Gradient flow and heavy-light physics

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color meets flavor

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P3H

Particle Physics Phenomenology after the Higgs Discovery

Gradient flow (GF)

- Standard tool for calculating scale setting ($\sqrt{8t_0}$), RG β-function, Λ parameter
  [Narayanan, Neuberger JHEP 03 (2006) 064] [Lüscher JHEP 08 (2010) 071][JHEP 04 (2013) 123], ...
- Introduce auxiliary dimension, flow time τ to regularize UV
  - $\rightarrow$ Well-defined smearing of gauge and fermion fields
  - $\rightarrow$ Smoothening UV fluctuations
- First order differential equation

$$\boxed{\begin{aligned} \partial_\tau B_\mu(\tau,x) &= \mathcal{D}_\nu(\tau) G_{\nu\mu}(\tau,x), \quad & B_\mu(0,x) &= A_\mu(x) \\ \partial_\tau \chi(\tau,x) &= \mathcal{D}^2(\tau)\chi(\tau,x), \quad & \chi(0,x) &= q(x) \end{aligned}}$$

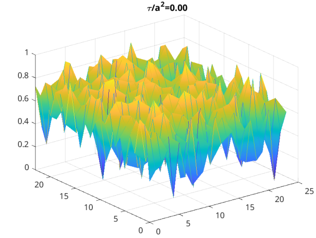
- Fermion fields need wavefunction renormalization

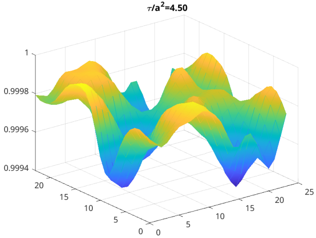
$$\chi_R(\tau;x) = Z_\chi^{1/2}(\tau)\chi(\tau;x) \quad \Rightarrow \quad O_R^{GF}(\tau;x) = Z_\chi^{n/2}(\tau) O(\tau;x)$$

- Match to $\overline{\text{MS}}$ scheme using short flow-time expansion (SFTX)

$$O^{\overline{MS}}(\mu;x) = \lim_{\tau\rightarrow 0} \zeta_O^{-1}(\mu;\tau) Z_\chi(\tau)^{n/2} O(\tau;x)$$

[Lüscher, Weisz JHEP 02 (2011) 051] [Suzuki PTEP 2013 (2013) 083B03]
[Lüscher PoS Lattice 2013 016] [Makino, Suzuki PTEP (2014) 063B02] ...





RBC-UKQCD's 2+1 flavor SDWF+Iwasaki gauge field configurations

	L	a^{-1}(GeV)	am_l	am_s	M_π (MeV)	$N_{src} \times N_{conf}$	
C1	24	1.785	0.005	0.040	340	32×101	[PRD 78 (2008) 114509]
C2	24	1.785	0.010	0.040	433	32×101	[PRD 78 (2008) 114509]
M1	32	2.383	0.004	0.030	302	32×79	[PRD 83 (2011) 074508]
M2	32	2.383	0.006	0.030	362	32×89	[PRD 83 (2011) 074508]
M3	32	2.383	0.008	0.030	411	32×68	[PRD 83 (2011) 074508]
F1S	48	2.785	0.002144	0.02144	267	24×98	[JHEP 1712 (2017) 008]

- Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]
- a: ~ 0.11 fm, ~ 0.08 fm, ~ 0.07 fm
- Measurements with strange and charm quark masses tuned to their physical value using D_s meson
- Well-studied ensemble set chosen for exploring heavy meson lifetimes calculation
- GF measurements: [M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW 2603.28616][2603.28617][Data] [M. Black, R. Harlander, A. Hasenfratz, A. Rago, OW PRD 113 (2026) 094504]

GF axial and vector schemes [M. Black, A. Hasenfratz, OW 2607.00493]

- Define GF $A-$ and $V-$schemes $\rightsquigarrow$ see slides by Anna Hasenfratz

$$\widetilde{\mathcal{Z}}_\chi^{(A)}(\tau)\,\langle A_0(\tau;t)\,A_0(0;0)\rangle = Z_A\,\langle A_0(\tau=0;t)\,A_0(0;0)\rangle\ , \qquad A^{\overline{MS}}(\mu) = A_{\text{bare}}$$

$$\widetilde{\mathcal{Z}}_\chi^{(V)}(\tau)\,\langle V(\tau;t)\,V(0;0)\rangle = Z_V\,\langle V(\tau=0;t)\,V(0;0)\rangle\ , \qquad V^{\overline{MS}}(\mu) = V_{\text{bare}}$$

- Calculate $\widetilde{Z}_\chi^{(A)}$ over $\widetilde{Z}_\chi^{(V)}$

$$\frac{\widetilde{Z}_\chi^{(A)}(\tau)}{\widetilde{Z}_\chi^{(V)}(\tau)} = \frac{Z_A}{Z_V}\,\frac{R_A(\tau)}{R_V(\tau)} \qquad \text{with} \quad R_A(\tau) = \lim_{t\to\infty}\frac{\langle A_0(\tau=0;t)\,A_0(0;0)\rangle}{\langle A_0(\tau;t)\,A_0(0;0)\rangle}$$

$$= 1 + \mathcal{O}(\tau m_q^2)$$

$\rightarrow$ Ratio is finite and dimensionless

$\Rightarrow$ Flow time dependence must manifest through only dimensionless combination: τm_q^2

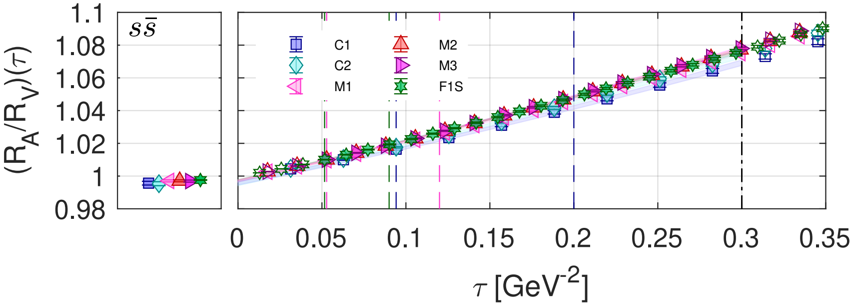
$\Rightarrow$ $\tau \to 0$ limit suppresses mass dimension $\Rightarrow \widetilde{Z}_\chi^{(A)}(0) = \widetilde{Z}_\chi^{(V)}(0)$

Extract Z_V/Z_A taking the $\tau \to 0$ limit [M. Black, A. Hasenfratz, OW 2607.00493]

- Obtain ratio of matching factors Z_V/Z_A from $\tau \to 0$ limit of flowed correlator ratios

$$\frac{Z_V}{Z_A} = \lim_{\tau \to 0} \frac{R_A(\tau)}{R_V(\tau)}$$

- For DWF: $Z_V = Z_A + \mathcal{O}(m_{\text{res}})$



	$\left[\frac{R_A}{R_V}\right]^{s\bar{s}}_{\tau=0}$
C1	0.9957(11)
C2	0.9953(12)
M1	0.99704(61)
M2	0.99707(59)
M3	0.99694(62)
F1S	0.99758(24)

Mixed fermion composite state

- Each flavor r, s has its own wavefunction renormalization: $\widetilde{Z}^r_\chi(\tau) = Z^r_A\, R^r_A(\tau)$, $\widetilde{Z}^s_\chi(\tau) = Z^s_A\, R^s_A(\tau)$
- GF mixed flavor renormalized operator: $$\widetilde{O}^{rs}_{GF}(\tau) = \sqrt{\widetilde{Z}^r_\chi(\tau)\widetilde{Z}^s_\chi(\tau)}\, O^{rs}(\tau)$$
- Axial charge matching factor Z^{rs}_A: $$\widetilde{A_0}^{rs}_{GF}(\tau) = Z^{rs}_A(\tau) A^{rs}_0(0) = \sqrt{\widetilde{Z}^r_\chi(\tau)\widetilde{Z}^s_\chi(\tau)}\; A^{rs}_0(\tau)$$
- So we obtain $$Z^{rs}_A = \lim_{\tau\to 0} \sqrt{\widetilde{Z}^r_\chi(\tau)\, \widetilde{Z}^s_\chi(\tau)}\, \frac{1}{R^{rs}_A(\tau)} = \lim_{\tau\to 0} \sqrt{Z^r_A Z^s_A}\, \frac{\sqrt{R^{rr}_A(\tau) R^{ss}_A(\tau)}}{R^{rs}_A(\tau)}$$

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- So we obtain

$$Z^{rs}_A = \lim_{\tau\to 0}\sqrt{\widetilde{Z}^r_\chi(\tau)\,\widetilde{Z}^s_\chi(\tau)}\,\frac{1}{R^{rs}_A(\tau)} = \lim_{\tau\to 0}\sqrt{Z^r_A Z^s_A}\,\boxed{\frac{\sqrt{R^{rr}_A(\tau)R^{ss}_A(\tau)}}{R^{rs}_A(\tau)}}$$

$$= \rho^{rs}_A(\tau)$$

$$= \sqrt{Z^r_A Z^s_A}\;\rho^{rs}_A$$

Mixed action/mixed mass renormalization

- Calculation of decay constants, form factors, bag parameters, ...in heavy-light systems requires heavy-light matching factors $Z^{hl}_{A,V}$ correcting for non-conserved lattice currents

$$Z_A^{hl} = \rho_A^{hl} \sqrt{Z_A^{ll} Z_A^{hh}}$$

- Flavor-diagonal contributions can be easily determined on the lattice
- Correction factor ρ_A^{hl} is expected to be ~ 1, mostly calculated perturbatively

  $\Rightarrow$ Mostly nonperturbative renormalization

  [Hashimoto et al. PRD 61 (1999) 014502] [El-Khadra et al. PRD 64 (2001) 014502]

  $\rightarrow$ Nonperturbatively tested for staggered fermions [Chakraborty et al. PoS LATTICE2013 (2014) 309]

- Particularly important when using effective actions for heavy quarks (e.g. Fermilab/RHQ)
- Maybe also relevant when combining e.g. Shamir DWF with stout-smeared Möbius DWF or largely different quark masses

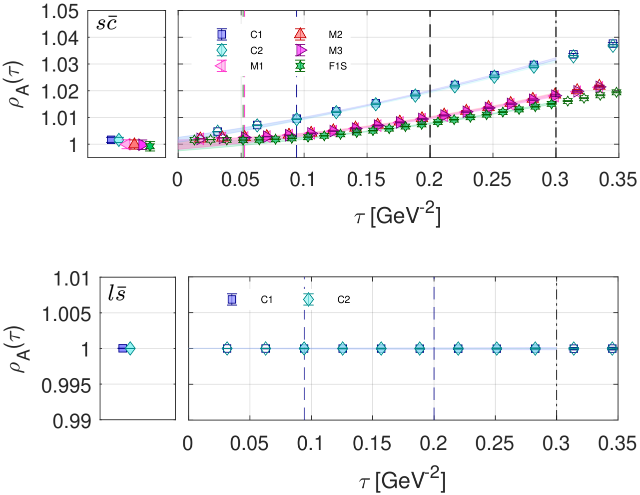
Nonperturbative determination of ρ_A^{rs} using gradient flow

- Calculate

$$\rho_A^{rs}(\tau) = \frac{\sqrt{R_A^{rr}(\tau) R_A^{ss}(\tau)}}{R_A^{rs}(\tau)}$$

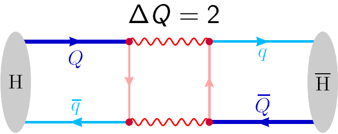
- Obtain ρ_A^{rs} for $\tau \to 0$

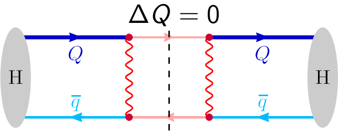
	$[\rho_A(\tau=0)]^{s\bar{c}}$	$[\rho_A(\tau=0)]^{l\bar{s}}$
C1	1.00164(95)	1.000022(13)
C2	1.00161(96)	1.0000073(44)
M1	1.0000(17)	—
M2	0.9999(17)	—
M3	0.9998(18)	—
F1S	0.9992(17)	—



Bag parameters for neutral meson mixing and heavy meson lifetimes

- Both processes are described by box diagrams in the SM





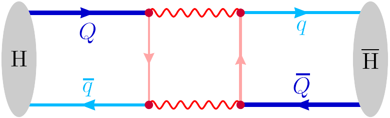
- Nonperturbative contribution parametrized in terms of bag parameters

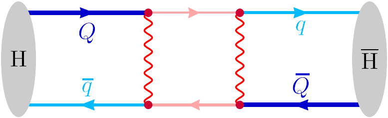
- $\Delta Q = 2$ calculations for short-distance contributions well established in lattice QCD
  $\rightarrow$ Demonstrate GF determination using a charm-strange setup
  [M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW 2603.28617]

- $\Delta Q = 0$ troubled by operator mixing under renormalization
  $\rightarrow$ First GF results for lifetime differences at the SU(3) symmetric point (extra slides)
  [M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW 2603.28616][2603.28617]

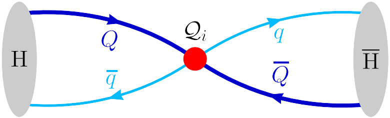
Lattice calculation of neutral $B^0_{(s)}$ meson mixing

- Standard Model process described by box diagrams





- Top quark contribution dominates $\Rightarrow$ short-distance process
  $\rightarrow$ Describe by point-like 4-quark operators



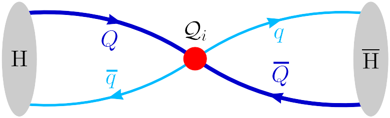
- Parameterize experimentally measured oscillation frequencies Δm_q by

$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 M_{B_q} f_{B_q}^2 \mathcal{B}_1 \left| V_{tq}^* V_{tb} \right|^2, \qquad q = d, s$$

$\rightarrow$ Nonperturbative contribution decay constant $f_{B_q}^2$ times bag parameter $\mathcal{B}_1$

$\rightarrow$ Dim-7 operators pioneered by HPQCD [HPQCD PRL 124 (2020) 082001]

$\Delta Q = 2$ mixing operators



- Standard Model process described by

$$\mathcal{Q}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta$$

  $\rightarrow$ General BSM considerations give rise to four additional dim-6 operators

- Calculate matrix element

$$\langle \mathcal{Q}_1^q \rangle = \langle \overline{B}_q | \mathcal{Q}_1^q | B_q \rangle = \tfrac{8}{3} f_{B_q}^2 M_{B_q} \mathcal{B}_1$$

- Match "lattice" bag parameter $\mathcal{B}_1$ to $\overline{MS}$

  $\rightarrow$ Different renormalization/matching procedures used in the literature

  Perturbative scheme: Fermilab/MILC, HPQCD

  Nonperturbative scheme: ETMC, RBC-UKQCD

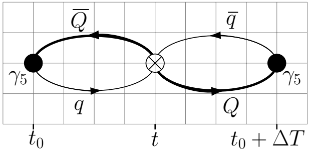
- Operator mixing occurs for non-chiral lattice fermions

  $\rightarrow$ GF may help here, too

GF determination of $\Delta Q = 2$ operators

- Calculate ratio of flowed matrix elements to extract bag parameters (only short distance)

$$R_{\mathcal{Q}_1}(t_0, t, \Delta T; \tau) = \frac{C^{\rm 3pt}_{\mathcal{Q}_1}(t_0, t, t_0 + \Delta T; \tau)}{\frac{8}{3} C^{2pt}(t_0, t; \tau)\, C^{2pt}(t_0 + \Delta T, t; \tau)} \to \mathcal{B}_1^{GF}(\tau)$$



- Match to $\overline{\text{MS}}$

$$\mathcal{B}_1^{\overline{\rm MS}}(\mu_{\rm UV}) = \lim_{\tau \to 0} (\zeta^{\rm imp}(\mu_{\rm UV}, \tau_\mu))^{-1} \mathcal{B}_1^{\rm GF}(\tau)$$

- Improve ζ coefficients by resuming logarithmic terms using RG equation

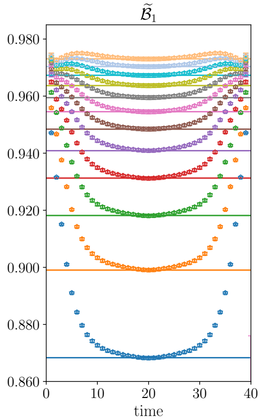
$$(\zeta^{\rm imp}(\mu_{\rm UV}, \tau_\mu))^{-1} = \zeta^{-1}(\mu_{\rm UV}, \tau_\mu) \times \exp\left(- \int_{\tau_\mu}^{\tau} d\tau' \frac{\widetilde{\gamma}^{\rm GF}(\tau')}{\tau'} \right)$$

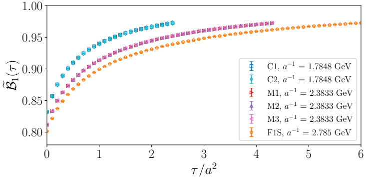
$\rightarrow \widetilde{\gamma}^{GF}$ gradient flowed anomalous dimension; $\tau_\mu = e^{-\gamma_{\rm E}}/(2\mu_{\rm UV}^2)$

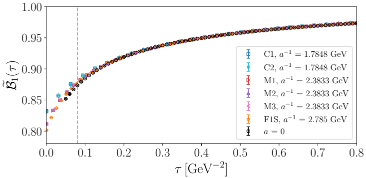
- For $\mathcal{B}_1$, ζ^{-1} is a scalar $\Rightarrow$ $a \to 0$ and $\tau \to 0$ limits can be interchanged

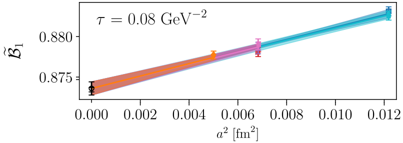
$\Delta Q = 2$ results for "neutral charm-strange meson"

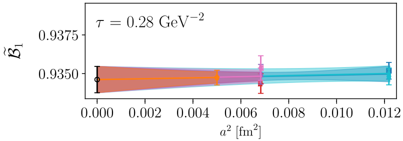
[M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW 2603.28617]







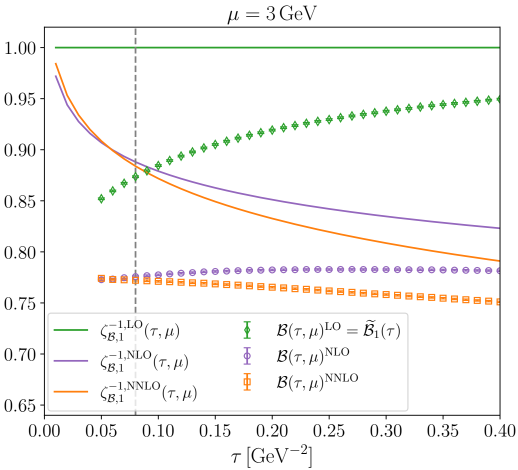




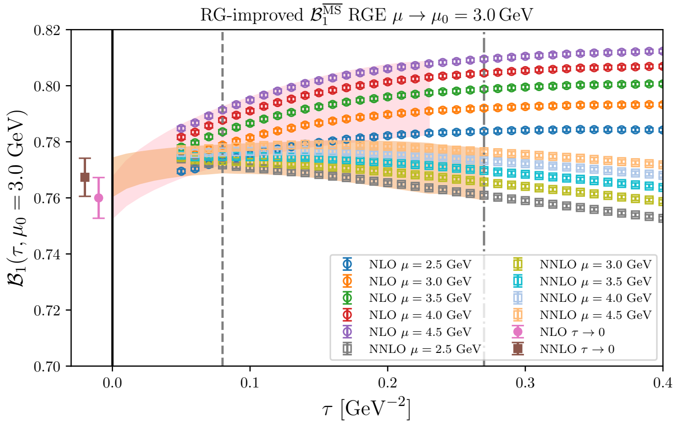
- F1S: $a^{-1} = 2.785$ GeV
  $M_\pi = 268$ MeV

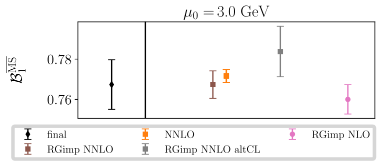
Multiplying SFTX coefficients ζ^{-1}

[M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW 2603.28617]



Zero flow time extrapolation [M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW 2603.28617]





$$\mathcal{B}_1^{\overline{\rm MS}}(3\text{GeV}) = 0.7673(123)$$
$$= 0.7673(68)_{\rm GF}(82)_{\rm CL}(37)_{\rm PT}(49)_{\rm OS}$$

- Perturbatively calculated $\widetilde{\gamma}^{GF}$ $\rightarrow$ vary $\mu = 2.5 - 4.5$ GeV
- Combined correlated fit varying τ

- Consistent with short-distance results for D^0 meson mixing
  - $\rightarrow$ ETM 2015: 0.757(27)
  - $\rightarrow$ FNAL/MILC 2017: 0.787(57)

Preview: improve further by nonperturbatively estimating $\widetilde{\gamma}^{GF}$

- Determine $\widetilde{\gamma}^{GF}$ nonperturbatively
  - $\rightsquigarrow$ Slides by Anna Hasenfratz; implementation/plots by Matthew Black

$$\widetilde{\gamma}^{A}_{\mathcal{B}_1}(\tau) = -2\tau \frac{d}{d\tau} \log \widetilde{Z}^{(A)}_{\mathcal{B}_1}(\tau)$$

- Knowing $\widetilde{\gamma}^{GF}$ in the continuum, we can calculate the flow time evolution

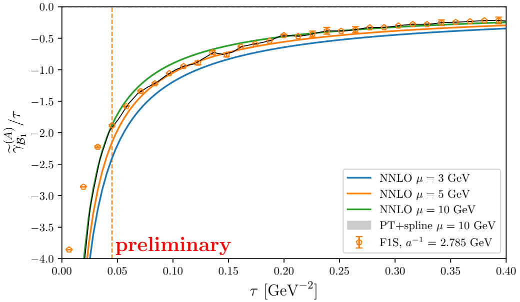
$$C_{\mathcal{B}_1}(\tau^*, \tau) = \exp\left\{ -\frac{1}{2} \int_{\tau}^{\tau^*} \frac{d\tau'}{\tau'} \, \gamma^{(A)}_{\mathcal{B}_1}(\tau) \right\} = \frac{\widetilde{Z}_{\mathcal{B}_1}(\tau^*)}{\widetilde{Z}_{\mathcal{B}_1}(\tau)}$$

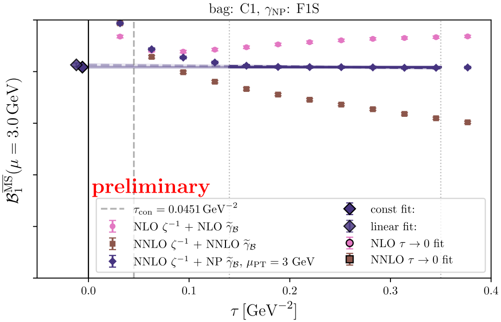
- Improved matching to $\overline{MS}$

$$\mathcal{B}_1^{\overline{MS}}(\mu_{UV}) = \lim_{\tau \to 0} \zeta^{-1}(\mu_{UV}, \tau^*) C_{\mathcal{B}_1}(\tau^*, \tau) \mathcal{B}_1^{GF}(\tau)$$

Preview: improve further by nonperturbatively estimating $\widetilde{\gamma}^{GF}$

Plots by Matthew Black





- Approximate $\widetilde{\gamma}^{(A)}_{\mathcal{B}_1}$ using $(s\bar{s})$-correlators on F1S
- Match to PT $\widetilde{\gamma}^{GF}$ at $\tau^* = 0.045$ GeV^{-2}

- C1 ($a^{-1} = 1.78$ GeV)
- $2/8 \lesssim \tau/a^2 \leq 0.35$

Summary

- GF+SFTX is a powerful concept and we are just starting to leverage its potential
- Prediction for the ratio of matching factors Z_V/Z_A
- Nonperturbative determination of the mixed action/mass correction factor ρ_A^{rs}
- GF results for $\Delta Q = 2$ bag parameters for a "neutral charm-strange meson"
  $\rightarrow$ First results for heavy meson lifetime differences at the SU(3) symmetric point ($\Delta Q = 0$)

Acknowledgment

- Grid [Peter Boyle et al.]
- Hadrons [Antonin Portelli et al.]
- Feyngame [Harlander et al.]
- OMNI, Universität Siegen
- HAWK, HLR Stuttgart
- LumiG, DEIC

Extra

Short flow-time expansion (SFTX)

- Re-express effective Hamiltonian in terms of ‘flowed’ operators

$$\mathcal{H}_{\text{eff}} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(\tau)\widetilde{\mathcal{O}}_n(\tau)$$

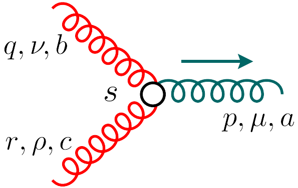
- Relate to regular operators in SFTX

ME of flowed operator (lattice)

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

PT calculated matching matrix

$$\sum_n \zeta^{-1}_{nm}(\mu,\tau)\langle\widetilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$



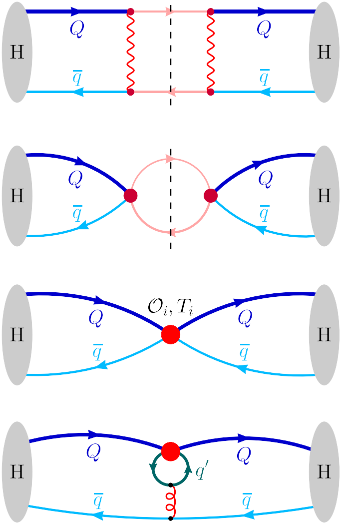
new Feynman diagrams

- Matrix element $\langle\mathcal{O}_m\rangle(\mu)$ in the $\overline{\text{MS}}$ scheme found after taking the $\tau \to 0$
  - $\to$ Large systematic effects at very small flow times
  - $\to$ Large flow time dominated by operators $\propto O(\tau)$

Heavy meson lifetimes ($\Delta Q = 0$ operators)

[M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW, arXiv:2603.28616][arXiv 2603.28617]

- Heavy quark expansion (HQE): lifetimes of heavy mesons described by 4-quark operators with $\Delta B = 0$
  - → Optical theorem: calculate imaginary part
  - → Integrate out W boson, double insertion $\Delta Q = 1$ eff. weak Hamiltonian
- Operators O_1, O_2, T_1, T_2, contribute
- $\Delta Q = 0$ operators mix under renormalization
  - → To date no complete LQCD determination (only exploratory work 20+ years ago)
  - → Mixing suppressed using GF
- Quark-line disconnected contributions
  - → Notoriously noisy, expensive to calculate on the lattice
  - → In principal we know what to do . . .



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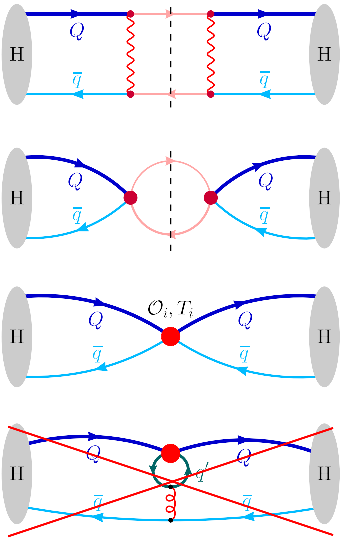


$\Delta Q = 0$ operators and bag parameters

[M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW, arXiv:2603.28616][arXiv 2603.28617]

- The four $\Delta Q = 0$ operators

$$O_1(\tau) = (\bar{Q}\gamma_\mu(1-\gamma_5)q)(\bar{q}\gamma_\mu(1-\gamma_5)Q)(\tau) \qquad T_1(\tau) = (\bar{Q}\gamma_\mu(1-\gamma_5)T^A q)(\bar{q}\gamma_\mu(1-\gamma_5)T^A Q)(\tau)$$

$$O_2(\tau) = (\bar{Q}(1-\gamma_5)q)(\bar{q}(1+\gamma_5)Q)(\tau) \qquad T_2(\tau) = (\bar{Q}(1-\gamma_5)T^A q)(\bar{q}(1+\gamma_5)T^A Q)(\tau)$$

- Determine bag parameters by calculating hadronic matrix elements using GF

$$\langle H|O_1|H\rangle(\tau) = f_H^2\, M_H^2\, B_1^H(\tau) \qquad \langle H|T_1|H\rangle(\tau) = f_H^2\, M_H^2\, \epsilon_1^H(\tau)$$

$$\langle H|O_2|H\rangle(\tau) = \frac{M_H^2}{(m_Q+m_q)^2} f_H^2\, M_H^2\, B_2^H(\tau) \qquad \langle H|T_2|H\rangle(\tau) = \frac{M_H^2}{(m_Q+m_q)^2} f_H^2\, M_H^2\, \epsilon_2^H(\tau)$$

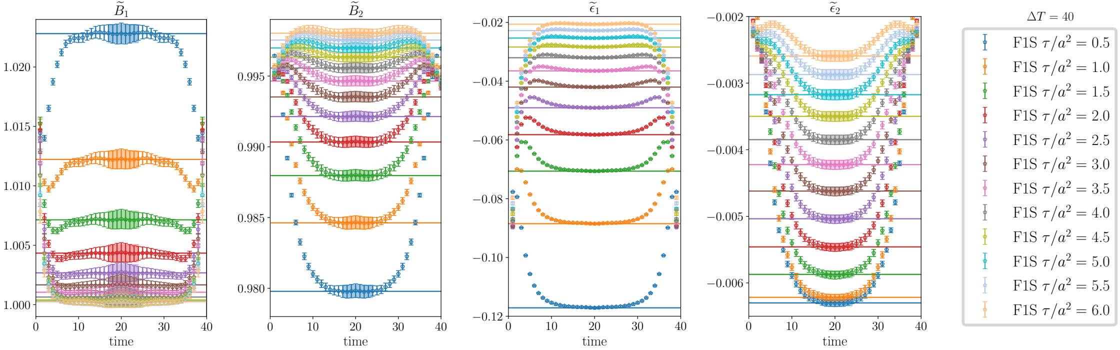
- Match to the $\overline{\text{MS}}$ scheme using SFTX

$$\begin{pmatrix} B_i(\tau,\mu) \\ \epsilon_i(\tau,\mu) \end{pmatrix} = \zeta_{B,i}^{-1}(\tau,\mu) \begin{pmatrix} B_i^{\text{GF}}(\tau) \\ \epsilon_i^{\text{GF}}(\tau) \end{pmatrix}$$

D_s meson lifetimes ($\Delta Q = 0$ operators)

[M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW, arXiv:2603.28616][arXiv 2603.28617]

- Extract bag parameters for all ensembles
  $\rightarrow$ F1S: $a^{-1} = 2.785$ GeV, $M_\pi = 268$ MeV



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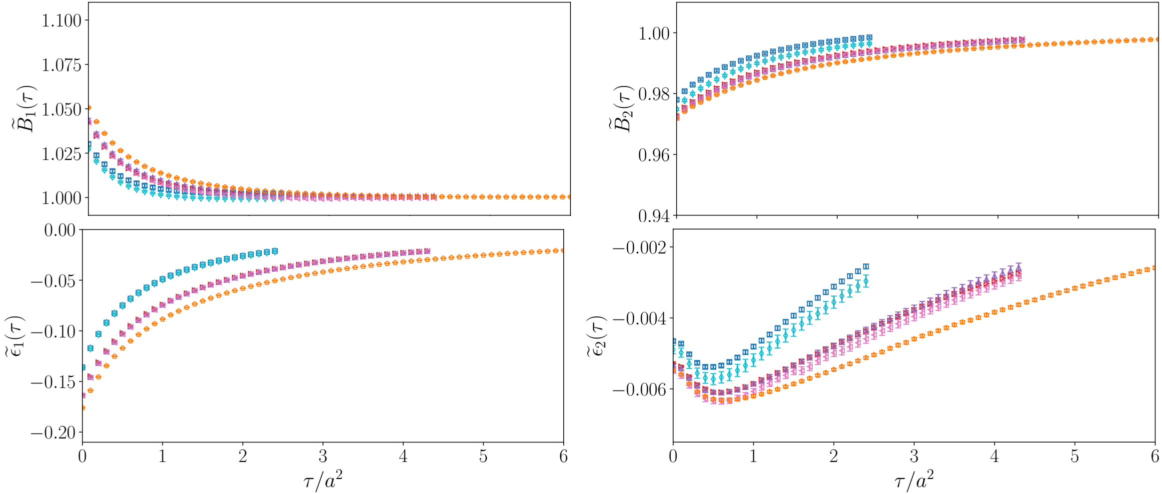
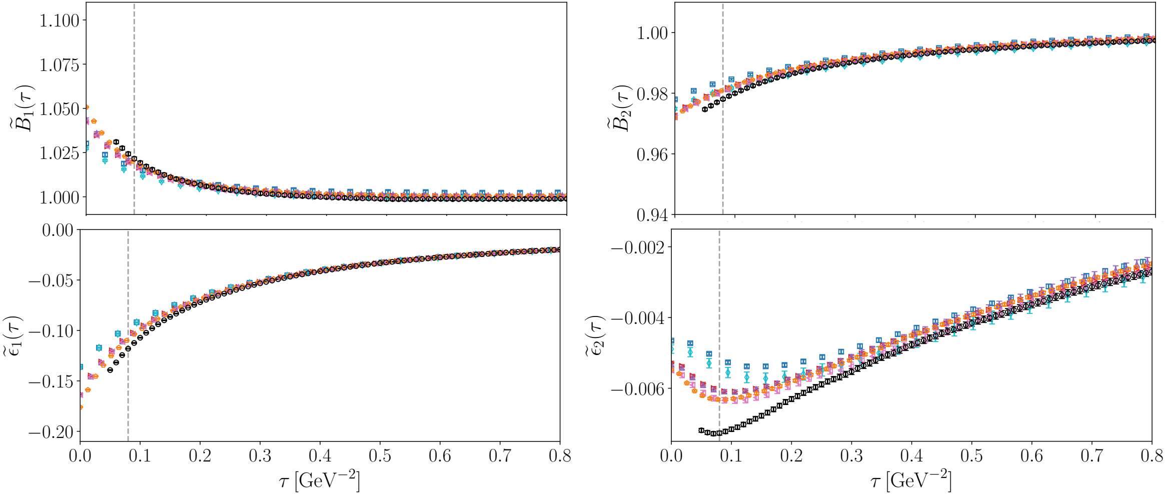
- Results for flow time in lattice units



D_s meson lifetimes ($\Delta Q = 0$ operators)

[M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW, arXiv:2603.28616][arXiv 2603.28617]

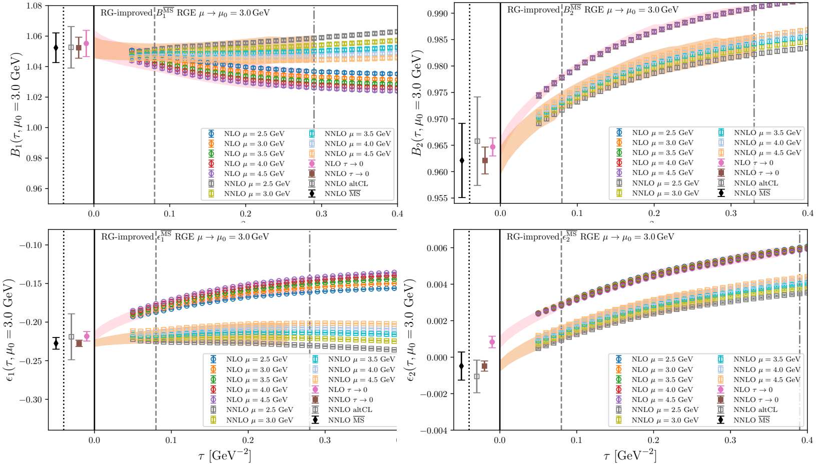
- Results for flow time in "physical" units (GeV^{-2})



D_s meson lifetimes ($\Delta Q = 0$ operators)

[M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, OW, arXiv:2603.28616][arXiv 2603.28617]

- $\tau \to 0$ limit



- Vary scale μ to compute ζ_{nm}
- Run back to $\mu_0 = 3$ GeV in $\overline{\text{MS}}$ scheme
- RG improve $\tau\partial_\tau \tilde{O}(\tau) = \tilde{\gamma}(\alpha_s(\mu)) L_{\mu\tau} \tilde{O}(\tau)$
- Final result: spread of fits w/ good p-val. btw. (τ_{min}, τ_{max})

- Half the spread of NNLO and NLO added as additional systematic error
- Alternative $a \to 0$ limit