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TOWARD LATTICE CALCULATION OF PION GLUON GPD

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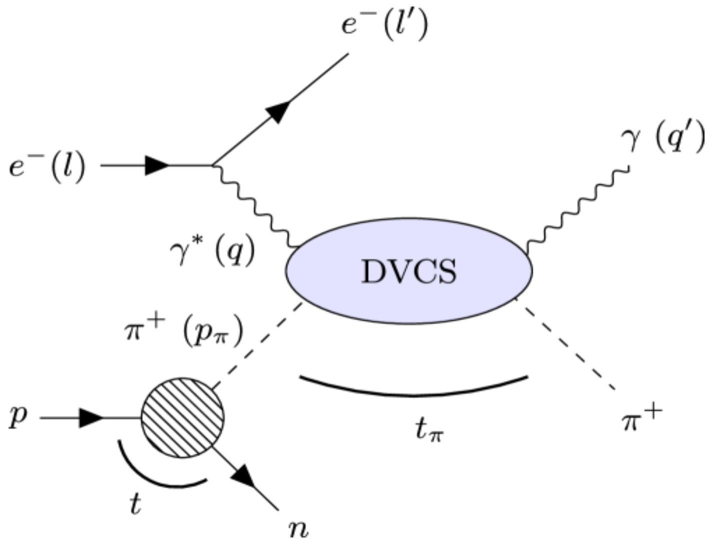
{Lattice} · {7/28/2026}

GLUON GENERALIZED PARTON DISTRIBUTION OF PION

Sullivan-process schematic; see J. M. Morgado Chávez et al., PRL 128:202501 (2022)



Light-cone definition of pion unpolarized gluon GPD

$$H_\pi^g(x,\xi,t) = \frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \left\langle \pi^+(p') \left| G^{+\mu}\left(-\frac{z}{2}\right) G_\mu^{\ +}\left(\frac{z}{2}\right) \right| \pi^+(p) \right\rangle \Big|_{z^+=0,\ \boldsymbol{z}_\perp=0}.$$

This light-cone-separated correlator cannot be calculated on Euclidean lattice

$$t^2 - \boldsymbol{x}^2 = 0 \xrightarrow{t=-i\tau} \tau^2 + \boldsymbol{x}^2 = 0$$

Experimental Challenge: No free pion, measurement relies on Sullivan process

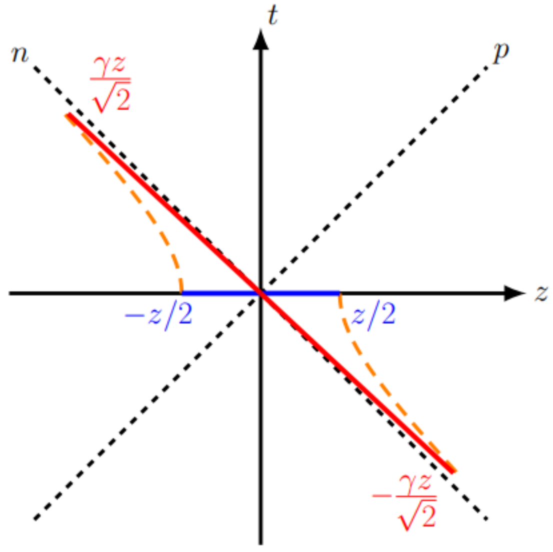
$$P = \frac{p'+p}{2}, \qquad \Delta = p' - p, \qquad \xi = -\frac{\Delta^+}{2P^+}, \qquad t = \Delta^2.$$

- Reduce to PDF in forward limit
- At zero skewness Fourier transform gives the impact parameter distribution

$$\mathrm{H}_\pi^{\mathrm{g}}(\mathrm{x},0,0) = \mathrm{x}\,\mathrm{g}_\pi(\mathrm{x}),$$

$$x g_\pi(x,\boldsymbol{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}\, H_\pi^g(x,0,-\boldsymbol{\Delta}_\perp^2).$$

CORRELATOR SEPARATED BY SPACE-LIKE INTERVAL

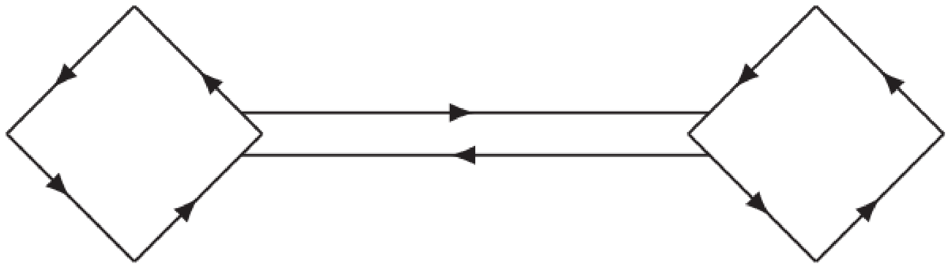


For unpolarized gluon PDF it has been proposed to calculate

X. Ji, Parton *Physics on a Euclidean Lattice*. *Phys. Rev. Lett.*, 110:262002, 2013]

$$M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p | \mathrm{F}_{\mu\alpha}(z)\, W[z,0]\, \mathrm{F}_{\lambda\beta}(0) | p \rangle$$

[X. Ji, SCPMA 57:1407 (2014)]



$$\times$$

$$C_{2\mathrm{pt}}(t,\mathbf{p}) = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}} \langle 0 | \chi_N(\mathbf{x},t)\, \overline{\chi}_N(\mathbf{0},0) | 0 \rangle$$

$$\langle 0 | O(G_{\mu\alpha}, G_{\lambda\beta}, z) | 0 \rangle = \langle 0 | G_{\mu\alpha}(z)\, U(z,0)\, G_{\lambda\beta}(0)\, U(0,z) | 0 \rangle$$

— Independent fluctuations

(correlation of gluon operator and 2pt)

LORENTZ-COVARIANT DECOMPOSITION

Unpolarized gluon PDF

$$\mathcal{M}_{pp}(\nu,0) = -\frac{1}{2}\int_{-1}^{1} dx\ e^{-ix\nu}\, xg(x)$$

$$M_{ti;it} + M_{ji;ij} = 2p_0^2\, \mathcal{M}_{pp}.$$

[I. Balitsky et al., Phys. Lett. B 808:135621 (2020)]

For nucleon gluon GPDs the projection is more complicated. But all the eight GPDs have been worked out

Unpolarized gluon GPD

$$H_{g/h}(x,\xi,t) = \int_{-\infty}^{\infty} d\omega\ e^{i\omega x}\, \mathfrak{F}(\omega,\xi,t)$$

$$(\mathcal{P}[\mathfrak{F}], M) = \frac{z_3^2\xi^2}{\eta^2}\left(M_{0i;0i} + M_{ij;ij}\right) + \frac{4\omega\xi^2(\xi-\eta)z_3}{\eta^2\Delta_1}\left(M_{02;12} + M_{12;02}\right)$$

Differ by the choice of basis

$$(\mathcal{P}'[\mathfrak{F}], M) = \frac{z_3^2\xi^2}{\eta^2} M_{0i;0i} + \frac{\xi^2[16\omega^2\xi(\xi-\eta) + z_3^2(4\xi^2P^2 - t)]}{2\eta^2\Delta_1^2} M_{ij;ij} + \frac{4\omega\xi^2(\xi-\eta)z_3}{\eta^2\Delta_1}\left(M_{02;12} + M_{12;02}\right)$$

[Jacob Schoenleber, Raza Sufian et al., PRD 111:094510 (2025)]

LATTICE SET-UP

- Wilson-clover valence quarks on MILC-generated lattice configurations (HISQ sea quarks)
- Lattice spacing $\mathrm{a} = 0.09\ \mathrm{fm}$
- pion mass $m_\pi = 310\mathrm{MeV}$
- 600 cfgs 1024 src positions per cfg

$$F_{\mu\nu}^{\mathrm{clov}}(x) = \frac{1}{8ig_0a^2}\left[Q_{\mu\nu}(x) - Q_{\mu\nu}^{\dagger}(x) - \frac{1}{3}\mathrm{Tr}\left(Q_{\mu\nu}(x) - Q_{\mu\nu}^{\dagger}(x)\right)\mathbf{1}\right]$$

$$C_\pi^{\mathrm{2pt}}(t_{\mathrm{sep}}, \mathbf{p}) = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x_{src}})}\langle 0|\mathcal{O}_\pi(\mathbf{x}, t_{\mathrm{src}} + t_{\mathrm{sep}})\mathcal{O}_\pi^{\dagger}(\mathbf{x_{src}}, t_{\mathrm{src}})|0\rangle$$

Pion 2pts and FF correlators

- momentum smearing smeared-smeared propagator

$$\rho = 3.25$$
$$p = 0.6p_{max}$$

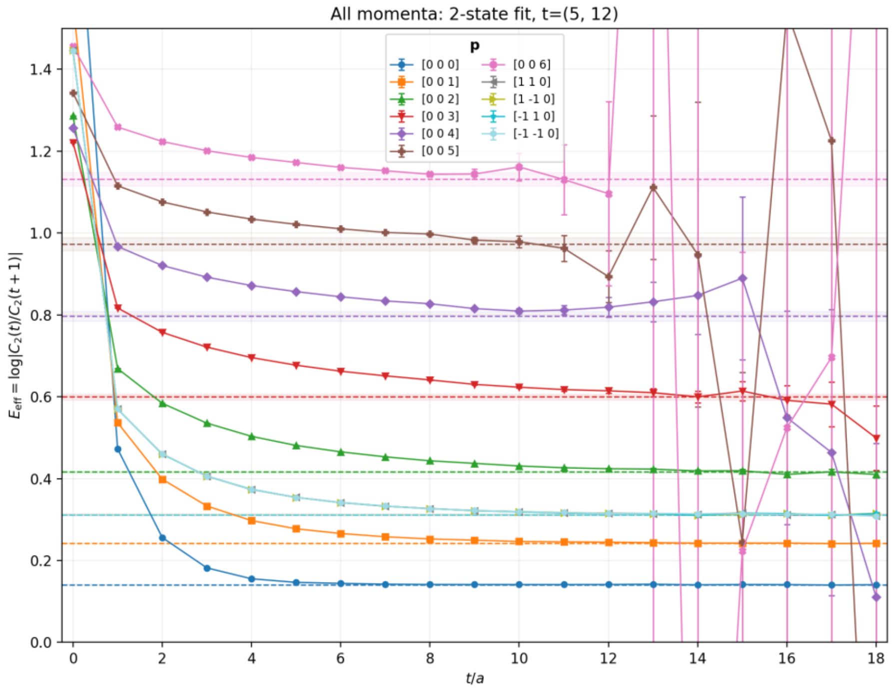
$\mathrm{p} = 0,1,2,3,4,5,6$ up to about 2.58GeV

- Kinematically enhanced interpolation operator $\gamma_4\gamma_5$
- Plaquette correlators calculated with gradient flow time up to $4a^2$

$$\mathcal{O}_{\mu\nu;\rho\sigma}(\mathbf{q}, \tau; z) = \sum_{\mathbf{x_{FF}}} e^{+i\mathbf{q}\cdot\mathbf{x_{FF}}}\,\mathrm{Tr}\left[\begin{array}{c} F_{\mu\nu}(\mathbf{x_{FF}} + z\hat{\mathbf{z}}, \tau) \\ W(\mathbf{x_{FF}} + z\hat{\mathbf{z}}, \mathbf{x_{FF}}; \tau)F_{\rho\sigma}(\mathbf{x_{FF}}, \tau)W^{\dagger}(\mathbf{x_{FF}} + z\hat{\mathbf{z}}, \mathbf{x_{FF}}; \tau)\end{array}\right]$$

2PT AND PLAQUETTE CORRELATOR

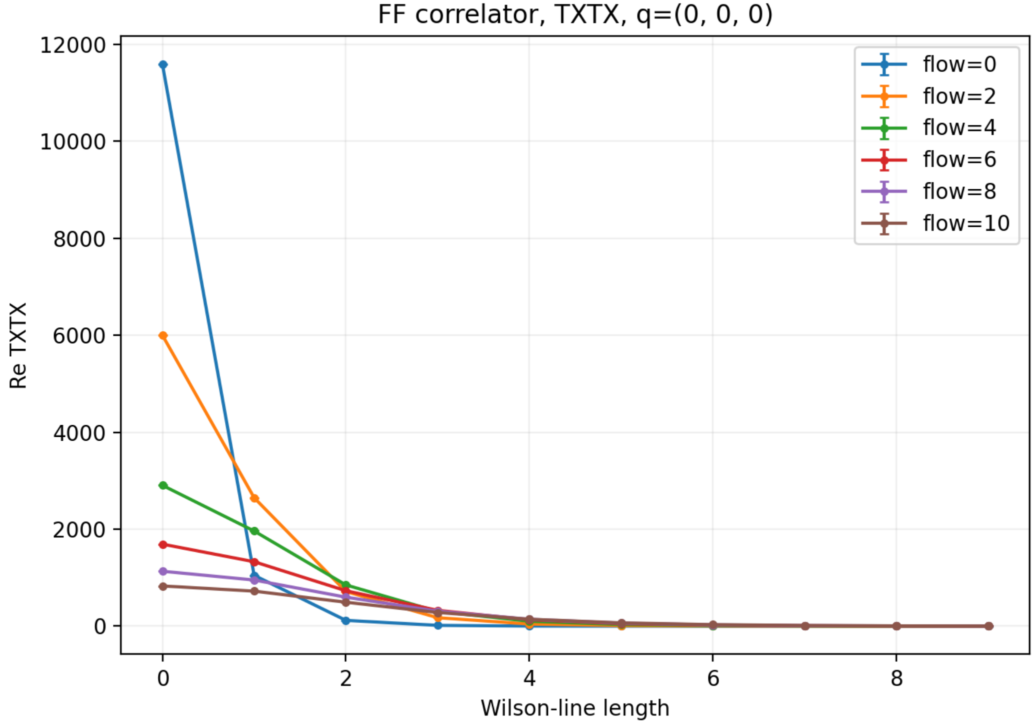
- Pion mas 310 Mev a=0.09fm 615k samples



- 2-state fit compared to effective energy

$$E_{\text{eff}} = \frac{1}{a}\log\left(\frac{C_{\pi\pi}(t)}{C_{\pi\pi}(t+a)}\right)$$

- Flow time 0-1 a^2



- FF correlator vs z, errorbar to small to be seen

$$\sum_z \langle 0| F_{txtx}(z)\, W(z,0)\, F_{txtx}(0)\, W(0,z) |0\rangle$$

3PT COMBINATION AND VACUUM SUBTRACTION

$$C^{3\mathrm{pt}}_{\mathrm{connected}}(t,\tau) = \frac{1}{N_{t\mathrm{src}}}\sum_{t_{\mathrm{src}}}\left[\frac{1}{N^{\mathrm{spatial}}_{\mathrm{src}}}\sum_{\mathbf{x}_{\mathrm{src}}} 2\mathrm{pt}(t;\mathrm{x}_{\mathrm{src}},t_{\mathrm{src}})e^{-\mathrm{iqxsrc}}\mathrm{FF}(\tau+t_{\mathrm{src}})\right]$$

$$C^{3\mathrm{pt}}_{\mathrm{subtraction}}(t,\tau) = \left[\frac{1}{N_{\mathrm{src}}}\sum_{t_{\mathrm{src}}}\sum_{\mathbf{x}_{\mathrm{src}}} 2\mathrm{pt}(t;\mathrm{x}_{\mathrm{src}},t_{\mathrm{src}})\right]\left[\frac{1}{N_{t\mathrm{src}}}\sum_{x_{src}}\sum_{t_{\mathrm{src}}}\mathrm{FF}(\tau+t_{\mathrm{src}})\,e^{-\mathrm{iqxsrc}}\right]$$

$$C^{(c)}_{3\mathrm{pt}}(t,\tau) = C^{(c)}_{\mathrm{connected}}(t,\tau) - C^{(c)}_{\mathrm{subtraction}}(t,\tau)$$

$$C^{(c)}_{3\mathrm{pt}}(t,\tau;\mathrm{P}_{\mathrm{sink}},\mathrm{q}) = \sum_{\mathbf{x}_{\mathrm{sink}},\mathbf{x}_{\mathrm{FF}}} e^{-i\mathrm{p}_{\mathrm{sink}}\cdot(\mathbf{x}_{\mathrm{sink}}-\mathbf{x}_{\mathrm{src}})}e^{+i\mathrm{q}\cdot(\mathbf{x}_{\mathrm{FF}}-\mathbf{x}_{\mathrm{src}})}C^{(c)}_{3\mathrm{pt}}(t,\tau;\mathrm{x}_{\mathrm{sink}},\mathrm{x}_{\mathrm{FF}})$$

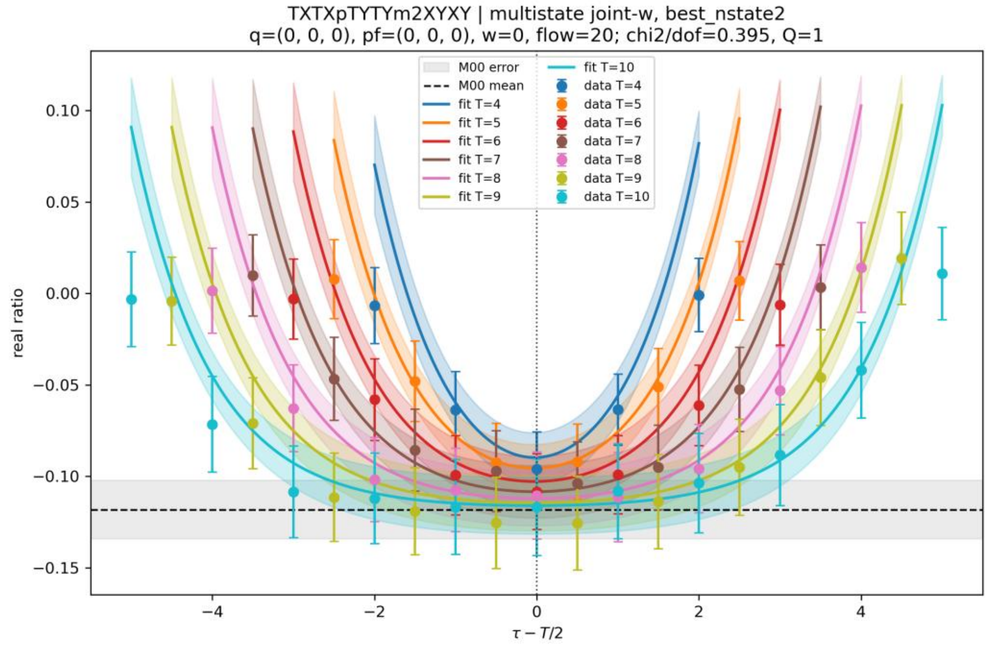
psink=q+psource

TWO-STATE FIT OF BARE MATRIX ELEMENT

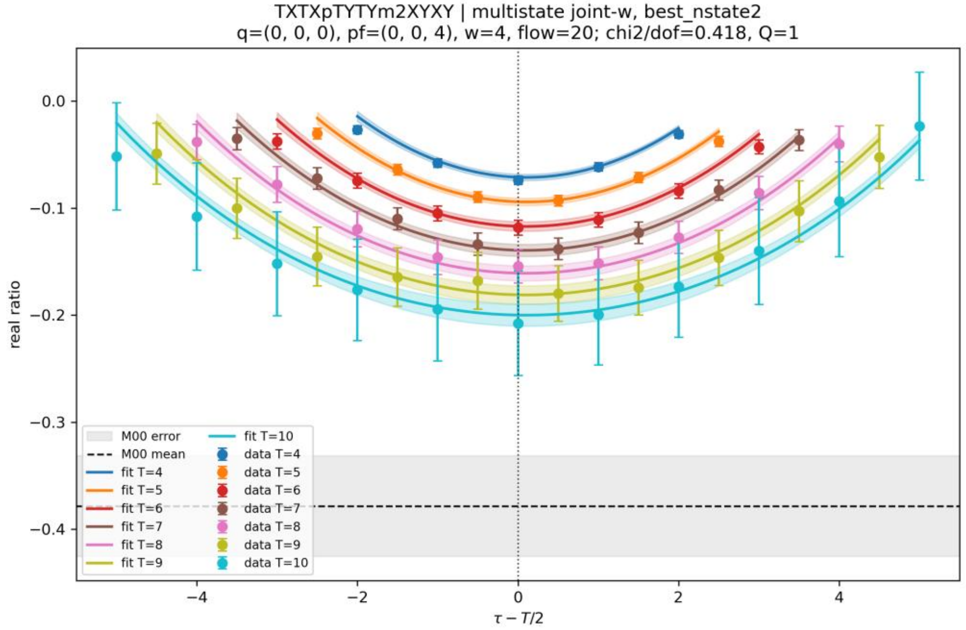
Total 6 parameters for off-forward

$$R(t_{\mathrm{sep}}, t_{\mathrm{ins}}; \mathbf{p_f}, \mathbf{p_i}) = \frac{C_{3\mathrm{pt}}(t_{\mathrm{sep}}, t_{\mathrm{ins}}; \mathbf{p_f}, \mathbf{p_i})}{C_{2\mathrm{pt}}(t_{\mathrm{sep}}, \mathbf{p_f})} \sqrt{\frac{C_{2\mathrm{pt}}(t_{\mathrm{sep}} - t_{\mathrm{ins}}, \mathbf{p_i}) C_{2\mathrm{pt}}(t_{\mathrm{ins}}, \mathbf{p_f}) C_{2\mathrm{pt}}(t_{\mathrm{sep}}, \mathbf{p_f})}{C_{2\mathrm{pt}}(t_{\mathrm{sep}} - t_{\mathrm{ins}}, \mathbf{p_f}) C_{2\mathrm{pt}}(t_{\mathrm{ins}}, \mathbf{p_i}) C_{2\mathrm{pt}}(t_{\mathrm{sep}}, \mathbf{p_i})}}$$

$$R(t_{\mathrm{sep}}, t_{\mathrm{ins}}) = M_{00} + A_i e^{-\Delta E_i t_{\mathrm{ins}}} + A_f e^{-\Delta E_f (t_{\mathrm{sep}} - t_{\mathrm{ins}})} + A_{if} e^{-\Delta E_i t_{\mathrm{ins}} - \Delta E_f (t_{\mathrm{sep}} - t_{\mathrm{ins}})}$$



- 2-state fit for p=0 q=0 z=0, flow=$2a^2$



- 2-state fit for p=4 q=0 z=4, flow=$2a^2$

RENORMALIZATION OF BARE MATRIX ELEMENTS

The nonlocal operator $O_{\mu\nu;\rho\sigma}(z) = F_{\mu\nu}(z)\, W(z,0)\, F_{\rho\sigma}(0)\, W^{\dagger}(z,0)$ suffers from linear and logarithmic divergence

X. Ji, J.-H. Zhang, and Y. Zhao, PRL 120:112001 (2018)

$$O_{\mathrm{g}}(z)_R = Z_O^{-1} e^{\delta \bar{m} z} O_{\mathrm{g}}(z)$$

Green, K. Jansen, and F. Steffens, PRL 121:022004 (2018)

Z_O^{-1} removes the divergence from the vertex diagrams

T. Ishikawa, Y.-Q. Ma, J.-W. Qiu, and S. Yoshida, PRD 96:094019 (2017)

$e^{\delta \bar{m} z}$ removes the divergence from the Wilson line self energy

To avoid infrared effects, we will not use ratio scheme at large distance

A possible candidate is the hybrid-ratio method which can be applied on one ensemble

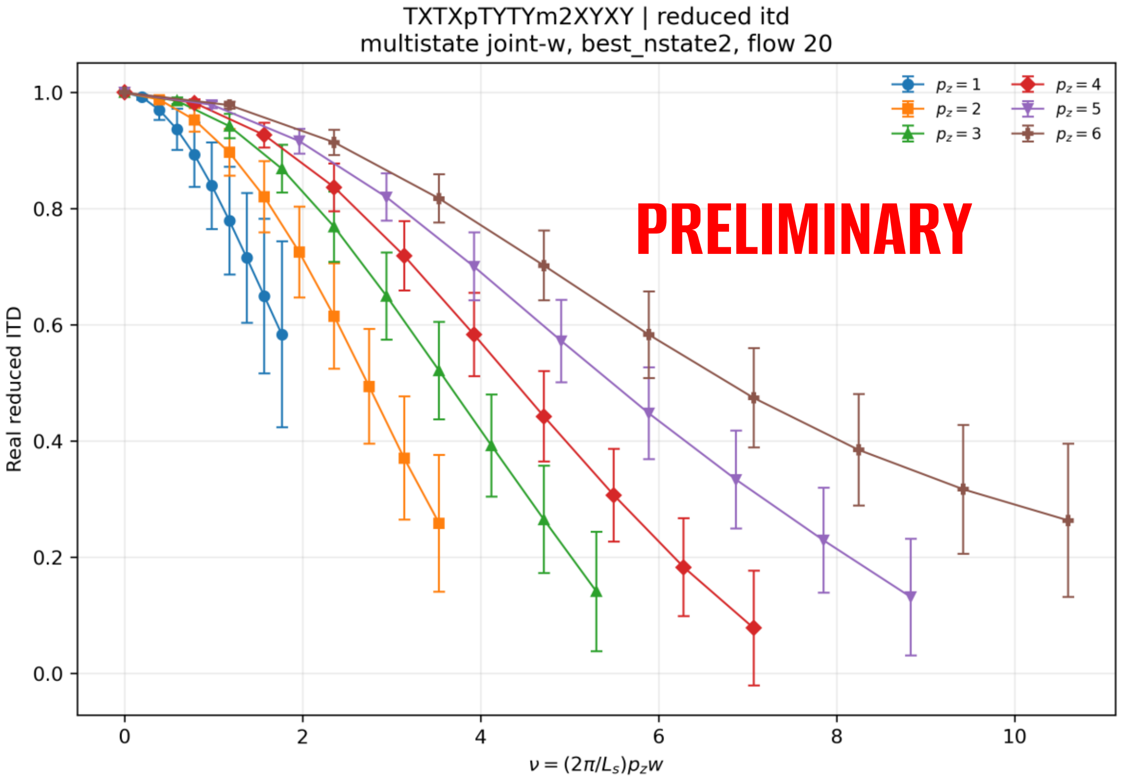
$$h^{\mathrm{R}}(z, P_z, Q^2, \xi) = \begin{cases} \frac{h^{\mathrm{B}}(z,P_z,Q^2,\xi)}{h^{\mathrm{B}}_{\pi}(z,P_z=0)} & z < z_s \\ e^{(\delta m + m_0)(z - z_s)} \frac{h^{\mathrm{B}}(z,P_z,Q^2,\xi)}{h^{\mathrm{B}}_{\pi}(z_s,P_z=0)} & z \geqslant z_s \end{cases}$$

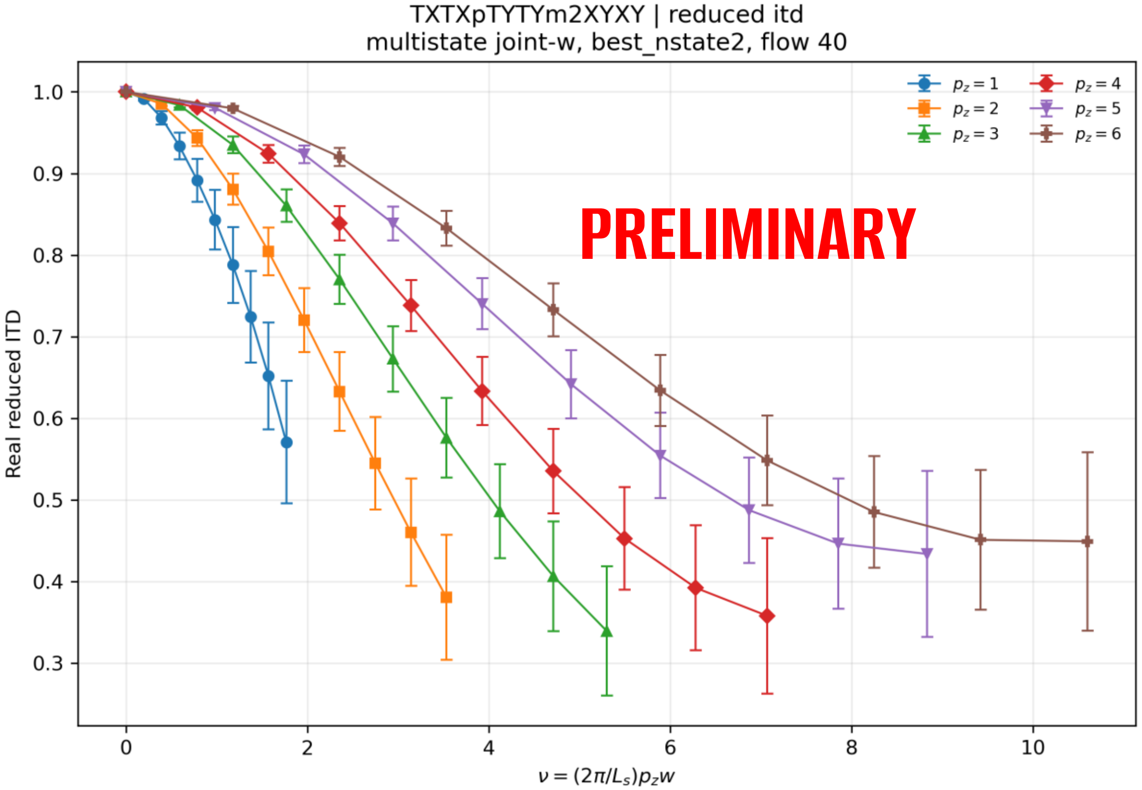
X. Ji et al., Nucl. Phys. B 964:115311 (2021)

δm determined from zero-momentum matrix element
m_0 determined from fitting

$$C_{0,\mathrm{NLO}}(z,\mu) = 1 + \frac{\alpha_s(\mu) C_A}{4\pi}\left[\frac{5}{3}\ln\left(\frac{z^2\mu^2}{4e^{-2\gamma_E}}\right) + 3\right].$$

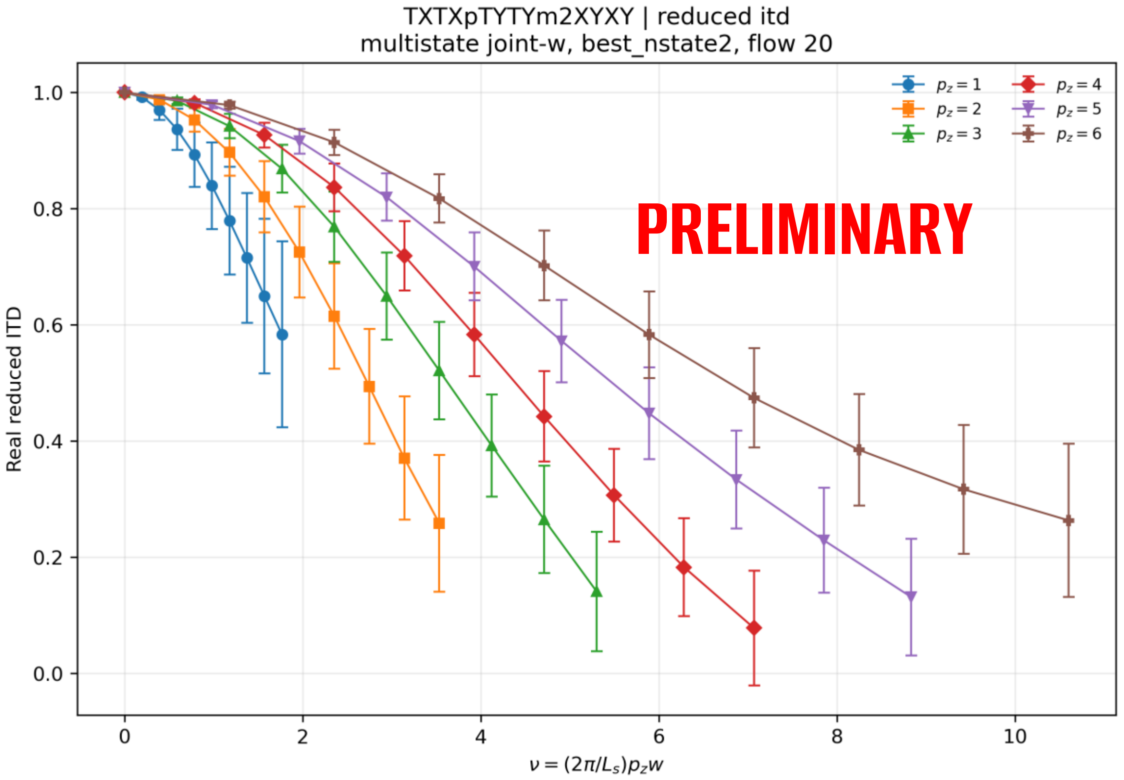
M(P,Z)/M(0,Z) NORMALIZED BY Z=0





- Ioffe time distribution at $\tau = 2a^2$
- Ioffe time distribution at $\tau = 4a^2$

M(P,Z)/M(0,Z) NORMALIZED BY Z=0



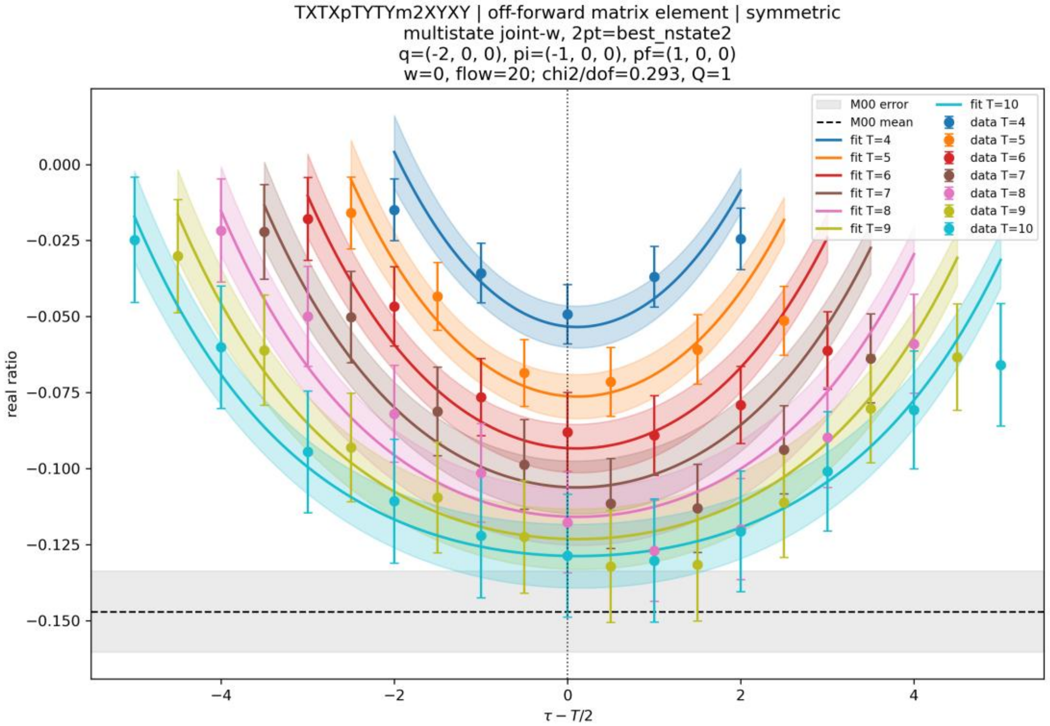
Pz dependence – evidence of higher-twist effects? Similar behavior seen in [arXiv:2510.26425] [arXiv:2409.02750]

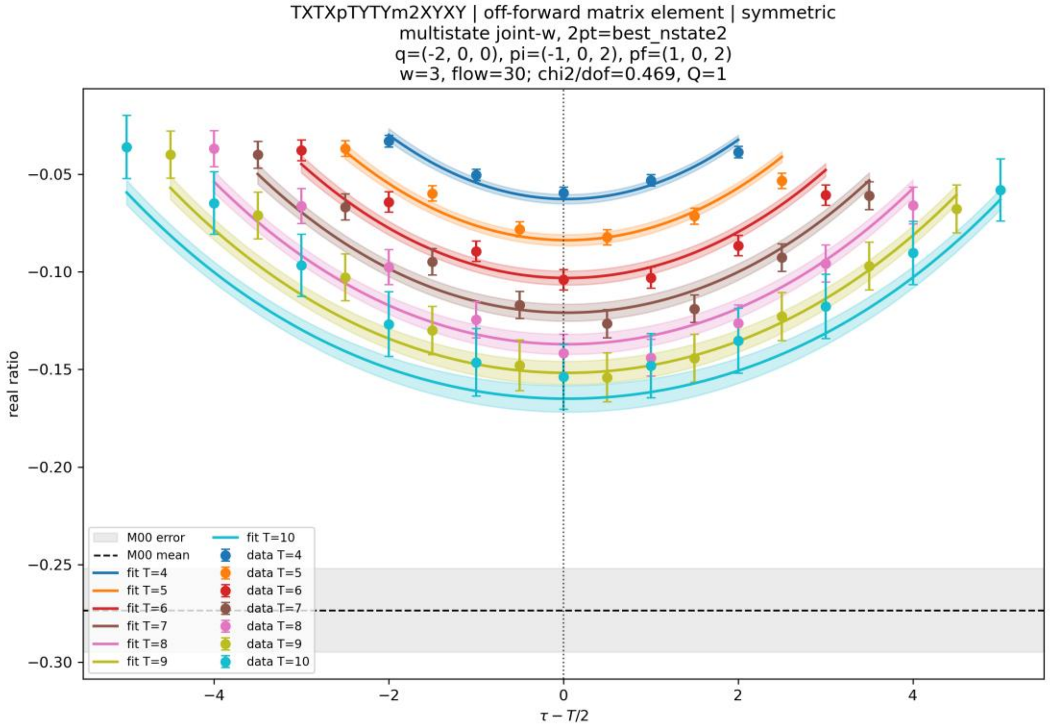
GOING FORWARD:

- INCREASE STATISTICS
- BETTER EXCITED STATE FITS
- PARAMETERIZE HIGHER TWIST EFFECTS (Z^2- DEPENDENCE)
- INVESTIGATE DIFFERENT RENORMALIZATION

- Ioffe time distribution at $\tau = 2a^2$

OFF-FORWARD MATRIX ELEMENT





- Symmetric frame q=(-2,0,0) pz=0 flow=$2a^2$
- Symmetric frame q=(-2,0,0) pz=2 flow=$3a^2$

OUTLOOK

- With more statistics (15% of planned statistics for now), the fit will be much better controlled.
- New technologies for large Ioffe time and z extrapolation could be applied.

  Generative ML for extracting PDF:

  T. A. Chowdhury, T. Izubuchi, M. Kamruzzaman, N. Karthik, T. Khan, T. Liu, A. Paul, J. Schoenleber, and R. S. Sufian, Phys. Rev. D 111, 074509 (2025).

  Large-z model from first principle:

  X. Ji, Y. Liu, and Y. Su, arXiv:2601.12189 (2026)

BACK-UP SLIDE FOR LORENTZ BASIS OF SPIN-0 CASE

$$
\begin{aligned}
\mathcal{T}_{gg} &= g^{\alpha\nu}g^{\beta\mu} - g^{\alpha\mu}g^{\beta\nu}, \\
\mathcal{T}_{PP} &= P^\alpha P^\mu g^{\beta\nu} - P^\beta P^\mu g^{\alpha\nu} - P^\alpha P^\nu g^{\beta\mu} + P^\beta P^\nu g^{\alpha\mu}, \\
\mathcal{T}_{P\Delta} &= P^\alpha \Delta^\mu g^{\beta\nu} - P^\beta \Delta^\mu g^{\alpha\nu} - P^\alpha \Delta^\nu g^{\beta\mu} + P^\beta \Delta^\nu g^{\alpha\mu}, \\
\mathcal{T}_{\Delta P} &= P^\mu \Delta^\alpha g^{\beta\nu} - P^\nu \Delta^\alpha g^{\beta\mu} - P^\mu \Delta^\beta g^{\alpha\nu} + P^\nu \Delta^\beta g^{\alpha\mu}, \\
\mathcal{T}_{\Delta\Delta} &= \Delta^\alpha \Delta^\mu g^{\beta\nu} - \Delta^\beta \Delta^\mu g^{\alpha\nu} - \Delta^\alpha \Delta^\nu g^{\beta\mu} + \Delta^\beta \Delta^\nu g^{\alpha\mu}, \\
\mathcal{T}_{PP\Delta\Delta} &= P^\beta P^\nu \Delta^\alpha \Delta^\mu - P^\alpha P^\nu \Delta^\beta \Delta^\mu - P^\beta P^\mu \Delta^\alpha \Delta^\nu + P^\alpha P^\mu \Delta^\beta \Delta^\nu, \\
\mathcal{T}_{Pz} &= P^\alpha z^\mu g^{\beta\nu} - P^\beta z^\mu g^{\alpha\nu} - P^\alpha z^\nu g^{\beta\mu} + P^\beta z^\nu g^{\alpha\mu}, \\
\mathcal{T}_{\Delta z} &= \Delta^\alpha z^\mu g^{\beta\nu} - \Delta^\alpha z^\nu g^{\beta\mu} - \Delta^\beta z^\mu g^{\alpha\nu} + \Delta^\beta z^\nu g^{\alpha\mu}, \\
\mathcal{T}_{PP\Delta z} &= P^\beta P^\nu \Delta^\alpha z^\mu - P^\beta P^\mu \Delta^\alpha z^\nu - P^\alpha P^\nu \Delta^\beta z^\mu + P^\alpha P^\mu \Delta^\beta z^\nu, \\
\mathcal{T}_{P\Delta\Delta z} &= -P^\beta \Delta^\alpha \Delta^\mu z^\nu + P^\alpha \Delta^\beta \Delta^\mu z^\nu + P^\beta \Delta^\alpha \Delta^\nu z^\mu - P^\alpha \Delta^\beta \Delta^\nu z^\mu, \\
\mathcal{T}_{zP} &= P^\mu z^\alpha g^{\beta\nu} - P^\nu z^\alpha g^{\beta\mu} - P^\mu z^\beta g^{\alpha\nu} + P^\nu z^\beta g^{\alpha\mu}, \\
\mathcal{T}_{z\Delta} &= \Delta^\mu z^\alpha g^{\beta\nu} - \Delta^\mu z^\beta g^{\alpha\nu} - \Delta^\nu z^\alpha g^{\beta\mu} + \Delta^\nu z^\beta g^{\alpha\mu}, \\
\mathcal{T}_{PPz\Delta} &= P^\beta P^\nu \Delta^\mu z^\alpha - P^\alpha P^\nu \Delta^\mu z^\beta - P^\beta P^\mu \Delta^\nu z^\alpha + P^\alpha P^\mu \Delta^\nu z^\beta, \\
\mathcal{T}_{P\Delta z\Delta} &= -P^\nu \Delta^\alpha \Delta^\mu z^\beta + P^\nu \Delta^\beta \Delta^\mu z^\alpha + P^\mu \Delta^\alpha \Delta^\nu z^\beta - P^\mu \Delta^\beta \Delta^\nu z^\alpha, \\
\mathcal{T}_{zz} &= z^\alpha z^\mu g^{\beta\nu} - z^\beta z^\mu g^{\alpha\nu} - z^\alpha z^\nu g^{\beta\mu} + z^\beta z^\nu g^{\alpha\mu}, \\
\mathcal{T}_{PPzz} &= P^\beta P^\nu z^\alpha z^\mu - P^\alpha P^\nu z^\beta z^\mu - P^\beta P^\mu z^\alpha z^\nu + P^\alpha P^\mu z^\beta z^\nu, \\
\mathcal{T}_{P\Delta zz} &= -P^\beta \Delta^\mu z^\alpha z^\nu + P^\alpha \Delta^\mu z^\beta z^\nu + P^\beta \Delta^\nu z^\alpha z^\mu - P^\alpha \Delta^\nu z^\beta z^\mu, \\
\mathcal{T}_{\Delta Pzz} &= -P^\nu \Delta^\alpha z^\beta z^\mu + P^\mu \Delta^\alpha z^\beta z^\nu + P^\nu \Delta^\beta z^\alpha z^\mu - P^\mu \Delta^\beta z^\alpha z^\nu, \\
\mathcal{T}_{\Delta\Delta zz} &= \Delta^\alpha \Delta^\mu z^\beta z^\nu - \Delta^\beta \Delta^\mu z^\alpha z^\nu - \Delta^\alpha \Delta^\nu z^\beta z^\mu + \Delta^\beta \Delta^\nu z^\alpha z^\mu.
\end{aligned}
$$

$$
\begin{aligned}
0 = &-\Delta_1^2\eta^2\omega^2\mathcal{T}_{gg} + \left(4\xi^2\omega^2 - \Delta_1^2 z_3^2\right)\xi^2\mathcal{T}_{PP} + 2\eta\xi^2\omega^2(\mathcal{T}_{P\Delta} + \mathcal{T}_{\Delta P}) + \eta^2\omega^2\mathcal{T}_{\Delta\Delta} - \xi^2 z_3^2\mathcal{T}_{PP\Delta\Delta} \\
&- \xi^2 t\omega\mathcal{T}_{Pz} + 2\eta\xi^2 P^2\omega\mathcal{T}_{\Delta z} - 2\eta\xi^2\omega\mathcal{T}_{PP\Delta z} - \xi^2\omega\mathcal{T}_{P\Delta\Delta z} - \xi^2 t\omega\mathcal{T}_{zP} + 2\eta\xi^2 P^2\omega\mathcal{T}_{z\Delta} \\
&- 2\eta\xi^2\omega\mathcal{T}_{PPz\Delta} - \xi^2\omega\mathcal{T}_{P\Delta z\Delta} - \xi^2 P^2 t\mathcal{T}_{zz} + \xi^2 t\mathcal{T}_{PPzz} + \xi^2 P^2\mathcal{T}_{\Delta\Delta zz},
\end{aligned}
$$

For instance, if we eliminate the $\mathcal{T}_{\Delta\Delta}$ structure, we obtain

$$\mathfrak{F} = -2\omega^2\mathcal{M}_{PP}|_{z^2=0} + 4\xi\omega^2\left(\mathcal{M}_{P\Delta}|_{z^2=0} + \mathcal{M}_{\Delta P}|_{z^2=0}\right) + \omega^2\Delta_1^2\mathcal{M}_{PP\Delta\Delta}|_{z^2=0},$$

whereas if we eliminate the $\mathcal{T}_{gg}$ structure we obtain

$$\mathfrak{F} = -2\omega^2\mathcal{N}_{PP}|_{z^2=0} + 4\xi\omega^2\left(\mathcal{N}_{P\Delta}|_{z^2=0} + \mathcal{N}_{\Delta P}|_{z^2=0}\right) + \omega^2\Delta_1^2\mathcal{N}_{PP\Delta\Delta}|_{z^2=0} - 8\xi^2\omega^2\mathcal{N}_{\Delta\Delta}|_{z^2=0}.$$

$$(\mathcal{P}[\mathfrak{F}], M) = \frac{z_3^2\xi^2}{\eta^2}(M_{0i;0i} + M_{ij;ij}) + \frac{4\omega\xi^2(\xi-\eta)z_3}{\eta^2\Delta_1}(M_{02;12} + M_{12;02})$$

and

$$(\mathcal{P}'[\mathfrak{F}], M) = \frac{z_3^2\xi^2}{\eta^2}M_{0i;0i} + \frac{\xi^2[16\omega^2\xi(\xi-\eta) + z_3^2(4\xi^2P^2 - t)]}{2\eta^2\Delta_1^2}M_{ij;ij} + \frac{4\omega\xi^2(\xi-\eta)z_3}{\eta^2\Delta_1}(M_{02;12} + M_{12;02}).$$

[Jacob Schoenleber, Raza Sufian et al., PRD 111:094510 (2025)]