Update on the gradient flow scale on CLS ensembles

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Scale setting

Scale setting is mandatory when:

- $\rightarrow$ converting dimensionful quantities computed on the lattice to physical units
- $\rightarrow$ taking the continuum limit for dimensionful quantities

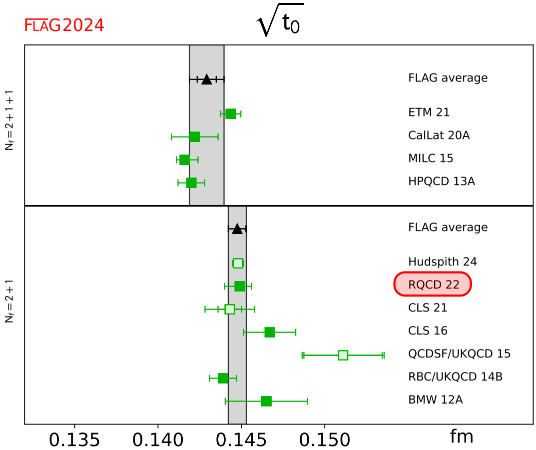
Setting the scale with the gradient flow parameter t_0 [Lüscher 2010]

- properties: mild dependence on quark masses, high precision, easy to compute
  $\rightarrow$ but large autocorrelations
- t_0 cannot be measured directly in experiments
  $\rightarrow$ connection to an experimental quantity is needed

Precision in scale setting

- for dimensionful quantities error from scale setting comes on top of statistical/systematic error of quantities computed from lattice
- high precision is needed in several cases, e.g.:
  $\rightarrow$ hadronic vacuum polarization contribution to the anomalous mag. moment of the muon
  $\rightarrow$ electroweak matrix elements that are required to constrain CKM parameters

Current Status: t_0 in FLAG 2024



Note also recent determinations by [HotQCD 25], [ALPHA 26], [NME 26]

Scale setting on $N_f = 2+1$ CLS ensembles

RQCD 2022 determination of t_0 on CLS ensembles

- RQCD 22: $\sqrt{t_{0,\text{phys}}} = 0.1449^{(7)}_{(9)}$ fm
- employ the light baryon spectrum using Ξ mass as input
- no CKM matrix elements needed and el. mag. corrections are less of an issue (compared to determinations using f_K,...)
- almost all available CLS ensembles plus some additional ensembles are utilized

Other determinations of t_0 on CLS ensembles

Reference	Input	$\sqrt{t_{0,\text{phys}}}$ in fm
[ALPHA 26]	$f_{\pi K}$	0.1440(6)(4)
[Hudspith *et al.* 24]	Ω	0.14480(32)(6)
[Strassberger *et al.* 2021]	$f_{\pi K}$	0.1443(7)(13)
[Bruno *et al.* 2016]	$f_{\pi K}$	0.1467(16)

see also Mainz 2026 $\rightarrow$ PhD thesis by Segner using light baryon spectrum

Note: only subsets of available CLS ensembles have been used in these calculations

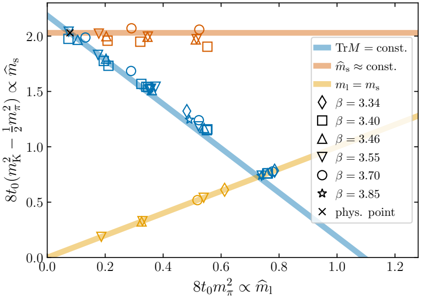
Simulation setup and extrapolation strategy

Simulation details of CLS ensembles

← many thanks to our CLS colleagues!

Utilize $N_f = 2 + 1$ ensembles obtained within the CLS (Coordinated Lattice Simulations) effort

- $N_f = 2 + 1$ non-perturbatively improved Wilson action (clover), tree-level improved Symanzik gauge action
- implementing open boundaries (at small lattice spacings) → avoid topological freezing



⇒ very good coverage of the quark mass plane

⇒ controlled chiral extrapolation towards the physical point

Simulation strategy: quark mass plane

simulations along 3 different trajectories:

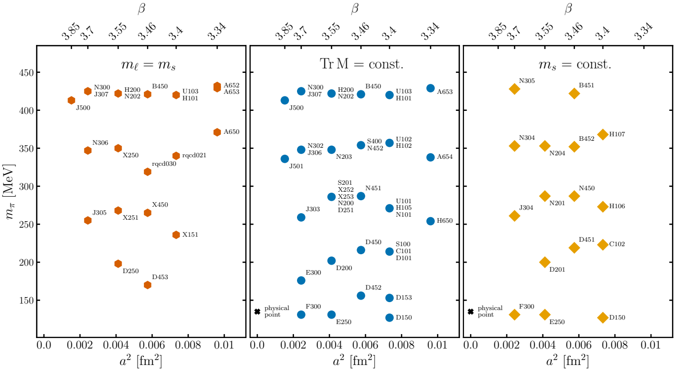
- $\mathrm{Tr}M = \text{const.}$ (sum of bare quark masses kept const.)
- $\widehat{m}_s \approx \text{const.}$ (approx. const. strange quark mass)
- $m_l = m_s$ (equal light and strange quark mass)

Light baryon masses: extrapolation strategy

- consider full mass-dependence of baryon octet/decuplet
- rescale quantities with t_0 ($m_B \to \sqrt{t_0}m_B,\ m_\pi \to \ldots$)
- parametrize quark mass + a dependence
- simultaneous extrapolation to continuum/physical point

  → extract physical t_0 using Ξ mass as input

Ensemble overview (2026)



- 6 different lattice spacings ($a \approx 0.1 - 0.04$ fm)
- 3 ensembles at/close to the physical point
- dedicated ensembles to study possible finite volume effects
- 69 ensembles in total with large statistics (1000-20000 MDUs)

What is new in 2026 compared to 2022?

New simulation points

- 2 new (near) physical ensembles:
  $\rightarrow$ D153: $a \approx 0.085$ fm, $m_\pi \approx 153$ MeV, 128×64^3
  $\rightarrow$ F300: $a \approx 0.049$ fm, $m_\pi \approx 136$ MeV, 256×128^3
- F300, J305/6/7, N306 ($a \approx 0.049$ fm$/\beta = 3.7$), D250/1, X252/3 ($a \approx 0.064$ fm$/\beta = 3.55$) D452, N451/2 ($a \approx 0.075$ fm$/\beta = 3.46$), D153, X151 ($a \approx 0.085$ fm$/\beta = 3.4$), H650 ($a \approx 0.098$ fm$/\beta = 3.34$)

Additional measurements/configurations/ensembles

- E250, E300, J500, J501, C102,...
- dedicated new finite volume ensembles (X252, X253, D251,...)
- longer temporal lattice extent: N302 $\rightarrow$ J306, N300 $\rightarrow$ J307, N401 $\rightarrow$ N451, rqcd017 $\rightarrow$ X151

isoQCD Physical Point Definition

- RQCD 22: $m_\pi = 134.8(3)$ MeV, $m_K = 494.2(3)$ MeV, $m_\Xi = 1316.9(3)$ MeV
- RQCD 26: $m_\pi = 135$ MeV and $m_K = 494.6$ MeV (Edinburgh Cons.), $m_\Xi = 1317.7$ MeV
  $\rightarrow$ will also provide derivatives of observables in order to be able to change the scheme afterwards (without re-fitting)

Extraction of the masses

Correlators

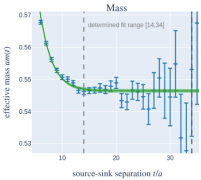
- point-to-all propagators with optimized Gaussian smearing on 3D-APE-smoothed links
  $\rightarrow$ 3-64 sources per configuration
- average correlators over all sources per configuration
  $\rightarrow$ ensembles with open boundary condition: only average on time slices in bulk region
- perform uncorrelated (plus correlated) fit to correlator
- autocorrelations are taken into account by means of a bin size analysis

Determination of the ground-state fit range

- range of ground-state dominance is explored by means of a two-state fit

$\rightarrow$ time slices where excited state contribution is negligible determines ground-state fit range

$\rightarrow$ correlated and uncorrelated fits basically give the same ground-state fit ranges



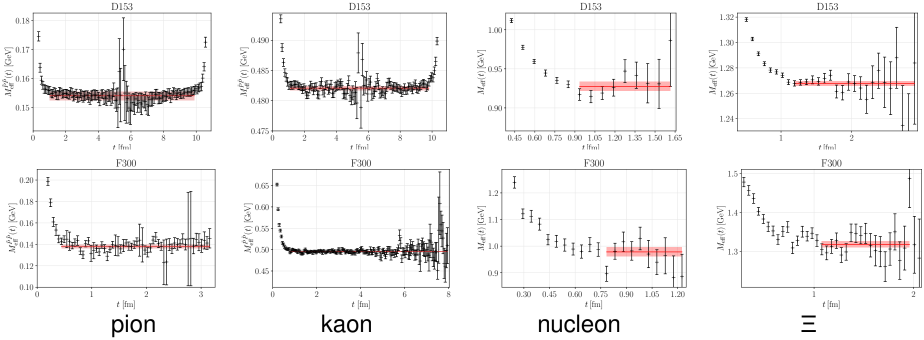
Ground-state fit

- perform one-state fit with the above determined (fixed) fit range

$\rightarrow$ no need to consider quality of fit by means of a corr. fit! (corr. χ^2 is monitored nevertheless)

- masses extracted from uncorrelated fits

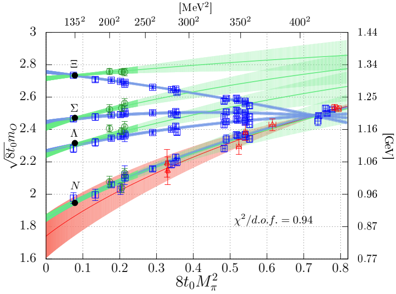
Effective Masses: D153 and F300



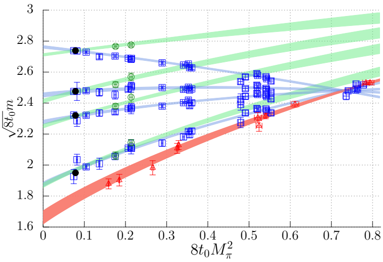
Effective masses for D153 (top row) and F300 (bottom row)

- D153: $a \approx 0.085$ fm, $m_\pi \approx 153$ MeV, 128×64^3; F300: $a \approx 0.049$ fm, $m_\pi \approx 136$ MeV, 256×128^3
- note StN-problem for kaon(!) for F300
- note peaks for F300 for pion $\rightarrow$ artefact for eff. mass due to specific source av. for open b.c.

Chiral and continuum extrapolation baryon spectrum



2022 data [RQCD 22]



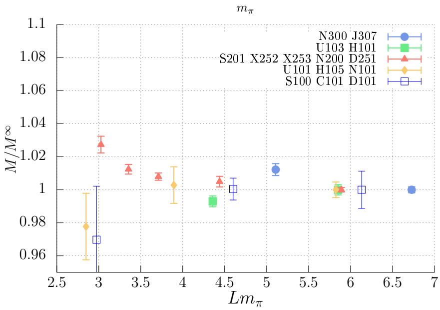
2026 data (PRELIMINARY)

"Best fit result" for baryon octet

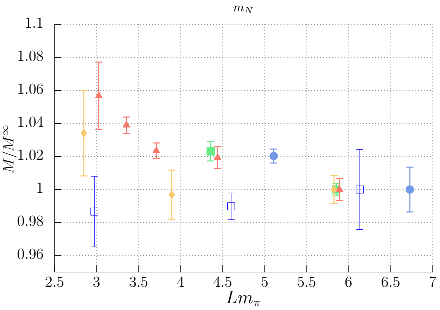
($m = N, \Lambda, \Sigma, \Xi$)

- apply the same fit form from best fit of 2022 to new data basis of 2026
- masses rescaled with $\sqrt{8t_0}$ and perform combined chiral and continuum fit
- curves show the continuum part utilizing NNLO baryon chiral perturbation theory (BChPT)
  $\rightarrow$ other fits have been/will be investigated including Taylor expansion with group theoretical constraints
- data points are projected accordingly based on the fit, black points are physical values
- all three trajectories are utilized: $\widehat{m}_s \approx$ const. (green), $\mathrm{Tr}M =$ const. (blue), $m_s = m_\ell$ (red)

Systematics: Finite volume effects (FSE)



pion mass (2026 data, PRELIMINARY)

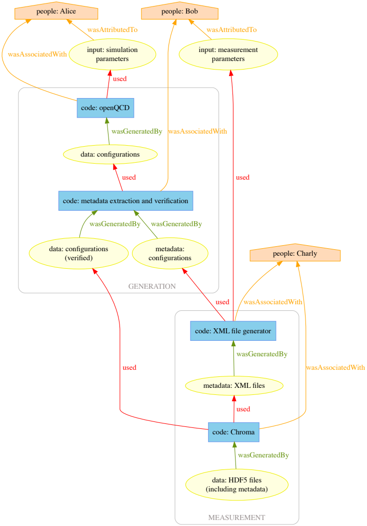


nucleon mass (2026 data, PRELIMINARY)

Finite volume effects: dedicated ensembles with small/large volumes

- 2022: small FSE for $m_\pi L > 4$ $\rightarrow$ exclude ensembles with $m_\pi L < 3.5$ and/or $L < 2.3$ fm
  $\rightarrow$ given the large number of ensembles: small FSE add up!
  $\rightarrow$ include FSE for baryons for all ens.: χ^2/dof reduced significantly (without extra parameters!)
  $\rightarrow$ FSE for mesons are also included in the analysis but they are negligible
- 2026: more data available in general, see plots above
  $\rightarrow$ in particular, (new) ensembles S201/X252/. . . (red points) allow for more detailed checks

Data management: Lattice QCD workflow provenance



Data management

$\rightarrow$ large amount of data needs to be organized

Generic Lattice QCD workflow with provenance

Including provenance allows us to answer provenance-related research questions such as

- **Q1** Which datasets are affected by an error or bug?
- **Q2** How are datasets affected by modifying a parameter?
- **Q3** Who was involved in generating the data?
- **Q4** Which codes and experts are needed to repeat a workflow?
- **Q5** Which data/parameters are needed to (re-) produce a result?

- develop suitable W3C PROV extensions
- define comprehensive standards for data management, incl. provenance metadata
- for more details see [T. Auge *et. al.* (2025)]

Outlook and Conclusions

Current status → work in progress

- update on the value of $t_{0,\mathrm{phys}}$ from RQCD 2022
- improved data basis 2026
- with the same analysis strategy first fits indicate that error goes down
- preliminary value for physical t_0 from new analysis is very similar to 2022 value

Outlook

- add remaining existing ensembles/additional measurements, improve statistics (F300/D153)
- consider further improvements on the extraction of the masses
- investigate finite volume effects in more detail using the new dedicated ensembles
- perform full AIC average: check for adjustments for cuts, higher order fit forms, etc.

→ goal: improve precision on $\sqrt{t_{0,\mathrm{phys}}}$ to below 0.3%

→ RQCD plans: investigate isospin-breaking corrections → see also Mainz effort [Segner 2023]

- update also LECs and sigma terms within this work, provide result for w_0