Highly improved staggered quarks on anisotropic lattices

Alexei Bazavov
Michigan State University

with Yannis Trimis (MSU), Johannes Weber (TU Darmstadt)

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Contents

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  - one-component spinor,
  - remnant chiral symmetry.

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  - MILC program: 2+1+1 flavor, mainly for zero temperature,
  - HotQCD program: 2+1 flavor, mainly for finite temperature and density.
- The existing ensembles are not optimal for spectral reconstruction type of problems.
- Anisotropic ensembles are preferable, the renormalized anisotropy $\xi \equiv a_\sigma / a_\tau > 1$ (even $\gg 1$).

Questions

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- How to tune the gauge and fermion anisotropy?
- What happens to the spectrum, in particular, to the staggered taste multiplets (i.e., unphysical states labeled by an additional quantum number, *taste*, that are the dominant discretization effects in staggered actions) with anisotropy?

Tuning

- Assume a single mass parameter, the strange quark mass m_s in the fermion part of the action (light and charm masses can be taken as a fixed fraction).

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- Anisotropic parameters: need four parameters, choose $(\beta, \xi_0(\beta), m_s(\beta), \xi_0^f(\beta))$ — lines of constant renormalized anisotropy (LCRA).

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- Anisotropic parameters: need four parameters, choose $(\beta, \xi_0(\beta), m_s(\beta), \xi_0^f(\beta))$ — lines of constant renormalized anisotropy (LCRA).
- Quenched ensembles: the tuning is simplified,
  - first, $(\beta, \xi_0(\beta))$,
  - then, $(m_s(\beta), \xi_0^f(\beta))$.

Anisotropic gauge action

- Anisotropic tree-level Symanzik-improved gauge action:

$$
\begin{aligned}
S_g \; &= \; \beta \frac{1}{\xi_0} \left(c_P \sum_{P_{\sigma\sigma}} \left(1 - \frac{1}{3} \mathrm{ReTr}(P_{\sigma\sigma}) \right) + c_R \sum_{R_{\sigma\sigma}} \left(1 - \frac{1}{3} \mathrm{ReTr}(R_{\sigma\sigma}) \right) \right) \\
&+ \; \beta \xi_0 \left(c_P \sum_{P_{\sigma\tau}} \left(1 - \frac{1}{3} \mathrm{ReTr}(P_{\sigma\tau}) \right) + c_R \sum_{R_{\sigma\tau}} \left(1 - \frac{1}{3} \mathrm{ReTr}(R_{\sigma\tau}) \right) \right),
\end{aligned}
$$

- $\beta = 10/g^2$, $c_P = 1$, $c_R = -1/20$ and ξ_0 is the bare gauge anisotropy.

Karsch, NPB (1982)

Lüscher, Weisz, CMP (1985)

Anisotropic gradient flow

Lüscher, CMP (2010)
Borsanyi et al., 1205.0781
Borsanyi et al., PRD (2018)

- The flow equation:

$$\frac{dU_\mu}{dt} = Z_\mu U_\mu$$

is modified to include flow anisotropy ξ_0^{gf}:

$$Z_\mu(x,t) = -\sum_{\nu\neq\mu} \rho_{\mu\nu} \mathscr{P}_A \Big[U_\mu(x,t) S^\dagger_{\mu\nu}[U](x,t) \Big],$$

where

$$\rho_{i4} = (\xi_0^{gf})^2, \quad \rho_{ij} = \rho_{4i} = 1.$$

Anisotropic gradient flow

- Spatial and temporal action density:

$$S_{\sigma\sigma}(t) = \frac{1}{4}\sum_{x,i\neq j} F_{ij}^2(x,t), \quad S_{\sigma\tau}(t) = \frac{1}{2}\sum_{x,i} F_{i4}^2(x,t).$$

- The scale setting conditions:

$$\left[t\frac{d}{dt}t^2\langle S_{\sigma\sigma}(t)\rangle\right]_{t=w_{0,\sigma}^2} = 0.15, \quad (\xi_0^{gf})^2\left[t\frac{d}{dt}t^2\langle S_{\sigma\tau}(t)\rangle\right]_{t=w_{0,\tau}^2} = 0.15$$

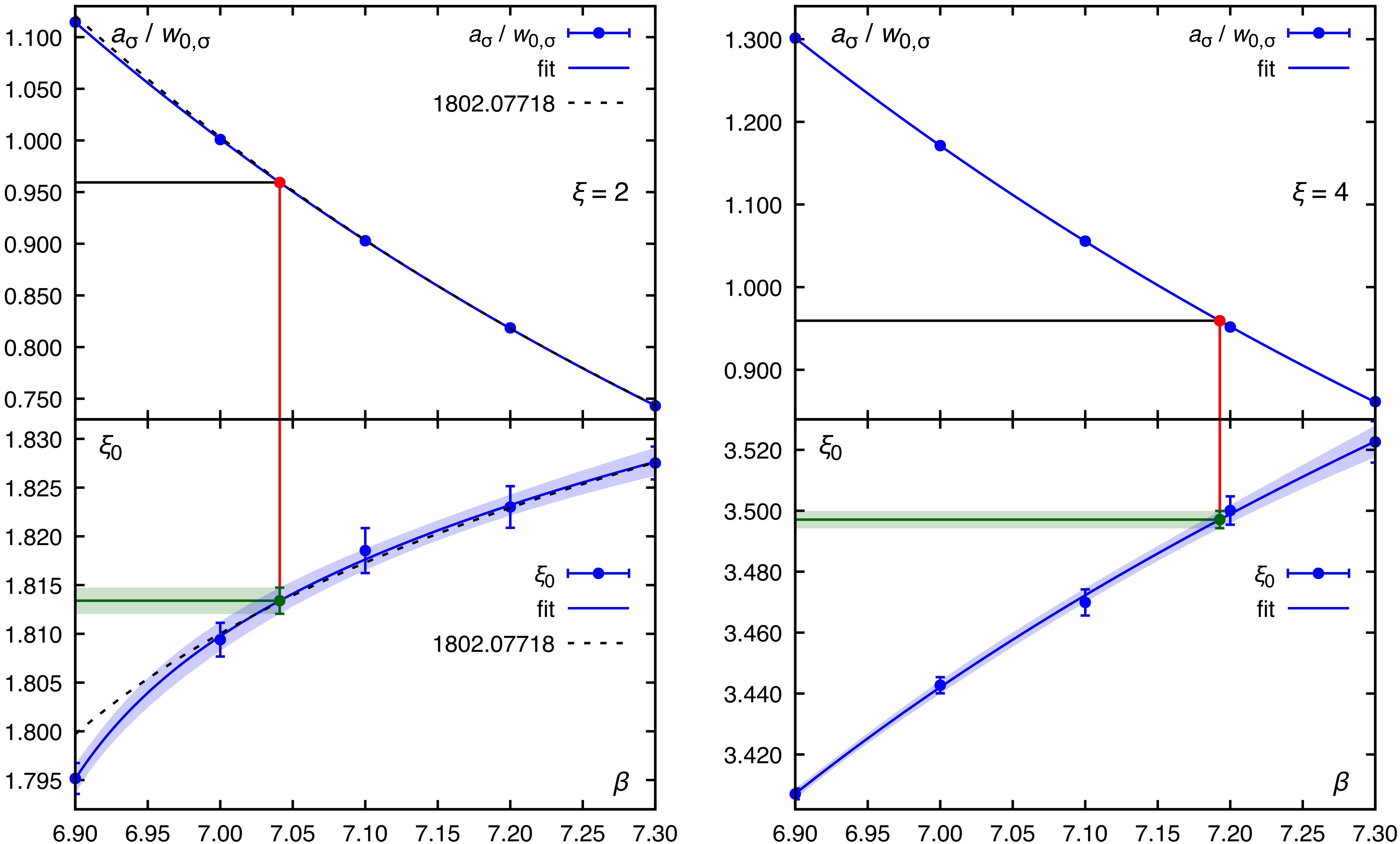
and

$$\frac{w_{0,\sigma}}{w_{0,\tau}} = 1.$$

Lüscher, JHEP (2010)
Borsanyi et al., JHEP (2012)

- Various choices for flow/observable action discretization, SWC appears to be the optimal one with anisotropy.

Lines of constant renormalized anisotropy



Fermion action

- Anisotropic fermion action:

$$S_f = a_\sigma^3 a_\tau \sum_{x,y} \bar{\tilde{\psi}}_x \tilde{M}_{xy} \tilde{\psi}_y, \qquad \tilde{M}_{xy} = \tilde{D}_{xy} + m\delta_{xy}.$$

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- Rescaling:

$$\psi = a_\sigma \sqrt{a_\tau}\, \tilde{\psi}, \quad D_{xy,\sigma} = a_\sigma \tilde{D}_{xy,\sigma}, \quad \xi D_{xy,\tau} = \xi a_\tau \tilde{D}_{xy,\tau} = a_\sigma \tilde{D}_{xy,\tau}\ .$$

- Dimensionless form of the action:

$$S_f = \sum_{x,y} \bar{\psi}_x M_{xy} \psi_y = \sum_{x,y} \bar{\psi}_x \left[D_{xy,\sigma} + \xi_0^f D_{xy,\tau} + a_\sigma m \delta_{xy} \right] \psi_y\ ,$$

where ξ_0^f is the bare fermion anisotropy.

- Early attempts with naive staggered.

Nomura, Matsufuru, Umeda, PTP (2004)

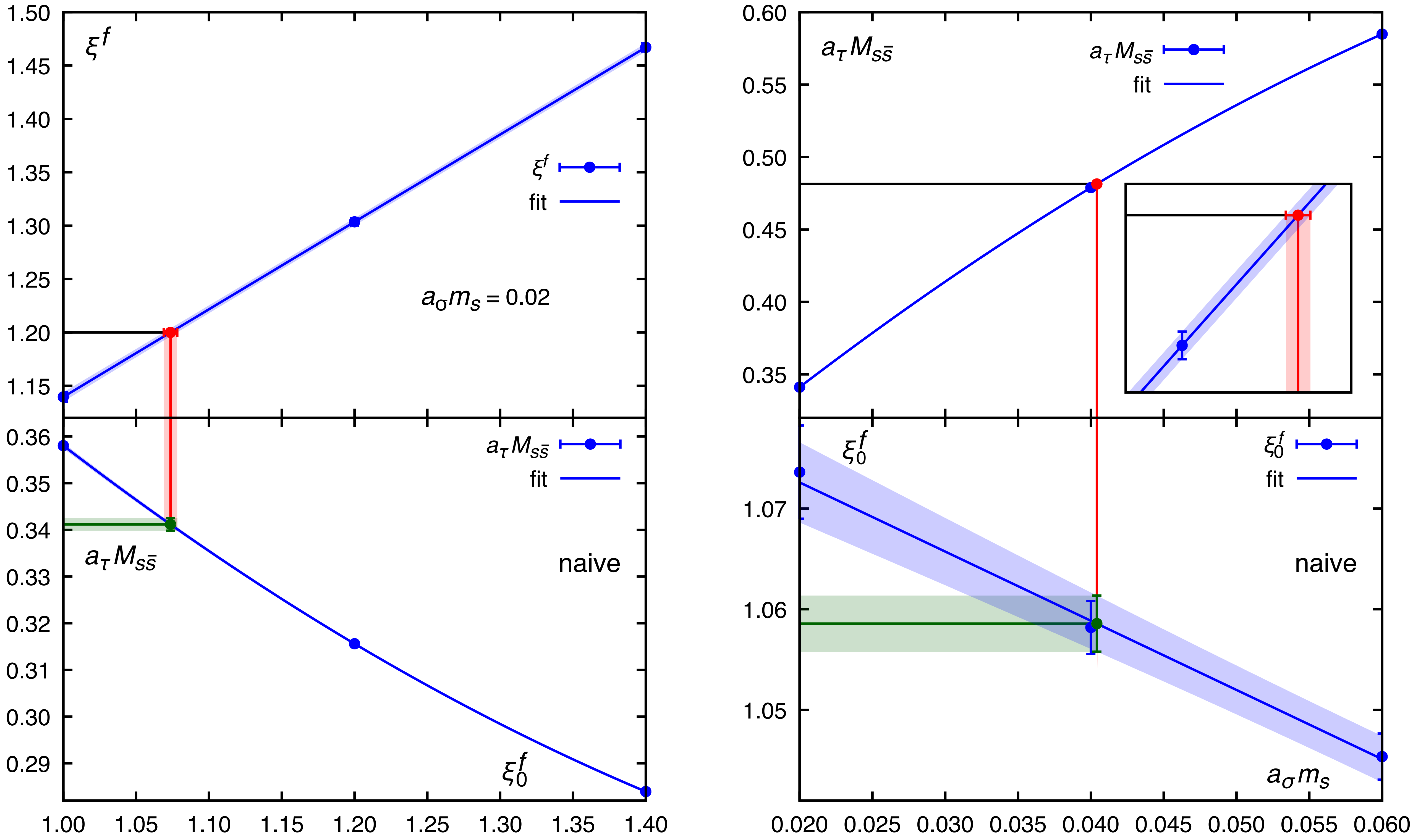
Levkova, Manke, Mawhinney, PRD (2006)

aHISQ

- Anisotropic HISQ action:

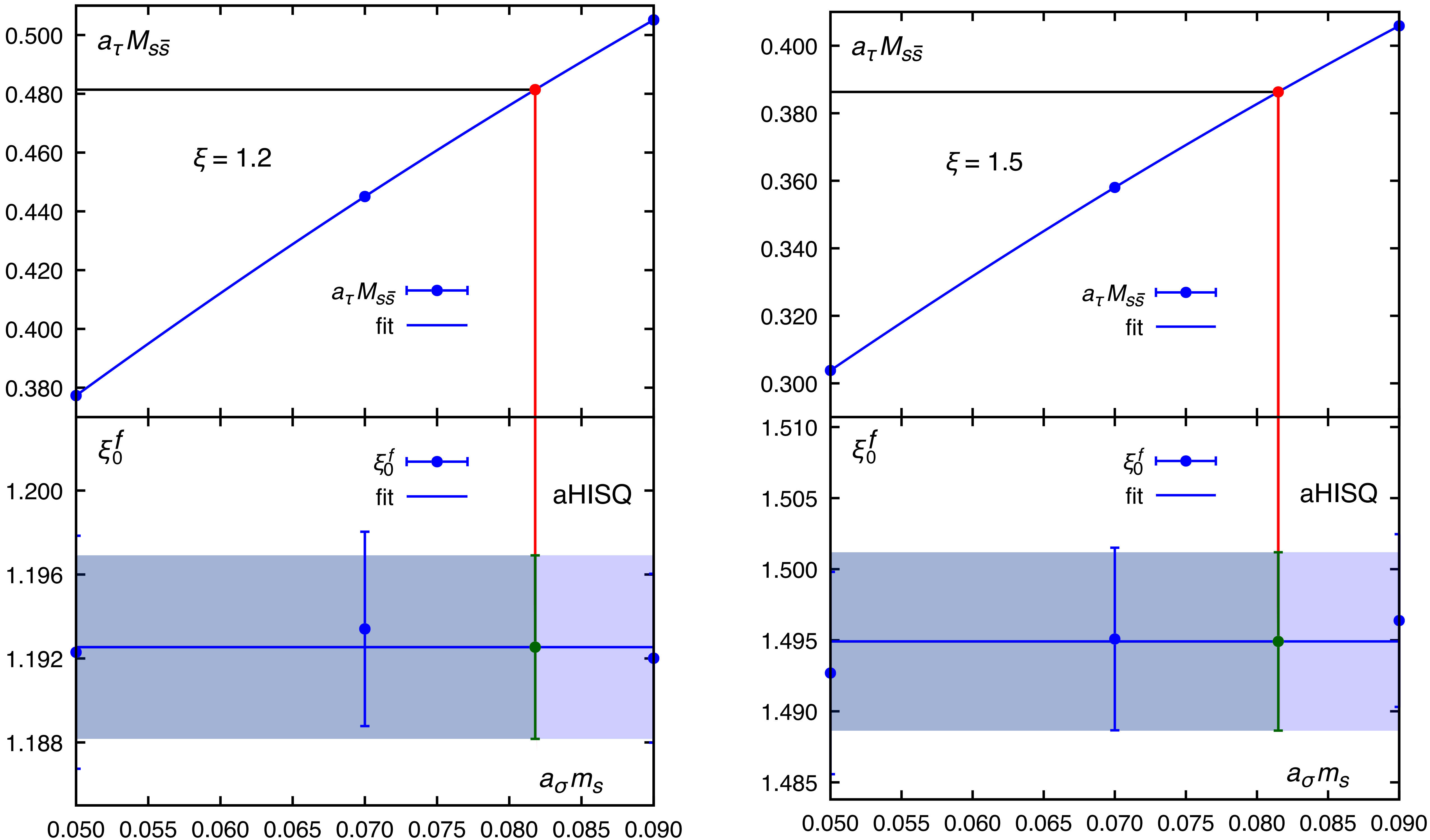
$$
\begin{aligned}
2D^{aHISQ}_{xy}[U] \;=\; & \sum_{\mu=1}^{3} \eta_{x,\mu} \left\{ X^{F}_{x,\mu}[U]\delta_{x+\hat{\mu},y} - X^{F\dagger}_{x-\hat{\mu},\mu}[U]\delta_{x-\hat{\mu},y} \right. \\
+ \; & \left. (1+\epsilon(a_\sigma m)) \left(X^{L}_{x,\mu}[U]\delta_{x+3\hat{\mu},y} - X^{L\dagger}_{x-3\hat{\mu},\mu}[U]\delta_{x-3\hat{\mu},y} \right) \right\} \\
+ \; & \xi_0^f \eta_{x,4} \left\{ X^{F}_{x,4}[U]\delta_{x+\hat{4},y} - X^{F\dagger}_{x-\hat{4},4}[U]\delta_{x-\hat{4},y} \right. \\
+ \; & \left. (1+\epsilon(a_\sigma m)) \left(X^{L}_{x,4}[U]\delta_{x+3\hat{4},y} - X^{L\dagger}_{x-3\hat{4},\mu}[U]\delta_{x-3\hat{4},y} \right) \right\} .
\end{aligned}
$$

Tuning fermion anisotropy (naive)



- Fixed $a_\sigma m = 0.02$, $\xi^f = 1.2$, then determine $a_\sigma m$ for physical $M_{s\bar{s}}$.

Tuning fermion anisotropy (aHISQ)



- For aHISQ ξ_0^f essentially does not depend on $a_\sigma m_s$.

Staggered pion taste spectrum

- With respect to the taste quantum number the pseudoscalar (γ_5) mesons are classified as
  - pseudoscalar, ξ_5,
  - axial vector, $\xi_\mu\xi_5$,
  - tensor, $\xi_\mu\xi_\nu$,
  - vector, ξ_μ,
  - scalar or singlet, 1.
- 1, 4, 6, 4, 1-fold degeneracy pattern.

Golterman, Smit, NPB (1984, 1985)
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Kilcup, Sharpe, NPB (1987)

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  - tensor, $\xi_\mu \xi_\nu$,
  - vector, ξ_μ,
  - scalar or singlet, 1.
- 1, 4, 6, 4, 1-fold degeneracy pattern.
- When $\xi \to \infty$, *i.e.*, $a_\tau = 0$, the shifts in the temporal direction are close to the identity, $\xi_4 \to 1$.
- New 2, 6, 6, 2-fold degeneracy pattern, splitting of the spatial and temporal components.

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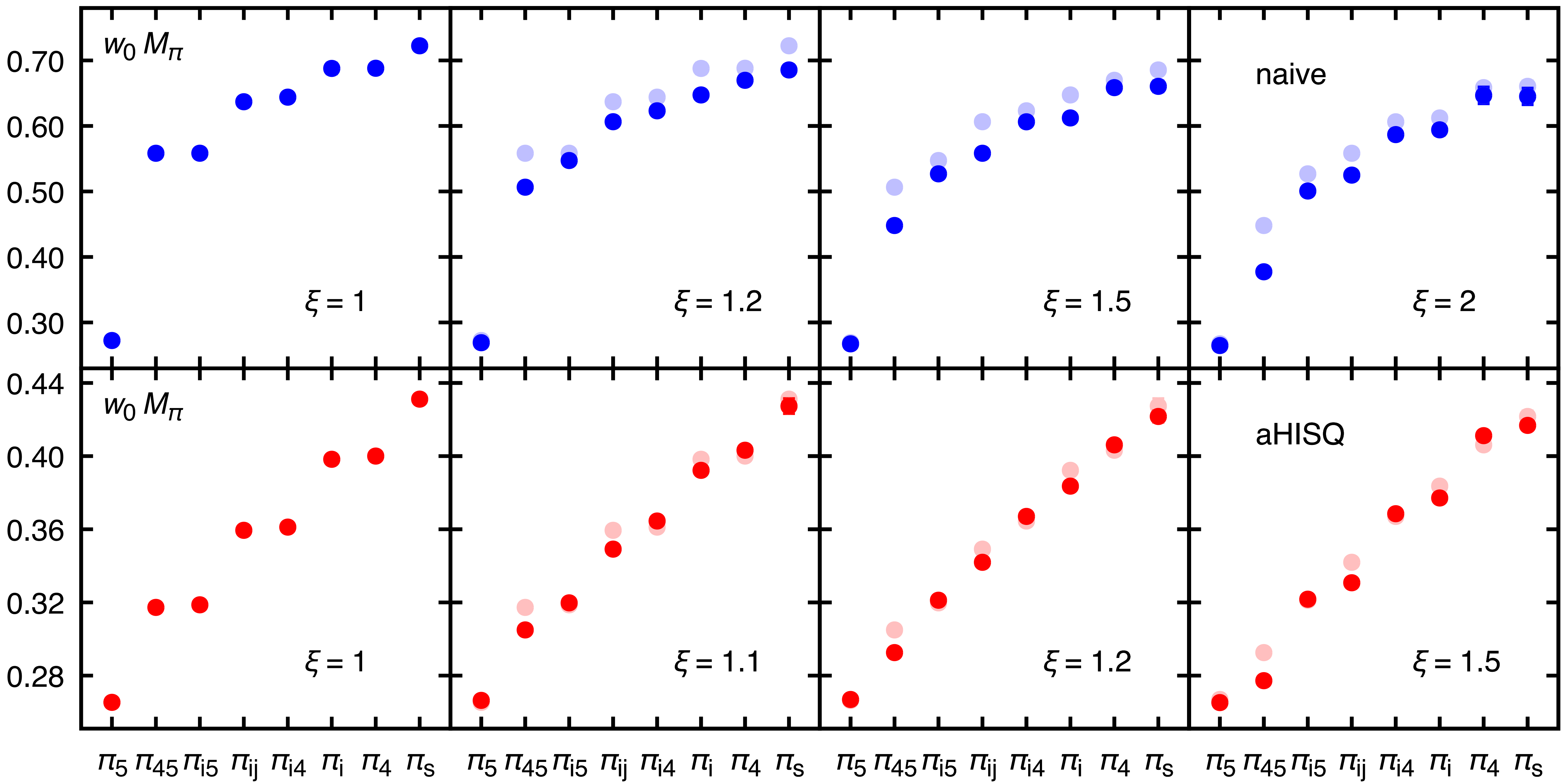
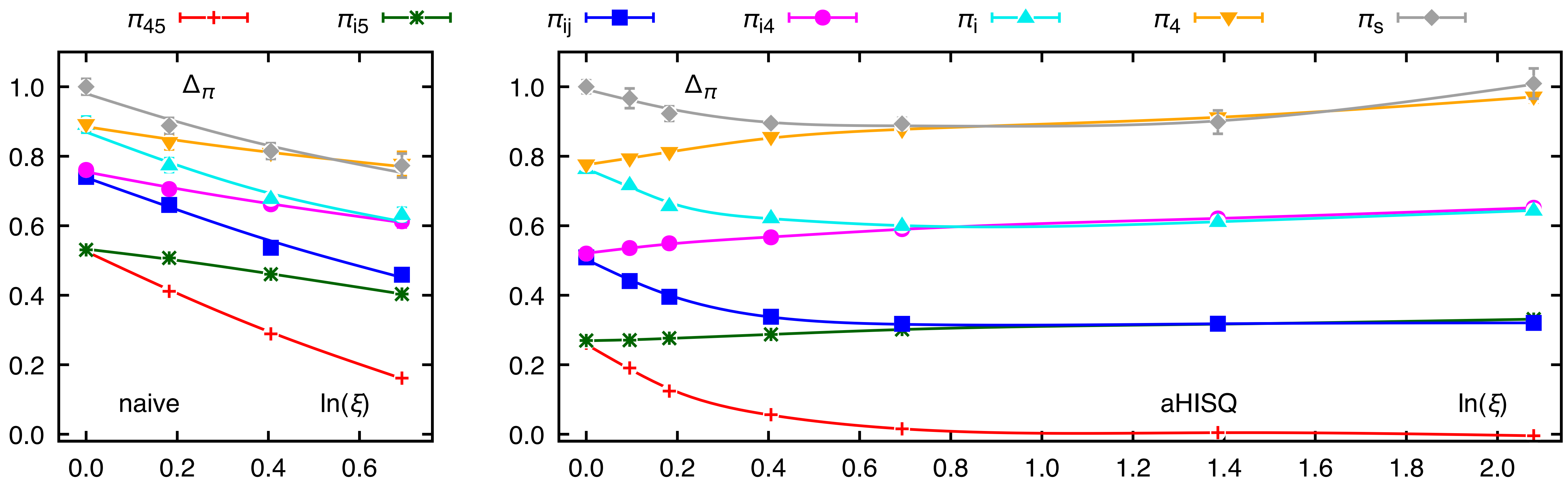
Golterman, NPB (1986)
Kilcup, Sharpe, NPB (1987)

Staggered pion taste spectrum



- Keep $a_\sigma = \text{const} \simeq 0.1665$ fm, vary ξ for naive staggered and aHISQ.

Staggered pion taste spectrum



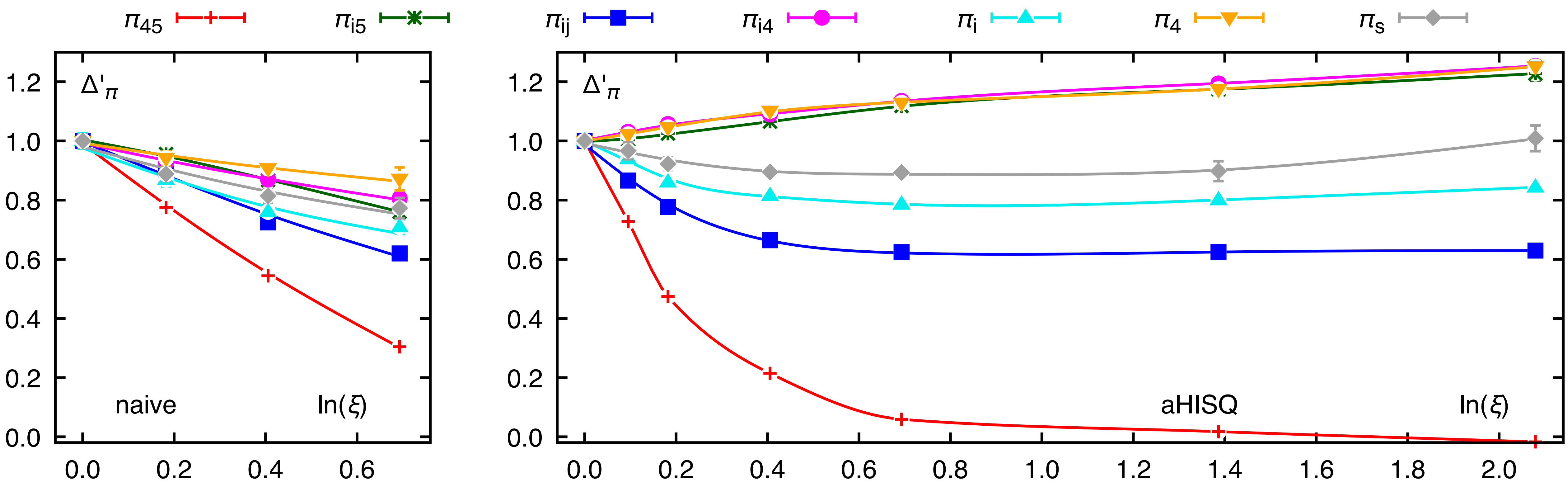
- Quadratic mass splittings:

$$\delta M_\pi \equiv M_\pi^2 - M_{\pi_5}^2 \, .$$

- Convenient normalization:

$$\Delta_\pi \equiv \frac{\delta M_\pi}{\delta M_{\pi_s}\Big|_{\xi=1}} \, .$$

Staggered pion taste spectrum



- Quadratic mass splittings:

$$\delta M_\pi \equiv M_\pi^2 - M_{\pi_5}^2 \,.$$

- Another normalization:

$$\Delta'_\pi \equiv \frac{\delta M_\pi}{\left.\delta M_\pi\right|_{\xi=1}} \,.$$

Staggered chiral perturbation theory

- Dimension-6 operators contributing to taste splittings:

Lee, Sharpe, PRD (1999)

$$
\begin{aligned}
-\mathcal{U} &= a^6 \sum_k C_k \mathcal{O}_k \\
&= a^6 C_1 \,\mathrm{Tr}\big(\xi_5 \Sigma \xi_5 \Sigma^\dagger\big) \\
&+ a^6 C_3 \frac{1}{2} \sum_\nu \Big[\mathrm{Tr}\big(\xi_\nu \Sigma \xi_\nu \Sigma\big) + \mathrm{Tr}\big(\xi_\nu \Sigma^\dagger \xi_\nu \Sigma^\dagger\big)\Big] \\
&+ a^6 C_4 \frac{1}{2} \sum_\nu \Big[\mathrm{Tr}\big(\xi_{\nu 5} \Sigma \xi_{5\nu} \Sigma\big) + \mathrm{Tr}\big(\xi_{\nu 5} \Sigma^\dagger \xi_{5\nu} \Sigma^\dagger\big)\Big] \\
&+ a^6 C_6 \sum_{\mu<\nu} \mathrm{Tr}\big(\xi_{\mu\nu} \Sigma \xi_{\nu\mu} \Sigma^\dagger\big).
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&+ a^6 C_4 \frac{1}{2} \sum_\nu \Big[\operatorname{Tr}\big(\xi_{\nu 5} \Sigma \xi_{5\nu} \Sigma\big) + \operatorname{Tr}\big(\xi_{\nu 5} \Sigma^\dagger \xi_{5\nu} \Sigma^\dagger\big)\Big] \\
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\end{aligned}
$$

- What is the effect of anisotropy on the level of the effective theory?

Staggered ChiPT inspired model

- Dimension-6 operators contributing to taste splittings:

$$
\begin{aligned}
-\mathcal{U}_{ani}/a_\sigma^6 \;=\;& C_1(\xi)\mathrm{Tr}\big(\xi_5\Sigma\xi_5\Sigma^\dagger\big) \\
+\;& C_3(\xi)\frac{1}{2}\sum_j\Big[\mathrm{Tr}\big(\xi_j\Sigma\xi_j\Sigma\big)+\mathrm{Tr}\big(\xi_j\Sigma^\dagger\xi_j\Sigma^\dagger\big)\Big] \\
+\;& D_3(\xi)\frac{1}{2}\Big[\mathrm{Tr}\big(\xi_4\Sigma\xi_4\Sigma\big)+\mathrm{Tr}\big(\xi_4\Sigma^\dagger\xi_4\Sigma^\dagger\big)\Big] \\
+\;& C_4(\xi)\frac{1}{2}\sum_j\Big[\mathrm{Tr}\big(\xi_{j5}\Sigma\xi_{5j}\Sigma\big)+\mathrm{Tr}\big(\xi_{j5}\Sigma^\dagger\xi_{5j}\Sigma^\dagger\big)\Big] \\
+\;& D_4(\xi)\frac{1}{2}\Big[\mathrm{Tr}\big(\xi_{45}\Sigma\xi_{54}\Sigma\big)+\mathrm{Tr}\big(\xi_{45}\Sigma^\dagger\xi_{54}\Sigma^\dagger\big)\Big] \\
+\;& C_6(\xi)\sum_{j<k}\mathrm{Tr}\big(\xi_{jk}\Sigma\xi_{kj}\Sigma^\dagger\big) \\
+\;& D_6(\xi)\sum_j\mathrm{Tr}\big(\xi_{j4}\Sigma\xi_{4j}\Sigma^\dagger\big).
\end{aligned}
$$

Staggered ChiPT inspired model

- Taste splittings:

$$
\begin{aligned}
\delta M_{45}(\xi) &= C_1(\xi) + 3C_3(\xi) + D_4(\xi) + 3D_6(\xi),\\
\delta M_{i5}(\xi) &= C_1(\xi) + 2C_3(\xi) + D_3(\xi) + C_4(\xi) + 2C_6(\xi) + D_6(\xi),\\
\delta M_{ij}(\xi) &= C_3(\xi) + D_3(\xi) + C_4(\xi) + D_4(\xi) + 2C_6(\xi) + 2D_6(\xi),\\
\delta M_{i4}(\xi) &= 2C_3(\xi) + 2C_4(\xi) + 2C_6(\xi) + 2D_6(\xi),\\
\delta M_{i}(\xi) &= C_1(\xi) + C_3(\xi) + 2C_4(\xi) + D_4(\xi) + 2C_6(\xi) + D_6(\xi),\\
\delta M_{4}(\xi) &= C_1(\xi) + D_3(\xi) + 3C_4(\xi) + 3D_6(\xi),\\
\delta M_{s}(\xi) &= 3C_3(\xi) + D_3(\xi) + 3C_4(\xi) + D_4(\xi).
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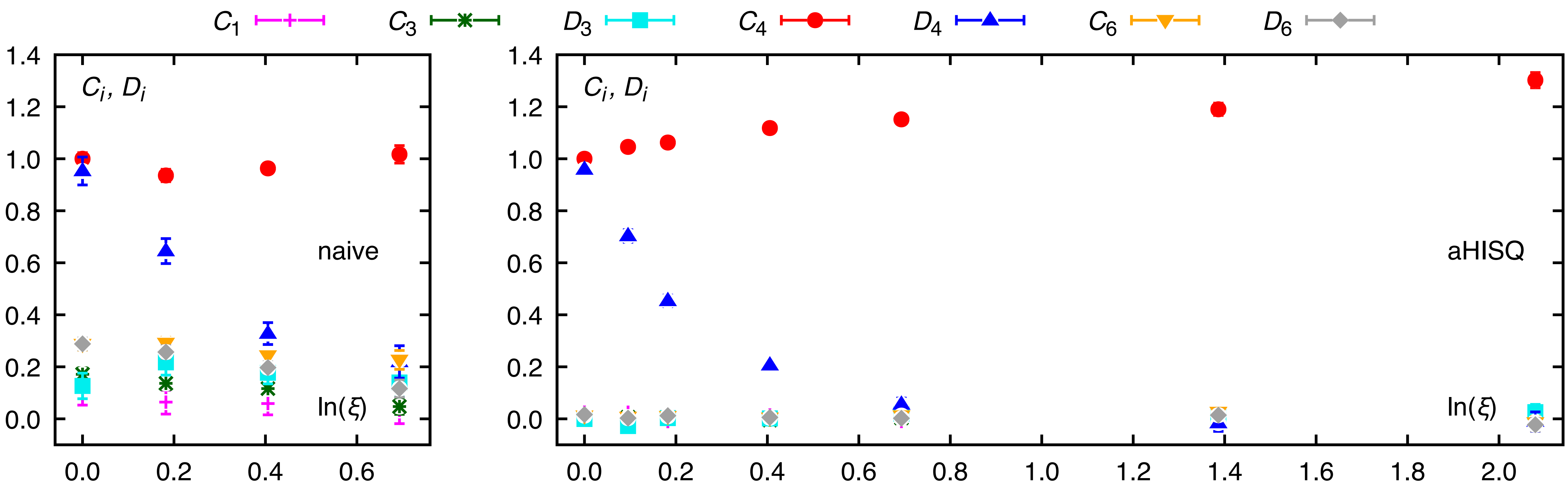
\delta M_{s}(\xi) &= 3C_3(\xi) + D_3(\xi) + 3C_4(\xi) + D_4(\xi).
\end{aligned}
$$

- Invert the dependence and determine the “low-energy constants” from the data.

"Low-energy constants"



- For aHISQ the splittings are dominated by C_4 and D_4 .
- For naive staggered C_4 and D_4 are dominant but the other operators also give a sizable contribution.

Staggered ChiPT inspired model

- Taste splittings for smeared actions:

$$
\begin{aligned}
\delta M_{45}(\xi) &= 0 \cdot C_4(\xi) + 1 \cdot D_4(\xi), \\
\delta M_{i5}(\xi) &= 1 \cdot C_4(\xi) + 0 \cdot D_4(\xi), \\
\delta M_{ij}(\xi) &= 1 \cdot C_4(\xi) + 1 \cdot D_4(\xi), \\
\delta M_{i4}(\xi) &= 2 \cdot C_4(\xi) + 0 \cdot D_4(\xi), \\
\delta M_{i}(\xi) &= 2 \cdot C_4(\xi) + 1 \cdot D_4(\xi), \\
\delta M_{4}(\xi) &= 3 \cdot C_4(\xi) + 0 \cdot D_4(\xi), \\
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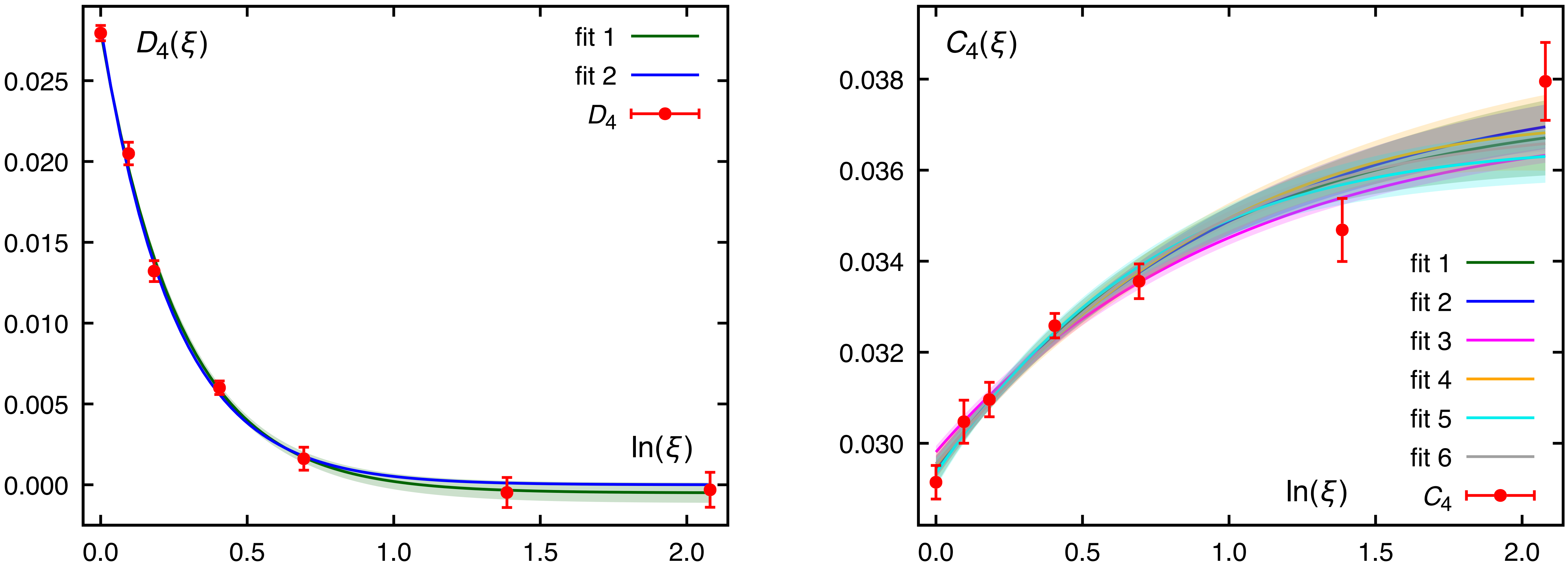
\delta M_{s}(\xi) &= 3\cdot C_4(\xi) + 1\cdot D_4(\xi).
\end{aligned}
$$

- The coefficients are equal to the number of gauge links in the interpolating operators.

Anisotropy dependence of "LECs"



- Preferred fit forms:

$$D_4(\xi) = \frac{d_2}{\xi^4}, \quad C_4(\xi) = c_1 \left(1 - (c_2 + c_3 \ln(\xi)) \frac{1}{\xi^2} \right),$$

$c_2 = c_3 = 1/5$ is favored by the data.

Anisotropy dependence of "LECs"

- Global fit:

$$\delta M_\pi(\xi) = c\left(l_\sigma \cdot \frac{5}{4}\left(1 - \frac{1}{5}\left(1 + \ln(\xi)\right)\frac{1}{\xi^2}\right) + l_\tau \cdot \frac{1}{\xi^4}\right).$$

Anisotropy dependence of “LECs”

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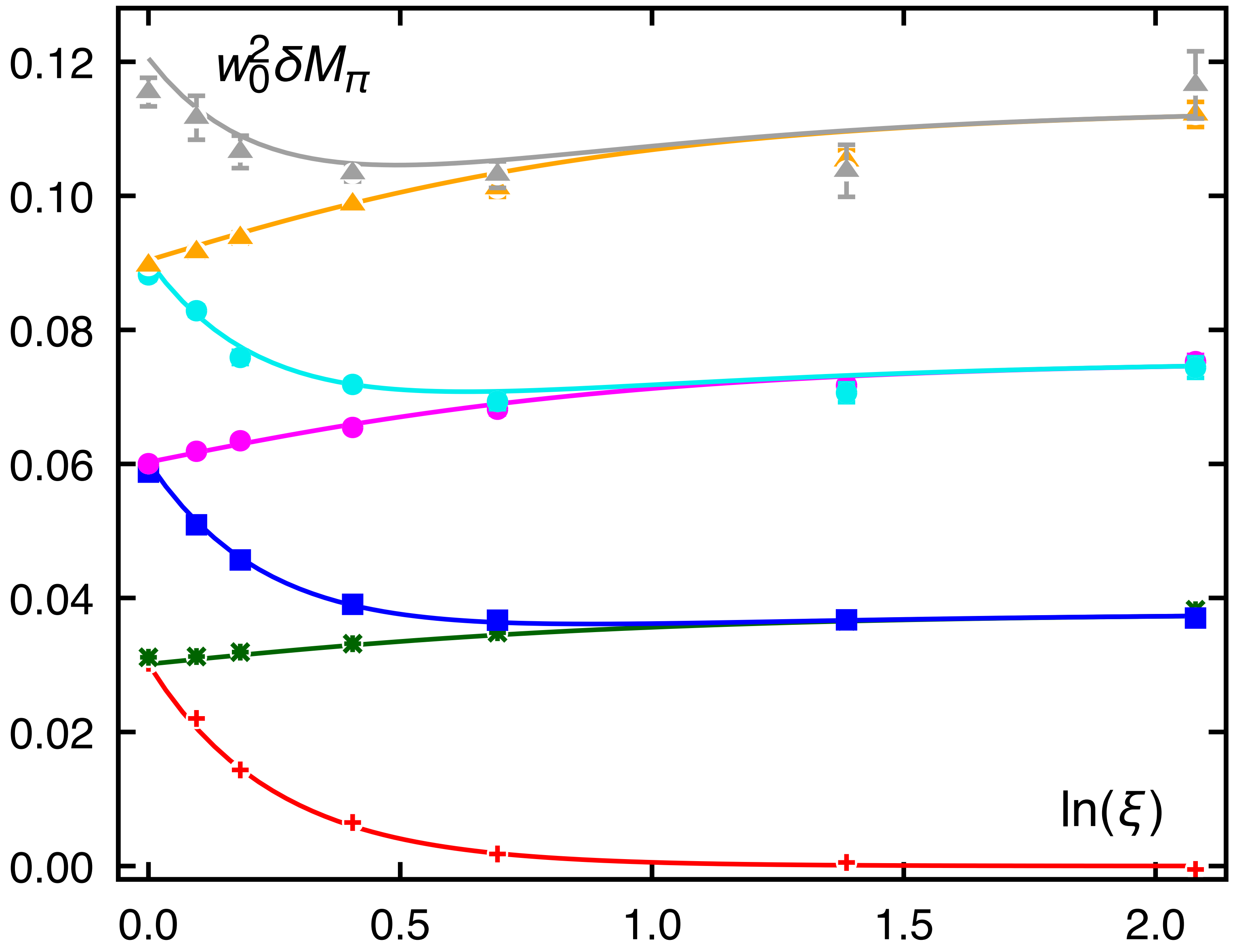

Conclusion

- For tuning the gauge anisotropy the Wilson flow/Clover observable (WC) is the optimal scheme.
- Staggered actions can work with anisotropy, for aHISQ the anisotropy is introduced in the Dirac operator after the smearing.
- For aHISQ the bare fermion anisotropy is almost mass independent and is very close to the target renormalized anisotropy.
- Staggered pion taste spectrum changes with anisotropy, a different degeneracy pattern emerges.
- An empirical model, inspired by the staggered chiral perturbation theory with additional "low-energy constants", describes the anisotropy dependence of the aHISQ taste spectrum very well.