QCD + QED thermodynamics from the lattice

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Why QED?

- QED interactions can usually be treated *perturbatively*

- Lattice QCD + QED[1] important for $T = 0$ precision physics:
  - Muon g-2
  - Hadron mass splittings
  - CKM matrix elements
  - ...

  More details in Xin-Yu Tuo's plenary

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- What about $T \neq 0$ phase transitions? $\rightarrow$ *Non-perturbative* phenomena
  See Jana Günter's plenary
- QCD + QED possesses **exact center symmetry**

  Preliminary hints of deconfinement phase transition!

  Scior, Edwards, von Smekal, Lattice '10,11,13

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Center symmetry

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  - **Pure SU(3) theory**: center symmetric, P order parameter, 1^{st}-order transition
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- Example: QCD $N_f = 3$ with imaginary isospin chemical potential:

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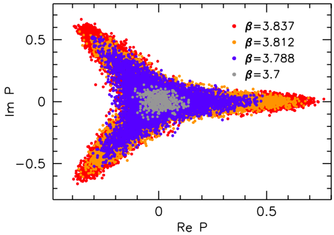
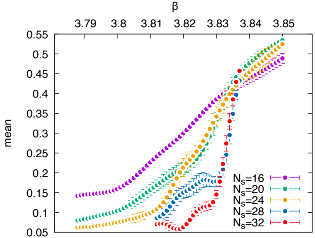
- 1^{st}-order deconfinement phase transition reappears!

Endrődi, Moore, Pieczynski, Sciarra, '26





Center symmetry

- QCD + QED also posses center symmetry:

$$\not{D}^{f}_{nl} = \frac{1}{2} \sum_{\mu} \gamma_\mu \left[U_\mu(n)\, u_{f,\mu}(n)\, \delta_{l,n+\hat{\mu}} - U^\dagger_\mu(n-\hat{\mu})\, u^\dagger_{f,\mu}(n-\hat{\mu})\, \delta_{l,n-\hat{\mu}} \right]$$

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- U(1) rotation cancel center transformation $z = e^{i2\pi/3}$ at $n = (\vec{n}, N_t - 1)$:

$$U_4(n) \to \boxed{z}\, U_4(n)$$

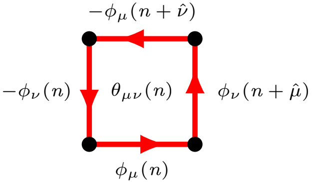
$$\phi_4(n) \to \phi_4(n) + 2\pi \Rightarrow \begin{cases} u_{u,4}(n) \to z^2\, u_{u,4}(n) = \boxed{z^{-1}}\, u_{u,4}(n) \\ u_{d/s,4}(n) \to \boxed{z^{-1}}\, u_{d/s,4}(n) \end{cases}$$

- Determinant is now Z_3 invariant

Compact QED

- **Compact QED** version plaquette action ($\beta_{\text{em}} = 1/e^2$)

$$S^{C}_{\mathrm{U}(1)} = \beta_{\text{em}} \sum_{\substack{n,\mu,\nu,\\ \mu<\nu}} \Big(1 - \cos\left[\theta_{\mu\nu}(n)\right]\Big)$$



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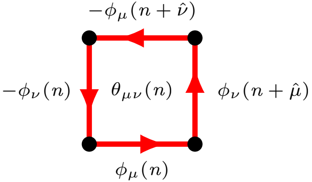

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- For U(1) is also possible to use **non-compact** action

$$S^{NC}_{\mathrm{U}(1)} = \frac{\beta_{\rm em}}{4} \sum_{n} \sum_{\mu,\nu} \left[A_\nu(n+\hat{\mu}) - A_\nu(n) - A_\mu(n+\hat{\nu}) + A_\mu(n)\right]^2$$

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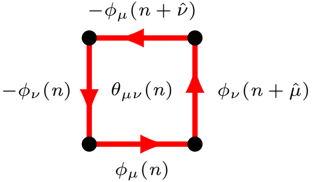

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- Different Polyakov loops influence the determinant:

$$P = \frac{1}{V} \sum_{\vec{n}} \mathrm{Tr} \prod_{n_t=0}^{N_t-1} U_4(\vec{n}, n_t) \qquad p_f = \frac{1}{V} \sum_{\vec{n}} \prod_{n_t=0}^{N_t-1} u_{f,4}(\vec{n}, n_t)$$

Lattice setup

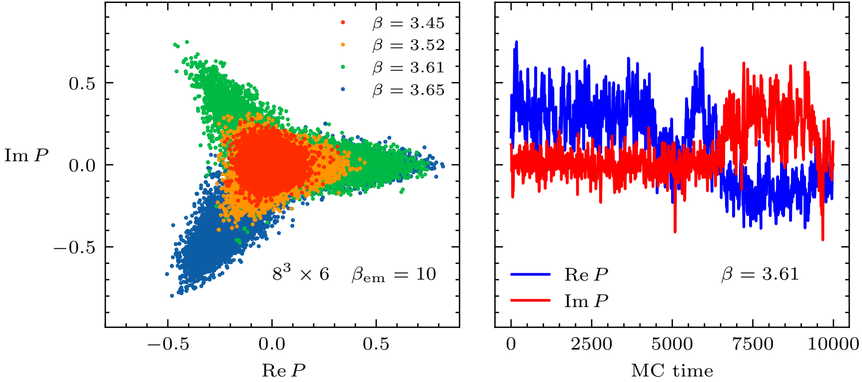
- Lattice QCD + QED $N_f = 2 + 1$ improved staggered quarks physical[2] masses
- Compact and non-compact QED actions
- Previous preliminary study: Scior, Edwards, von Smekal, Lattice '10,11,13
  - QCD_2 with strongly coupled compact QED $\beta_{\text{em}} \sim 1$
  - Evidence for 2^{nd}-order deconfinement transition (like SU(2) gauge theory)
- We mainly explore weak QED coupling regime $\beta_{\text{em}} = 10$

[2] QED effects not included in the LCP

Results: compact QED

Polyakov loop

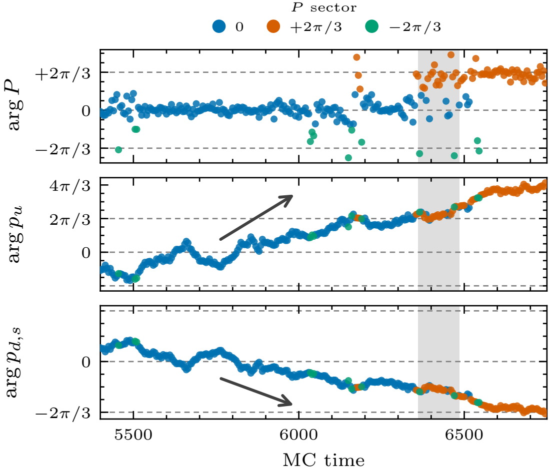
- Consider first small volume $8^3 \times 6$



- Sampling Polyakov sectors/phases is challenging

QED Polyakov loops

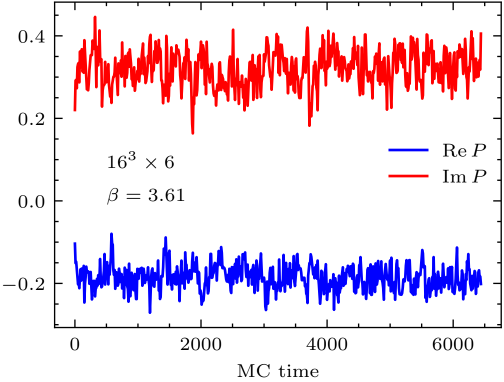
- Why is sampling different SU(3) Polyakov sectors difficult?
- Transition among P sectors achieved when $p_u, p_{d,s}$ rotate globally



- Tunneling is unlikely with usual sampling algorithms

Larger volumes

- In larger volumes tunneling is further suppressed



- Ergodicity problem?

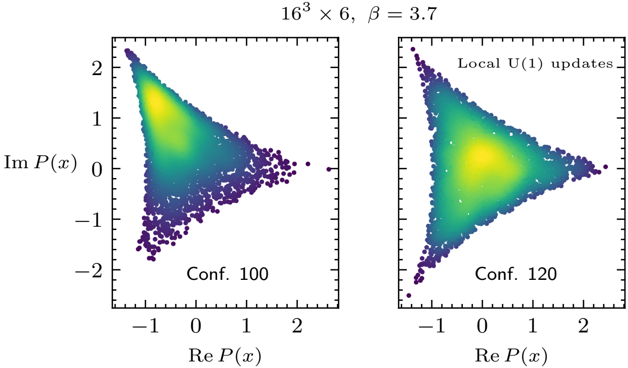
- Need mixture among phases for definite answer on the phase transition

Local updates: ongoing work

- Local U(1) update at given $n = (\vec{n}, N_t - 1)$

$$\phi_4(n) \to \phi_4(n) + 2\pi$$

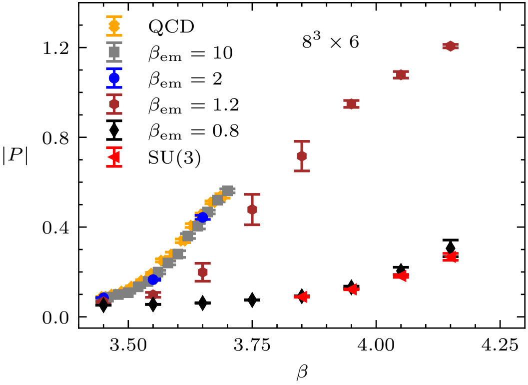
- Changes determinant, Metropolis accept-reject $\sim 50\%$ acceptance rate



- Local updates allow larger volumes to explore different configuration space
- Need to refine algorithms to solve ergodicity problems

QED coupling dependence

- What about previous results? $\rightarrow$ we also study β_{em} dependence
- Strong QED coupling shifts Polyakov loop towards SU(3) result
- Qualitatively consistent with Scior, Edwards, von Smekal, Lattice '13



- Possible explanation phase transition of lattice QED at $\beta_{\mathrm{em}} \sim 1$

Results: non-compact QED

Non-compact QED

- State-of-the-art $T = 0$ simulations use **non-compact QED**
- In addition:
  - Needs gauge fixing ✓
  - Fourier-accelerated HMC ✓
  - Zero mode removal

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- Finite volume QED faces several difficulties (review Patella '17)
- Intuitive idea: Gauss' law incompatible with periodic BC
- More precisely: in finite V large gauge transformations are a symmetry

$$A_\mu(x) \to A_\mu(x) - \frac{2\pi n_\mu}{N_\mu}$$
$$\psi(x) \to e^{2\pi i \sum_\mu (N^{-1} n)_\mu x_\mu}\, \psi(x)$$

- These constant shifts lead to IR divergences due to photon zero modes

Zero-mode removal

- Different ways of addressing the problem:
  - QED$_{TL}$:

$$\sum_n A_\mu(n) = 0 \quad \forall\mu \iff \tilde{A}_\mu(\vec{k}=0,\omega=0) = 0 \quad \forall\mu$$

  - QED$_L$:

$$\sum_{\vec{n}} A_\mu(\vec{n},\tau) = 0 \quad \forall\tau,\mu \iff \tilde{A}_\mu(\vec{k}=0,\omega) = 0 \quad \forall\omega,\mu$$

  - Massive photon QED, C^* boundary conditions, ...

- All these choices break Z_3 symmetry explicitly

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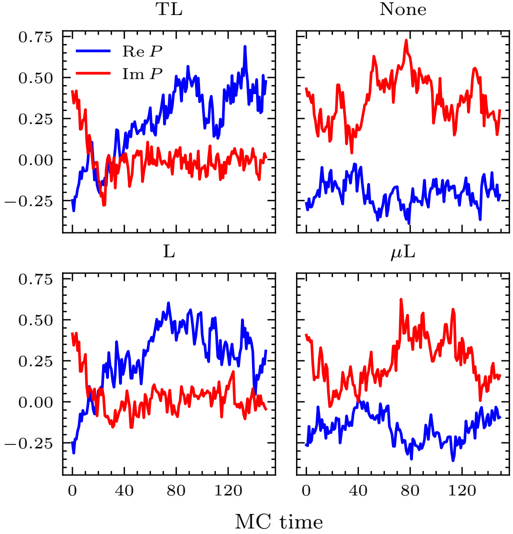
- We propose novel $\text{QED}_{\mu L}$:

$$\tilde{A}_\mu(\vec{k}=0, \omega) = 0 \quad \forall\omega, \mu \quad \text{except} \quad \tilde{A}_4(\vec{k}=0, \omega=0)$$

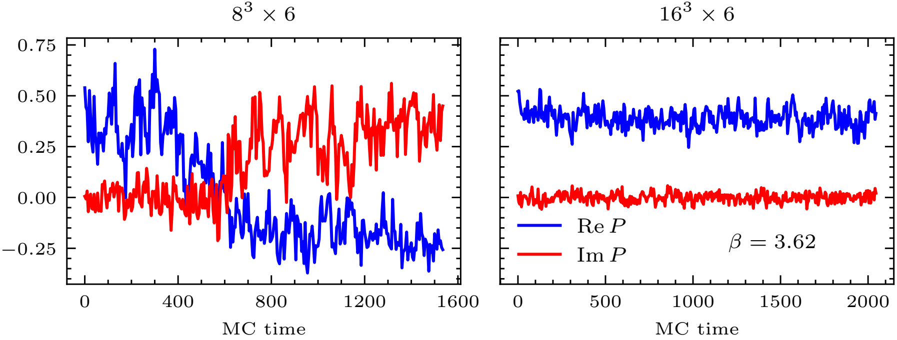
Zero-mode removal

- We start chains from configuration with imaginary P
- Only schemes compatible with Z_3 stay in imaginary Polyakov loop sectors



Non-compact QED+QCD

- Non-compact QED suffers from the same sampling issues as the compact case



Summary

Take-home messages

- QCD + QED has exact center symmetry $\rightarrow$ interesting for thermodynamics
- Sampling Polyakov loop sectors challenging, even in small volumes
- Only strong QED coupling takes P to pure SU(3) values

Outlook

- Improved algorithms: local updates, parallel tempering,...
- Continuum limit

Backup slides

Scale setting