Axial-vector form factors and EDM from Lattice QCD

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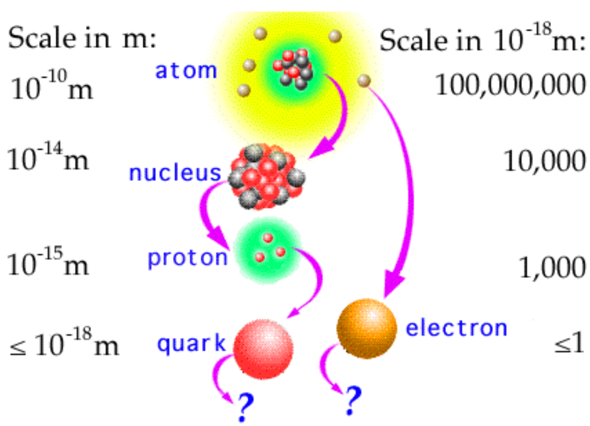
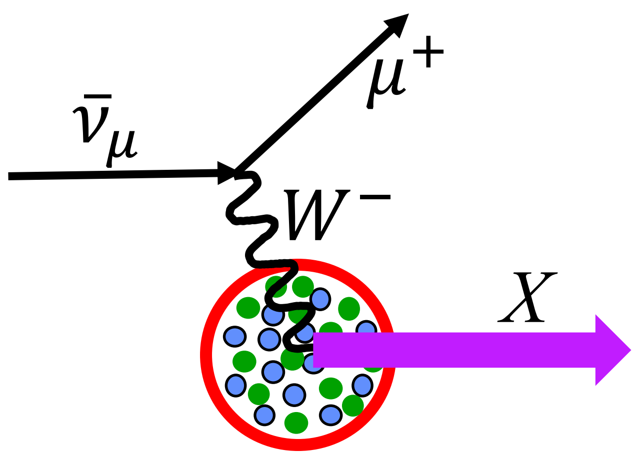
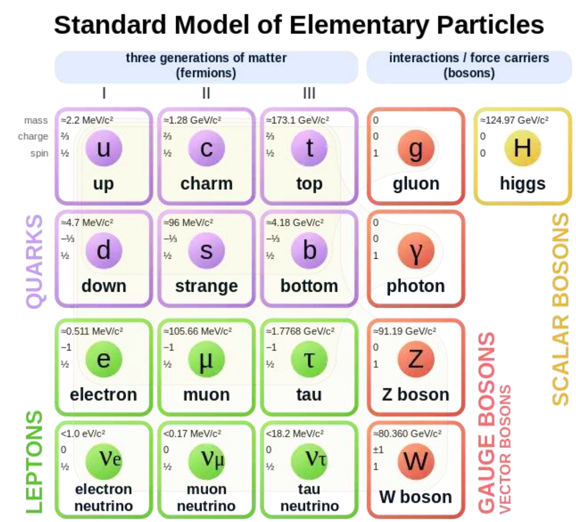






LA-UR: 26-26651

Lattice 2026, Univ of Maryland, July 27, 2026

Abstract

This talk gives an overview of the calculation of many properties of the nucleon that probe physics of the standard model and beyond using lattice QCD. The include the axial vector form factors for the international flagship experiment DUNE; the electric dipole moment of the nucleons and charge-conjugation violating pion-nucleon couplings, $\bar{g}_0, \bar{g}_1$; radiative corrections to neutron β −decay; and first moments of parton distribution functions.

Acknowledgements

Many thanks to my PNDME and NME collaborators

Tanmoy Bhattacharya, Vincenzo Cirigliano, Fangcheng He, Yong-Chull Jang, Balint Joo, Huey-Wen Lin, Emanuele Mereghetti, Santanu Mondal, Sungwoo Park, Oleksandr (Sasha) Tomalak, Frank Winter, Jun-sik Yoo, Boram Yoon

Computer Awards at

OLCF (INCITE HEP133)
OLCF (ALCC HEP145)
ERDCAP@NERSC (HEP, NP)
USQCD@JLAB, USQCD@BNL, USQCD@FNAL
LANL Institutional Computing

Acknowledge the MILC collaboration for HISQ ensembles

Outline

- High Statistics: MILC Ensembles
- Controlling excited states to get ground-state nucleon ME

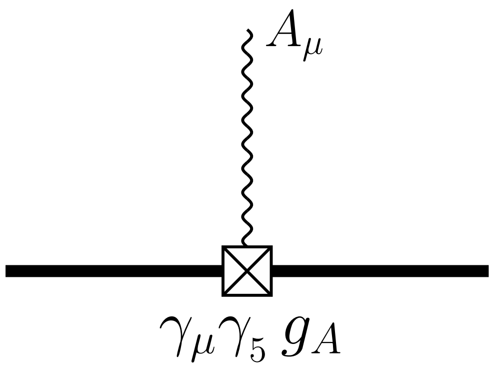
- Status of results for AVFF for the nucleon
- The pion-nucleon CP violating coupling $\bar{g}_0$
- Radiative corrections to neutron decay
- Second Mellin moments of the nucleon

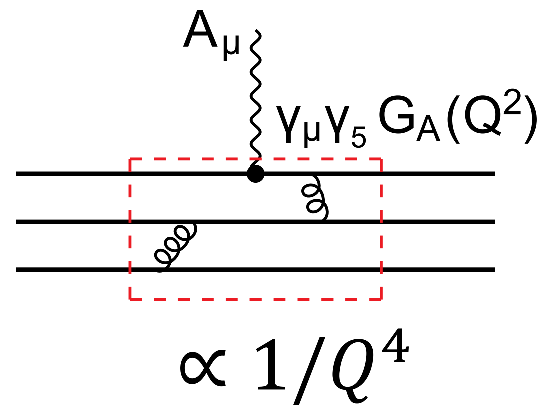
Also see talks by
Sungwoo Park: EM form factors: Th@2.40
Jun-sik Yoo: Isovector charges: Wed@10.20

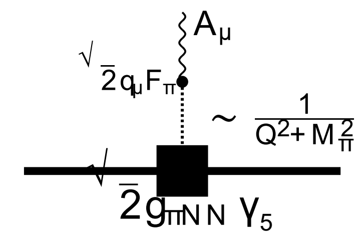
MILC: High statistics HISQ ensembles → new Clover-on-HISQ Calculation

	a	M_π	M_K	$M_\pi L$	$L^3 T$	# Conf
a09m135	0.0873	135	494	3.80	$64^3 96$	$\frac{6060}{8{,}000+}$
a06m135	0.0564	135	497	3.78	$96^3 192$	$\frac{8500}{10{,}000}$
a04m135	0.0426	136	495	4.21	$144^3 288$	$\frac{0}{2{,}000}$

1) $\Gamma^n \rightarrow ME \rightarrow$ Axial Form Factors, $G_A, \tilde{G}_P, G_P$







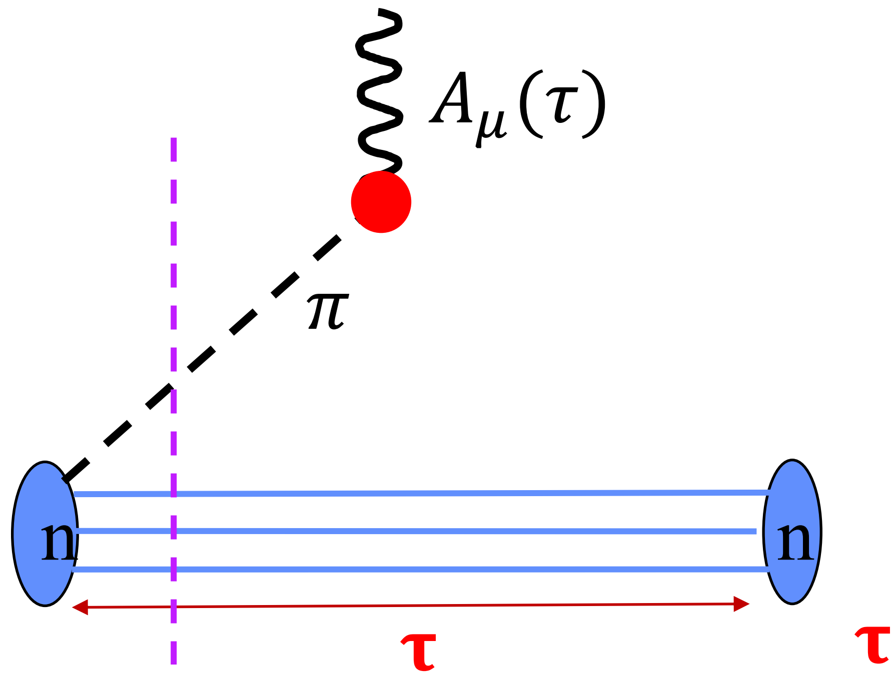
On each [iso-symmetric] ensemble characterized by $\{a, M_\pi, M_\pi L\}$
the ground state matrix element can be decomposed into FF as

$$\langle N(p_f) | A^\mu(q) | N(p_i) \rangle = \bar{u}(p_f) \left[\gamma^\mu G_A(q^2) + q_\mu \frac{\tilde{G}_P(q^2)}{2M} \right] \gamma_5 u(p_i)$$

$$\langle N(p_f) | P(q) | N(p_i) \rangle = \bar{u}(p_f) G_P(q^2) \gamma_5 \, u(p_i)$$

PCAC [$\partial_\mu A_\mu = 2mP$] relates $G_A, \; \tilde{G}_P, \; G_P$

χPT: coupling to $N\pi$ state is large in the axial channel



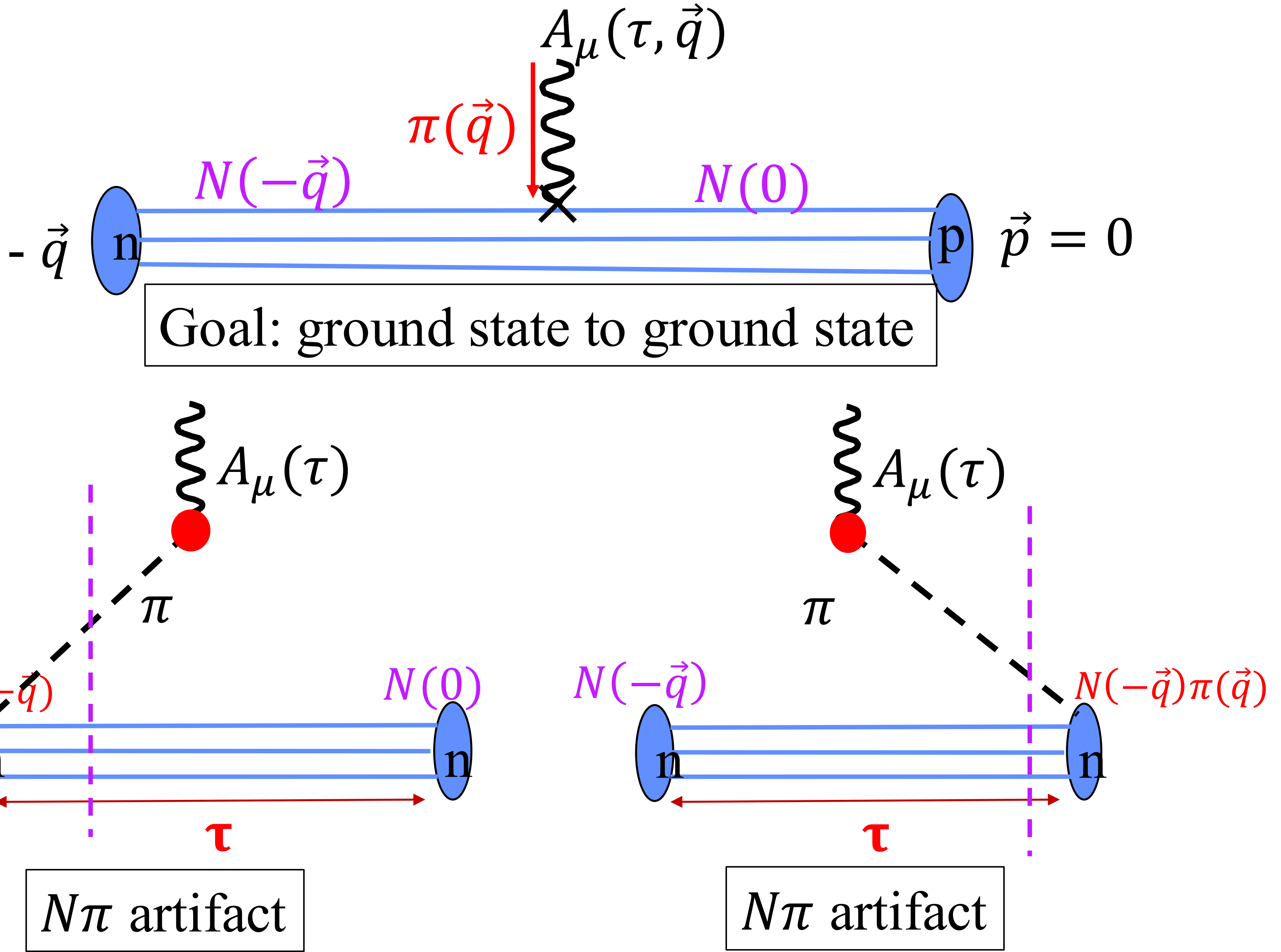
Enhanced coupling to $N\pi$ state: Since the pion is light, the vertex ● can be anywhere in the lattice 3-volume

$$A_i^* \quad \langle i|A_4|j\rangle$$

$\sim V^{-1}$ $\qquad\qquad$ $\sim V$

χPT: Oliver Bär: Phys. Rev. D 99, 054506 (2019), Phys. Rev. D 100, 054507 (2019)

$N\pi$ state in the axial channel



Data Driven Method: Simultaneous fit to all 4 correlators

PRL 124, 072002 (2020)

With "3" the direction of spin projection

$$\langle N(p_f)|A_{1,2}(q)|N(p_i)\rangle \to -\frac{q_{1,2}q_3}{2M}\tilde{G}_P$$

Gives $\tilde{G}_P$

$$\langle N(p_f)|A_3(q)|N(p_i)\rangle \to -[\frac{q_3^2}{2M}\tilde{G}_P - (M+E)G_A]$$

Gives both G_A, $\tilde{G}_P$

$$\langle N(p_f)|A_4(q)|N(p_i)\rangle \to -q_3\,[\frac{E-M}{2M}\tilde{G}_P - G_A]$$

Redundant. Dominated by excited states

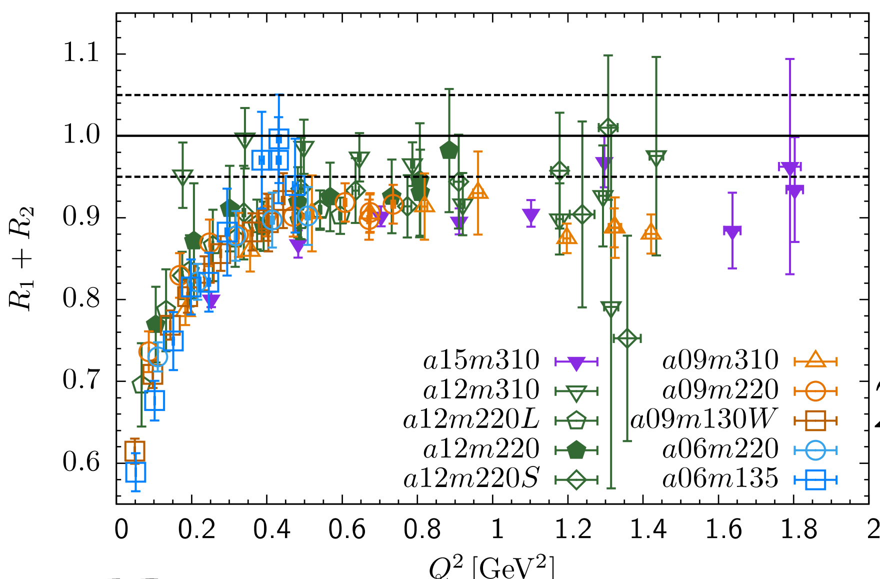
$$\langle N(p_f)|P(q)|N(p_i)\rangle = \bar{u}(p_f)G_P(q^2)\gamma_5\,u(p_i)$$

M_1 fixed by A_4: Resulting FF satisfy ($\partial_\mu A_\mu = 2\hat{m}$P)

$$2\hat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

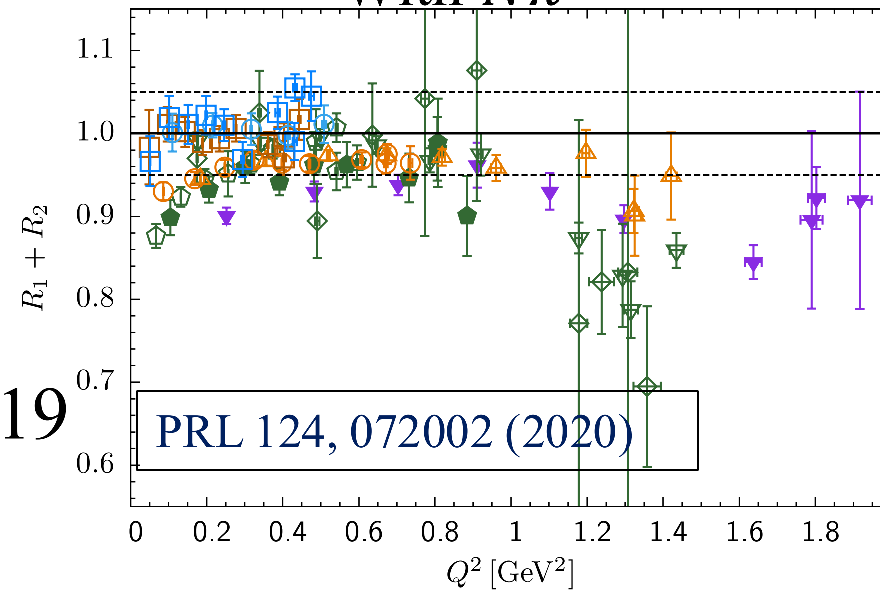
Check 1: FF with $N\pi$ satisfy PCAC relation

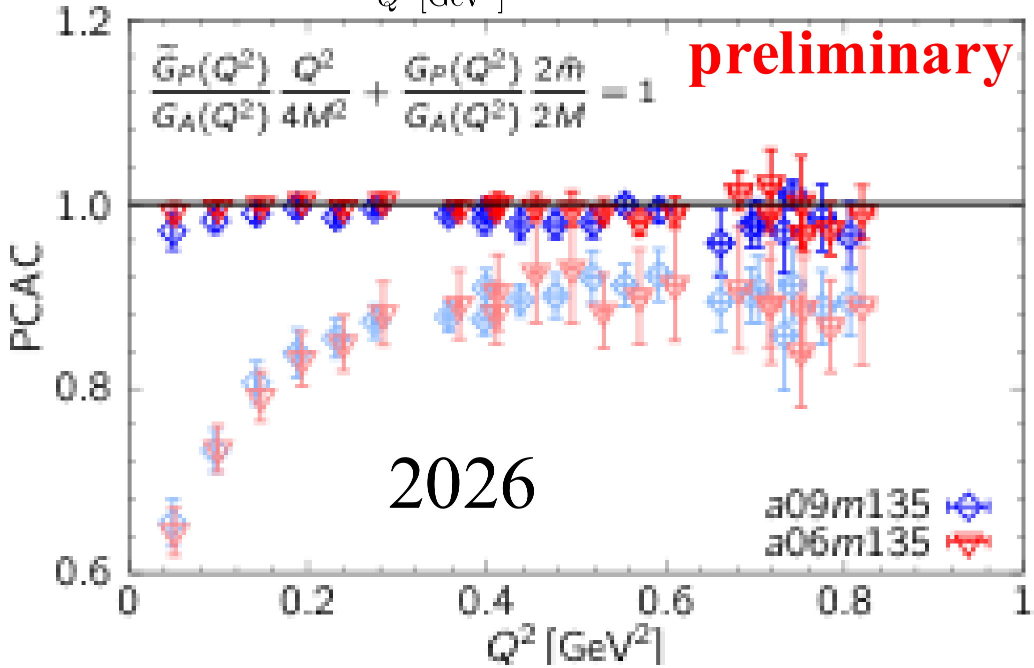
Standard Analysis



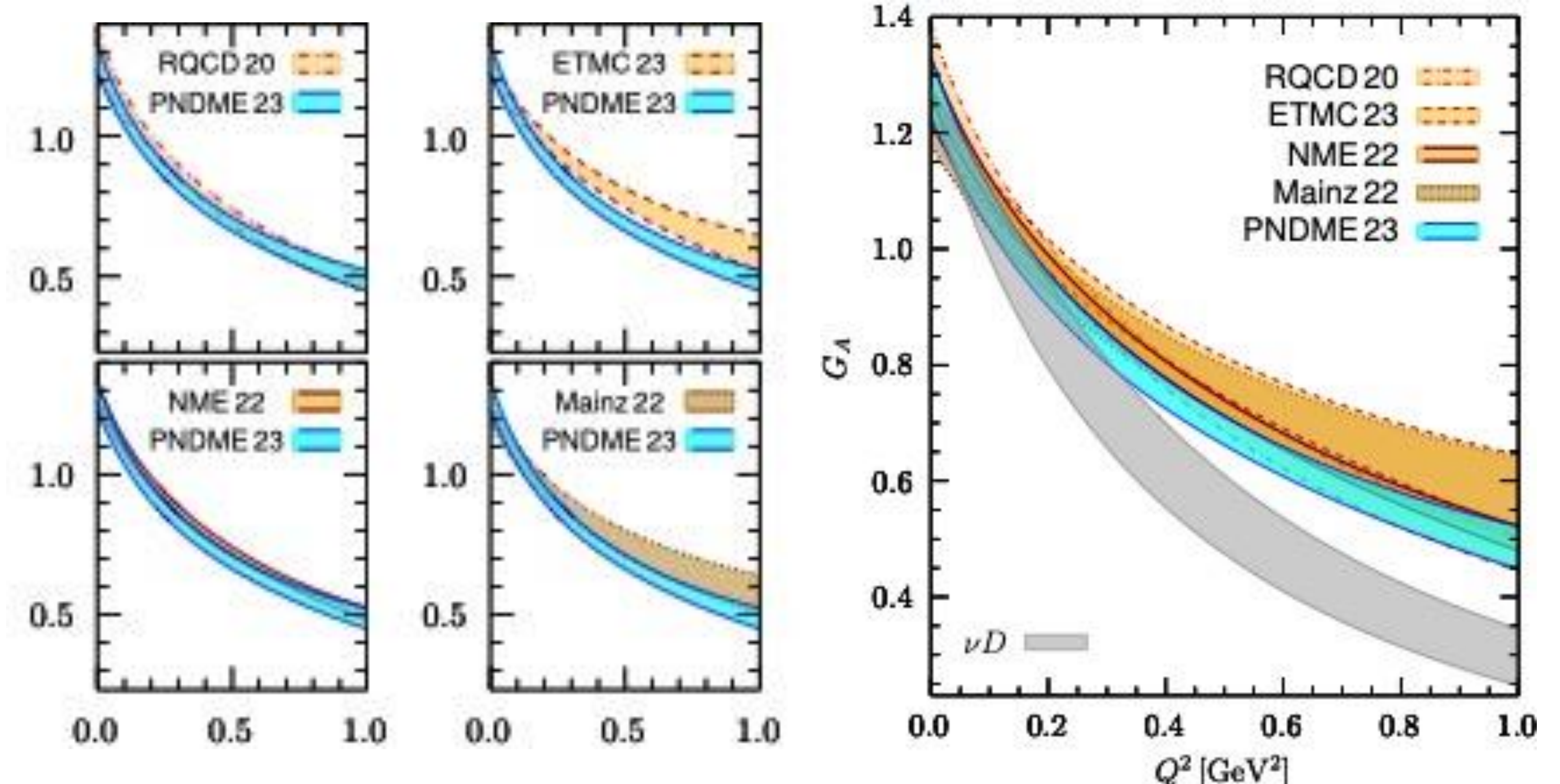
2019

With $N\pi$





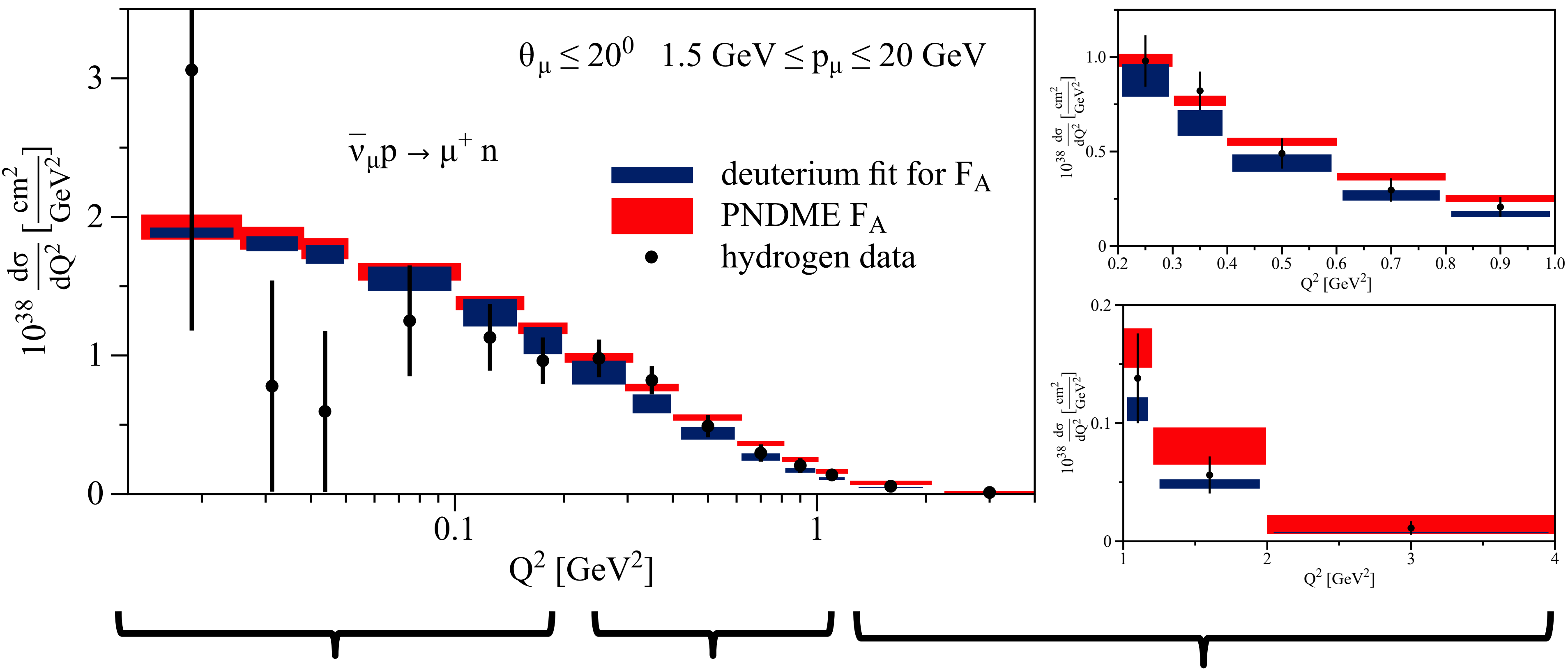
Comparing axial form factor from LQCD



PNDME:
$g_A = 1.281(53)$
$\langle r_A^2 \rangle = 0.498(56)\mathrm{fm}^2$

Status @ lattice2023

Comparing prediction of x-section using AFF from $\nu - D$ and PNDME with MINERvA data

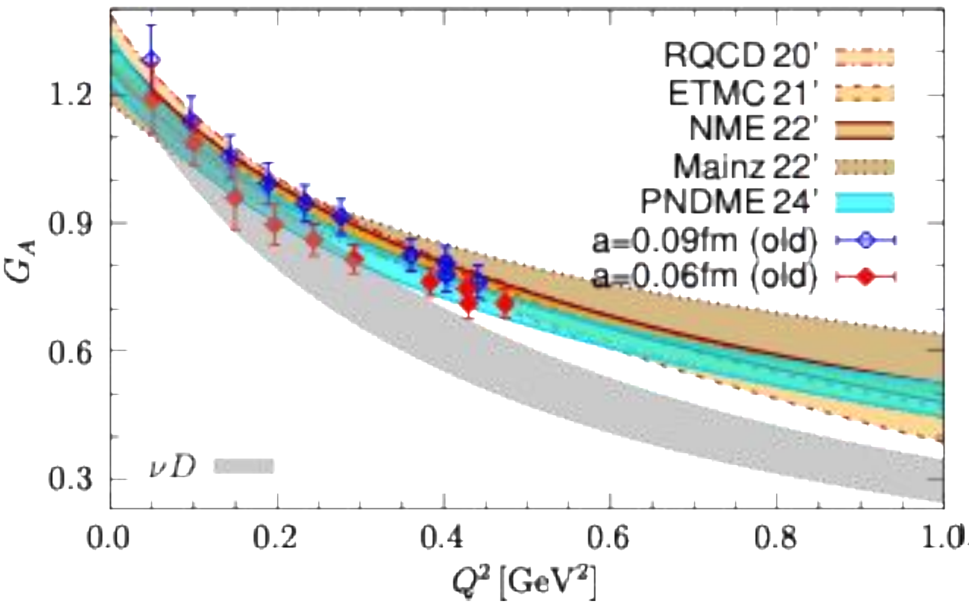


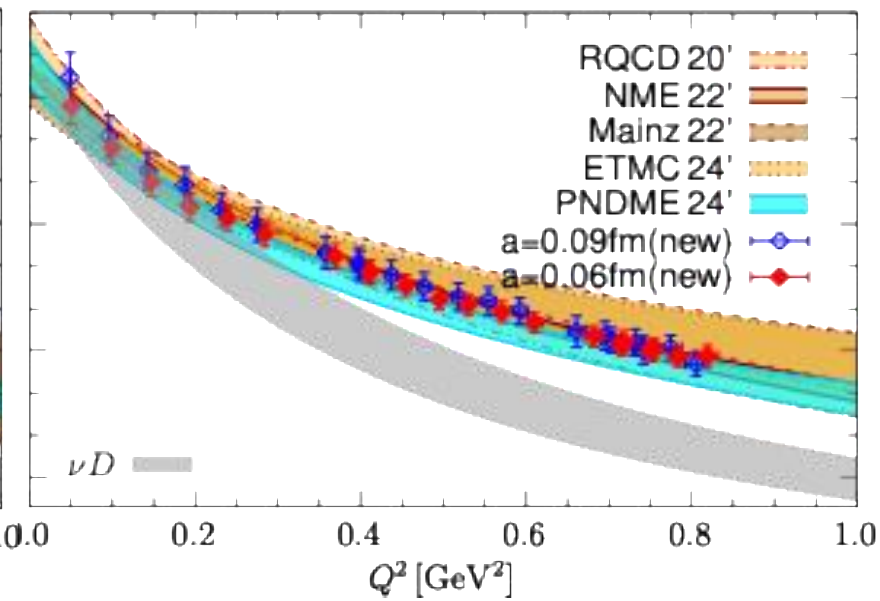
$Q^2 < 0.2\ \mathrm{GeV}^2$ $\quad 0.2 < Q^2 < 1$ $\quad Q^2 > 1\ \mathrm{GeV}^2$

T. Cai, et al., (MINERvA) Nature **volume 614**, pages 48–53 (2023); Phys. Rev. Lett. 130, 161801 (2023)

Oleksandr Tomalak, Rajan Gupta, Tanmoy Bhattacharya, PRD 108 (2023) 074514

IMPROVEMENT IN AVFF



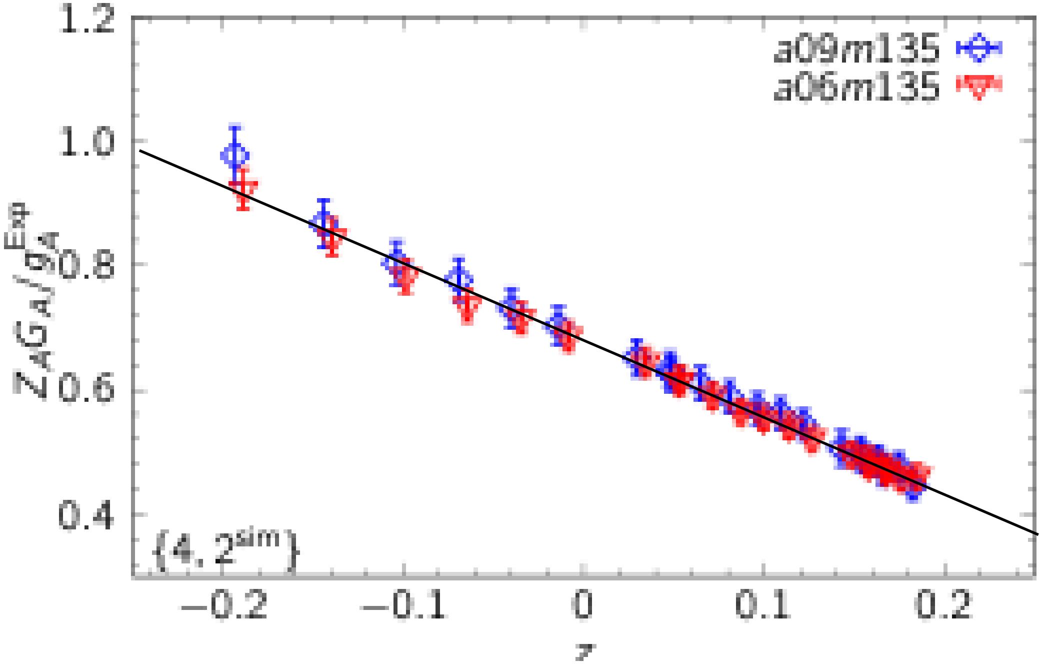


2019, 2024

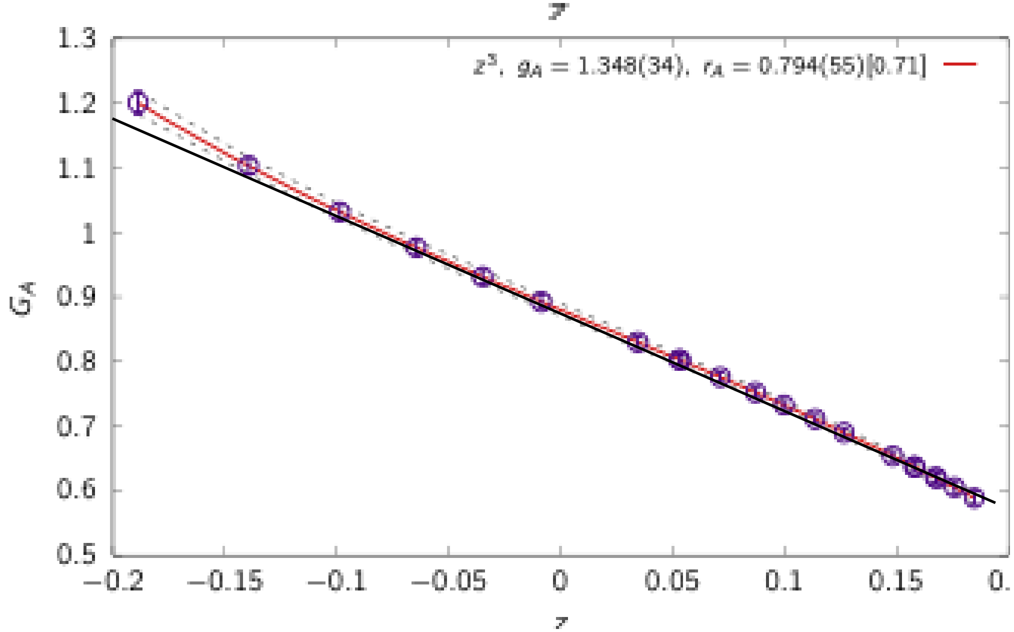
2026

form factor from νD data is not preferred

Z-expansion fit to the form factor



Data are almost linear in z

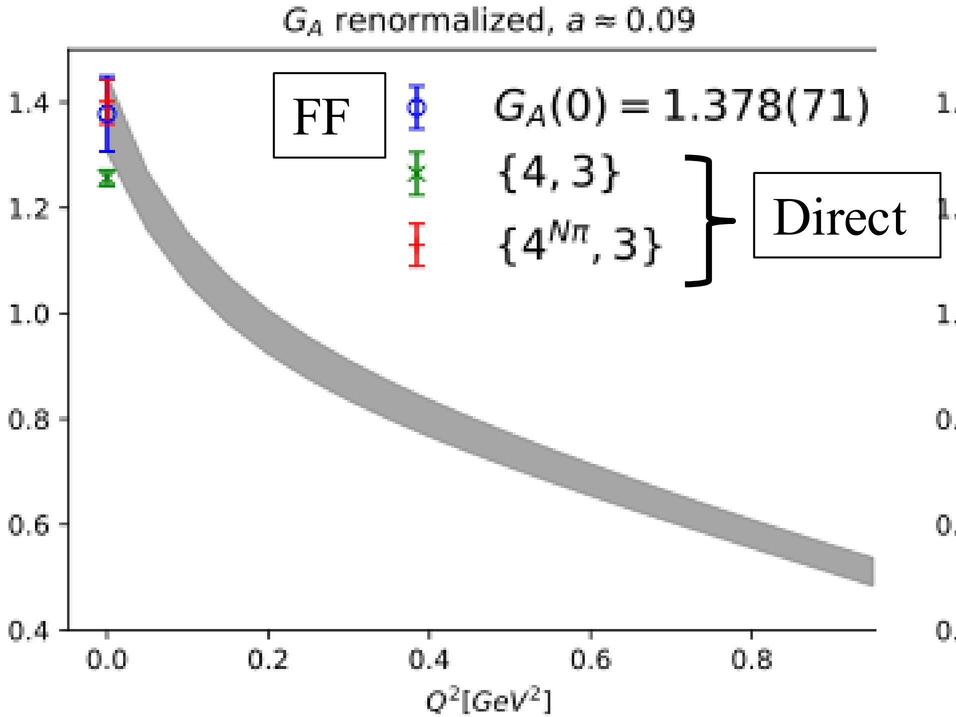


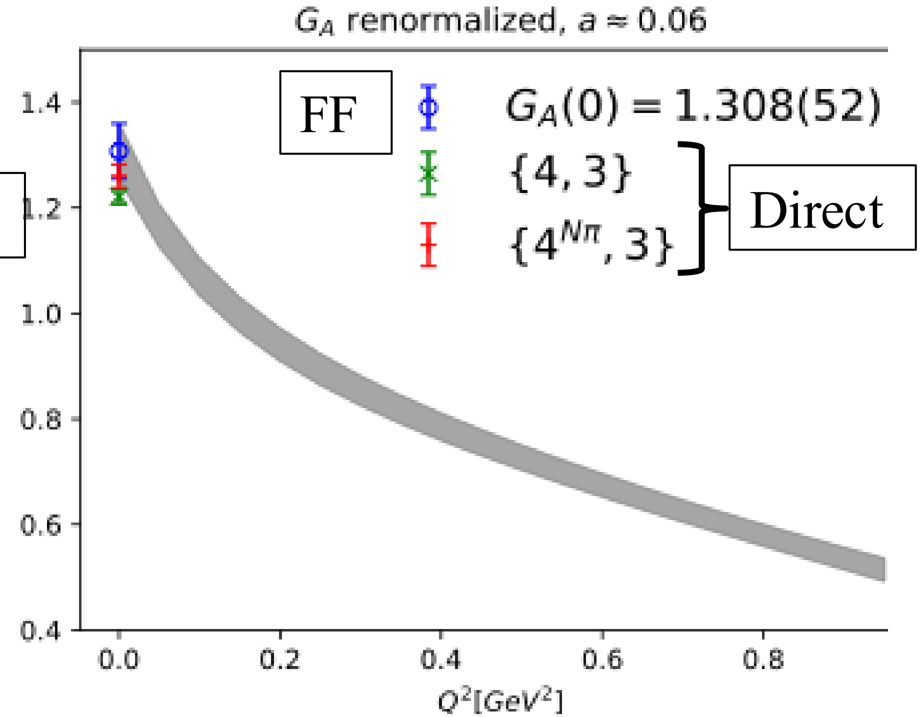
Z^3-expansion fits the data

Check 2:

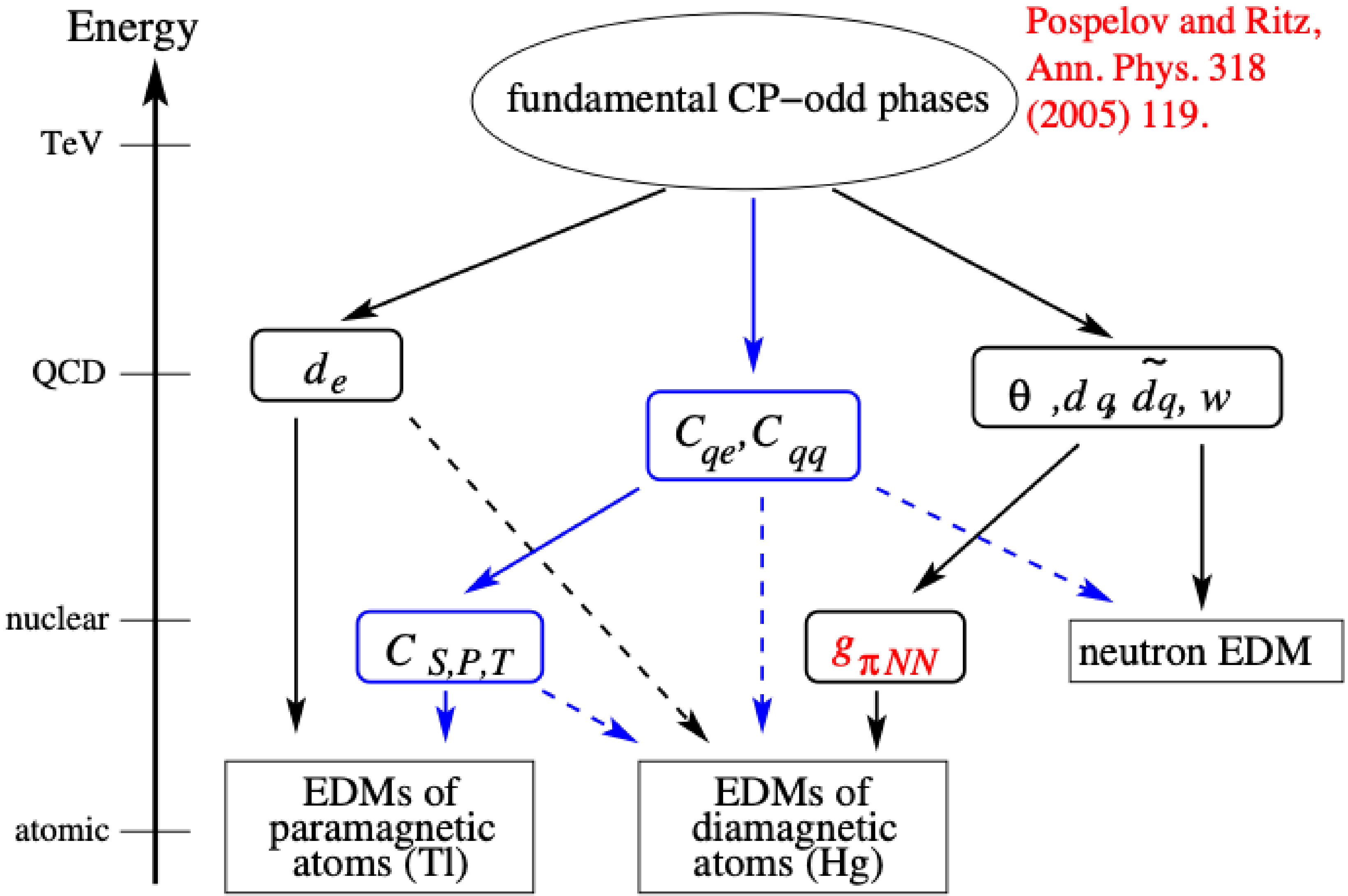
The 2 extractions of g_A must agree at $a = 0$

- Extrapolation of the FF to $Q^2 = 0$
- Direct calculation from the forward matrix element





2) EDMs as probes of BSM Physics



Pospelov and Ritz, Ann. Phys. 318 (2005) 119.

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Effective CPV Lagrangian at Hadronic Scale

$$
\begin{aligned}
\mathcal{L}_{\mathrm{CPV}}^{d\le 6} = &-\frac{g_s^2}{32\pi^2}\bar{\theta} G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
&-\frac{i}{2}\sum_{q=u,d,s,c} d_q \bar{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
&-\frac{i}{2}\sum_{q=u,d,s,c} \tilde{d}_q g_s \bar{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (qcEDM)} \\
&+ d_w \frac{g_s}{6} G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
&+ \sum_i C_i^{(4q)} O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

- $\bar{\theta} \le \mathcal{O}(10^{-8} - 10^{-11})$: Strong CP problem
- Dim=5 terms suppressed by $d_q \approx \langle \mathrm{v}\rangle/\Lambda_{BSM}^2$; effectively dim=6
- Dim=6 terms suppressed by $d_W \approx \langle 1\rangle/\Lambda_{BSM}^2$
- All terms up to $d = 6$ are leading order

Each CPV interaction contributes to:
EDMs of neutrons and protons, $\{d_n, d_p\}$
CPV pion-nucleon couplings $\bar{g}_0, \bar{g}_1, \bar{g}_2$

Constraining CPV in BSM models and Baryogenesis

At lowest order in chiral EFT, the CPV πNN Lagrangian is

$$L = -\frac{\bar{g}_0}{2F_\pi}\,\bar{N}\,\pi\cdot\tau N - \frac{\bar{g}_1}{2F_\pi}\,\pi_0\bar{N}\,N - \frac{\bar{g}_2}{2F_\pi}\,\pi_0\bar{N}\,\tau^3 N$$

$\{\bar{g}_0, \bar{g}_1, \bar{g}_2\}$ contribute to EDMs of diamagnetic atoms (e.g. Hg)

Need to measure EDMs of at least 4 complementary systems which can be analyzed in terms of 4 couplings $\{d_n, d_p, \bar{g}_0, \bar{g}_1\}$

$\{d_n, d_p, \bar{g}_0, \bar{g}_1\}$ get contributions from each of the EFT operators

Contributions of Effective CPV Lagrangian to $\{d_n, d_p\}$
Method known → first calculations

- Quark EDM: PHYSICAL REVIEW D 98, 091501 (2018)
  - The EM current acquires a CPV piece
  - $\{d_n, d_p\} = \sum_q d_q g_T^q$
- Θ-term: PHYSICAL REVIEW D 103, 114507 (2021)
  - Calculate 4-point function of correlation of Θ-term with J^μ_{EM}
  - $\{d_n, d_p\} = \bar{\Theta} \lim_{q^2 \to 0} \frac{F_3(q^2)}{2M_N \, \bar{\Theta}}$
- Quark chromo EDM: PHYSICAL REVIEW D 108, 074507 (2023)
  - Calculate 4-point function of correlation of qcEDM with J^μ_{EM}
  - $\{d_n, d_p\} = \tilde{d}_q \lim_{q^2 \to 0} \frac{F_3(q^2)}{2M_N \, \tilde{d}_q}$

Contributions of Effective CPV operators to $\{\bar{g}_0, \bar{g}_1\}$

- Operator = Θ-term or Quark chromo EDM:
  - Form factor method
  - Calculate 4-point function correlating CPV Operator with P or $\partial_4 A_4$
  - $[-\frac{Q^2}{2M_N}\tilde{G}_{S,A}^{0,1} + 2m_q\tilde{G}_S^{0,1}] = \bar{g}_{0,1}$

G_S^I is the scalar form factor in isospin channel I

- Problem with the form-factor method:
  - $\langle N\ \partial_\mu A_\mu\ O\ \bar{N}\rangle$ and $\langle N\ P\ O\ \bar{N}\rangle$ dominated by $N\pi$ excited state
  - Numerical data explained in χPT with large $N\pi$ contamination
  - The $N\pi$ cancel in the combination $2m_q P - \partial_\mu A_\mu$ (shown in χPT & data)
  - But the resulting statistical signal using $2m_q P - \partial_\mu A_\mu$ is POOR

Paper to appear in a few weeks

AWI (PCAC) to the rescue

- Θ-term (rotated to the imaginary quark mass basis):
- $L \to L_{QCD} - i\, m_* \, \overline{\Theta} \int d^4x\, \bar{\psi}\, \gamma_5\, \psi$
- Isovector AWI: $(\partial_\mu A_\mu^{u-d} - 2mP^{u-d})|_{CPV} = 2\, i\, m_*\, \overline{\Theta}\, S^{u-d}$
- The right-hand side is the isovector scalar charge g_S^{u-d} (No enhanced $N\pi$)
- $\bar{g}_0 = \overline{m}\,(1 - \epsilon^2)\, \overline{\Theta}\, g_S^{u-d}$
- $\frac{\bar{g}_0}{2F_\pi} = (17.4 \pm 1.9)10^{-3}\, \overline{\Theta}.$ (Taking m_* and S^{u-d} from FLAG 2024)

- In progress: Quark chromo EDM (beware complications of renormalization):
- $L \to L_{QCD} - \frac{i}{2} \sum_{q\in\{u,d,s,c,b\}} \tilde{d}_q \left[\bar{q}\, \sigma_{\mu\nu}\, G^{\mu\nu}\, \gamma_5\, q\right]$
- $$(\partial_\mu A_\mu^{u-d} - 2mP^{u-d})|_{CPV} = i\, \frac{\tilde{d}_u + \tilde{d}_d}{2} \left[\bar{q}\, \sigma_{\mu\nu}\, G^{\mu\nu}\, q\right]_{u-d} + i\, \frac{\tilde{d}_u - \tilde{d}_d}{2} \left[\bar{q}\, \sigma_{\mu\nu}\, G^{\mu\nu}\, q\right]_{u+d}$$

3) Test SM unitarity of CKM matrix

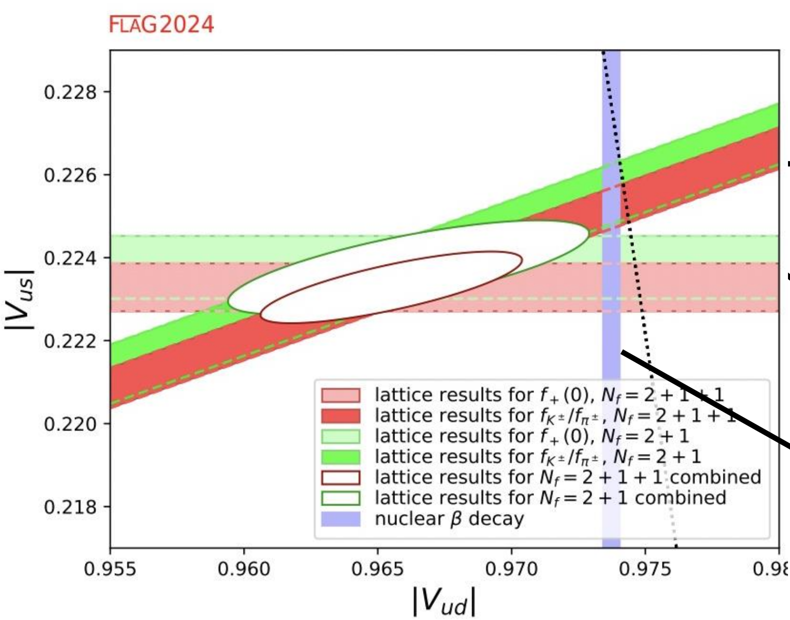
Radiative Corrections to Neutron $\boldsymbol{\beta}$ −Decay

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

Standard Model: 3-family unitary condition

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1,$$

$|V_{ub}|^2 \approx (2 \pm 0.4) \times 10^{-5}$ Is negligible



$|V_{us}|/|V_{ud}| = 0.23126(50)$ leptonic decays $K^+ \to \mu^+\nu|$

$|V_{us}| = 0.22328(58)$ kaon semileptonic decays $(K \to \pi\ell\nu_\ell)$

$|V_{ud}| = 0.97373(31).$ nuclear $(0^+ \to 0^+)$ beta decay $^{10}C \to ^{10}B + e^+ + \nu_e$

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$|V_{ud}|$ from neutron β –Decay

$$|V_{ud}|^2 = \frac{5099.3(4)\ \text{sec}}{\tau_n(1+3g_A^2)(1+RC)}$$

A. Sirlin, Rev. Mod. Phys. 50, 573 (1978)

A. Czarnecki, W. Marciano, and A. Sirlin PRD 70(2004)

Neutron lifetime (sec)

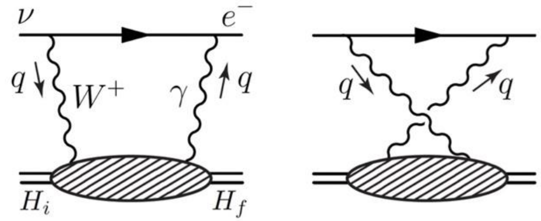
Axial charge

Radiative Correction)

$$RC = \frac{\alpha}{2\,\pi}\left(g(E) + 4\,ln\frac{m_Z}{m_p} + \Delta_{np}\right)$$

Not affected by
strong interaction.
Small uncertainty

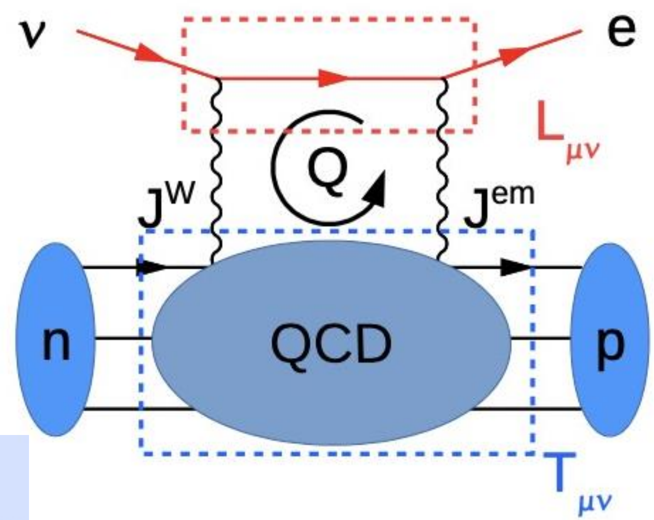
γW box contribution
(Dominates the uncertainty of RC)



First results from P. Ma, el.al, *Phys.Rev.Lett.* 132 (2024) 19, 191901

γW-Box diagram

$$\Box^{VA}_{\gamma W} = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2} M_n(Q^2).$$



Hadronic Form Factor

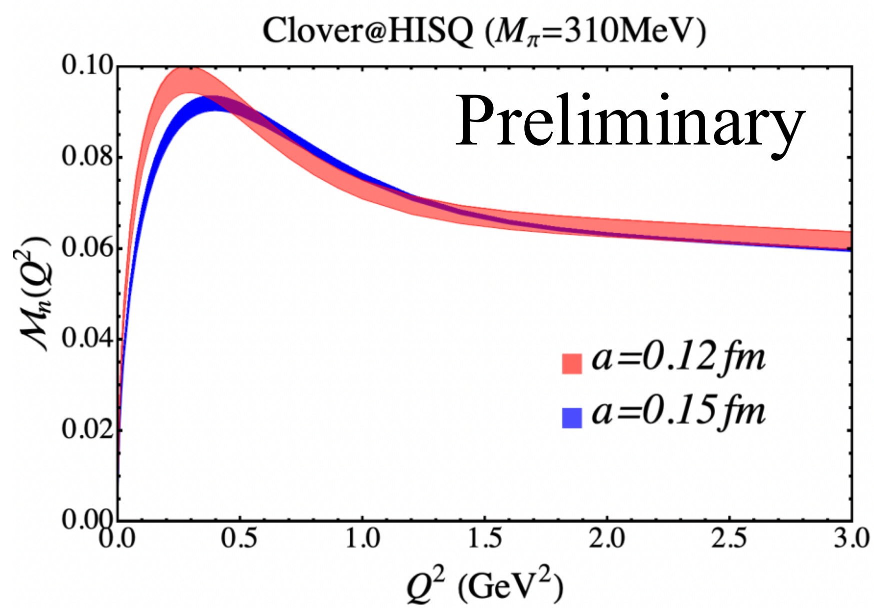
$$\mathcal{M}_H\left(Q^2\right) = -\frac{1}{6}\frac{1}{F_\perp^H}\frac{\sqrt{Q^2}}{M_H}\int d^4x\,\omega(\vec{x},t)\epsilon_{\mu\nu\alpha 0}\,x_\alpha\,\mathcal{H}^{VA}_{\mu\nu}(\vec{x},t)$$

Weighting function

$$\omega(t,\vec{x}) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^3\theta\,d\theta}{\pi}\,\frac{j_1\left(\sqrt{Q^2}|\vec{x}|\cos\theta\right)}{|\vec{x}|}\cos\left(\sqrt{Q^2}t\sin\theta\right).$$

Hadronic tensor

$$\mathcal{H}^{VA}_{\mu\nu}(t,\vec{x}) \equiv \langle H_f|T\left[J^{em}_\mu(t,\vec{x})J^{W,A}_\nu(0)\right]|H_i\rangle,$$



4) Moments of distributions

JHEP 21, 004 (2020);
arXiv:2608.02836

$$\langle x\rangle_q = \int_0^1 x[q(x)+\overline{q}(x)]dx$$

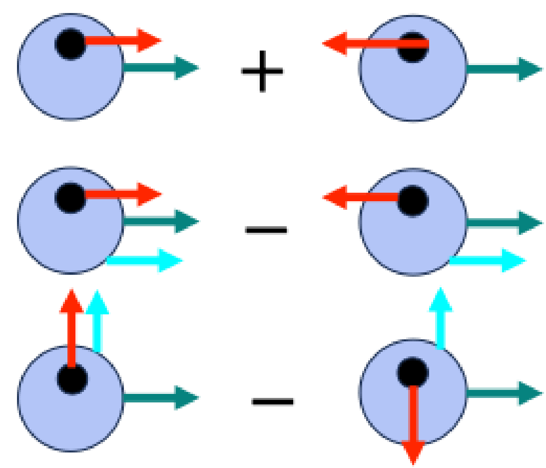
$$\langle x\rangle_{\Delta q} = \int_0^1 x[\Delta q(x)+\Delta\overline{q}(x)]dx$$

$$\langle x\rangle_{\delta q} = \int_0^1 x[\delta q(x)+\delta\overline{q}(x)]dx$$

Momentum fraction (unpolarized) $\langle x\rangle_q$

Helicity moment (polarized) $\langle x\rangle_{\Delta q}$

Transversity moment (polarized) $\langle x\rangle_{\delta q}$



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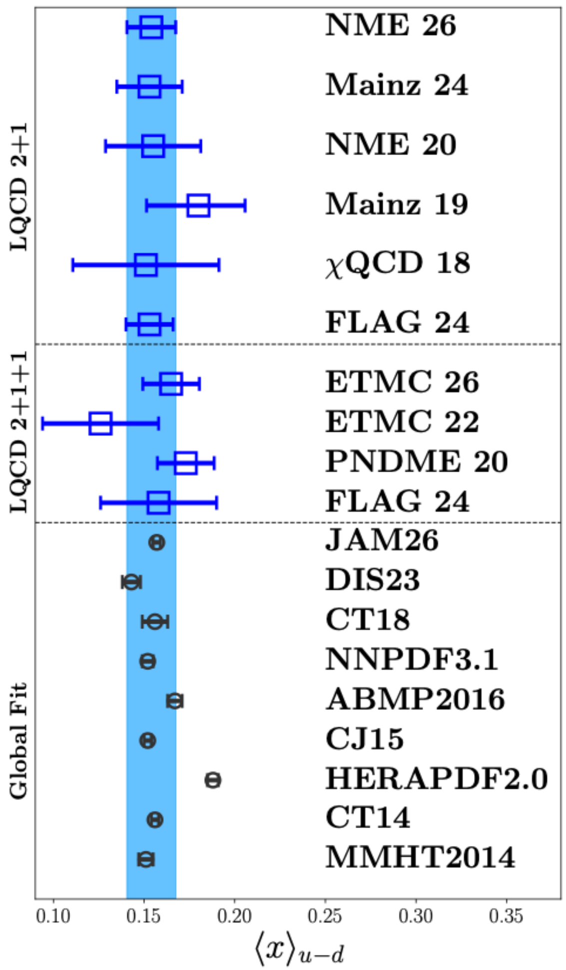
Status of results:

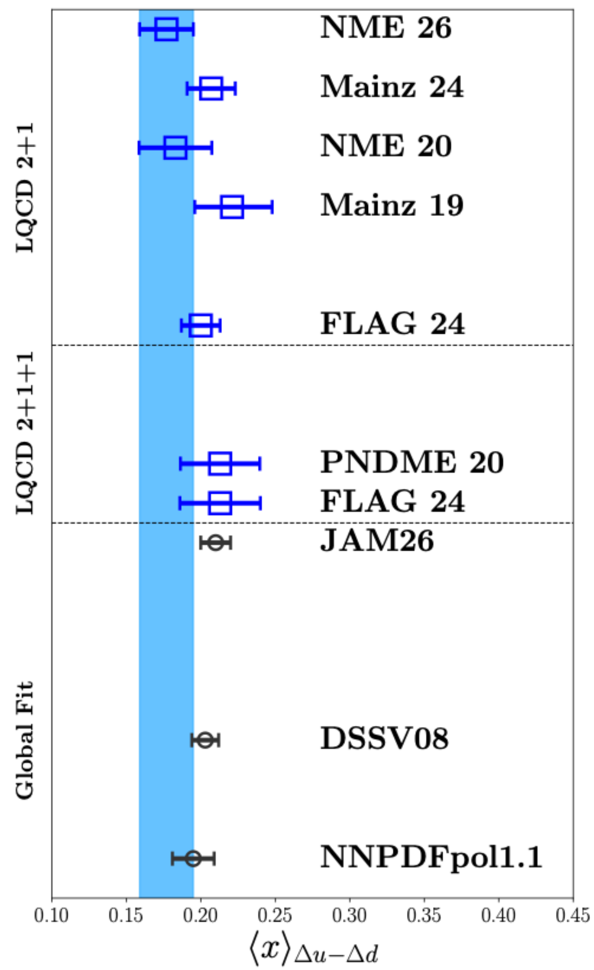
arXiv:2608.02836

$\langle x \rangle_q = 0.154(10)(9)$



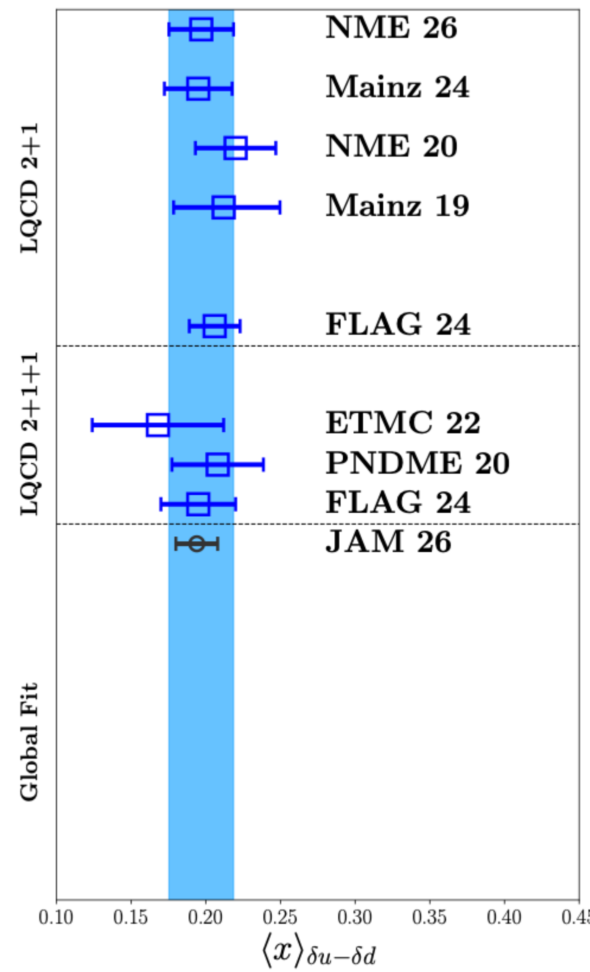
Momentum Fraction

$\langle x \rangle_{\Delta q} = 0.177(10)(15)$



Helicity Moment

$\langle x \rangle_{\delta q} = 0.197(12)(18)$



Transversity Moment

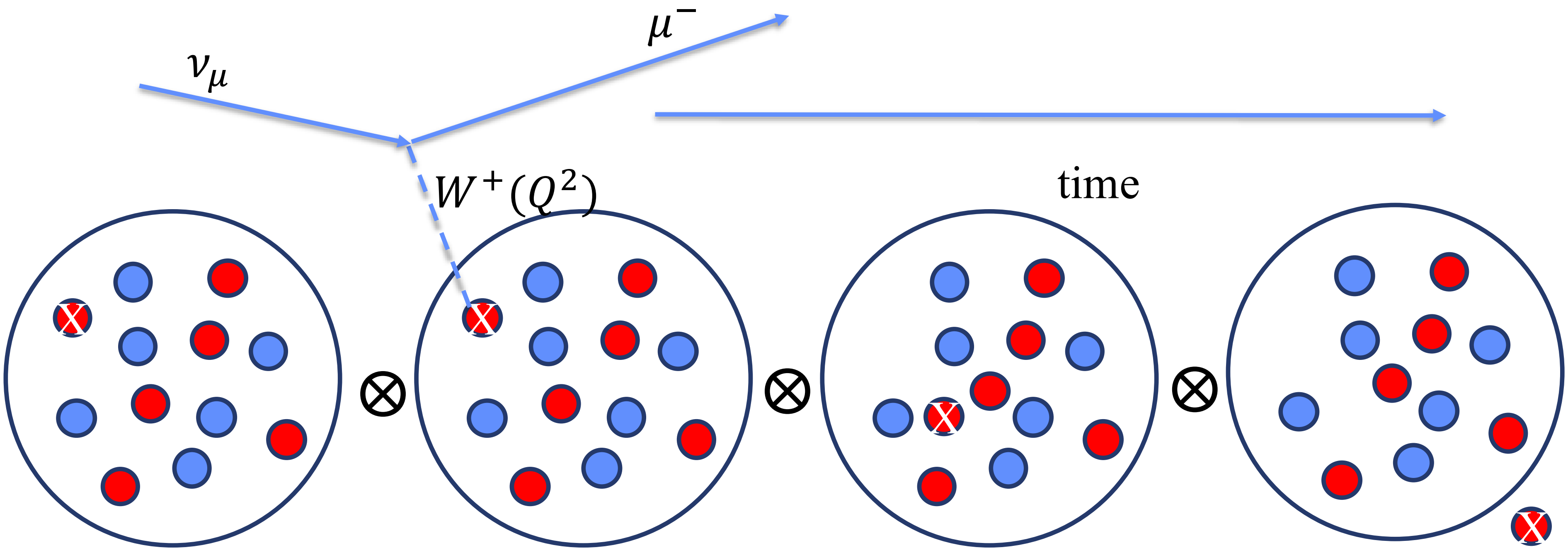
Summary

- High precision calculations of nucleon matrix elements probe many aspects of the standard model and physics beyond it
- Charges $g_{A,S,T}^{u,d,s,c}$
  - $\sigma_{\pi N} \equiv m_{ud}\, g_S^{u+d}$ is a fundamental parameter in nuclear physics
  - $g_{A,P,S,T}^{u,d,s,c}$: coupling of dark matter to nucleon in various channels
  - Contribution of quark spin [EDM] to nucleon spin [EDM] $g_A^{u,d,s,c}$ $[g_T^{u,d,s,c}]$
- Moments: $\langle x \rangle_q$; $\langle x \rangle_{\Delta q}$; $\langle x \rangle_{\delta q}$ provide constraints on PDFs
- Axial, Electric, Magnetic form factors (DUNE, EIC)
- Radiative corrections to pion, kaon, neutron β-decay: $\boldsymbol{V_{ud}}, \boldsymbol{V_{us}}$
- $\{d_n, d_p, \bar{g}_0, \bar{g}_1\}$: Connect EDMs to CPV in the SM and BSM

EXTRA

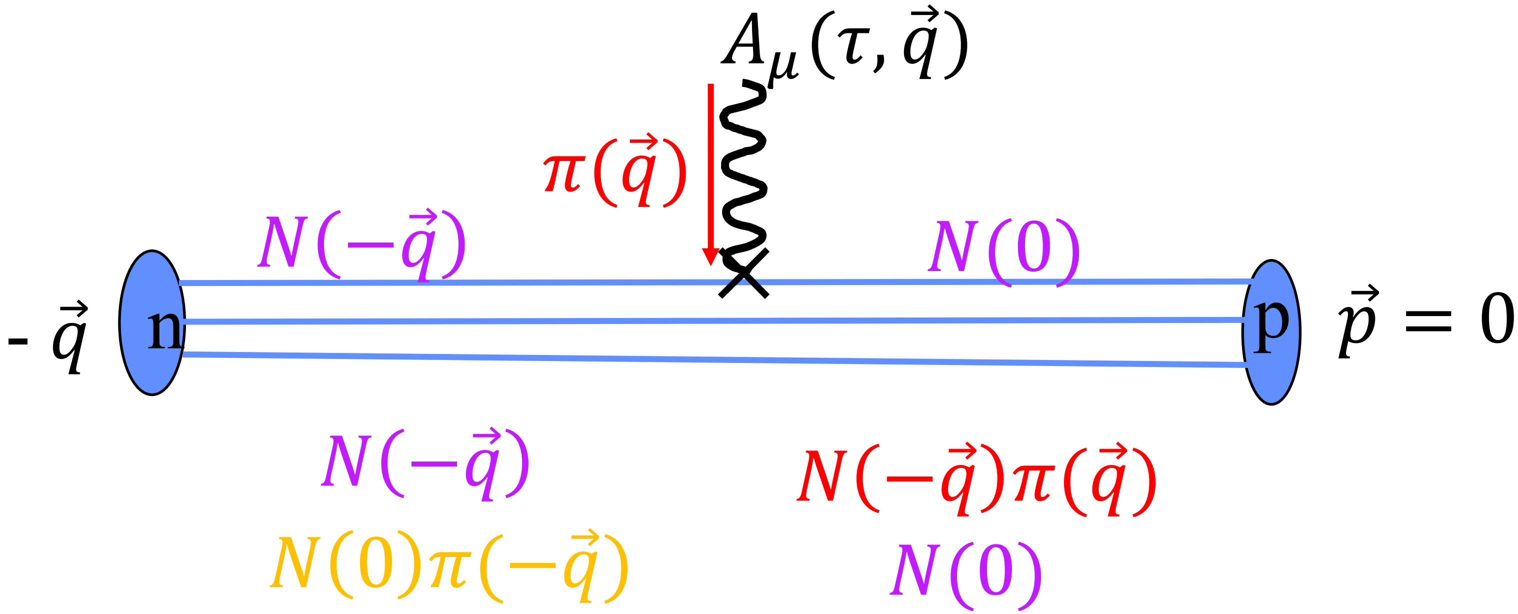
Neutrino-nucleus interaction involves convolution of 4 stages assuming factorization

Event generators provide a statistical description of various outcomes involving complex physics of quasi-elastic, resonant, …, DIS regions

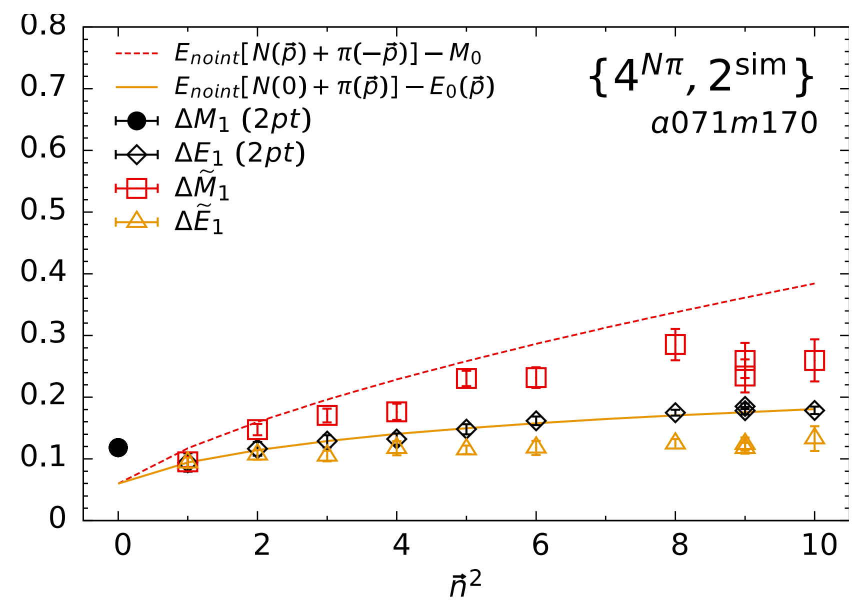


Nuclear theory	Lattice QCD	AFDMC	Calibration of signal
Determine the wavefunction of a nucleon within the target nucleus	Interaction with a nucleon is given by the axial vector form factor $G_A(Q^2)$	Evolution of struck nucleon within the nucleus governed by the nuclear force	Displaced nucleon once outside the nucleus produces signals picked up by detectors

$N\pi$ state in the axial channel



Mass gaps extracted from simultaneous fits to $A_{1,2}, A_3, A_4, P$ correlators match above picture



The pion-nucleon sigma term: Resolving tension between Lattice QCD and Phenomenology

$$\boxed{\sigma_{\pi N} \equiv m_{ud} g_S^{u+d} \equiv m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle}$$

FLAG Reports 2019, 2021, 2024:

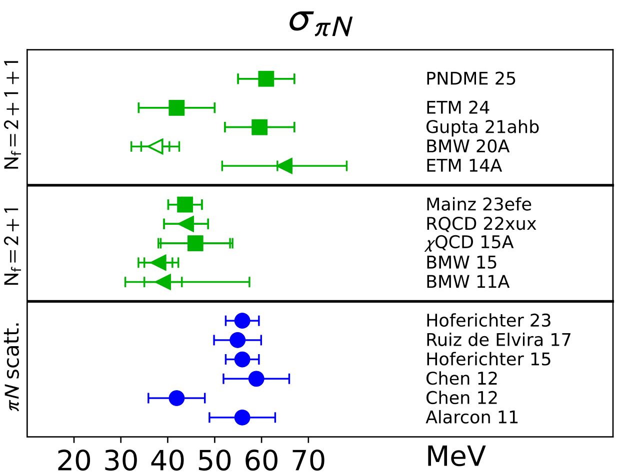
- Lattice results ~40 MeV
- Phenomenology favors ~60 MeV

BMW (arXiv:2007.03319) $\sigma_{\pi N}$ = 37.4(5.1) MeV (FH)

RQCD (JHEP 05 (2023) 035) $\sigma_{\pi N}$ = 43.9(4.7) MeV (FH)

Mainz (PRL 131 (2023) 261902) $\sigma_{\pi N}$ = 43.7(3.6) MeV (FH)

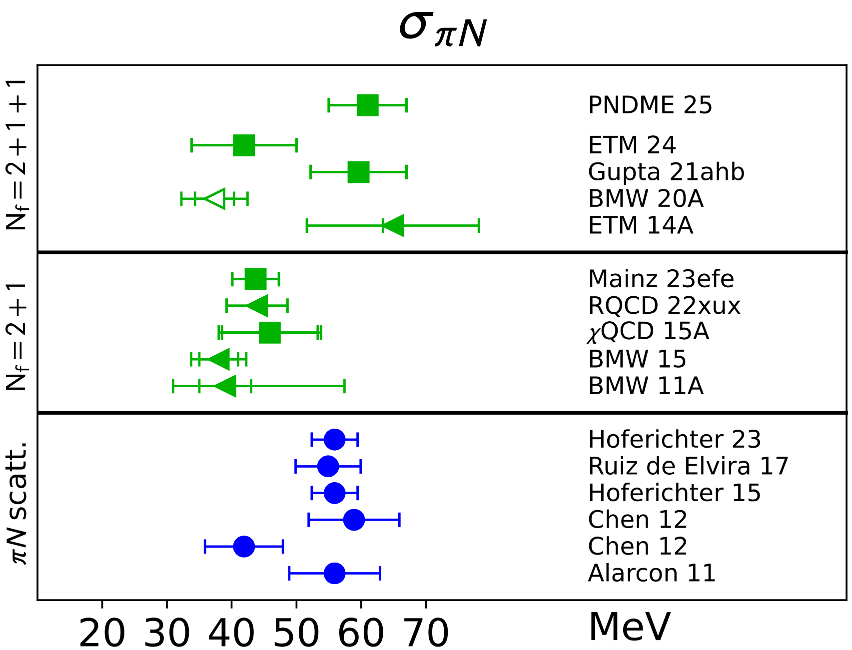
ETM (PRD **111**, 054505) $\sigma_{\pi N}$ = 41.9(8.1) MeV (Direct)

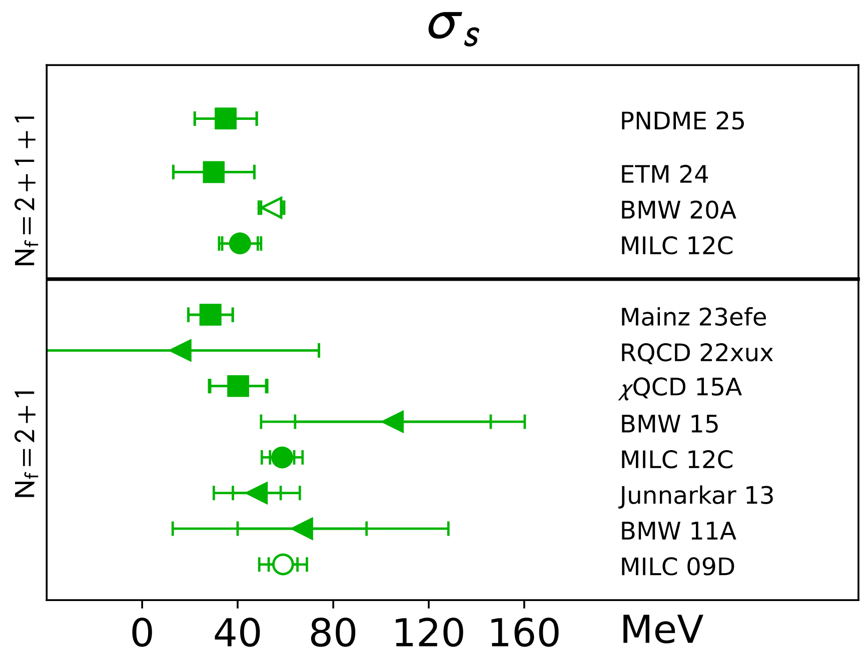


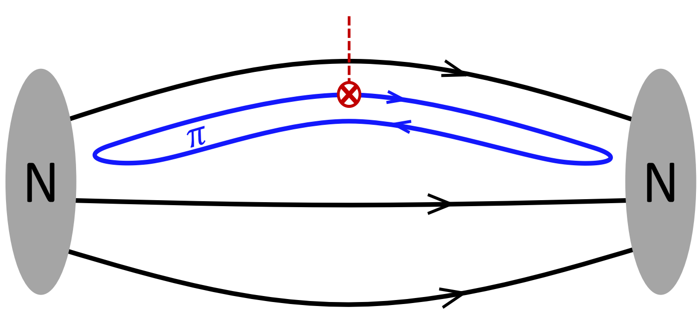
LANL Results: PRL 127 (2021) 242002; e-Print: 2503.07100

- Without including $N(\vec{k})\pi(-\vec{k})$ and $N(0)\pi(\vec{k})\pi(-\vec{k})$ states: = 42 (6) MeV
- Including $N(\vec{k})\pi(-\vec{k})$ and $N(0)\pi(\vec{k})\pi(-\vec{k})$ states: = 61 (6) MeV

Sigma terms







Enhanced contribution?

No enhancement expected as KΣ is the lowest state