



Berkeley
UNIVERSITY OF CALIFORNIA



UNIVERSITY OF
MARYLAND

Real-Time Estimators for Scattering Observables

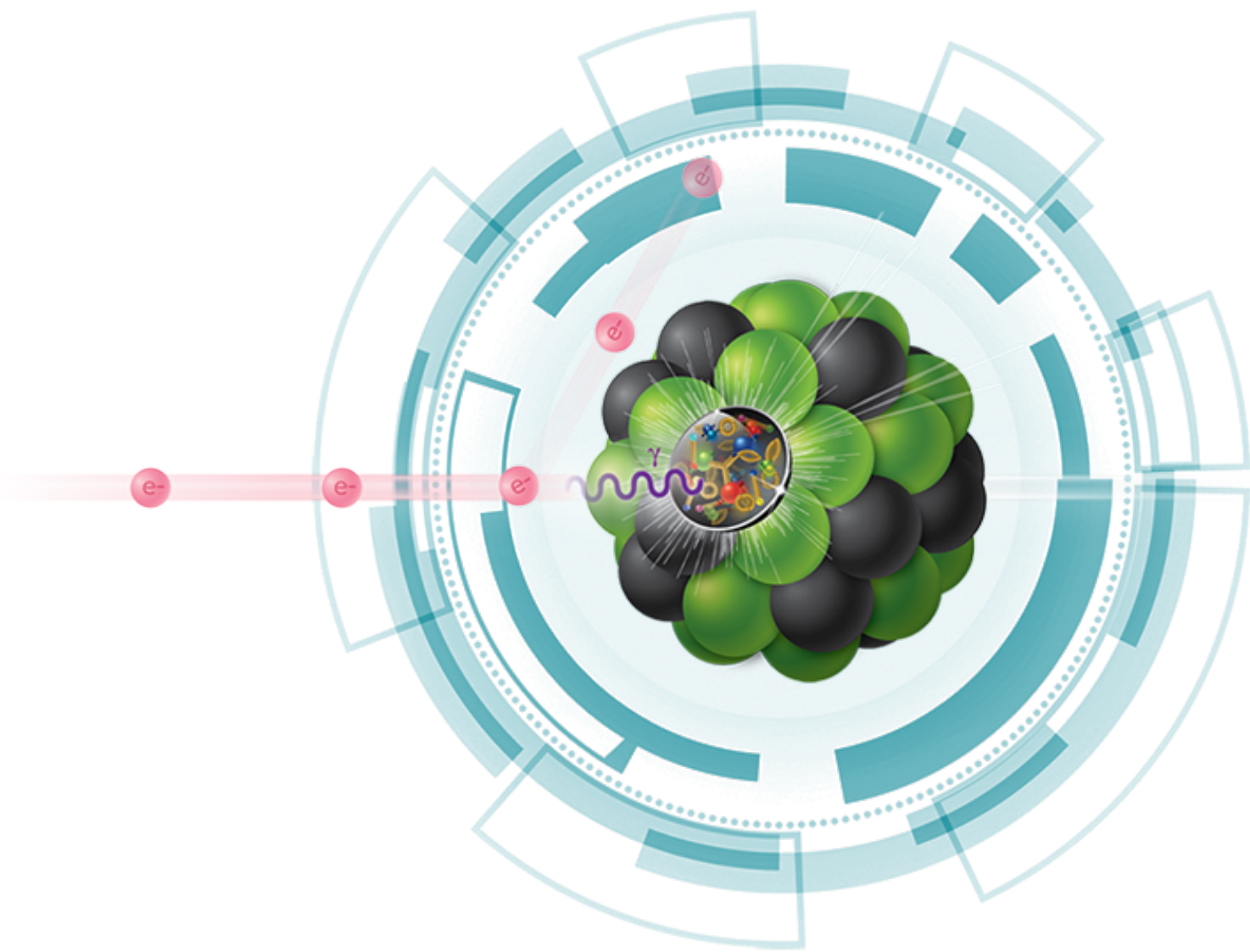
(RESOs)



Ivan Mauricio Burbano Aldana

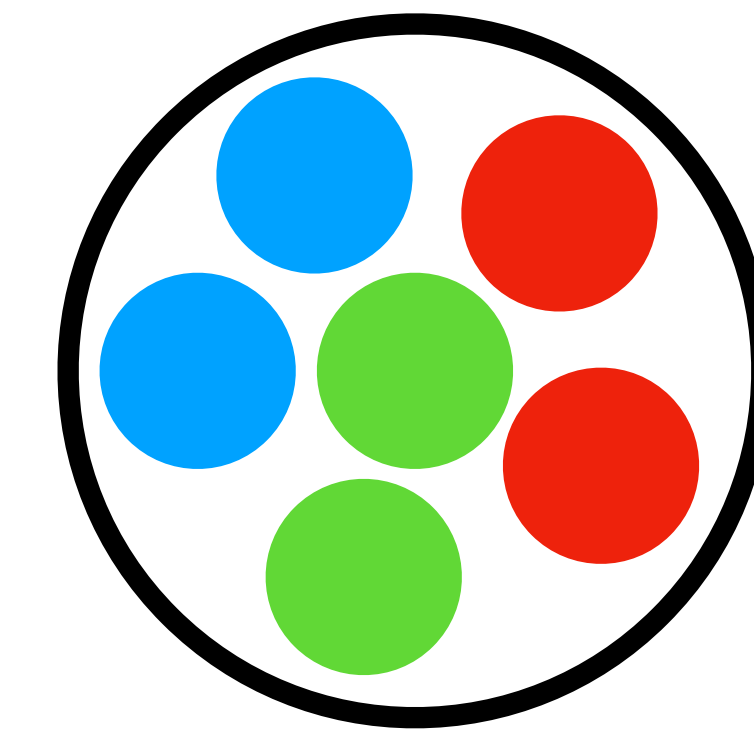
High Energies and Multiplicities

Nuclear Structure, Exotic Hadrons and Beyond the Standard Model

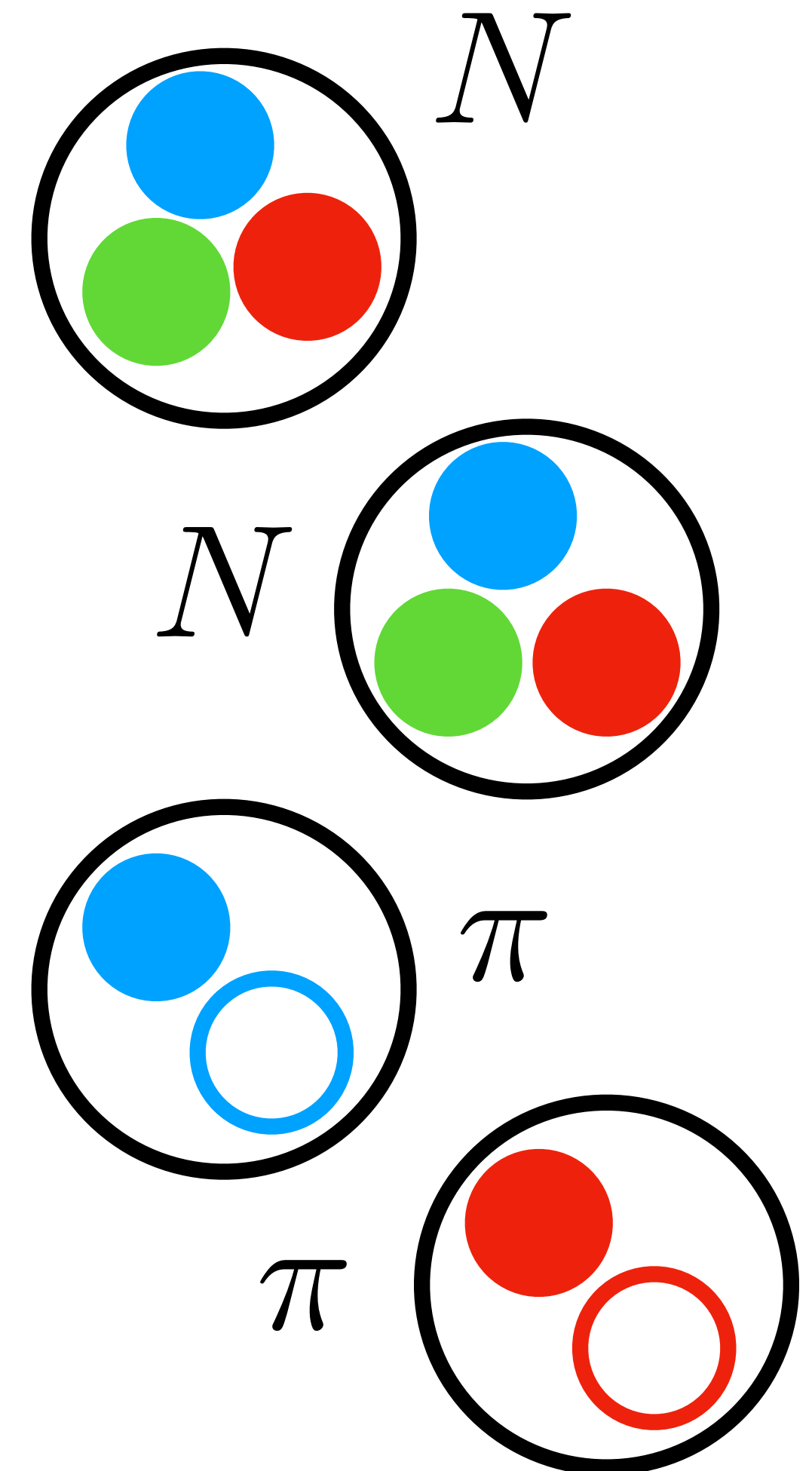


Electron-Ion Collider

(similar case for
Deep Underground
Neutrino Experiment)

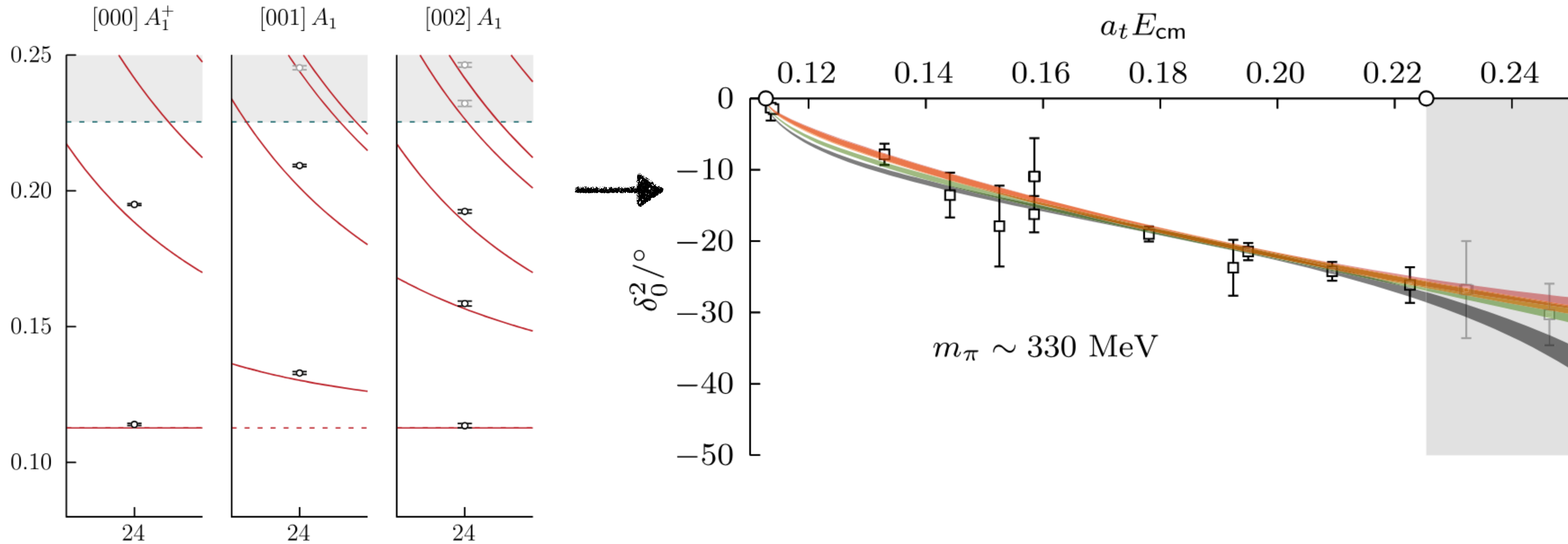
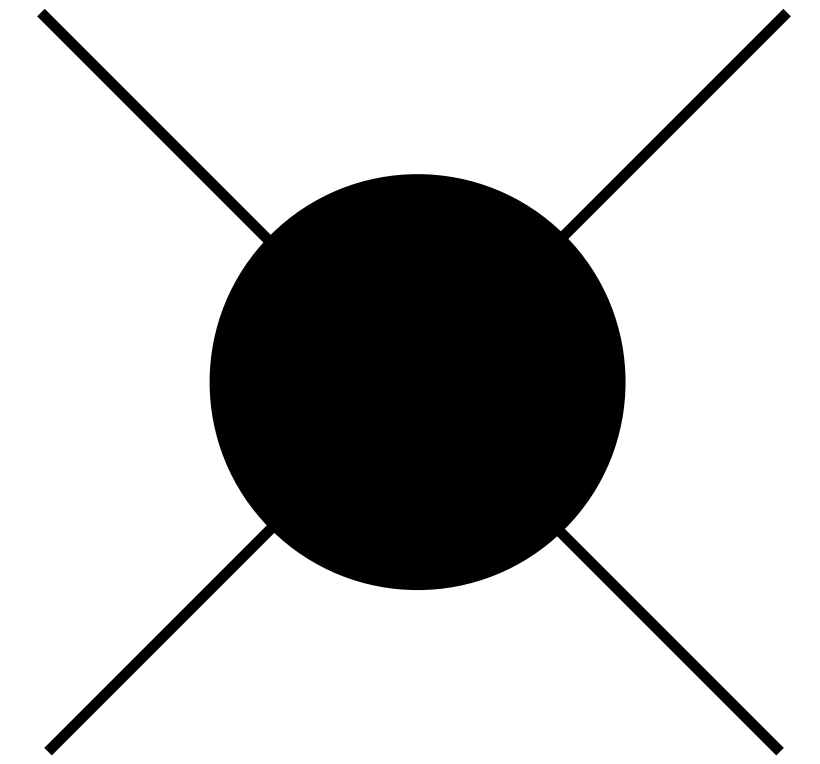


$d^*(2380)$



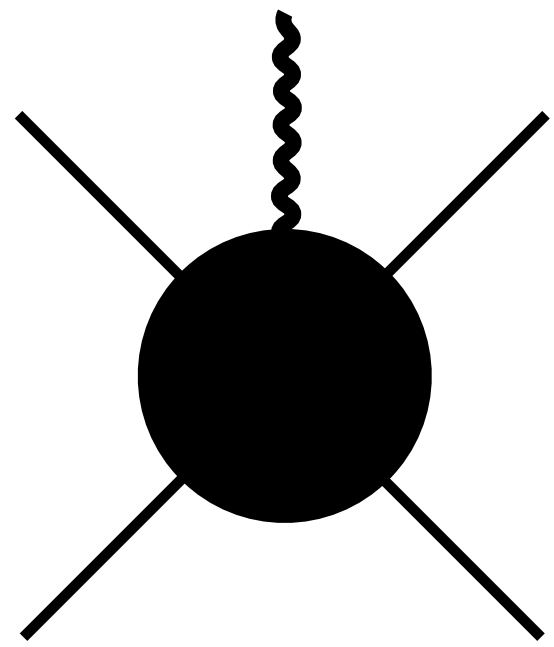
Luscher's Method

Some state of the art

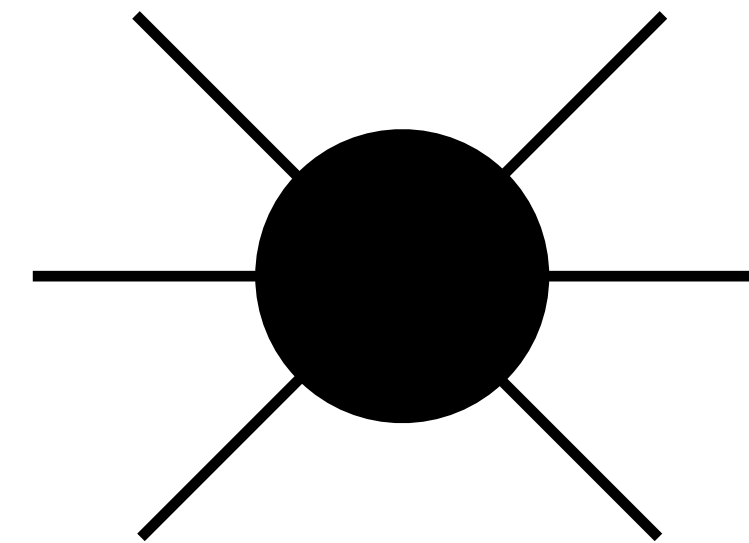
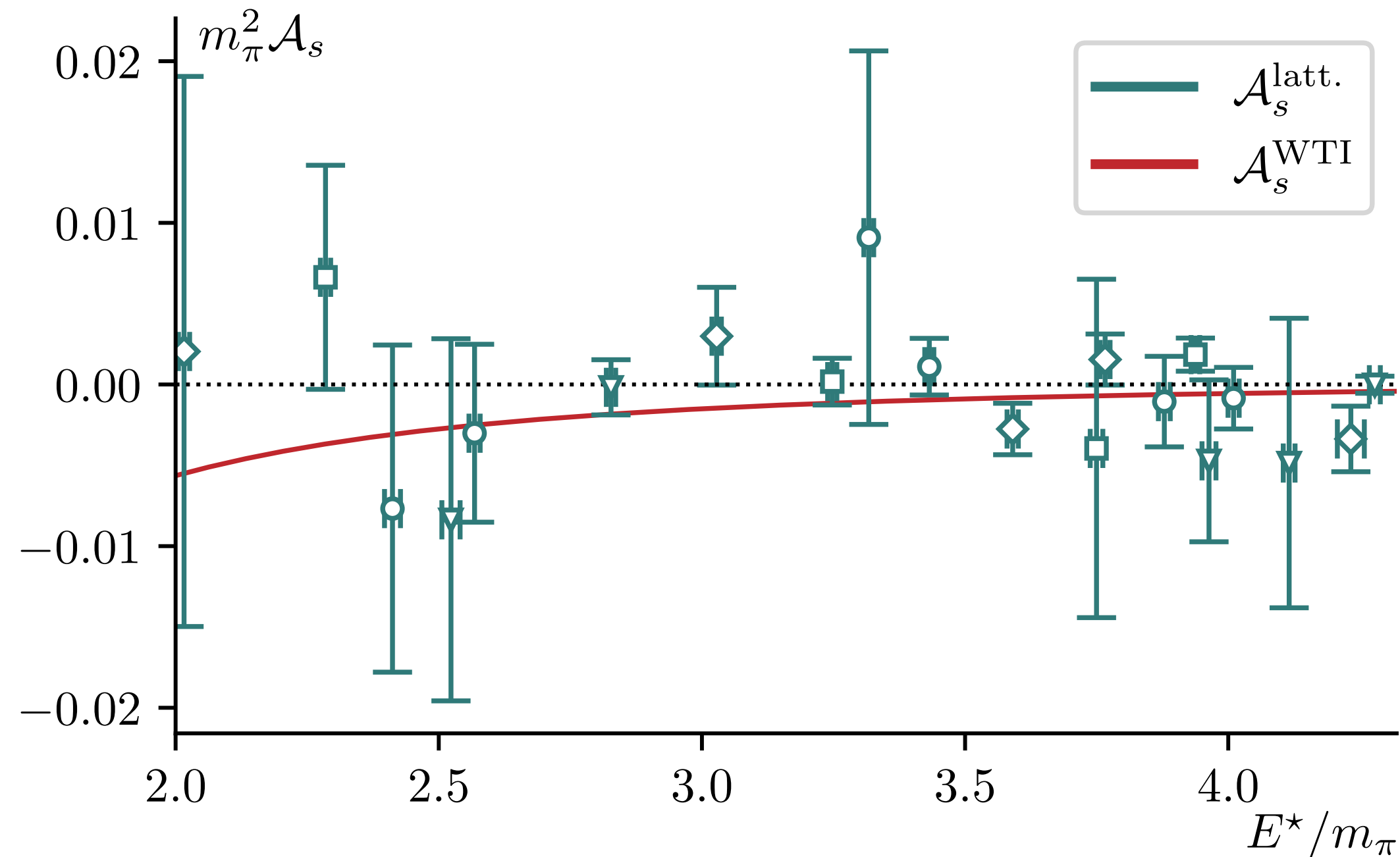


Luscher's Method

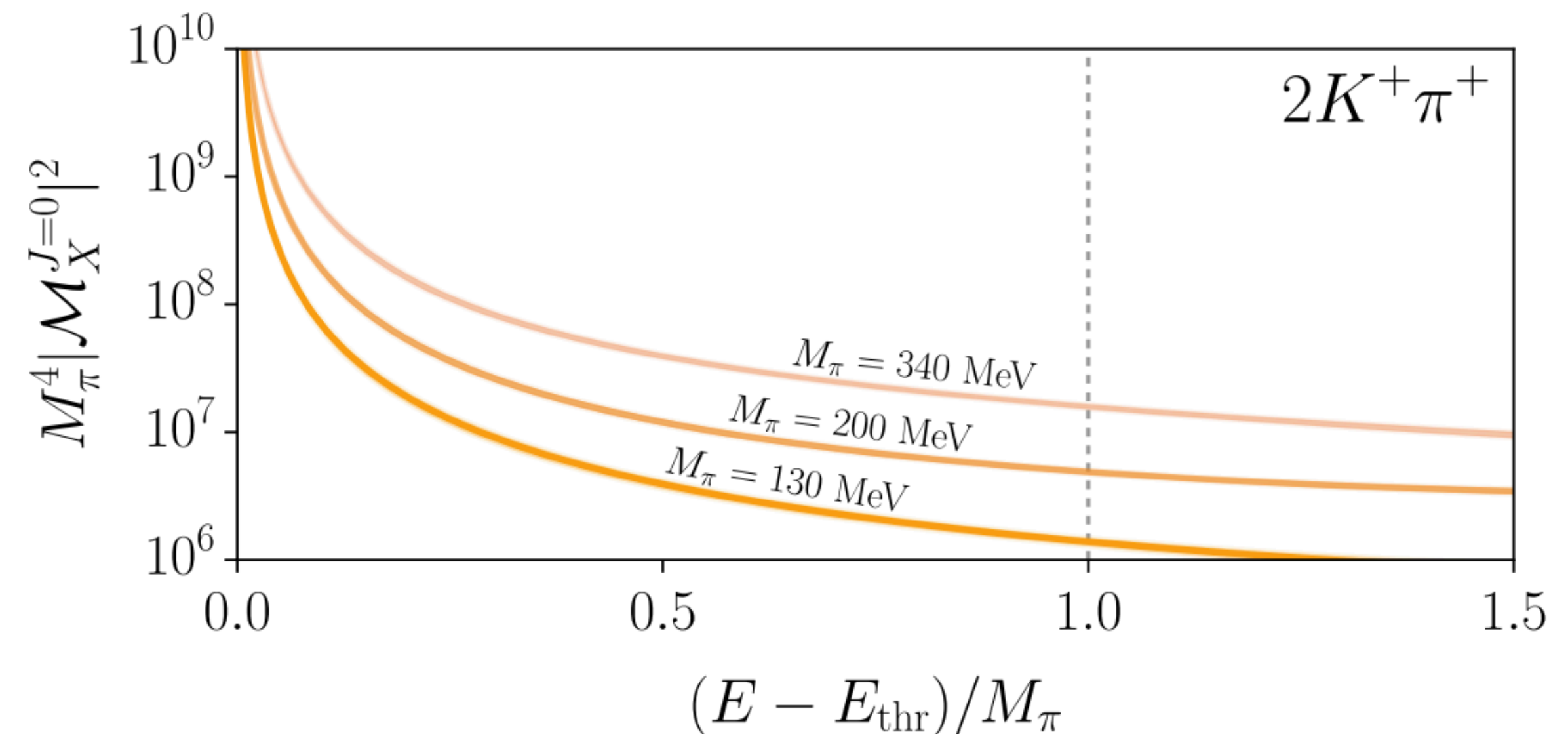
Some state of the art



F. Ortega-Gama, R. A. Briceño and IMBA, to appear (there will be a talk about this next in the Hadronic and Nuclear Structure path)

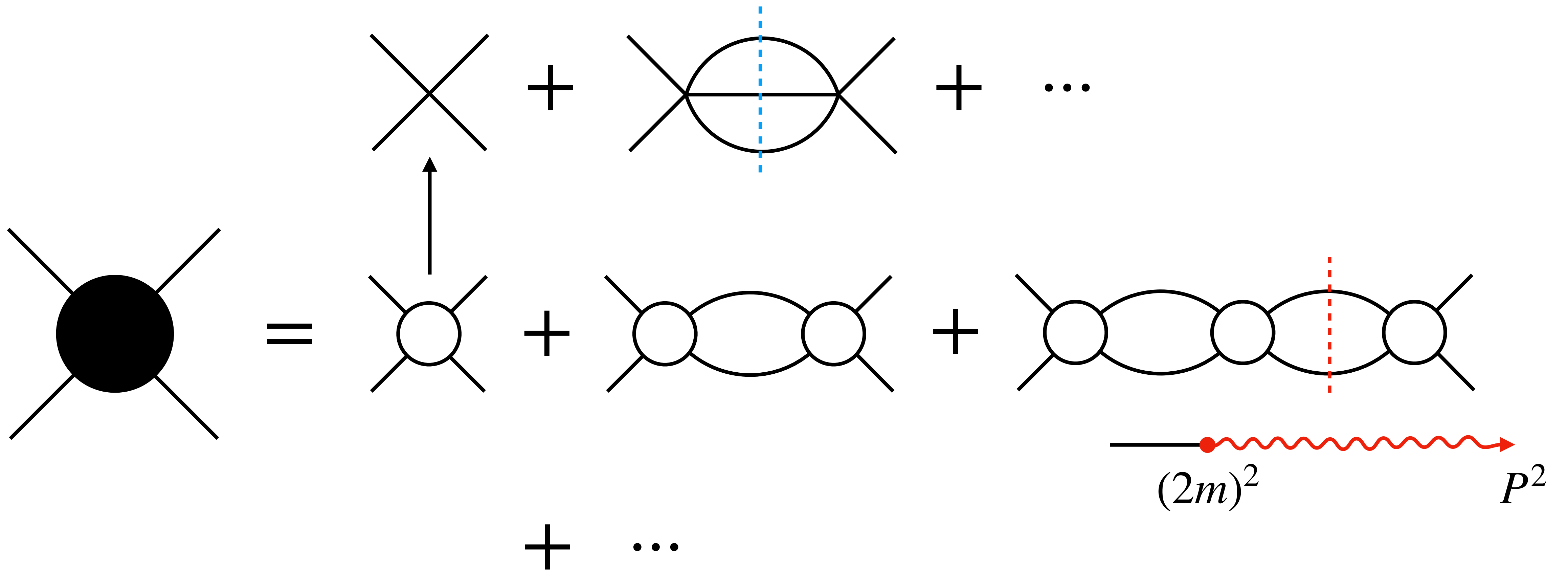


S. M. Dawid, Z. T. Draper, A. D. Hanlon, B. Hörz, C. Morningstar, F. Romero-López, S. R. Sharpe, and S. Skinner, “QCD Predictions for Physical Multimeson Scattering Amplitudes,” *Physical Review Letters* **135**, 021903 (2025),

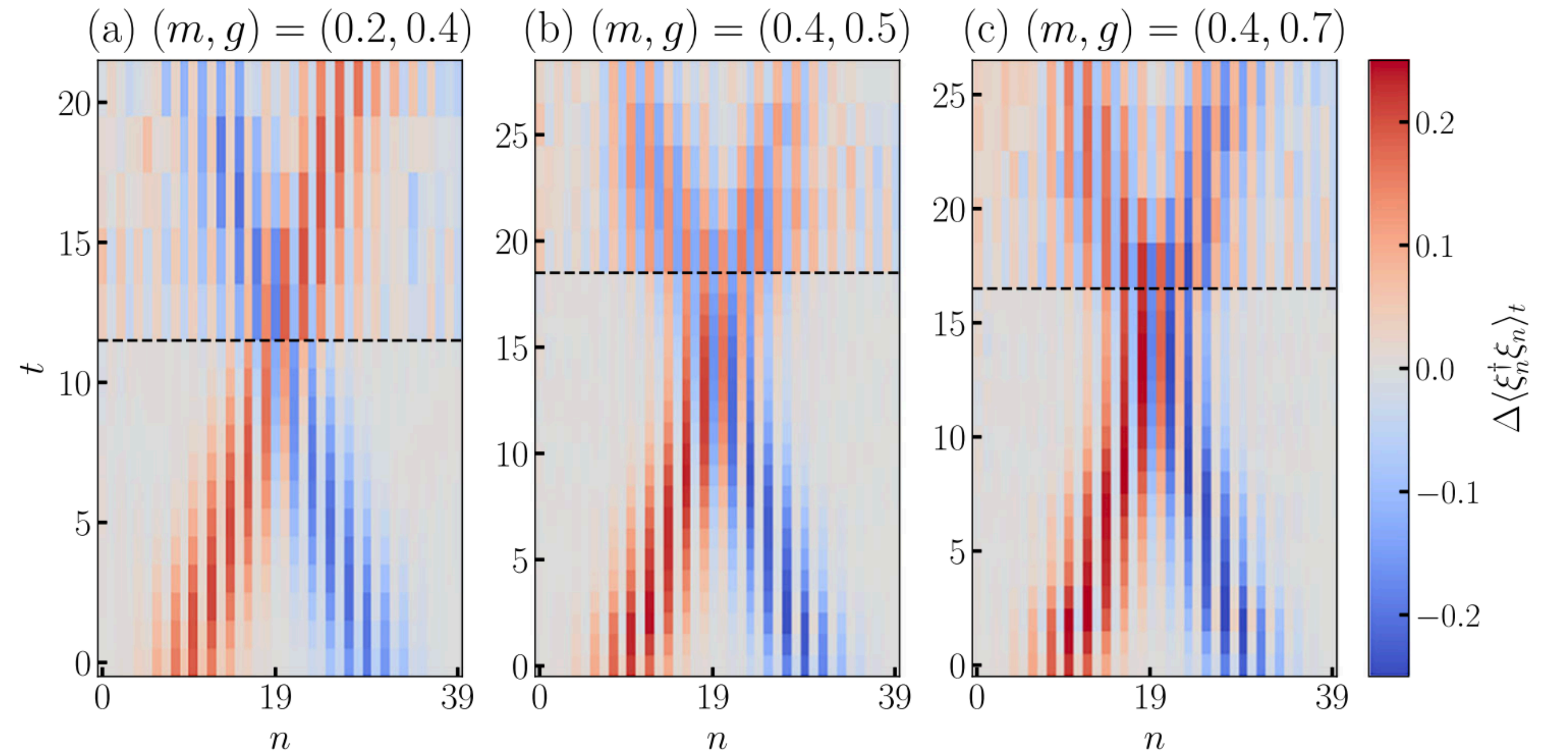
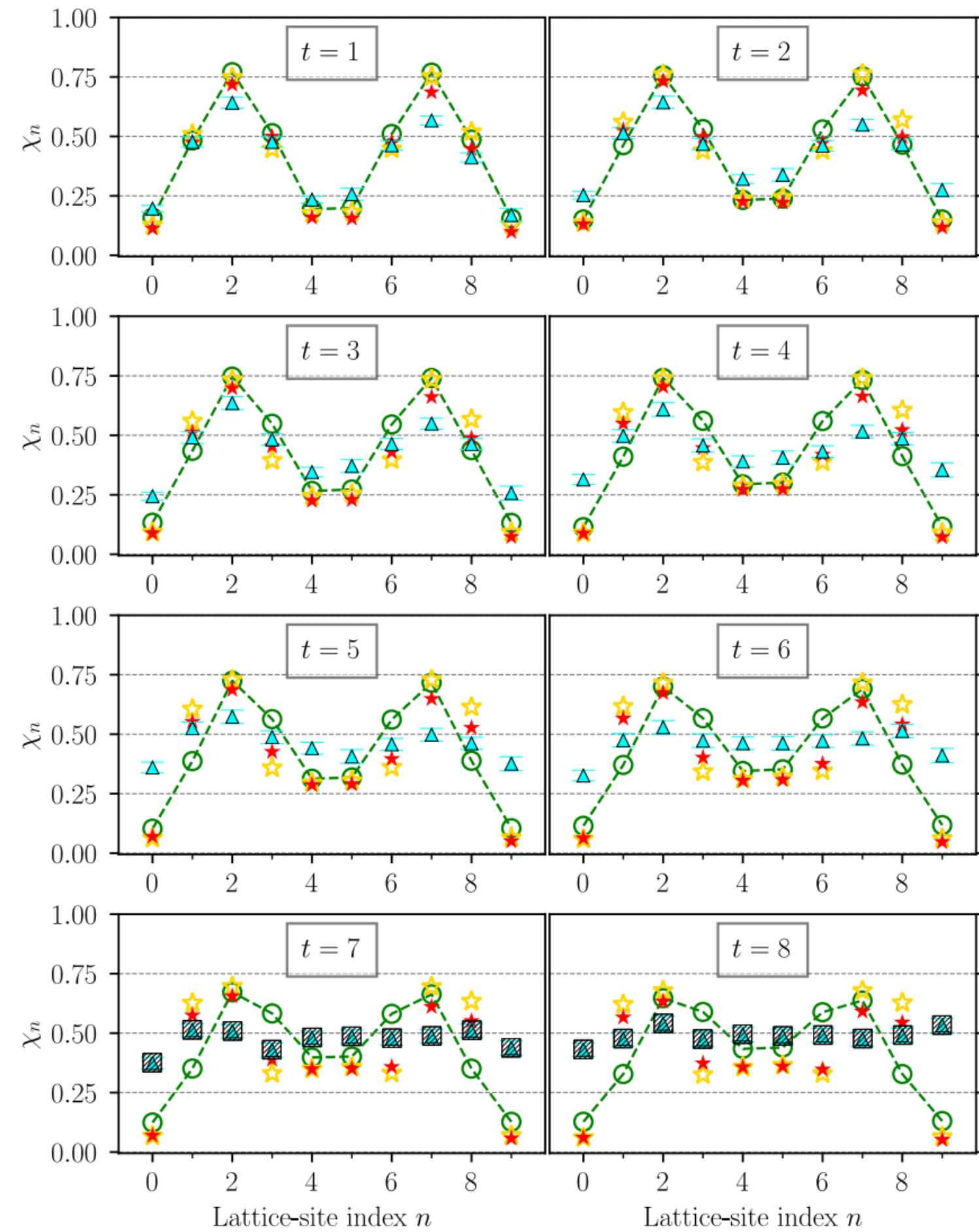


Luscher's Method

How does it work?



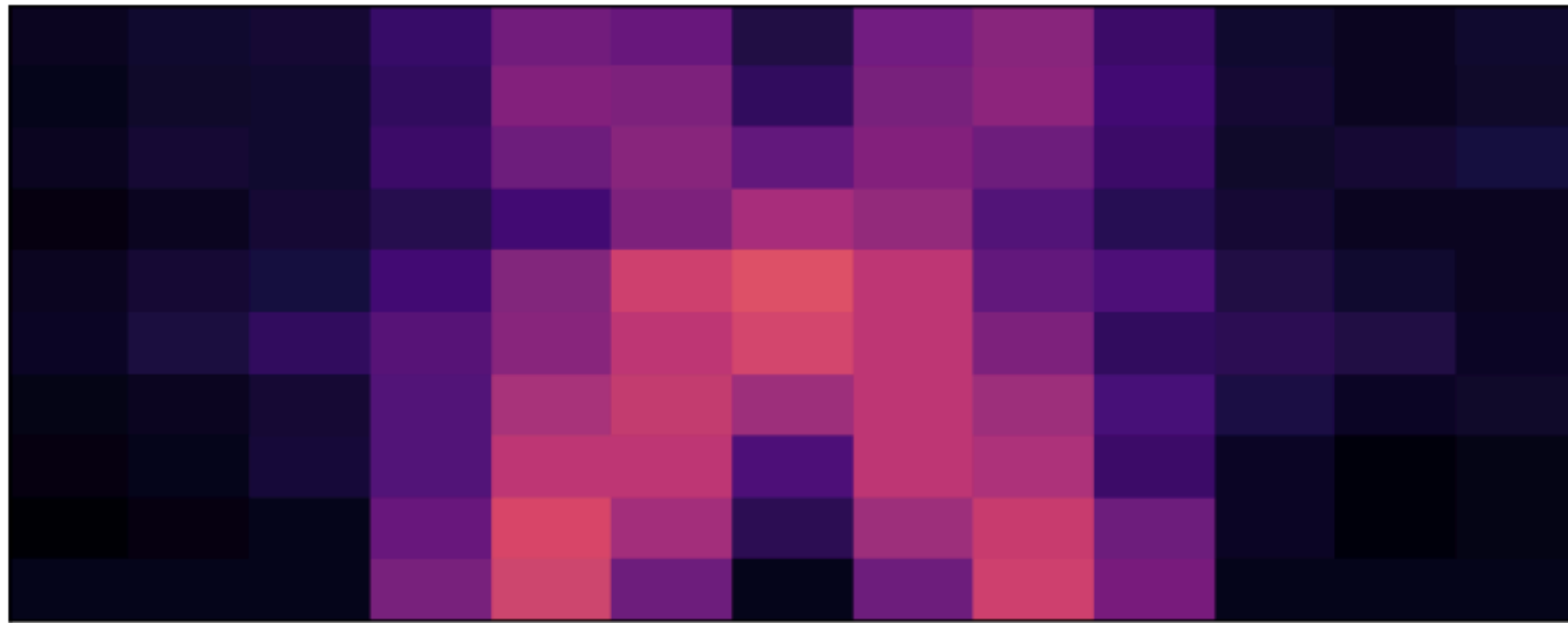
Simulations of Scattering on Quantum Computers



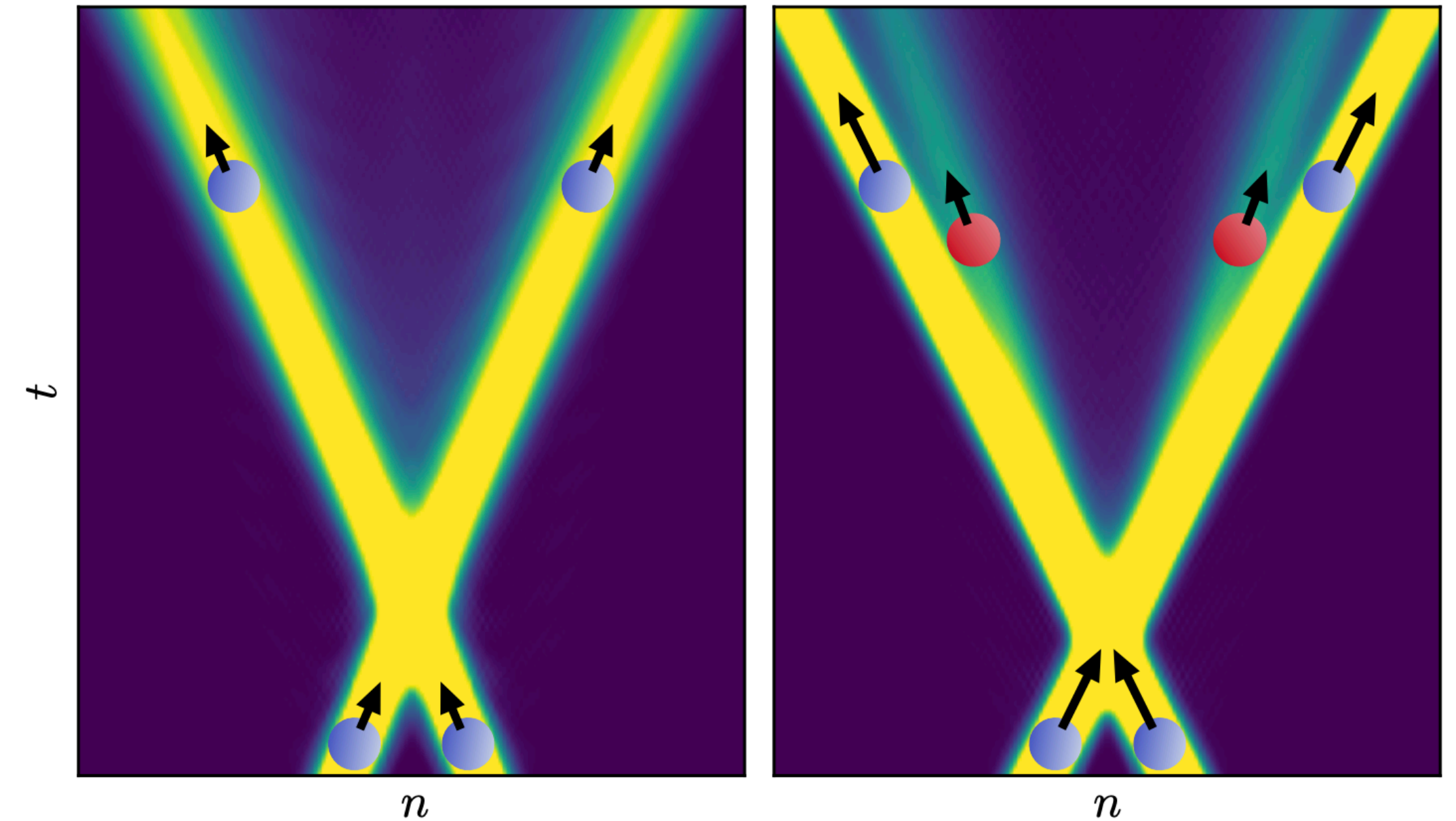
Y. Chai, J. Gibbs, V. R. Pascuzzi, Z. Holmes, S. Kühn, F. Tacchino, and I. Tavernelli, “Resource-efficient simulations of particle scattering on a digital quantum computer,” *npj Quantum Information* **12**, 100 (2026),

Z. Davoudi, C.-C. Hsieh, and S. V. Kadam, “Quantum computation of hadron scattering in a lattice gauge theory,” [arXiv:2505.20408 \[quant-ph\]](https://arxiv.org/abs/2505.20408) (2025).

Simulations of Scattering on Quantum Computers



N. A. Zemlevskiy, “Scalable quantum simulations of scattering in scalar field theory on 120 qubits,” *Phys. Rev. D* **112**, 034502 (2025), [arXiv:2411.02486](https://arxiv.org/abs/2411.02486).



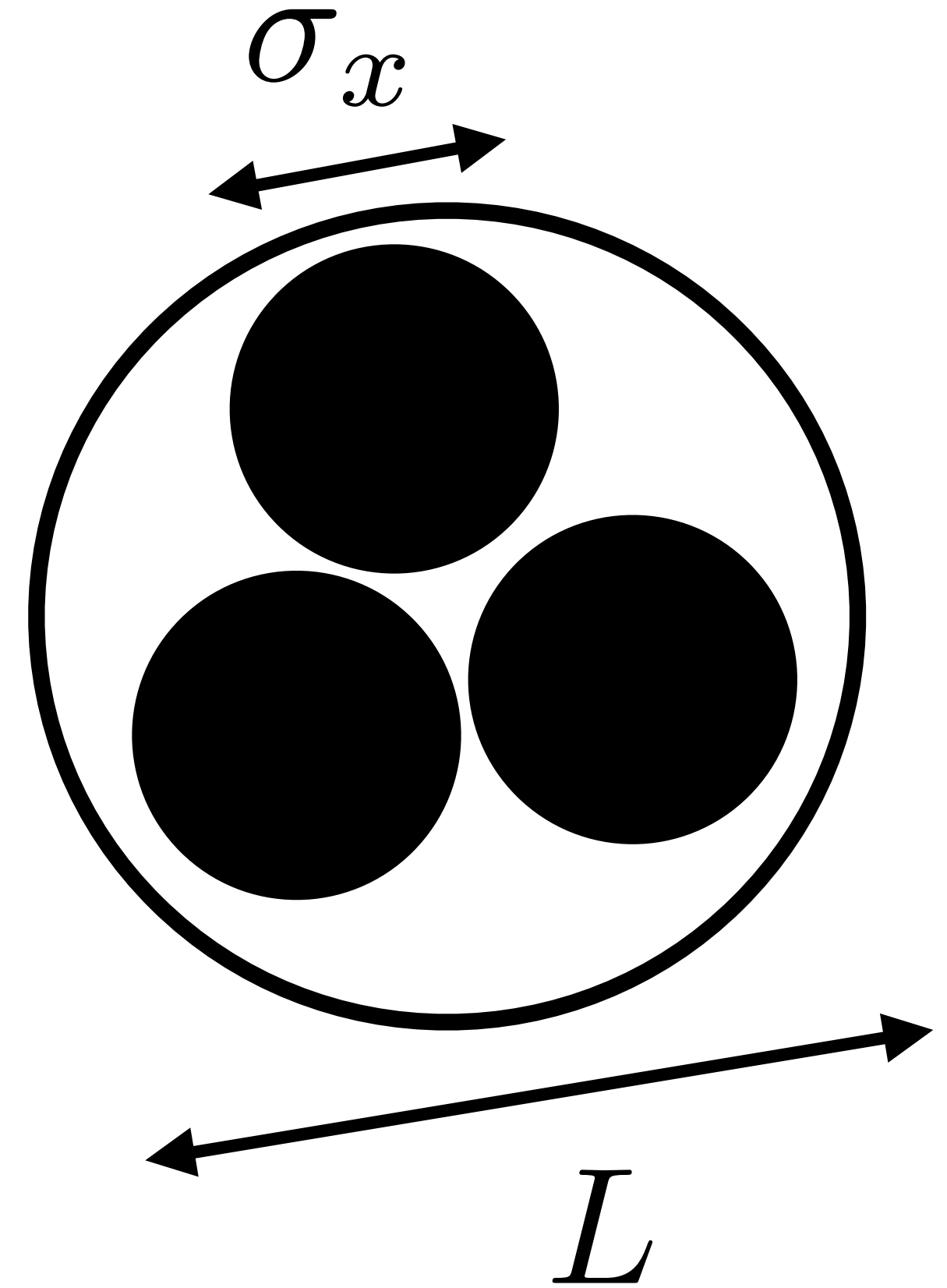
R. C. Farrell, N. A. Zemlevskiy, M. Illa, and J. Preskill, “Digital quantum simulations of scattering in quantum field theories using $\backslash(W\backslash)$ states,” [arXiv:2505.03111](https://arxiv.org/abs/2505.03111) (2025).

How about in 3+1 dimensions?

$$\sigma_x \sigma_p \propto 1$$

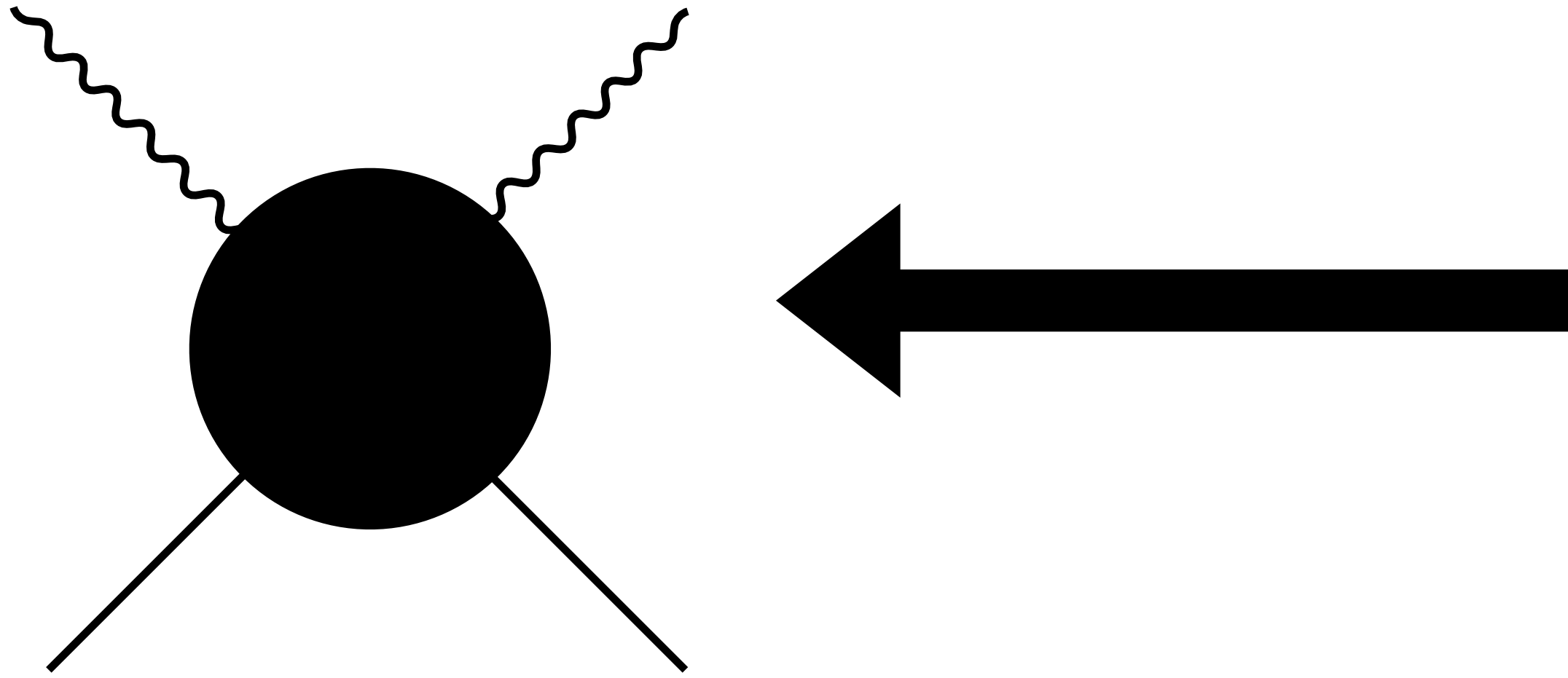
$$L \propto \sigma_x$$

$$\sigma_p \propto \frac{2\pi}{L} = \frac{2\pi}{5\text{fm}} \sim 250\text{MeV}$$

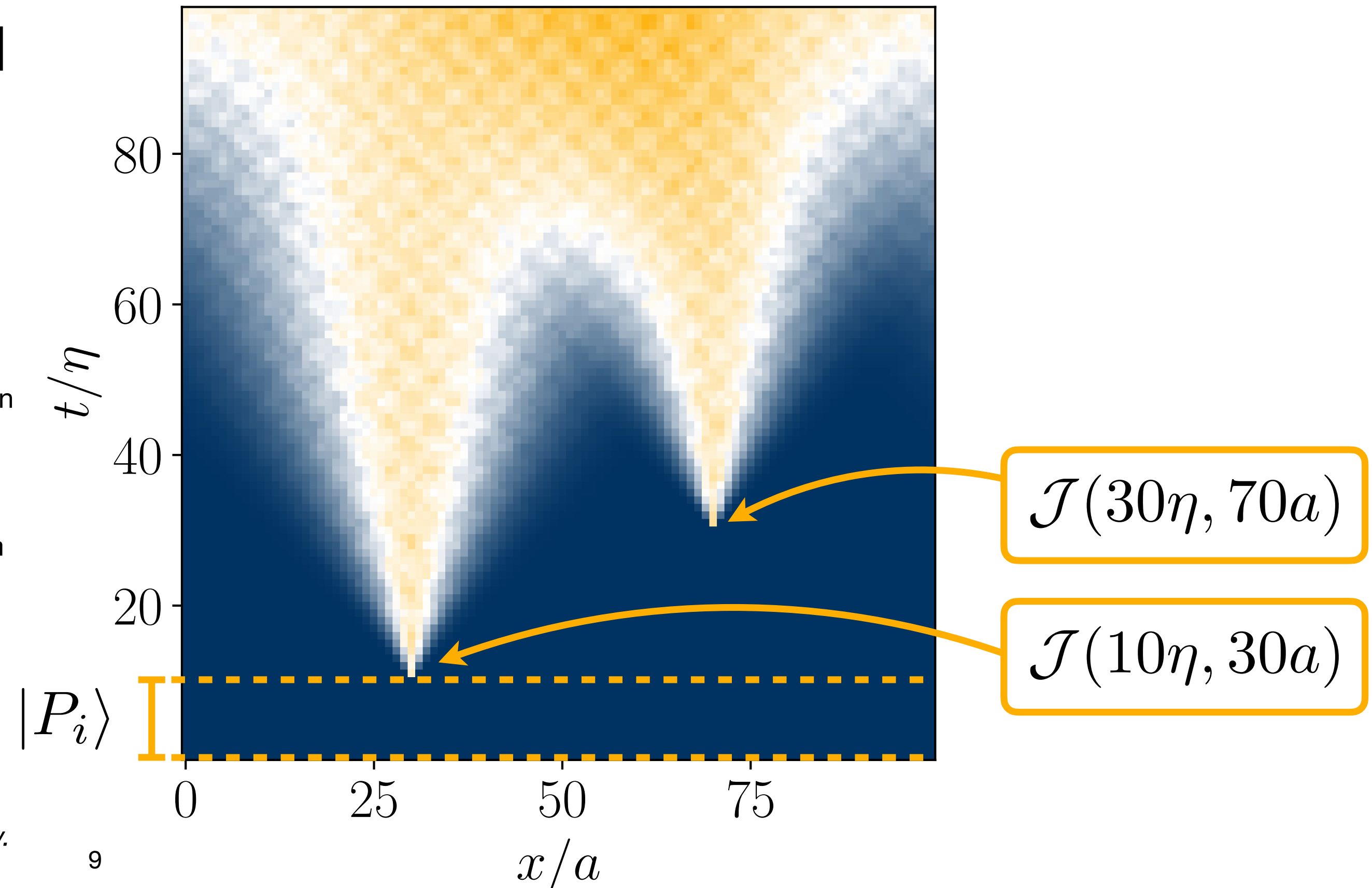


Real-Time Estimators for Scattering Observables

RESOs for Compton amplitudes



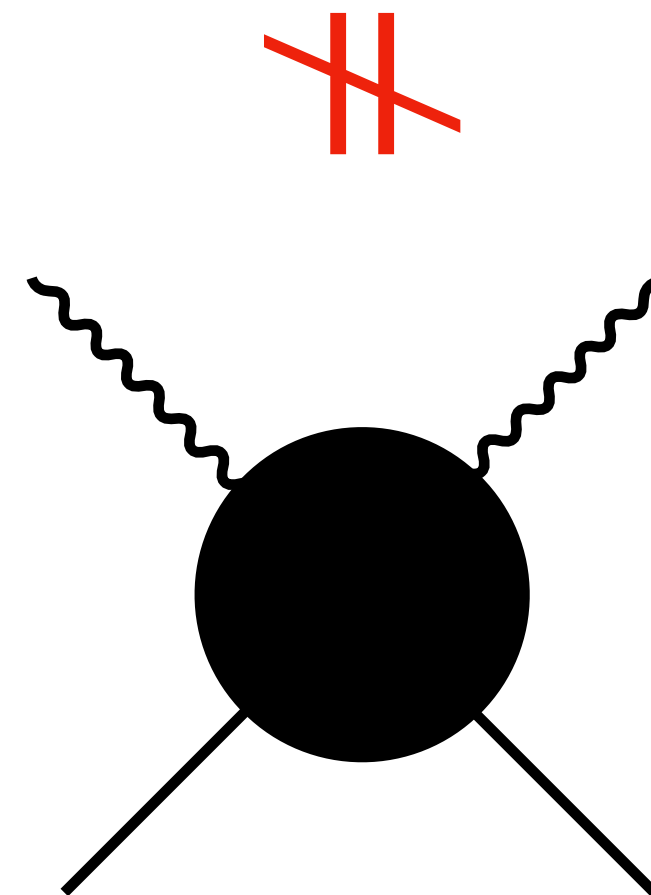
$$\langle P_f | T \mathcal{J}(t, \mathbf{x}) \mathcal{J}(0) | P_i \rangle$$



1. R. A. Briceño, J. V. Guerrero, M. T. Hansen, and A. M. Sturzu, “The role of boundary conditions in quantum computations of scattering observables,” *Phys. Rev. D* **103**, 014506 (2021)
2. R. A. Briceño, M. A. Carrillo, J. V. Guerrero, M. T. Hansen, and A. M. Sturzu, “Accessing scattering amplitudes using quantum computers,” *PoS LATTICE2021*, 315 (2022)
3. T. Li, W. K. Lai, E. Wang, and H. Xing (QuNu Collaboration), “Scattering amplitude from quantum computing with reduction formula,” *Phys. Rev. D* **109**, 036025 (2024)
4. M. A. Carrillo, R. A. Briceño, and A. M. Sturzu, “Inclusive reactions from finite Minkowski spacetime correlation functions,” *Phys. Rev. D* **110**, 054503 (2024)
5. R. A. Briceño, R. G. Edwards, M. Eaton, C. González-Arciniegas, O. Pfister, and G. Siopsis, “Toward coherent quantum computation of scattering amplitudes with a measurement-based photonic quantum processor,” *Phys. Rev. Research* **6**, 043065 (2024)
6. IMBA, M. A. Carrillo, R. Urek, A. N. Ciavarella, and R. A. Briceño, “Real-time estimators for scattering observables: A full account of finite-volume errors for quantum simulation,” *Phys. Rev. D* **113**, L071502 (2026)

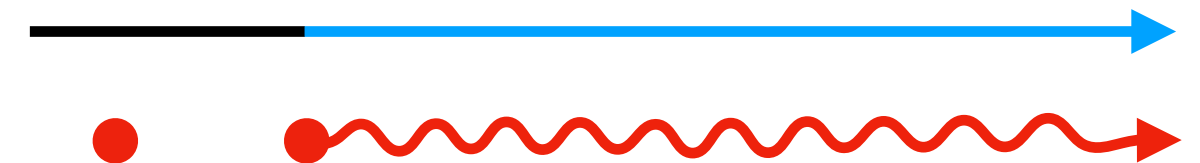
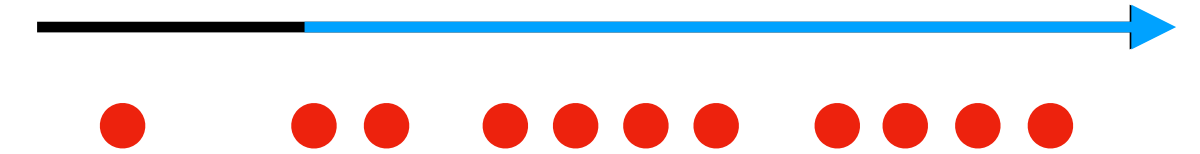
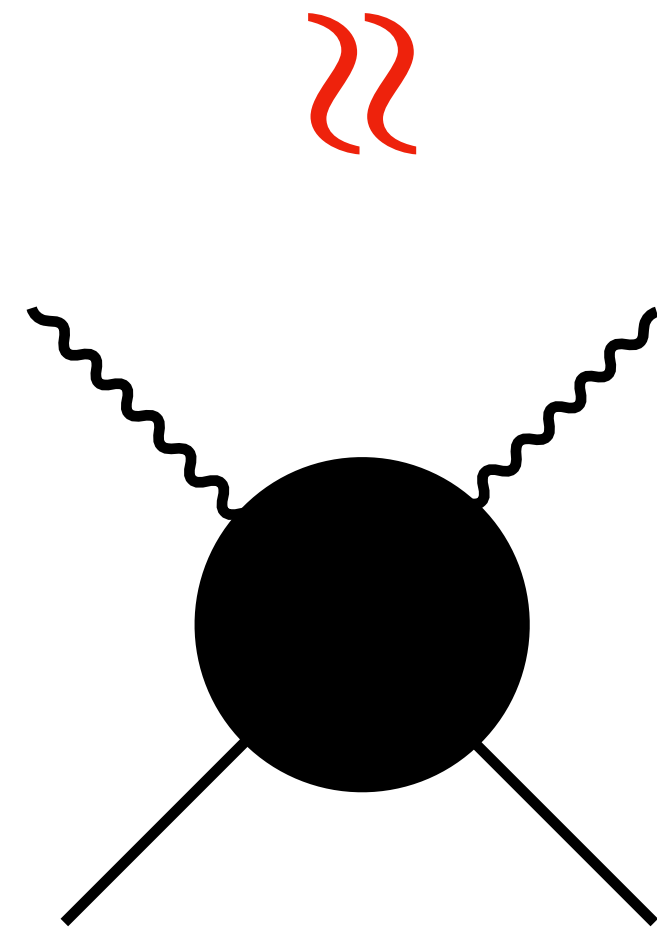
Extracting Compton-like Amplitudes

$$\int d^D x e^{iq \cdot x} \langle P_f | T \mathcal{J}(t, \mathbf{x}) \mathcal{J}(0) | P_i \rangle$$



Extracting Compton-like Amplitudes

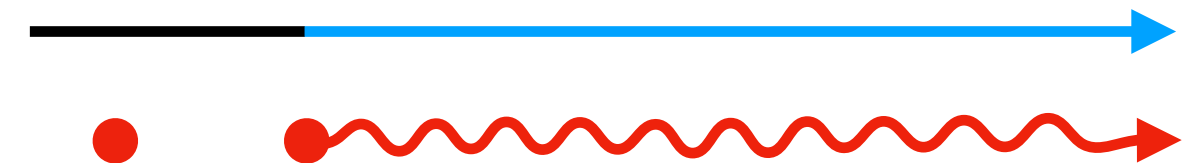
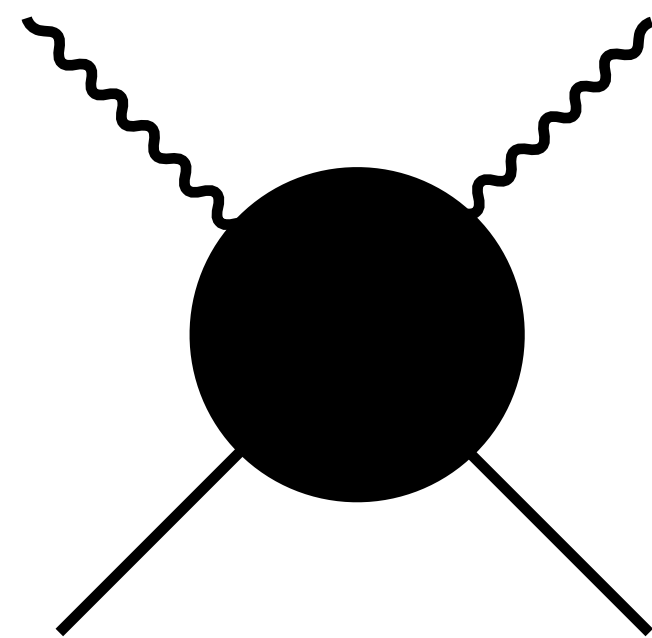
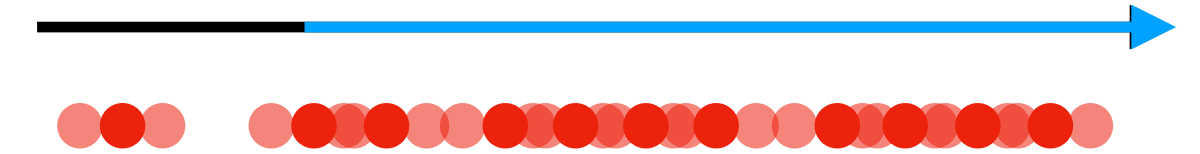
$$\int d^D x e^{iq \cdot x - \epsilon |t|} \langle P_f | T \mathcal{J}(t, \mathbf{x}) \mathcal{J}(0) | P_i \rangle$$



Extracting Compton-like Amplitudes

Restoring Lorentz Invariance

$$\left\langle \int d^D x e^{iq \cdot x - \epsilon |t|} \langle P_f | T \mathcal{J}(t, \mathbf{x}) \mathcal{J}(0) | P_i \rangle \right\rangle_{P_i, P_f, q}$$

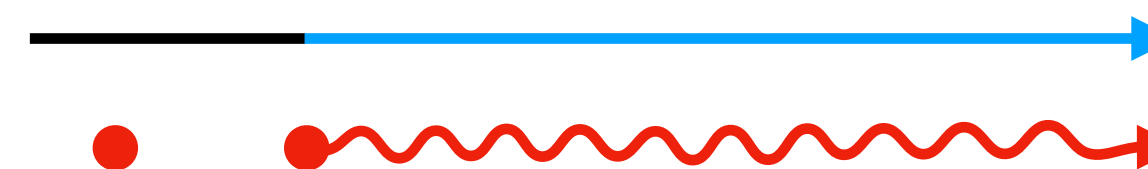
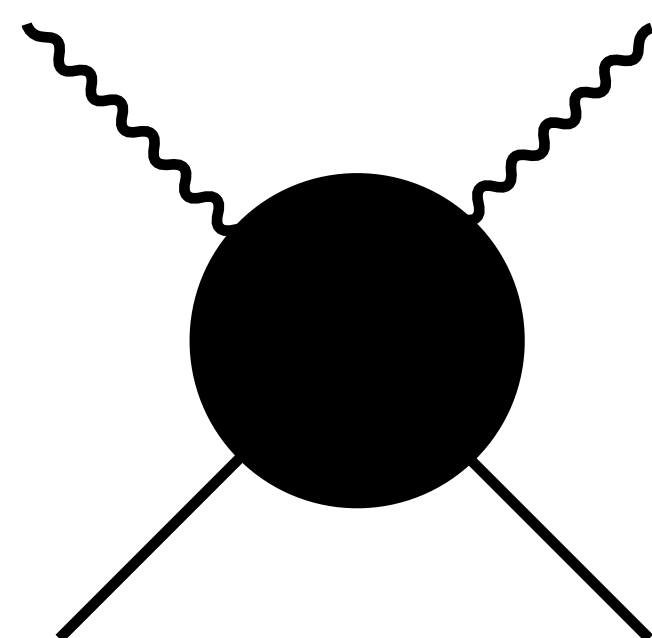
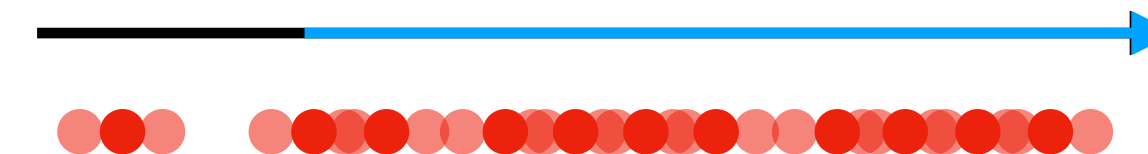


Extracting Compton-like Amplitudes

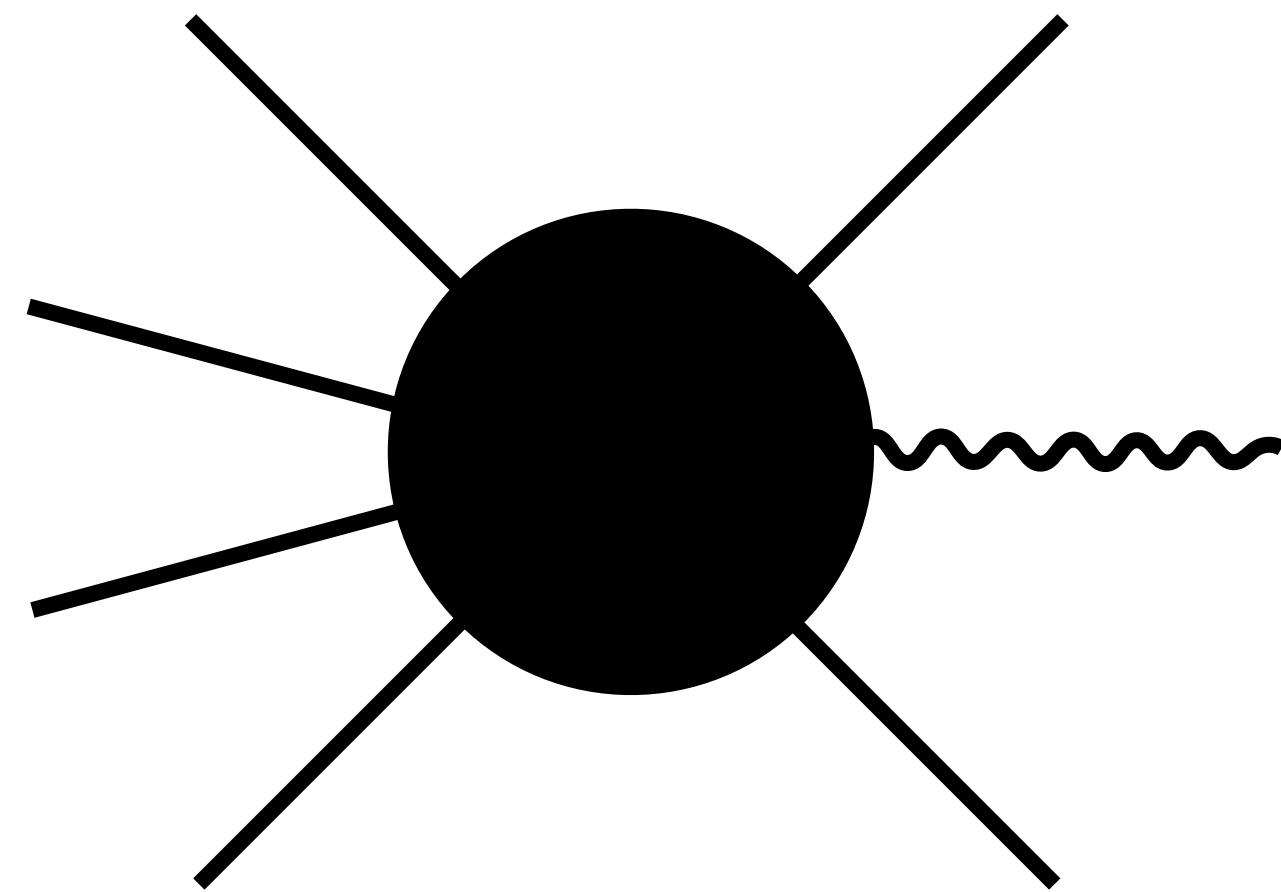
Restoring Lorentz Invariance



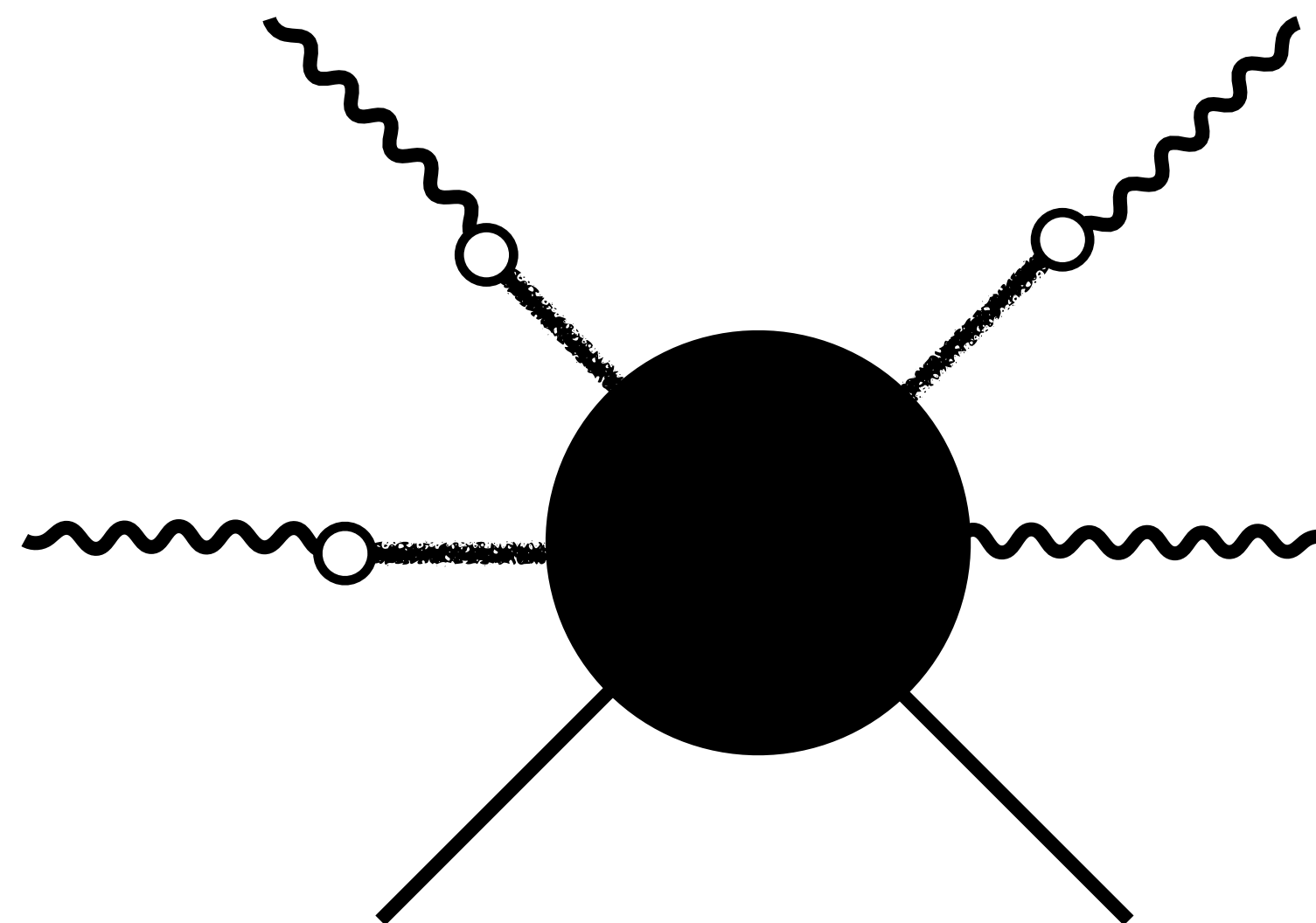
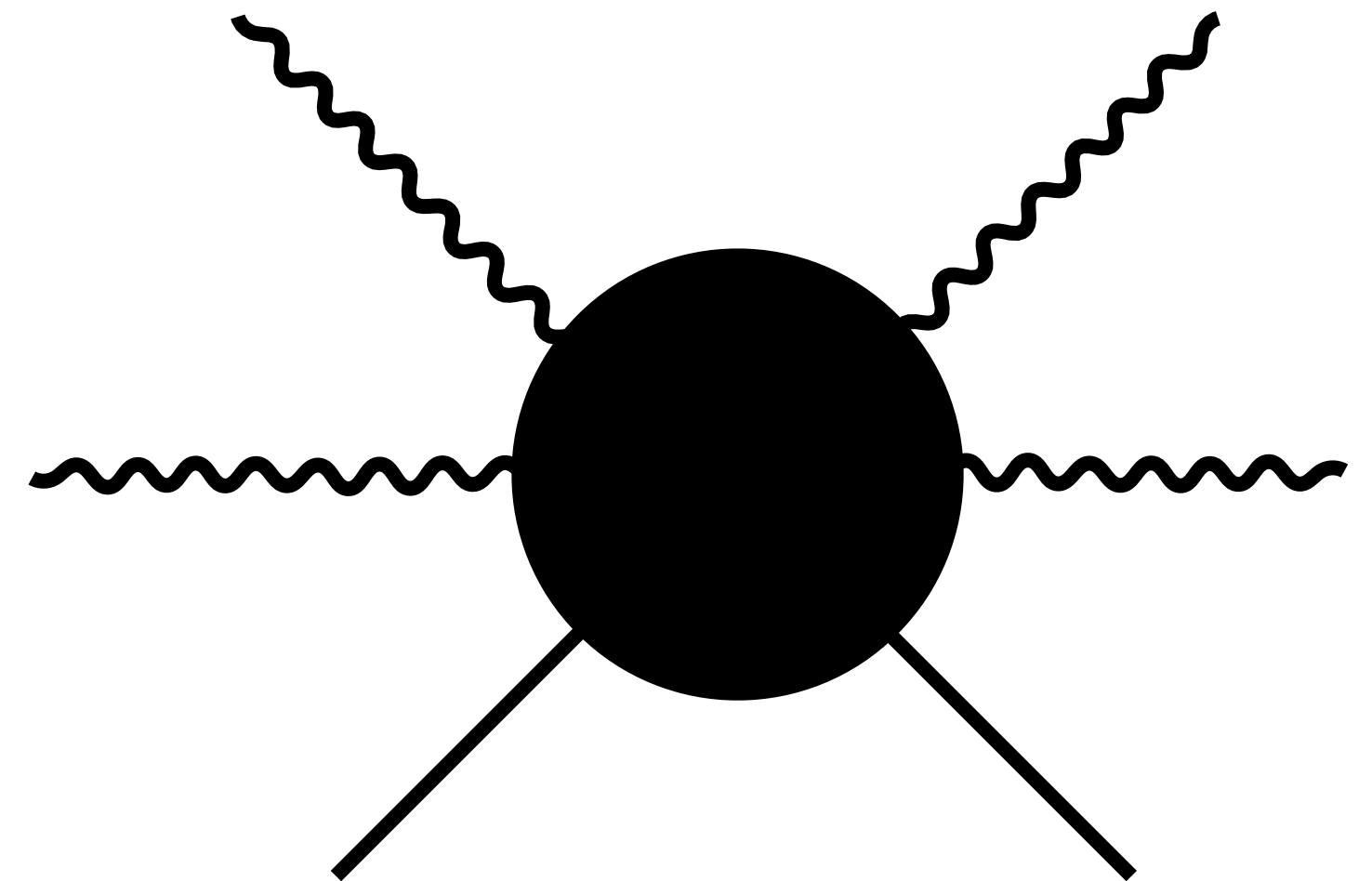
$$\left\langle \int d^D x e^{iq \cdot x - \epsilon |t|} \langle P_f | T \mathcal{J}(t, \mathbf{x}) \mathcal{J}(0) | P_i \rangle \right\rangle_{P_i, P_f, q}$$



Extracting all amplitudes

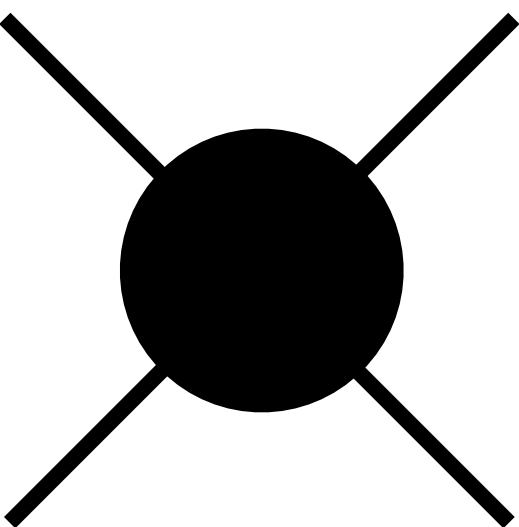
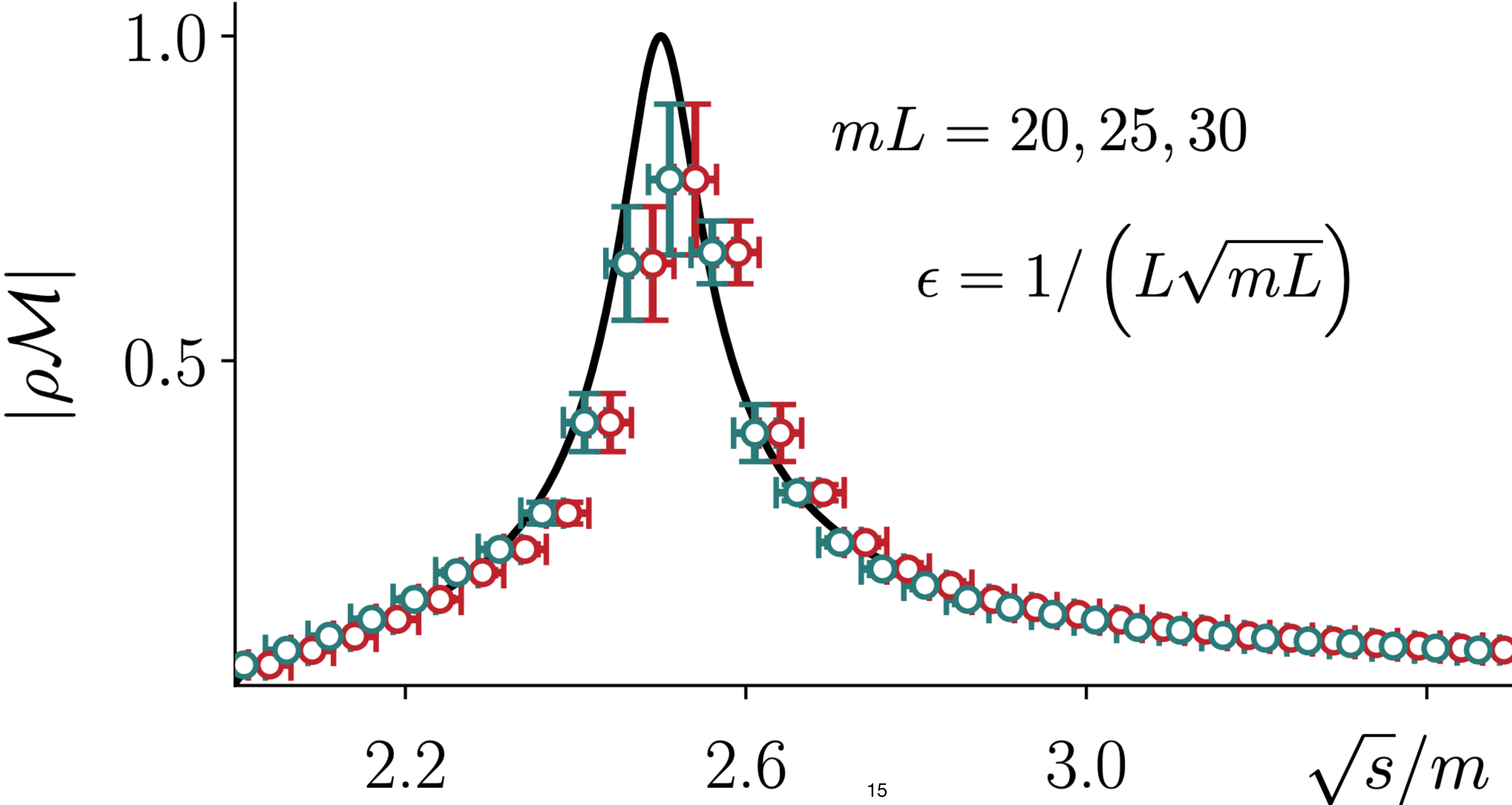


LSZ reduction



Numerical benchmarking

M. Carrillo, R. Briceño, A. Sturzu: arXiv: 2406.06877



Proof (without constants)

$$\left\langle \left(\text{Sun}(L) - \text{Sun}(\infty) \right)_n \right\rangle =$$

Lorentz invariant!

$$\int_{[0,1]^{N_P}} d^{N_P} u \, \delta \left(\sum_i u^i - 1 \right) \left\langle e^{i L n_a p^a} \right\rangle \int d^{N_L D} \ell \, e^{-i L n_a \ell^a} \left(\frac{i}{g_{ab} \ell^a \ell^b - \Delta} \right)^{N_P}$$

Proof (without constants)

$$\left\langle \left(\text{Diagram with } L \text{ and } \infty \right)_n \right\rangle =$$

Lorentz invariant!

$$\int_{[0,1]^{N_P}} d^{N_P} u \delta \left(\sum_i u^i - 1 \right) \left\langle e^{i L n_a p^a} \right\rangle \int d^{N_L D} \ell e^{-i L n_a \ell^a} \left(\frac{i}{g_{ab} \ell^a \ell^b - \Delta} \right)^{N_P}$$

$$\Delta \left(L \|n\| \sqrt{\Delta} \right)^{-\nu-1/2} e^{-L \|n\| \sqrt{\Delta}}$$

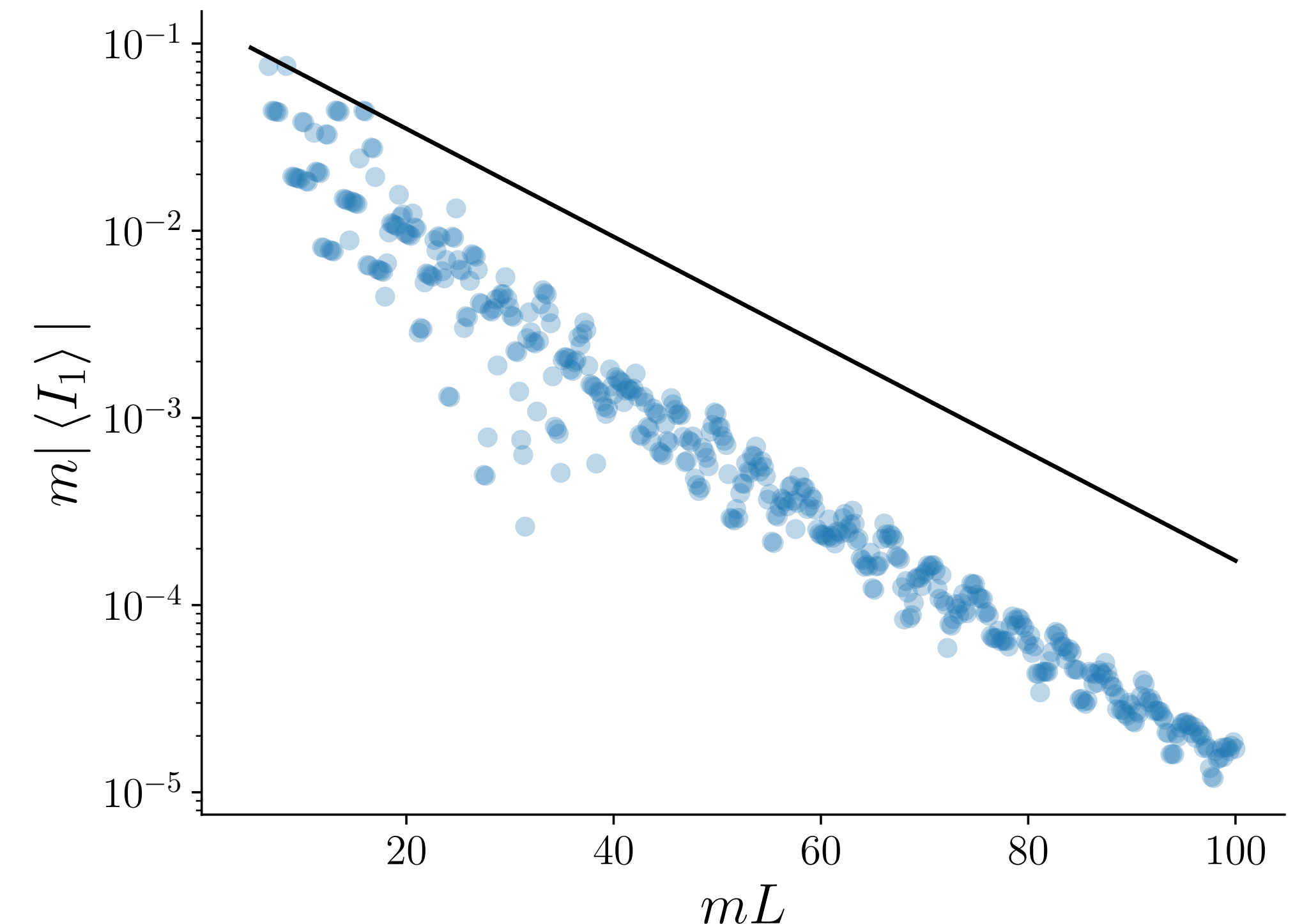
$$\left(\frac{\sqrt{\Delta}}{L \|n\|} \right)^\nu K_\nu (L \|n\| \sqrt{\Delta})$$

Conclusions



- Using input from quantum computers, there are workflows, uniform across all reactions and kinematics, that allow us to study general scattering processes.
- We have a simple to implement formula that allows us to estimate finite-volume errors due to arbitrary diagrams

$$\text{Error} = \sum_{\mathbf{n}} \frac{2}{(4\pi)^{N_L D/2}} \int_{[0,1]^{N_P}} d^{N_P} u \delta\left(\sum_i u^i - 1\right) \times \\ g^{-D/2} \left\langle e^{iL n_a \cdot p^a} \right\rangle \left(\frac{2\sqrt{\Delta}}{L\|n\|} \right)^\nu K_\nu(L\|n\|\sqrt{\Delta})$$



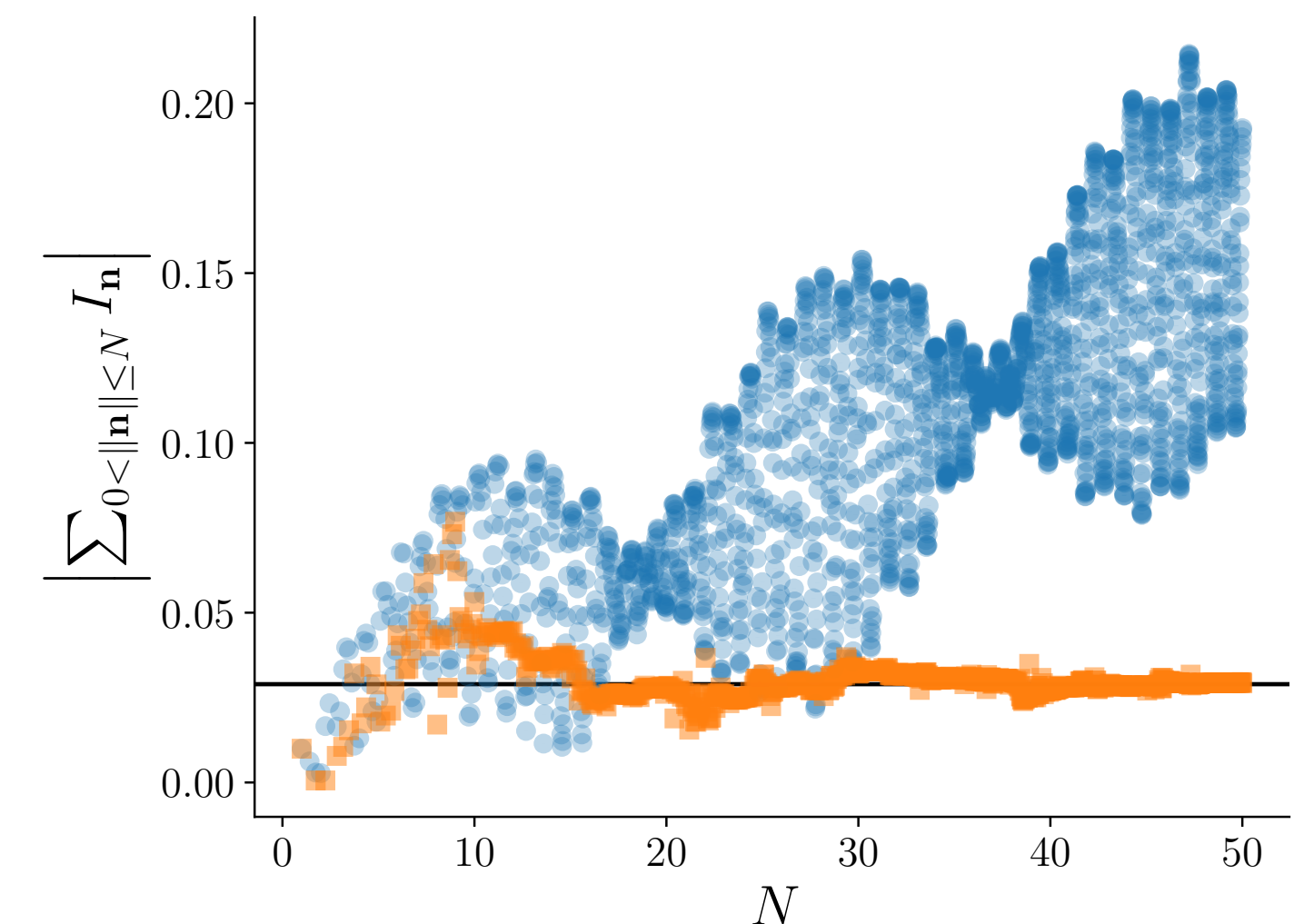
Conclusions

- Depending on the kinematic region of interest, there can be different types of error suppression

$$K_\nu(L\|n\|\sqrt{\Delta}) \sim e^{-L\|n\|\sqrt{\Delta}} \sim \begin{cases} e^{-L\|n\|\sqrt{\operatorname{Re} \Delta}}, & \operatorname{Re} \Delta > 0, \\ e^{-L\|n\|\epsilon/\sqrt{|\operatorname{Re} \Delta|}}, & \operatorname{Re} \Delta < 0. \end{cases}$$

- Polynomial errors are also under control

$$L^{N_P - (N_L D + 1)/2}$$



Thanks!



And be on the lookout for...

