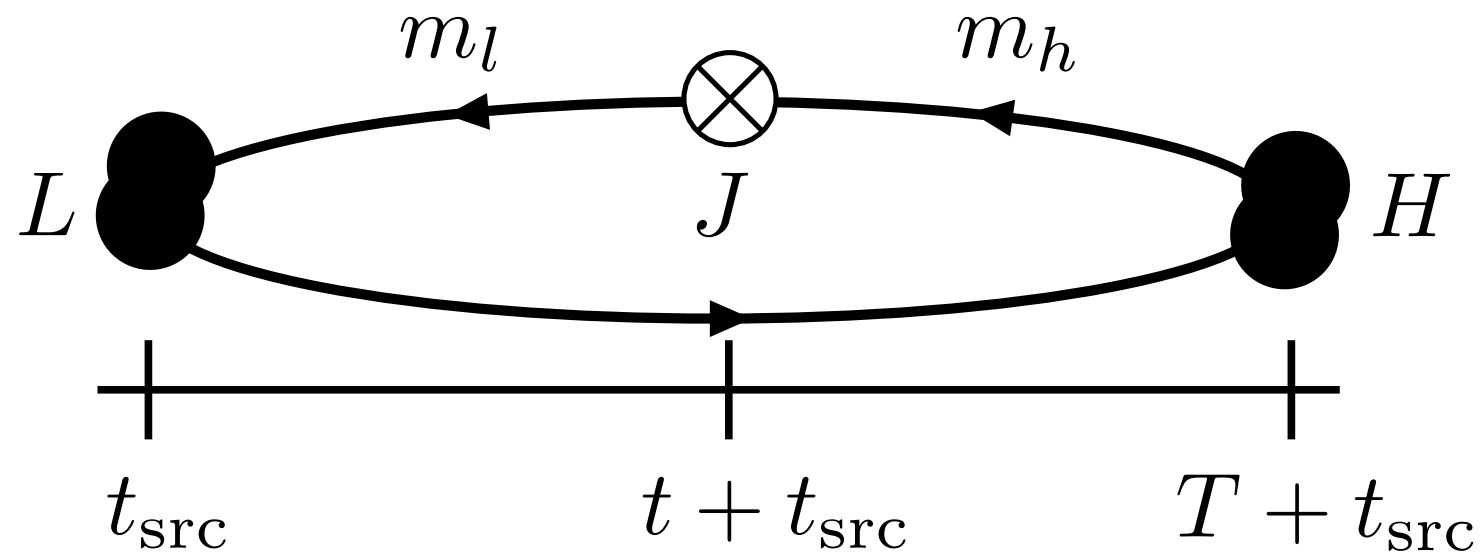
$B \rightarrow \pi l \nu$ Semileptonic Decay

- Exclusive decay



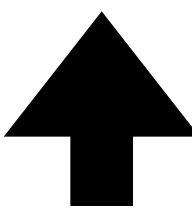
$$d\Gamma \propto |V_{ub}|^2 \times (\text{Kinematics}) \times (\text{QCD})$$

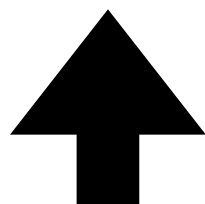
Weak Decay on the Lattice



$$C_{3pt}(t, T) = \langle \pi(t_{src}, \mathbf{p}) J(t_{src} + t, -\mathbf{p}) B(T + t_{src}, \mathbf{0}) \rangle$$

Insert complete sets
of states to extract
matrix elements


 $|L\rangle\langle L|$


 $|H\rangle\langle H|$

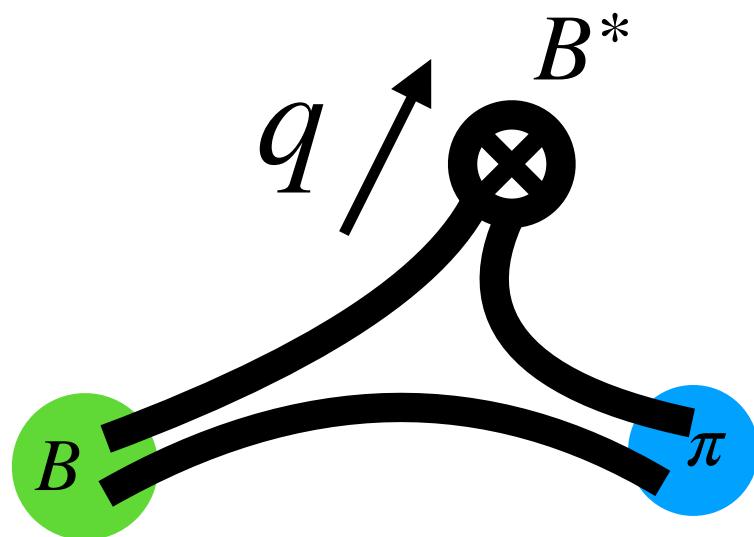
Form Factor Definitions

$$\langle \pi | V^\mu | B \rangle \propto (\text{kinematics}) \times f_{\parallel}(q^2) + (\text{kinematics}) \times f_{\perp}(q^2)$$

$$f_{\parallel}(q^2) \propto \langle L | V^0 | H \rangle$$

$$f_{\perp}(q^2) \propto \langle L | V^i | H \rangle$$

$$f_+(q^2) = (\text{kinematics}) \times f_{\perp}(q^2) + (\text{kinematics}) \times f_0(q^2)$$



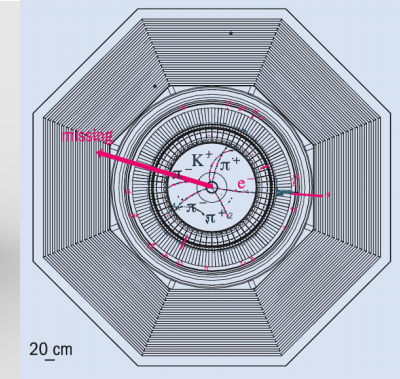
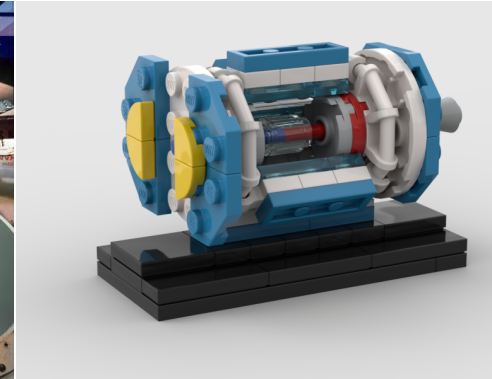
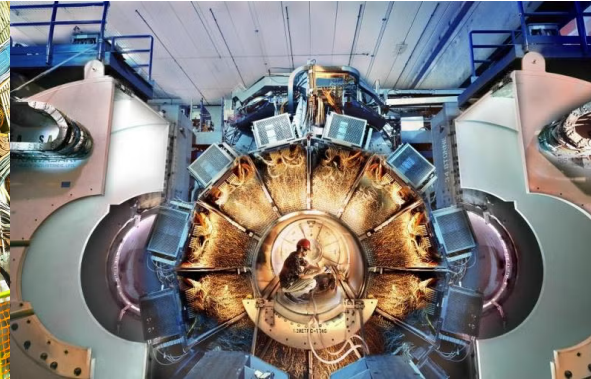
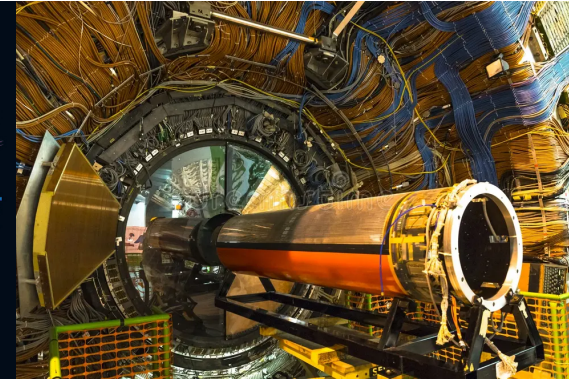
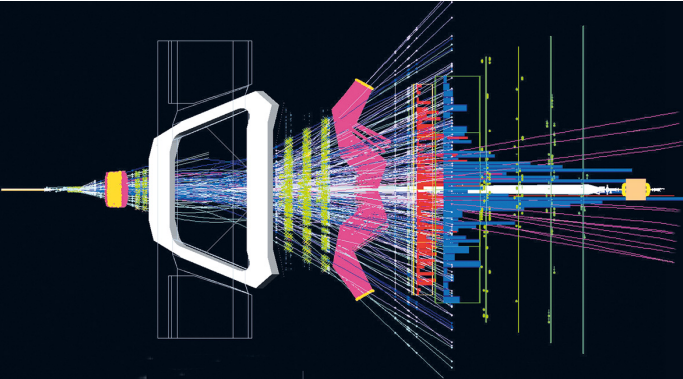
$$\propto \frac{1}{M_{B^*}^2 - q^2}$$

$$f_0(q^2) \propto \langle L | S | H \rangle$$

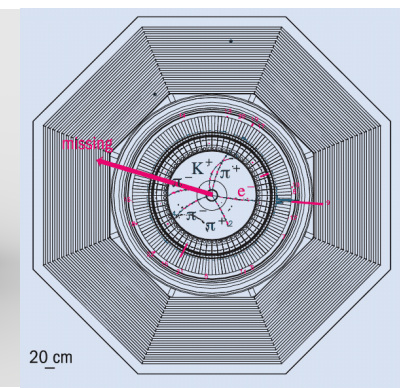
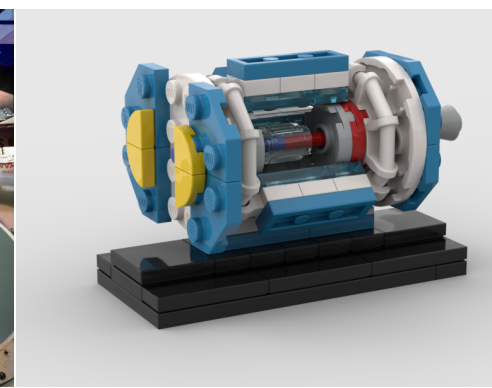
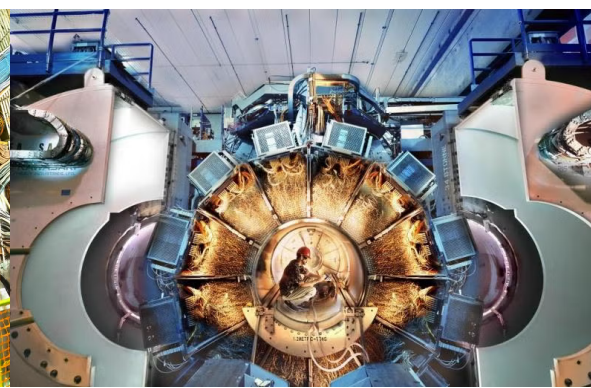
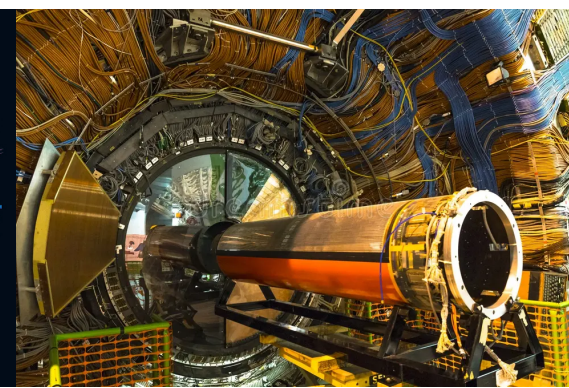
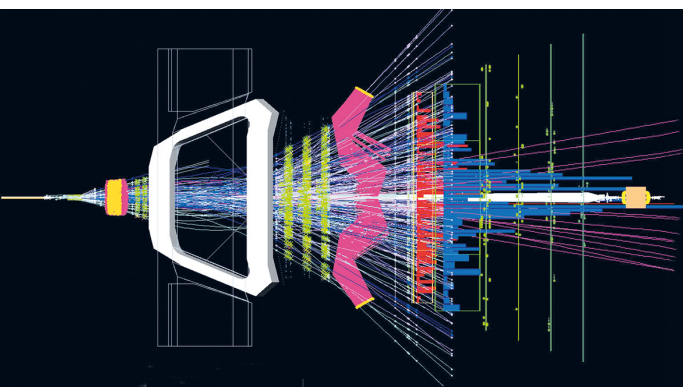
$$B : J^P = 0^-$$

$$B^* : J^P = 1^-$$

$$? : J^P = 0^+$$

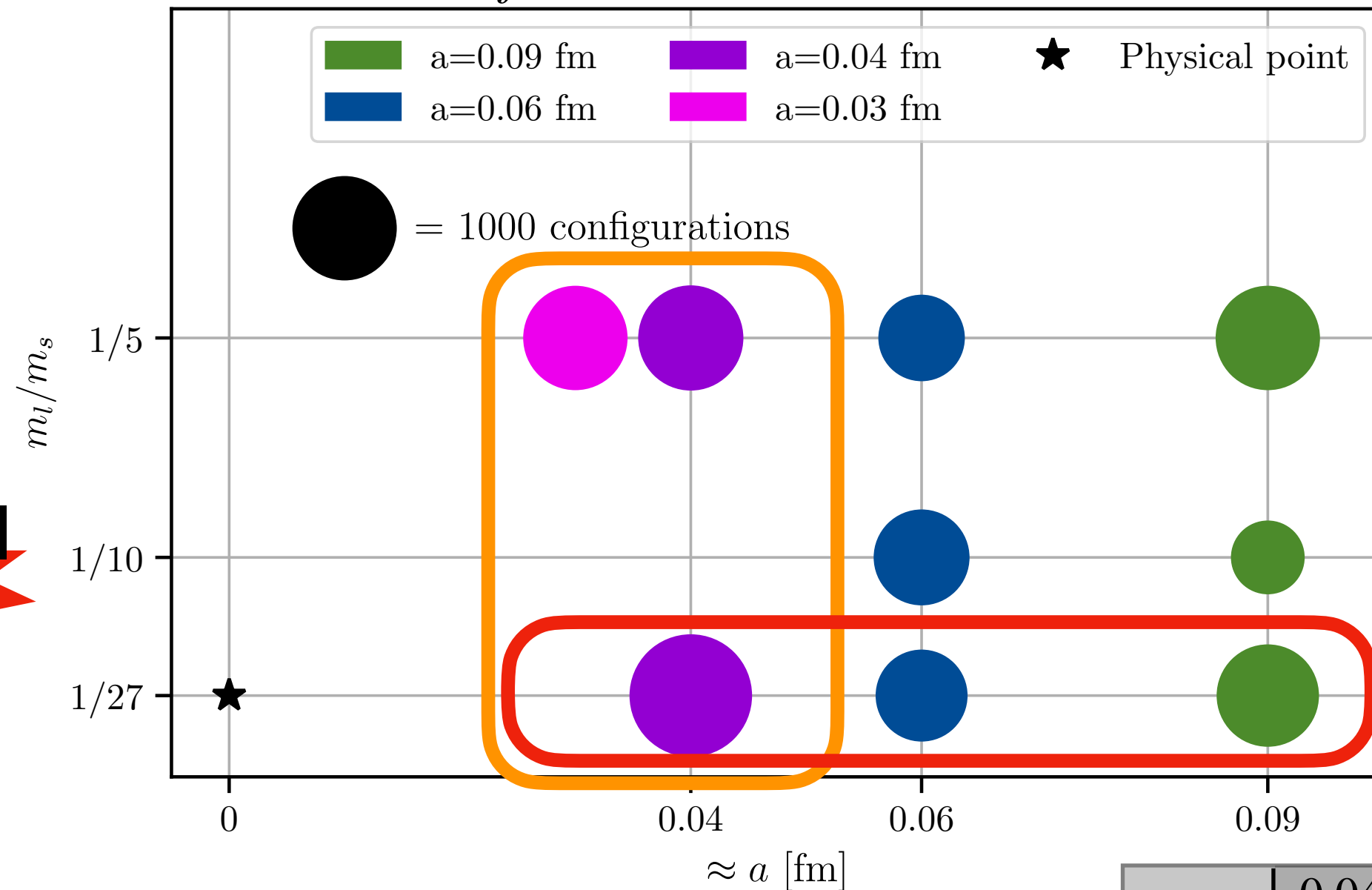


Fit Procedure



Gauge-Field Ensembles

$$n_f = 2 + 1 + 1 \text{ HISQ}$$

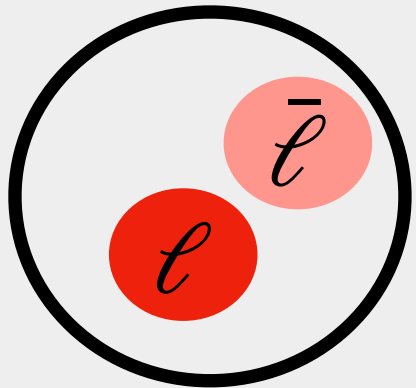


Physical
 M_π

Physical m_b below lattice cutoff $\rightarrow$

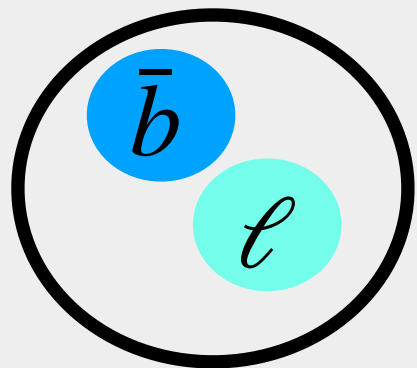
	0.04fm	0.03fm
am_b	0.843	0.6084

Meet the Players



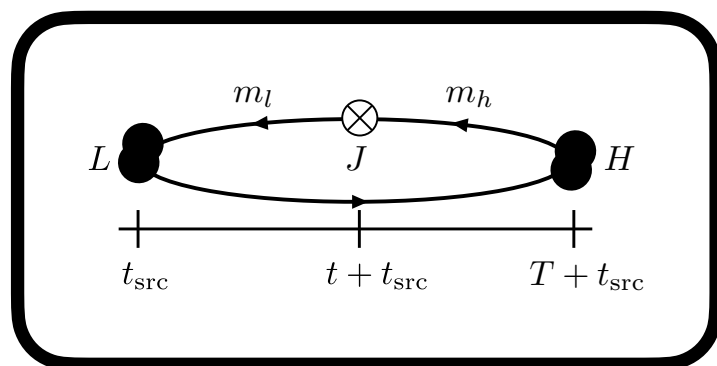
Pion 2pt function!

- computed at 8 momenta



B meson 2pt function!

- computed at rest
- heavy quark masses ranging from $\sim m_c$ up to and past m_b



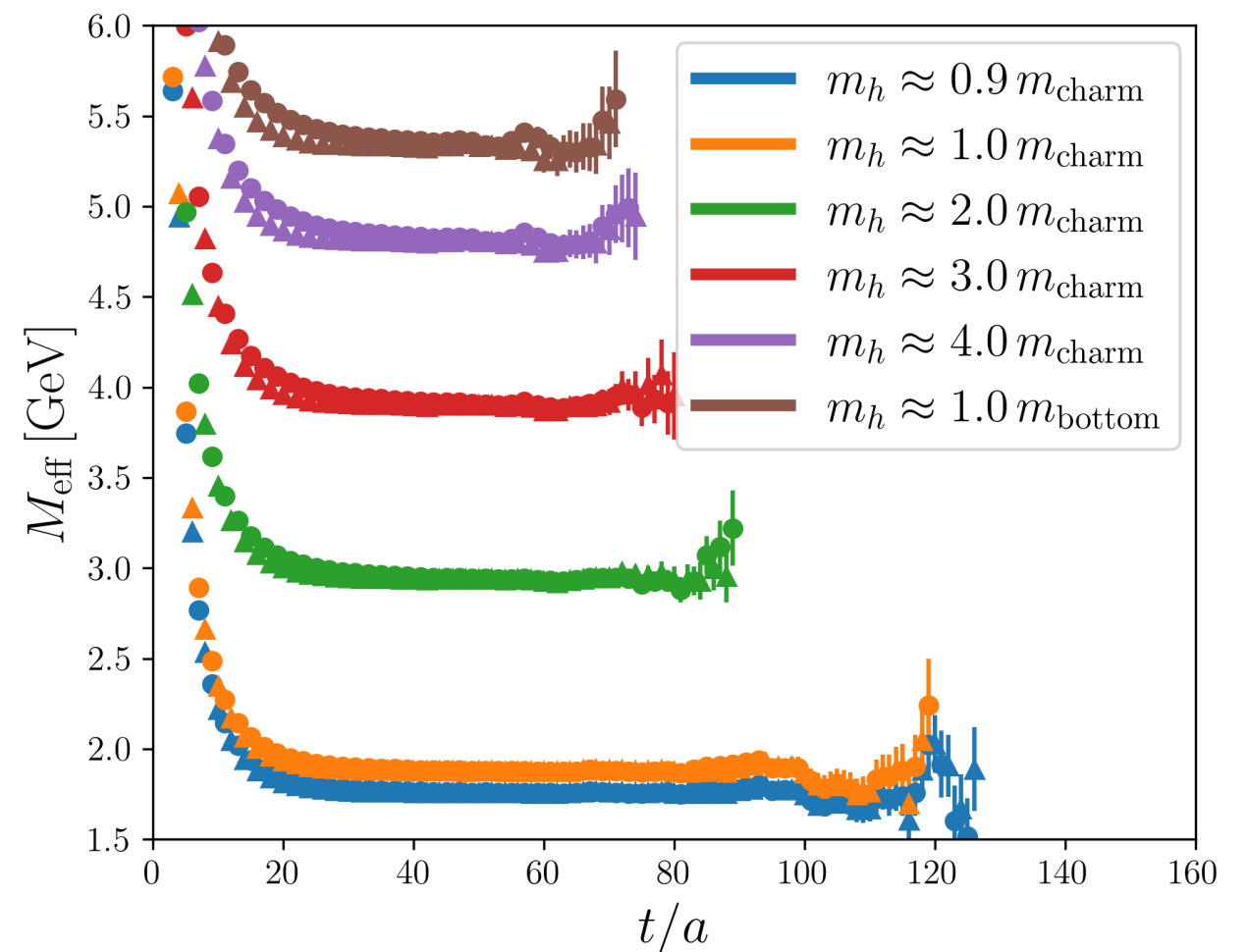
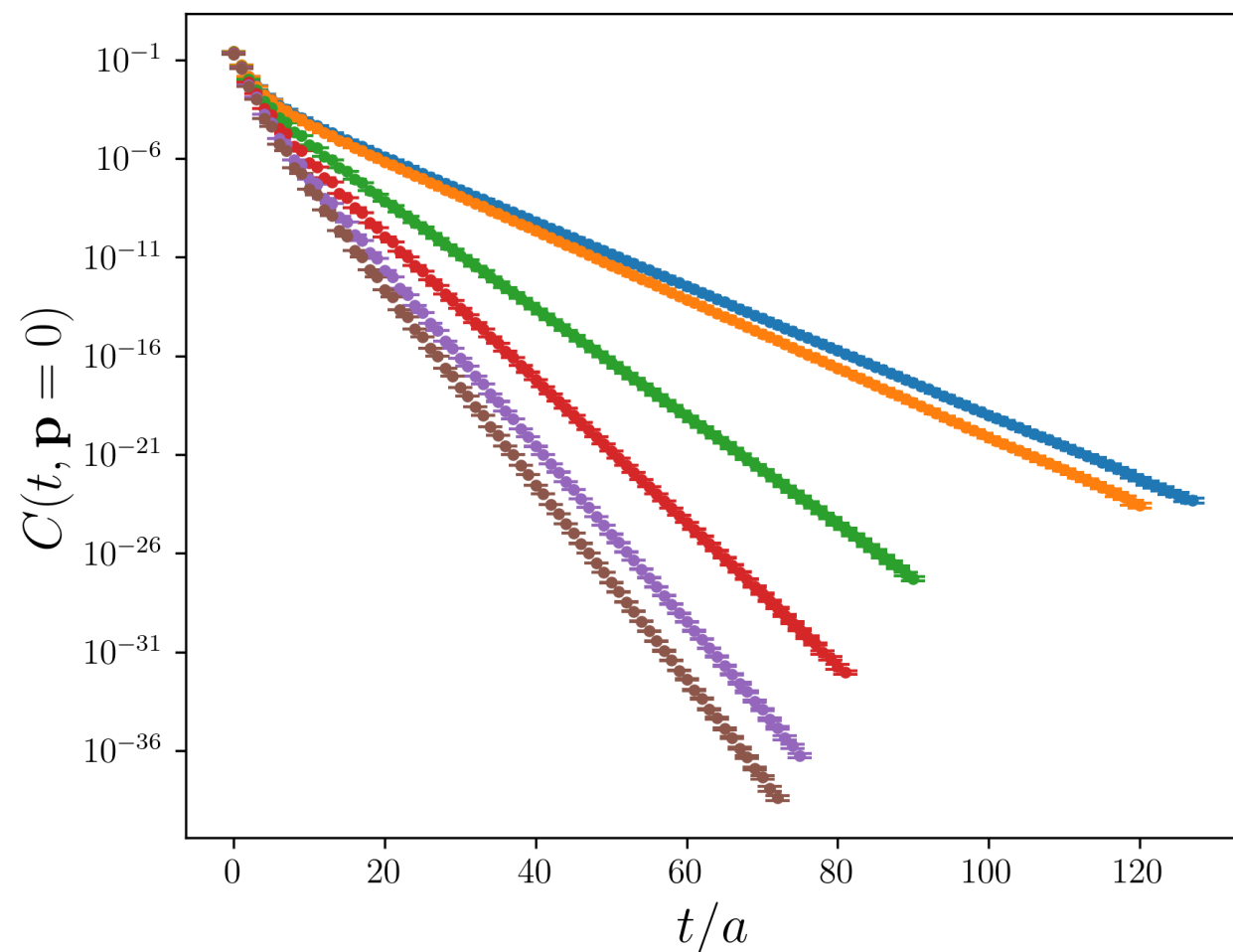
Heavy to Light 3pt function!

- 5 source-sink separations, up to $\sim 3\text{fm}$
- computed at 8 momenta, 5 heavy quark masses

Initial 2pt Fits

- We do Bayesian modeling to fit our correlator data to truncated spectral decompositions.
- Values for priors are based on the PDG, effective mass plateaus of the data, and naive multiparticle states
- We test for stability in the desired posteriors by choosing reasonable variations of t_{min} , as well as the number of states in the truncated spectral decomposition
- We also test for stability under inflation of prior widths, and shifting of the central values.

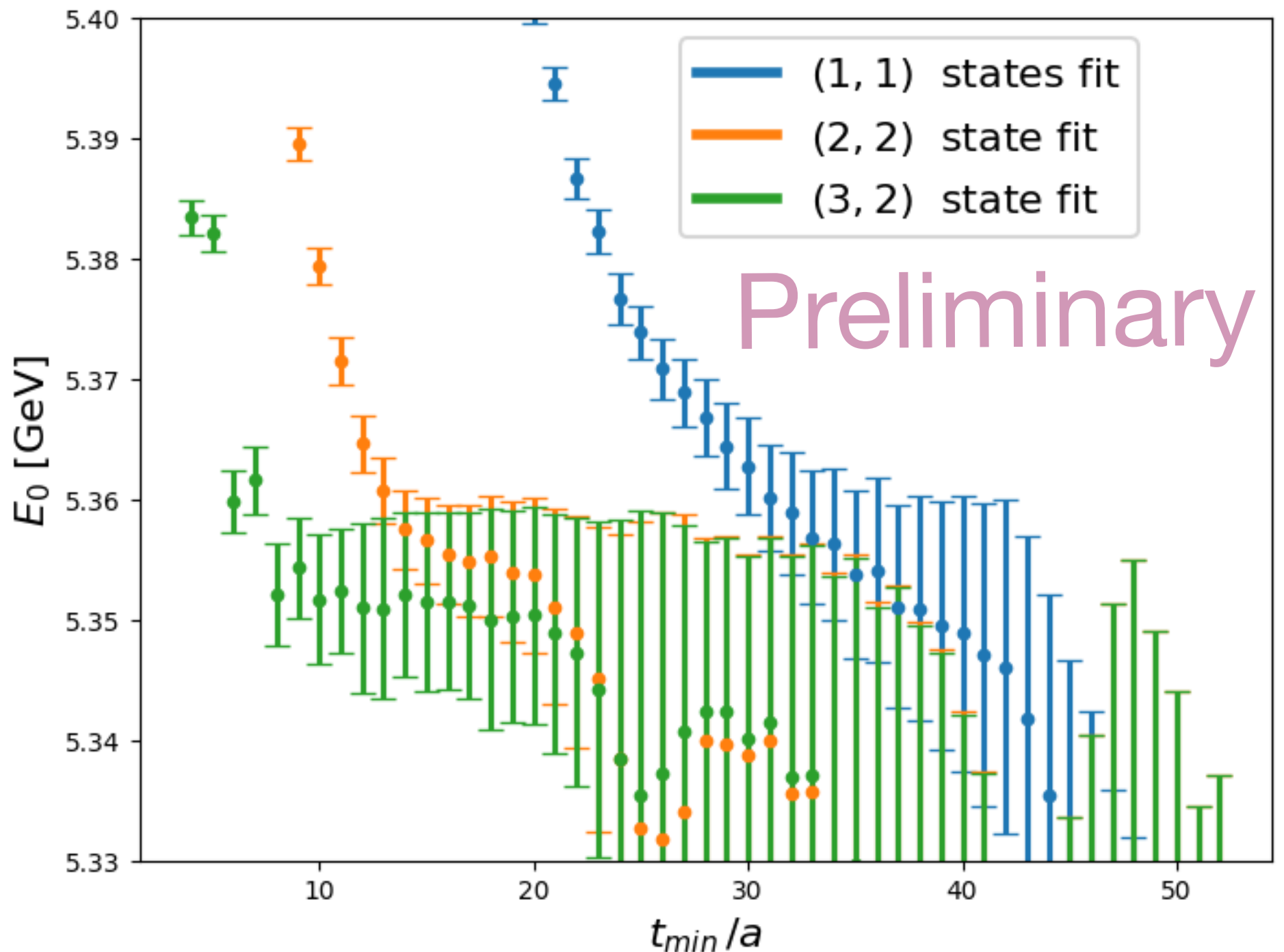
B-meson 2pt Functions



$$a = 0.042 \text{ fm}$$

$$M_\pi = 135 \text{ MeV}$$

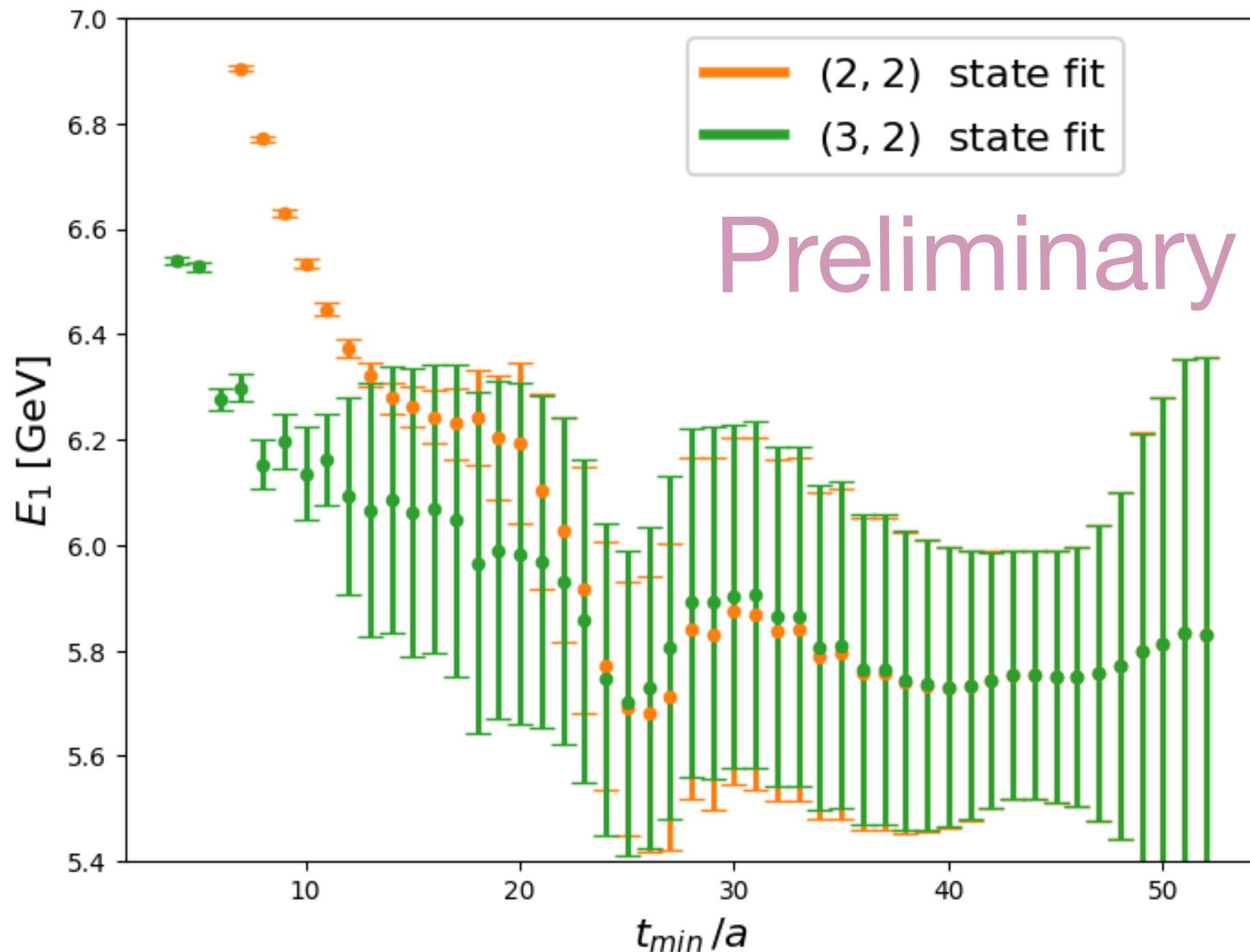
B-meson Ground State



$a=0.042\text{fm}$

$M_\pi = 135\text{ MeV}$

B-meson Excited State



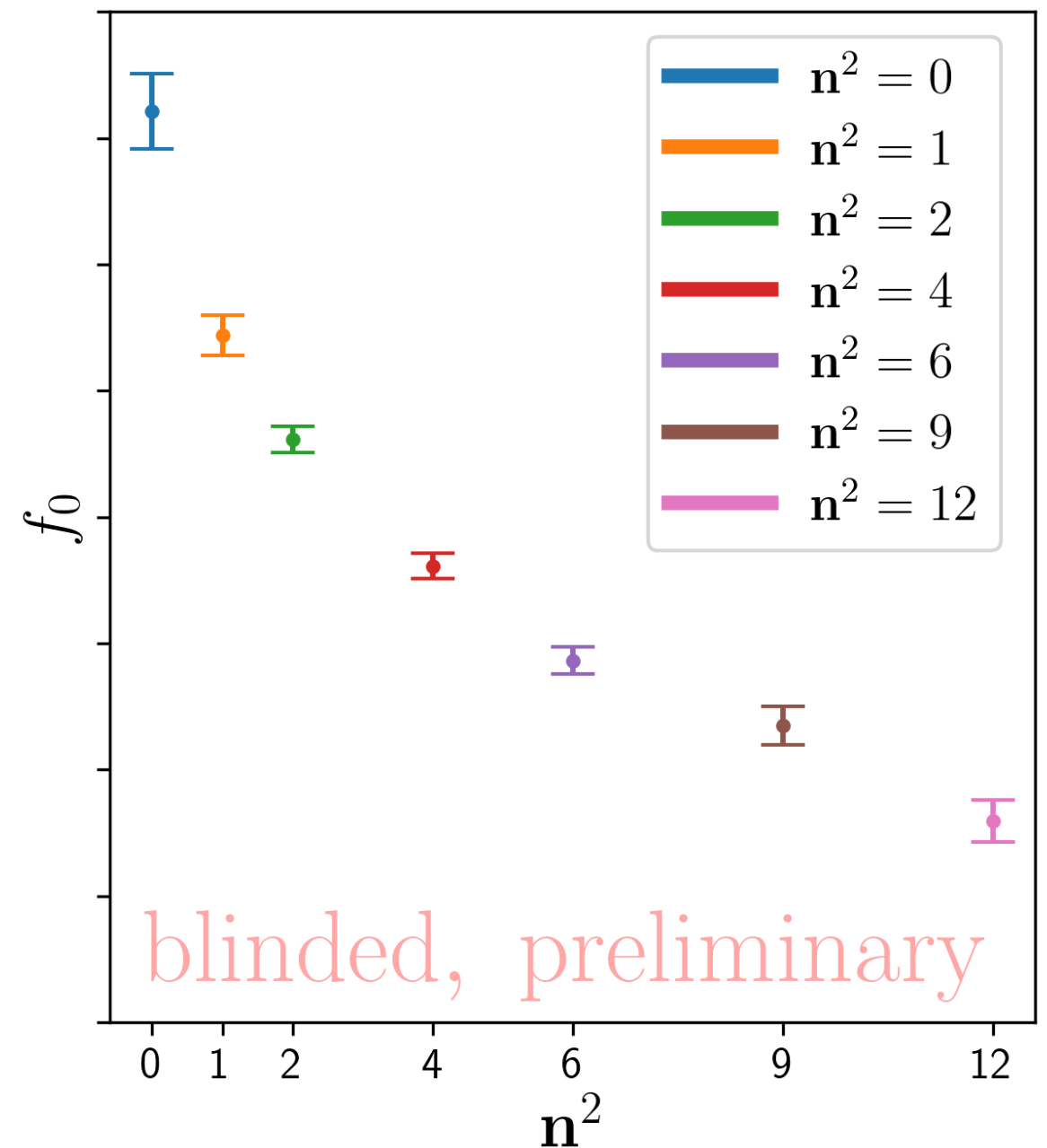
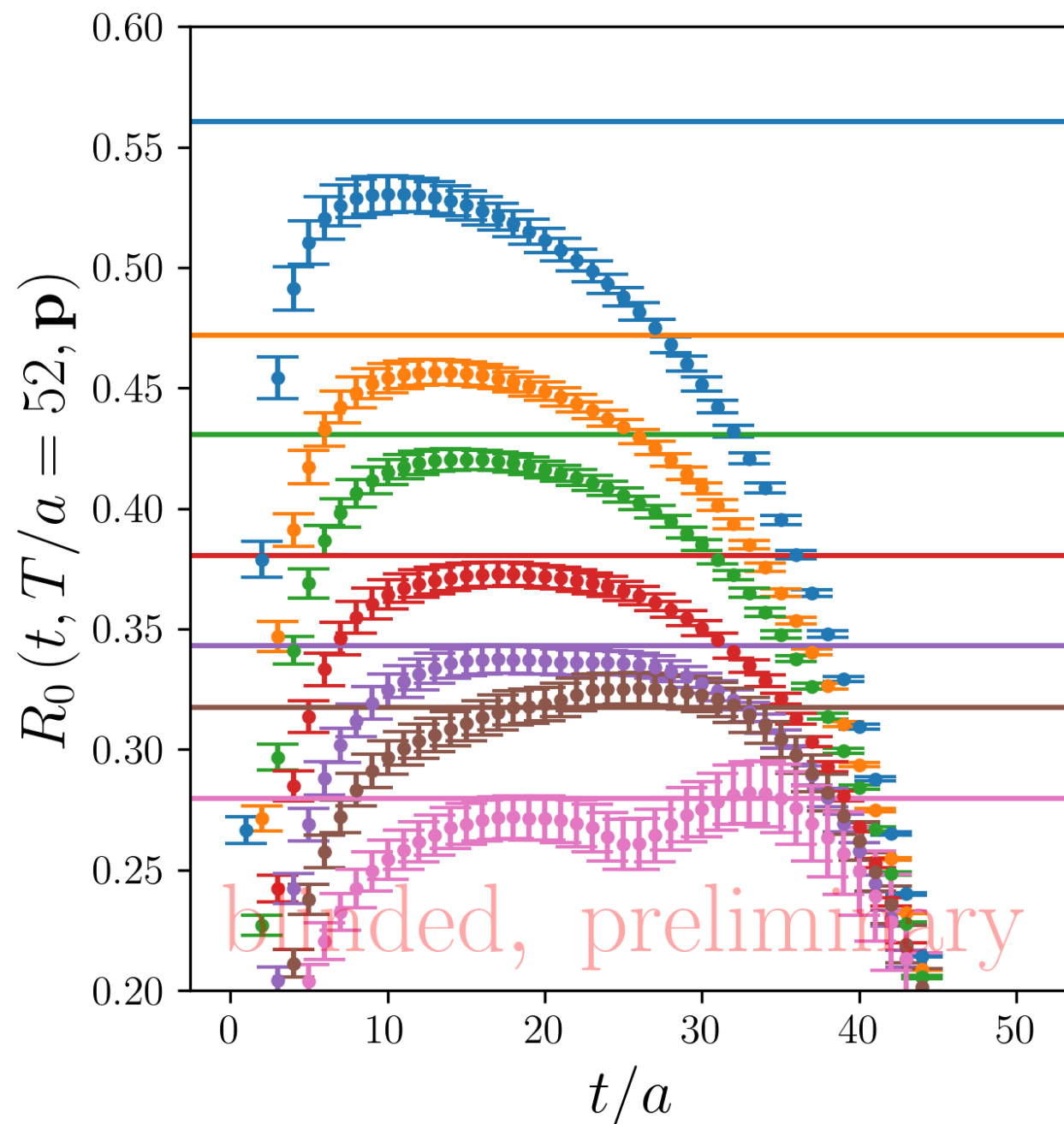
Joint 2pt + 3pt Fits

1. Do isolated 2pt fits
2. Assemble priors for joint fits
3. Simultaneously fit the pion 2pt function, B-meson 2pt function, and 3pt functions to truncated spec. decomp.
4. 2pt spectrum and 3pt spectrum enforced to be equivalent in joint fits
5. Extract matrix element for ground state

$$\left\{ \begin{array}{l} C_{\pi}(t, \mathbf{p}) \\ C_B(t, \mathbf{0}) \\ C_{3pt}(t, T_1, \mathbf{p}) \\ C_{3pt}(t, T_2, \mathbf{p}) \\ C_{3pt}(t, T_3, \mathbf{p}) \\ C_{3pt}(t, T_4, \mathbf{p}) \end{array} \right.$$

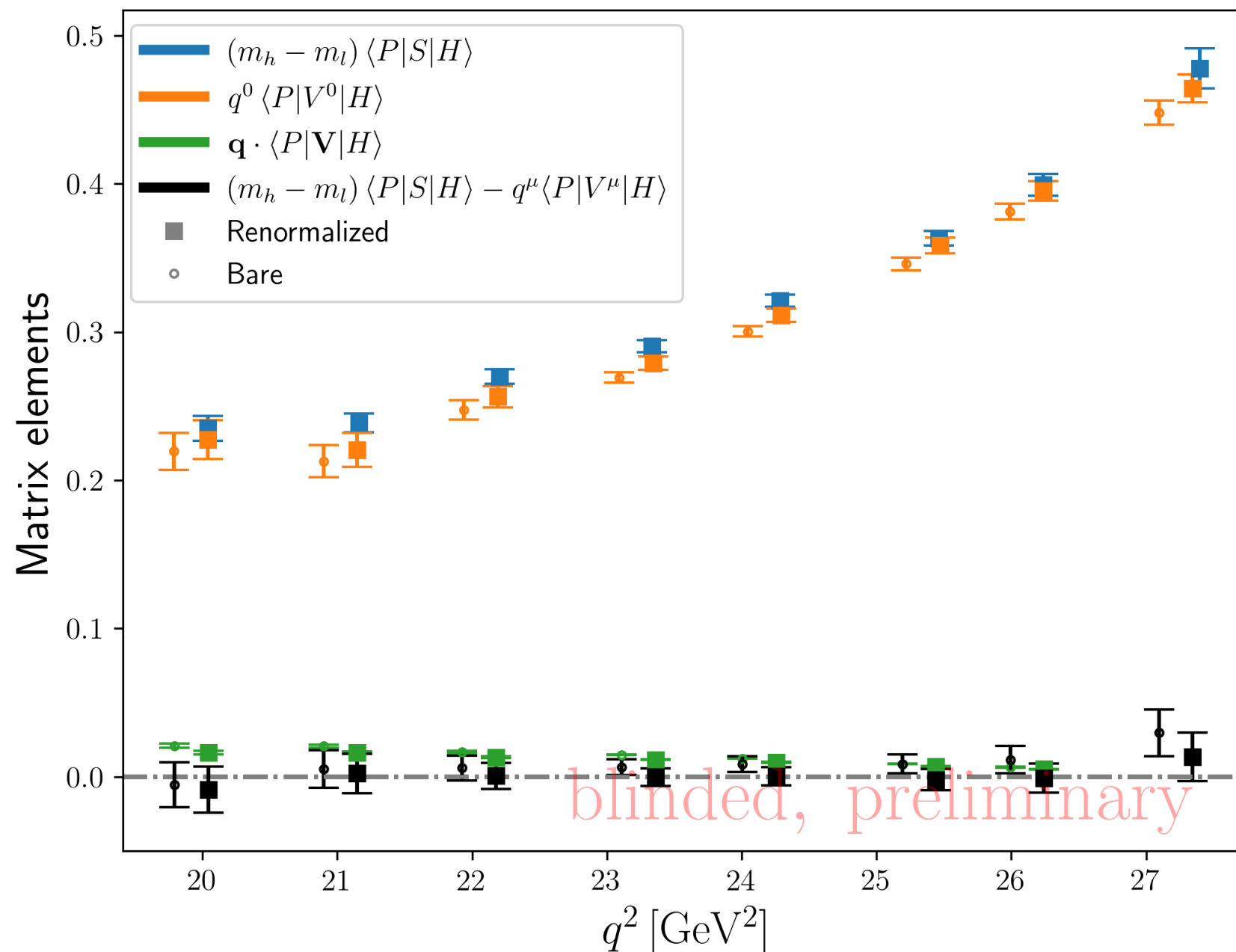
Extracting Matrix Elements

$$R_0(t, T, \mathbf{p}) \propto \frac{C_{H \rightarrow P}^S(t, T, \mathbf{p})}{\sqrt{C_P(t, \mathbf{p})C_H(T - t)}}$$



Partial Conservation of the Vector Current

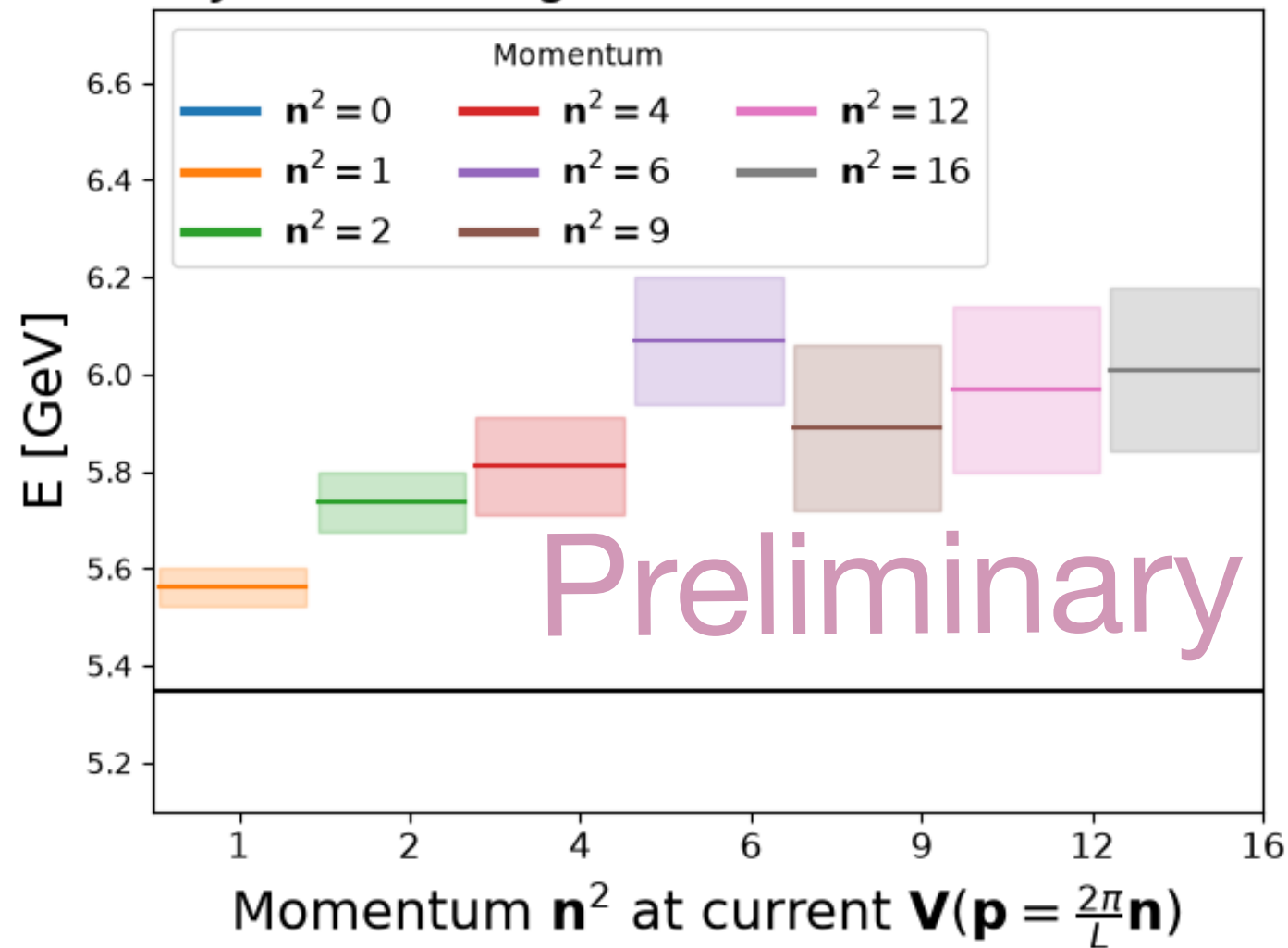
$$Z_{V^0} (M_H - E_P) \langle P | V^0 | H \rangle + Z_{V^i} \mathbf{q} \cdot \langle P | \mathbf{V} | H \rangle = Z_M Z_S (m_h - m_x) \langle P | S | H \rangle$$



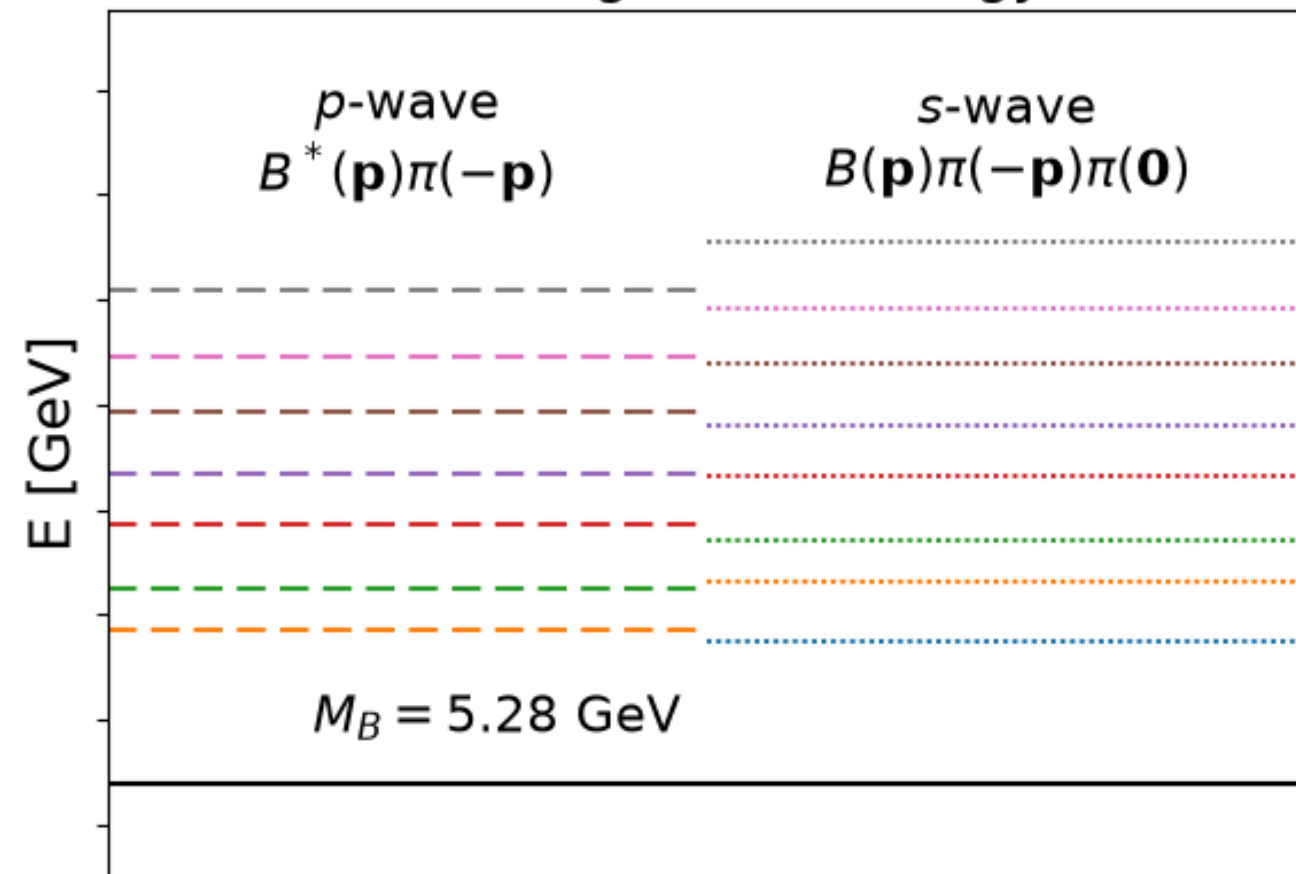
B-meson 0^- spectrum

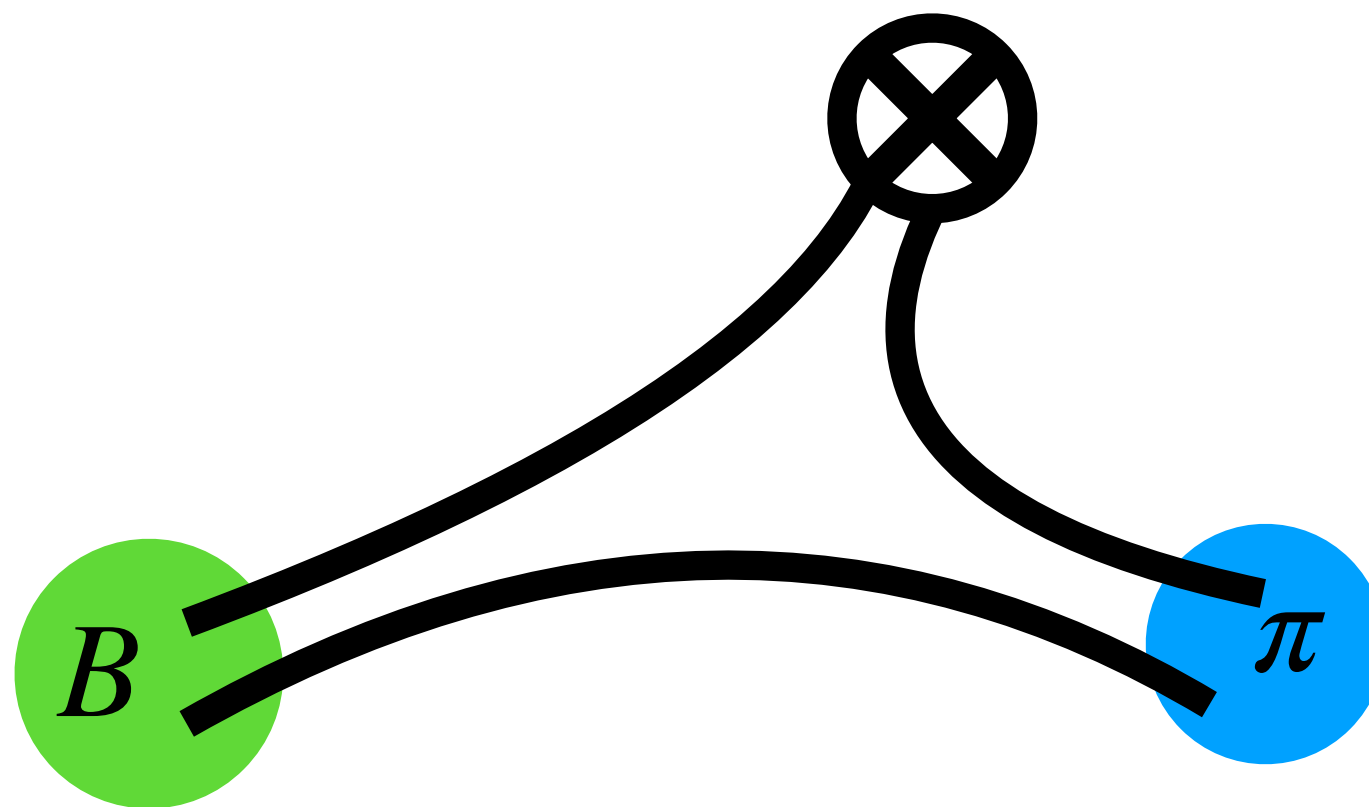
* Work in Progress

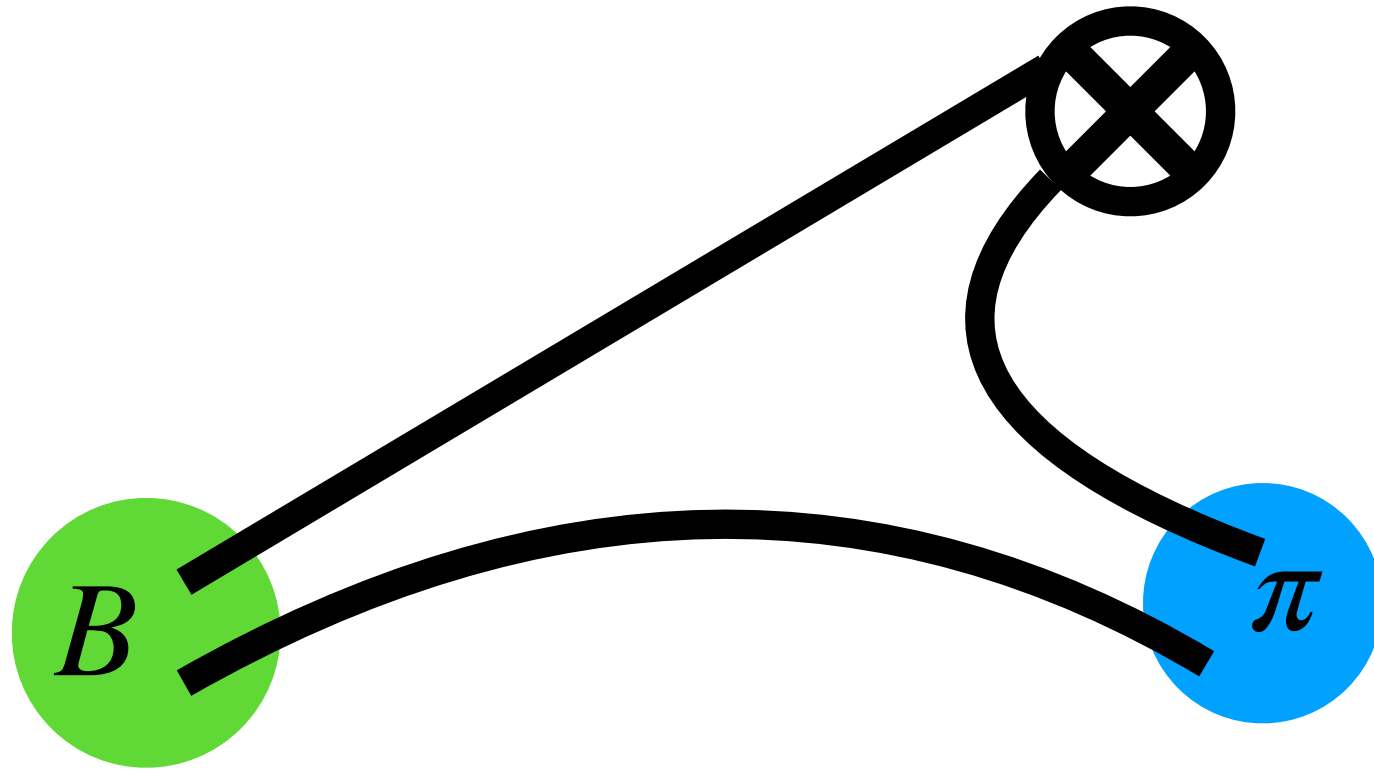
Joint fit: 0^- ground and excited state

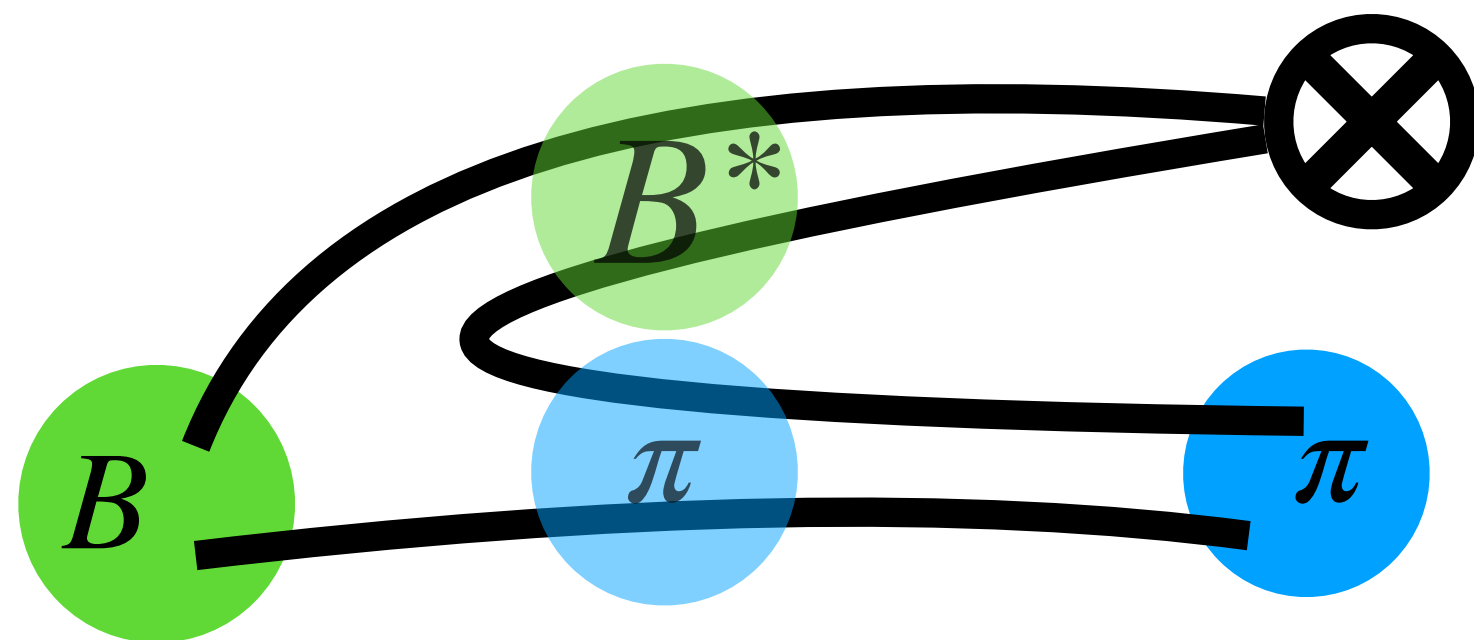


Non-interacting 0^- FV energy levels

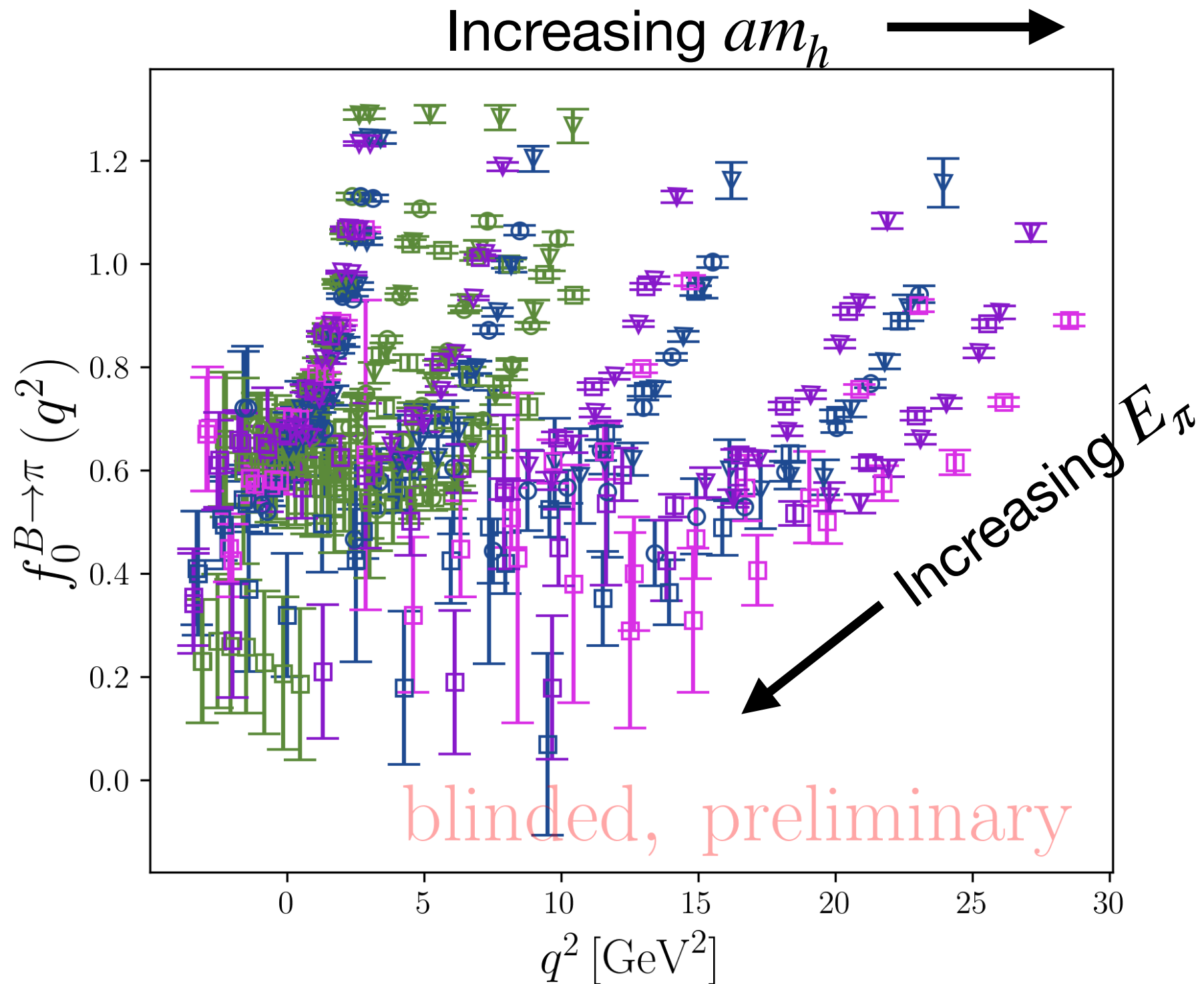


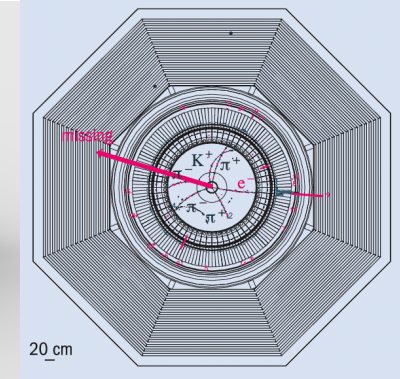
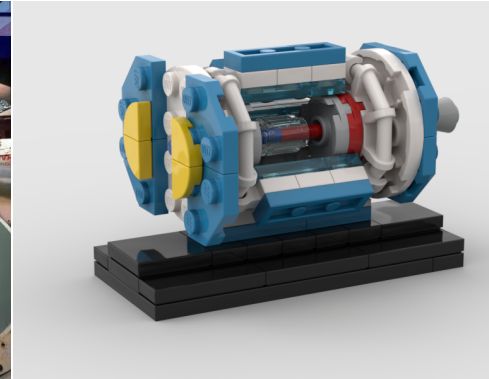
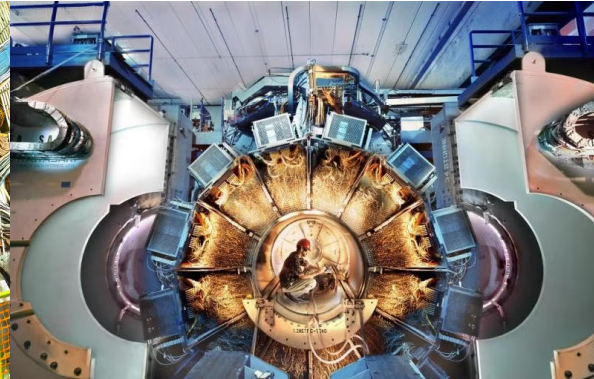
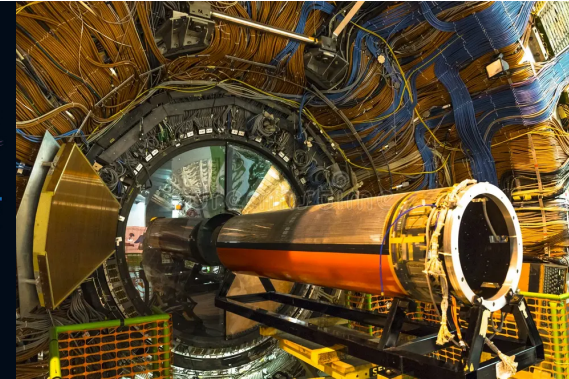
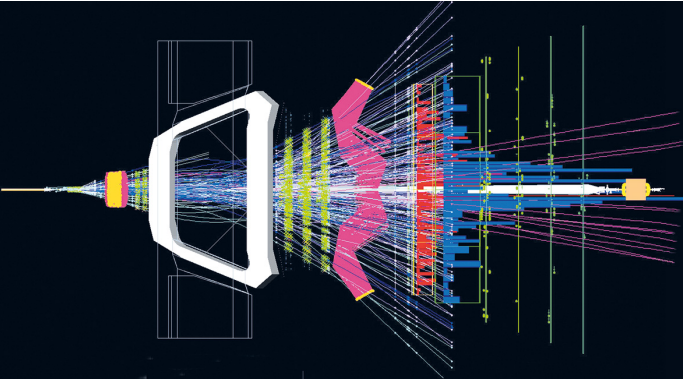




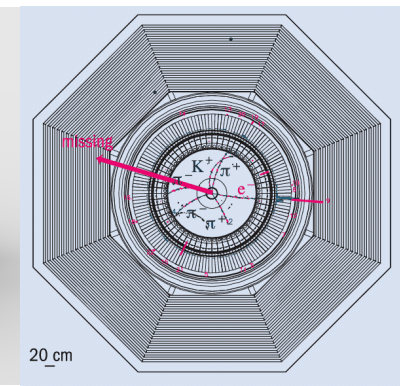
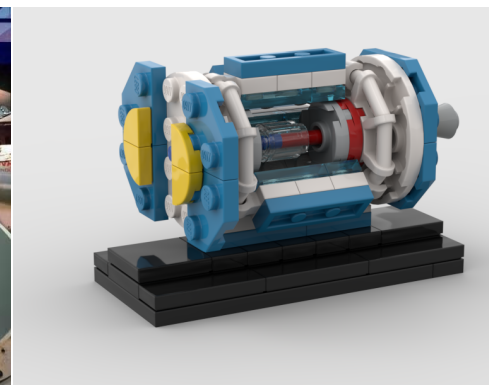
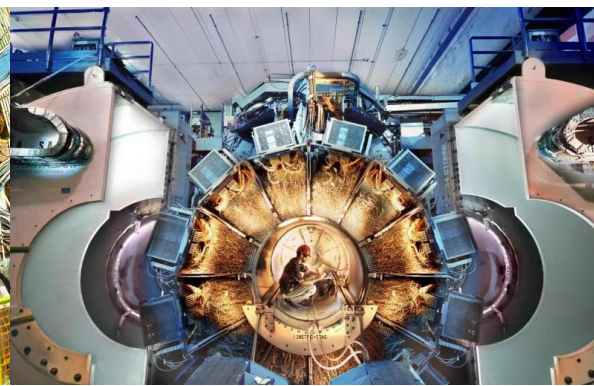
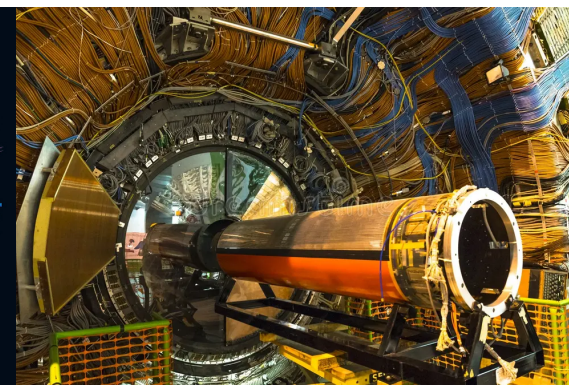
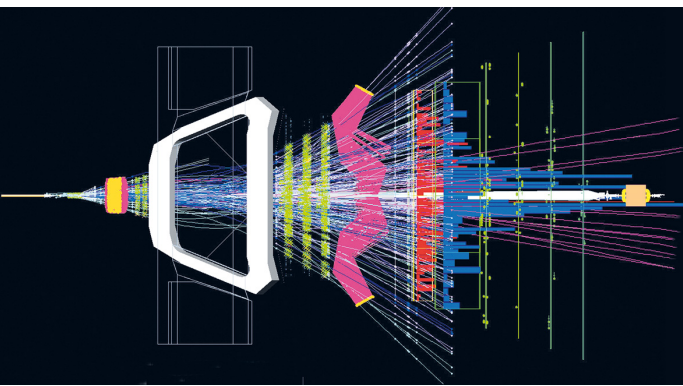


Form Factor Fits





Journey to the Continuum



Continuum Fit Form

$$w_0^{d_P} f_P = \frac{c_0}{w_0 (E + \Delta_{xy,P})} \times \left[1 + \delta f_{\text{logs}} + \text{discretization} + \text{mistunings} \right]$$

- Heavy Meson Rooted Staggered χ PT

Continuum Fit Form

$$w_0^{d_P} f_P = \frac{c_0}{w_0 (E + \Delta_{xy,P})} \times \left[1 + \delta f_{\text{logs}} + \text{discretization} + \text{mistunings} \right]$$

$$\frac{1}{E + \Delta_{xy,P}} = \frac{2M_{B_y}}{M_{B_x^*} - q^2} \quad \longrightarrow \quad \Delta_{xy,P} = \frac{M_{B_x^*(J^P)}^2 - M_B^2 - M_\pi^2}{2M_B}$$

- The angular momentum that appears must match the current

$$0^+ : B_0^* = ??$$

$$1^- : M_{B^*(PDG)} \approx 5325 \text{ [MeV]}$$

Continuum Fit Form

$$w_0^{d_P} f_P = \frac{c_0}{w_0 (E + \Delta_{xy,P})} \times \left[1 + \delta f_{\text{logs}} + \boxed{\text{discretization}} + \boxed{\text{mistunings}} \right]$$

Heavy Quark Discretization:

$$x_h^2 = \frac{2}{\pi} (am_h)^2$$

Can be suppressed
by α_s , velocity, $\frac{\Lambda_{HQET}}{M_H}$,
or combinations thereof

Heavy Quark Mistuning:

$$\chi_H = \frac{\Lambda_{HQET}}{M_H^{\text{sim}}} - \frac{\Lambda_{HQET}}{M_B^{\text{PDG}}}$$

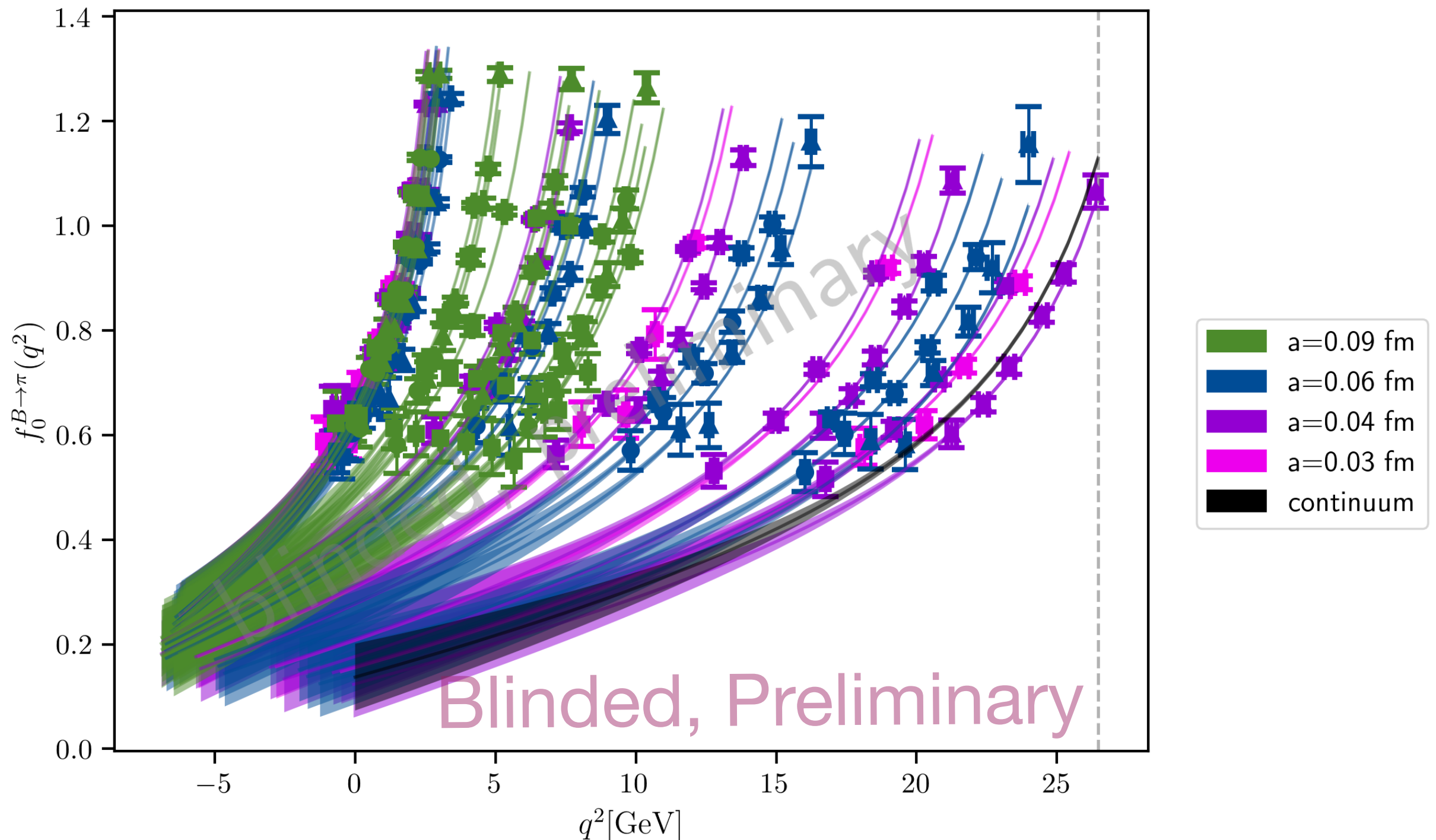
Others:

$$\chi_l = \frac{(M_\pi^{\text{sim}})^2}{8\pi^2 f_\pi^2}$$

$$\chi_E = \frac{\sqrt{2}E}{4\pi f_\pi}$$

$$\chi_S = \frac{(M_K^{\text{sim}})^2 - (M_K^{\text{PDG}})^2}{8\pi^2 f_\pi^2}$$

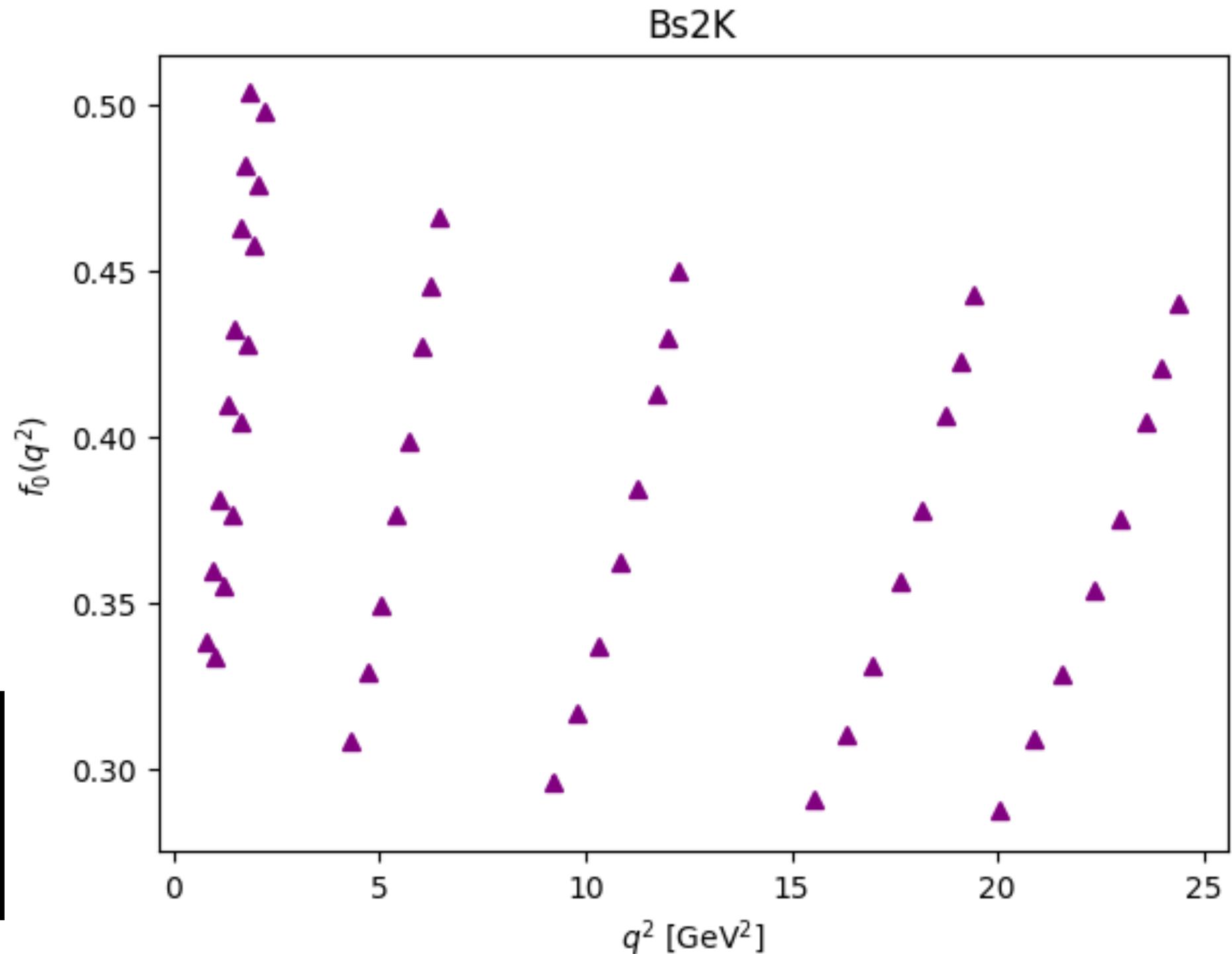
Chiral-Continuum Fit



Conclusions

- Simulations are now running with 4 flavors
(2x light, strange, charm) HISQ with fine enough lattice spacings to reach the physical bottom quark mass
- We are currently working on taking the continuum limit
- Next steps will be combining our continuum form factors with experimental data to calculate $|V_{ub}|$
- Extract form factors for $B_S \rightarrow K\ell\nu$ and repeat

$B_S \rightarrow K$ Correlator Fits

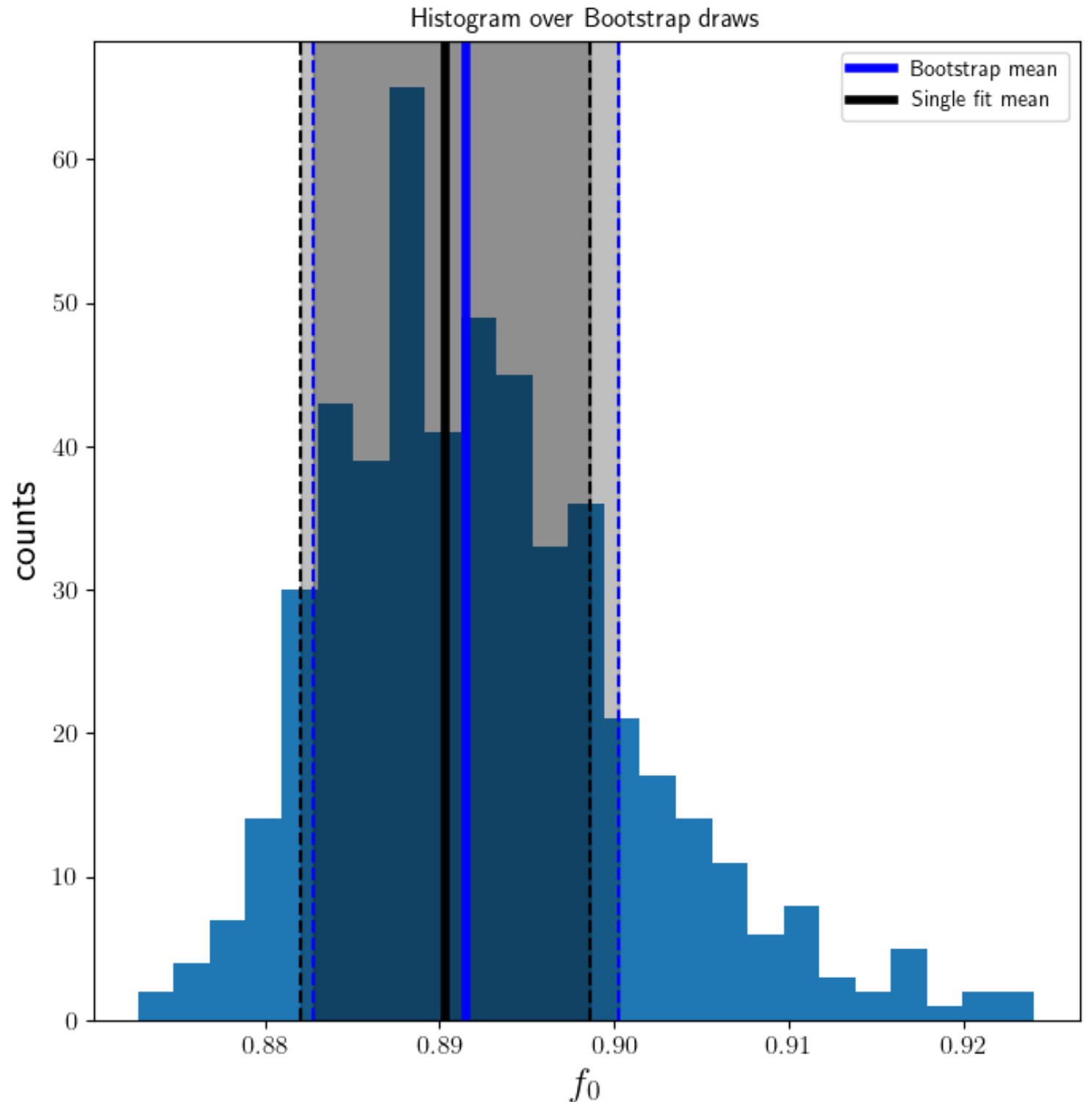


$a=0.042\text{fm}$

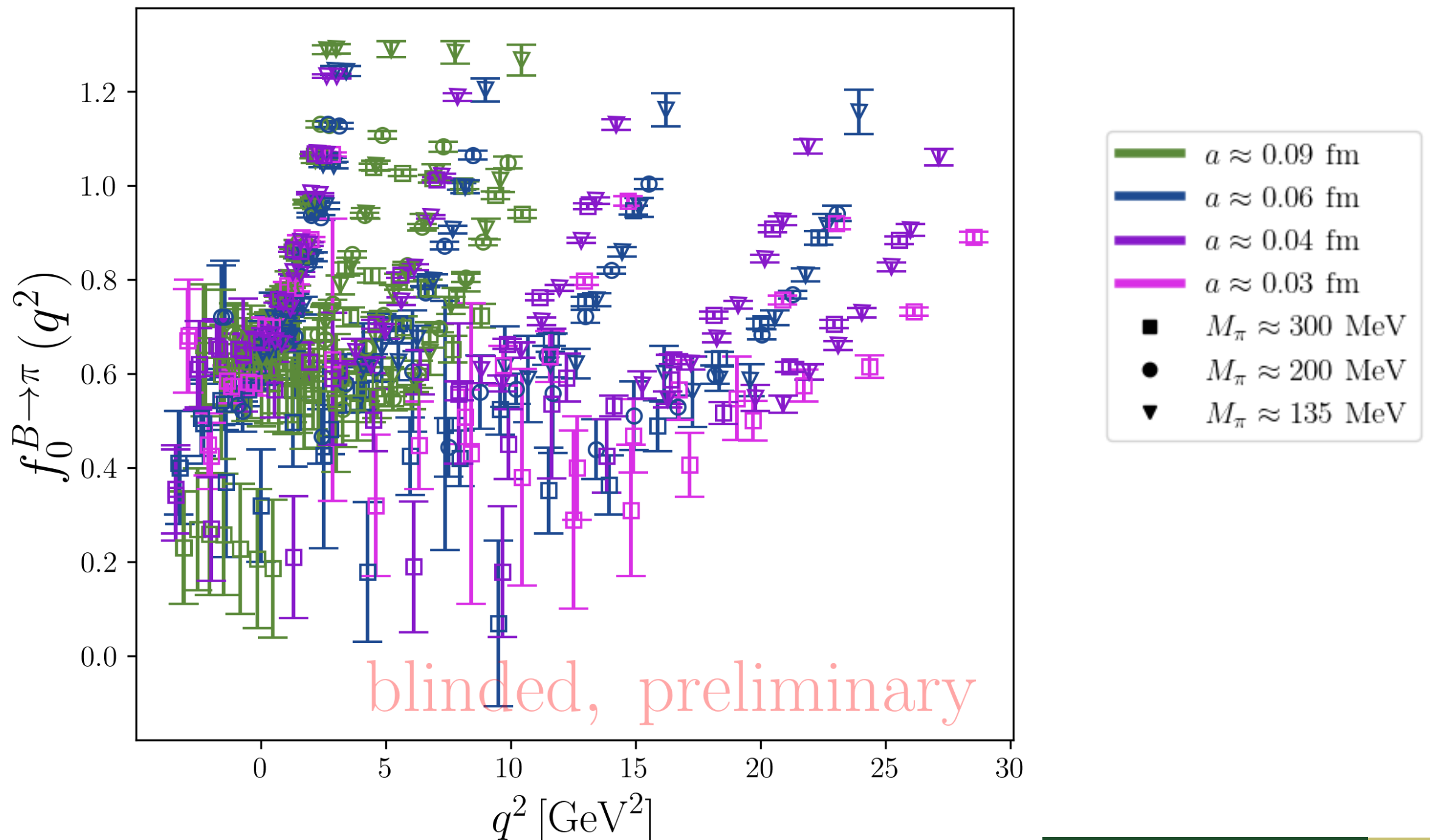
$M_\pi = 135\text{ MeV}$

Bootstraps

- Correlations are included in the error propagation via bootstrapping with 500 draws
- Single fit posteriors are seen to be statistically consistent with a similar size error bar
- $a = 0.042 \text{ fm}$,
 $m_l = 0.2 m_s$,
 $m_h = 1.0 m_b$



All Form Factors: f_0



All Form Factors: f_+

