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First-principles determination of anomaly-induced pion decay beyond the chiral limit

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CONTENTS

Theory vs. experiment: current status

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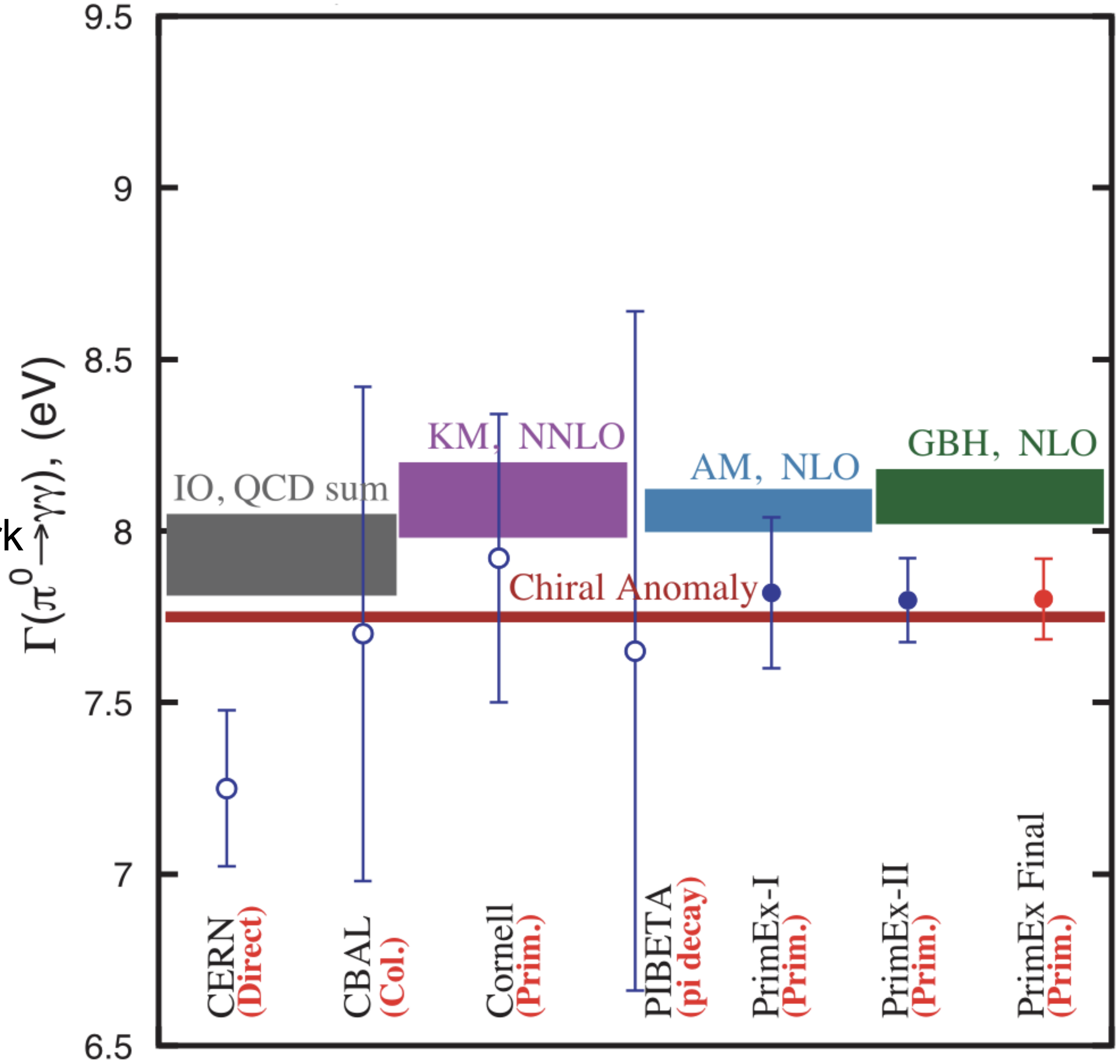
$\Gamma(\pi^0 \to \gamma\gamma)$ is a percent level test of QCD in confinement region

- Chiral-limit: ABJ anomaly gives:
  $\Gamma_{ABJ} = 7.75eV$.

- ChPT predicts an enhancement due to nonzero quark mass to roughly $8.1eV$.
  - π^0 mass correction $(m_l \neq 0)$
  - π^0-η-η' mixing $(m_u \neq m_d)$

- PrimEx final results:
  $7.80(12)eV$, 1.5% precision,

$1\sim2\ \sigma$ discrepancy between ChPT and Experiment.



The experimental target is moving below one percent

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Ø Precision is improving step by step, turning $\pi^0 \to \gamma\gamma$ into a sharper test of QCD.

PrimEx I		PrimEx II		JLab 22 GeV upgrade
Γ = 7.82(22) eV		Γ = 7.80(12) eV		future program
2.8%	→	**1.5%**	→	**sub-percent**
first high-precision result		systematics reduced		new precision benchmark
Phys.Rev.Lett. 106 (2011) 162303		*Science* 368 (2020) 6490, 506-509		A. Accardi, et al., Eur.Phys.J.A 60 (2024) 173

The uncertainty has moved from few-percent to percent level; the next experimental program aims at sub-percent control.

Can lattice QCD resolve the small correction compared to anomaly with comparable precision?

π^0 Transition form factor

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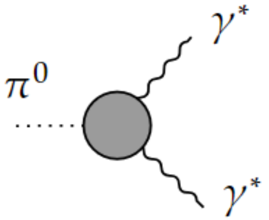
- In the continuum theory, $\pi \to \gamma\gamma$ transition form factor is defined in Euclidean space as



$$\varepsilon_{\mu\nu\alpha\beta}Q^\alpha Q'^\beta \mathcal{F}_{\pi^0\gamma^*\gamma^*}(Q^2, Q'^2) = \int d^4x\, e^{-i(Q-\frac{1}{2}P)\cdot x}\mathcal{H}_{\mu\nu}(x)$$

$Q = (iE, \vec{Q})$ is arbitrary 4-momentum for one off-shell photon

$\mathcal{H}_{\mu\nu}(x) = \langle 0|T\{J_\mu(\frac{x}{2})J_\nu(-\frac{x}{2})\}|\pi(P)\rangle$ is the hadronic function

- Lorentz decomposition

$$\mathcal{H}_{\mu\nu}(x) = \varepsilon_{\mu\nu\alpha\beta}x^\alpha P^\beta H(x^2, P\cdot x)$$

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(Q^2, Q'^2) = \int d^4x\, \omega(Q,P,x) H(x^2, P\cdot x)$$

$$\langle \omega(Q,P,x)\rangle_{SO(3)} = -i\, e^{(E-\frac{1}{2}m_\pi)t}\frac{|\vec{x}|}{|\vec{Q}|}j_1(|\vec{Q}||\vec{x}|)$$

- Decay width

$$\Gamma_{\pi^0\to\gamma\gamma} = \frac{\pi\alpha^2 m_{\pi^0}^3}{4}\left|F_{\pi^0\gamma^*\gamma^*}(0,0)\right|^2$$

Direct computation

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- Current status

  - Direct lattice widths at isospin-symmetric limit

    e.g. RBC-UKQCD 2024, 7.60(27) eV, 3.6%.

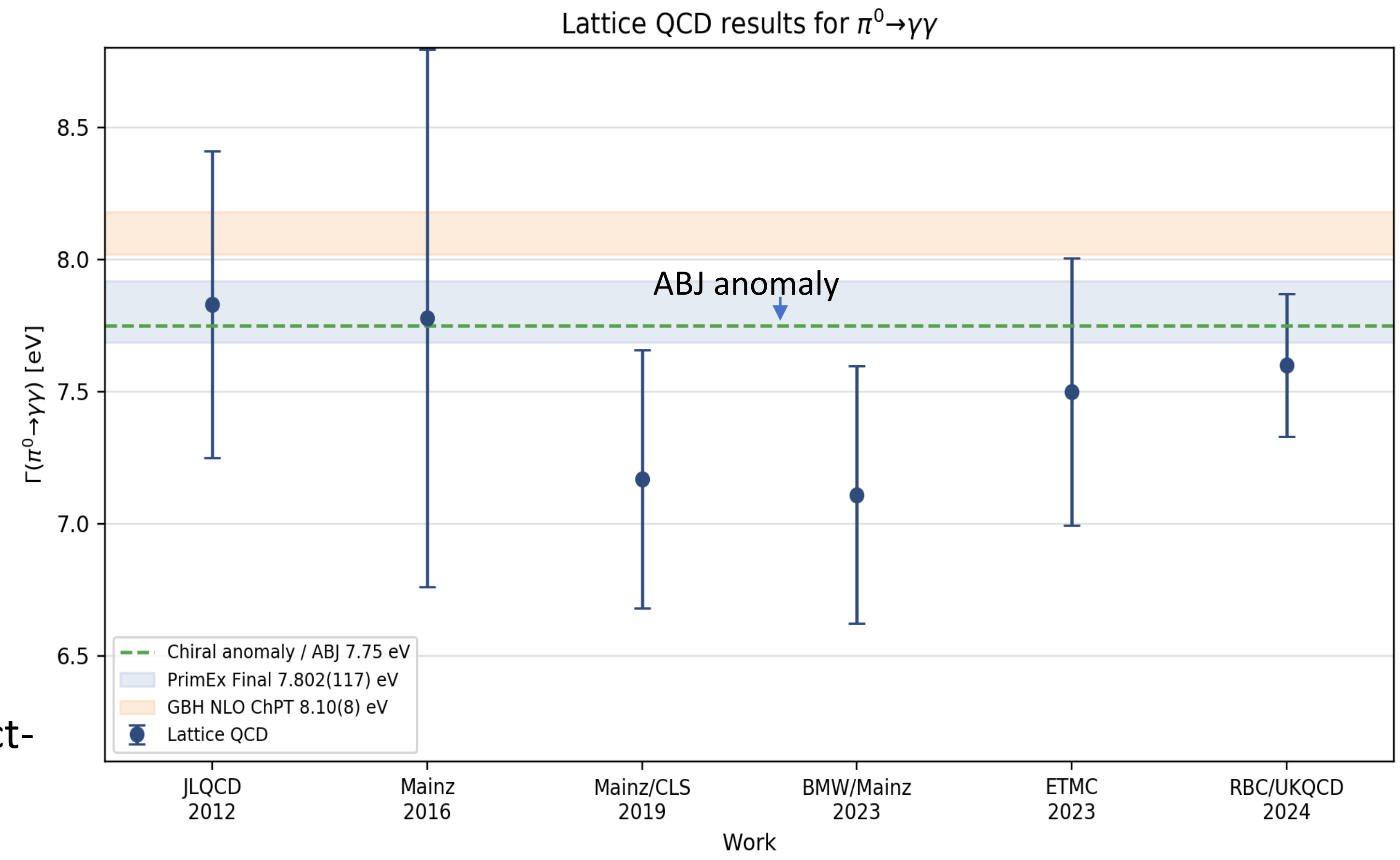
    Disconnected diagrams are evaluated

  - PrimEx II: 7.80(12) eV, 1.5%.

- $\Gamma_{\pi^0 \to \gamma\gamma} = \Gamma_{ABJ}$ + mass correction

  $\Gamma_{ABJ} \approx 7.75\ eV$

  The pion-mass correction is small, while direct-width errors are already at O(few %).



Direct Γ is a useful consistency check, but it is not yet a clean way to compare with experiment and ChPT at percent-level precision

Computation of mass correction

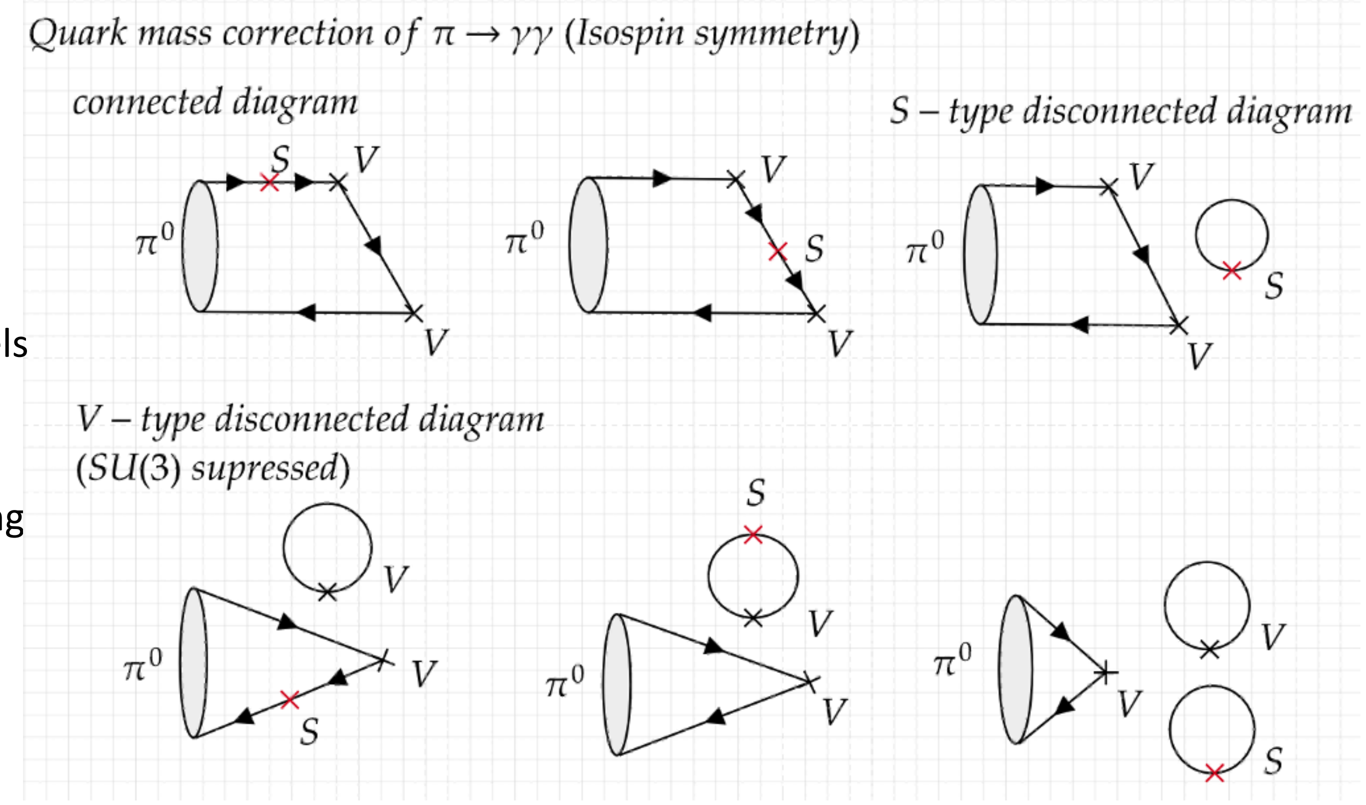
- Mass correction naturally yields a computation of 4-point function

$$\frac{\partial}{\partial m_q}\langle J_\mu J_\nu P\rangle \sim -\int d^4z\,\langle J_\mu J_\nu P\,\bar{q}q(z)\rangle.$$

- Original thought is that mass correction itself is about O(5%) compared to anomaly

If one can calculate it with $\sim 10\%$ uncertainty → then obtain $\Gamma_{\pi^0\gamma\gamma}$ with ~0.5% error

- However …



  - The operator $O_{IS} = \frac{1}{2}(\bar{u}u + \bar{d}d)$ yields disconnected diagrams, which are quite noisy.

  - The derivatives $\frac{df_\pi}{dm_\pi^2}, \frac{dF_{\pi\gamma\gamma}}{dm_\pi^2}$ contains chiral log, which cancels when the two terms combined

    Adding $\frac{df_\pi}{dm_\pi^2}, \frac{dF_{\pi\gamma\gamma}}{dm_\pi^2}$ together, large signals cancel, but leaving uncertainties

  - Finally, the result carries a $\sim 4\%$ error

    → Much larger than ~0.5% as anticipated

We need to seek for a better solution! → ABJ anomaly is exactly known. How do we make use of it?

Anomalous ward identity

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- Original mass correction treats the ABJ anomaly at chiral limit as a reference

$$\frac{\partial}{\partial m_q}\langle J_\mu J_\nu P\rangle \sim -\int d^4z\,\langle J_\mu J_\nu P\,\bar{q}q(z)\rangle.$$

- However, ABJ anomaly is valid even at nonzero quark mass

  - Define $A_\mu^3 = \frac{1}{2}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$

    $P^3 = \frac{1}{2}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) \quad P^0 = \frac{1}{2}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)$

  - Anomalous PCAC relation in QCD reads $\partial_\mu A_\mu^3 = \hat{P}^3 + \hat{P}^0 - \pi K_0 \alpha F_{\mu\nu}^{em}\tilde{F}_{\mu\nu}^{em}$

    with $\hat{P}^3 = 2m_l P^3$ $\quad \hat{P}^0 = \delta m\, P^0$ $\quad K_0 = \frac{N_c(Q_u^2 - Q_d^2)}{4\pi^2} = \frac{1}{4\pi^2}$

  - Construct the AVV and PVV correlator, define the amplitude related to $\pi^0 \to \gamma\gamma$

$$q^\mu A_{\mu 5\rho\sigma}(p,q) = q_\mu \int d^4x d^4z\, e^{ipx} e^{iqz} \langle T[J_\rho^{em}(x) J_\sigma^{em}(x) A_\mu^3(z)]\rangle = \epsilon_{\rho\sigma\lambda\alpha} p_\lambda q_\alpha \mathcal{W}(-q^2)$$

$$A_{\mu\rho\sigma}^{(a)}(p,q) = i\int d^4x d^4z\, e^{ipx} e^{iqz} \langle T[J_\rho^{em}(x) J_\sigma^{em}(x) \hat{P}(z)]\rangle = \epsilon_{\rho\sigma\lambda\alpha} p_\lambda q_\alpha \hat{\mathcal{F}}^{(a)}(-q^2)$$

$$A_{\rho\sigma}^{anom}(p,q) = -\epsilon_{\rho\sigma\lambda\alpha} p_\lambda q_\alpha K_0$$

Mass correction of decay width

➢ Determine quark mass correction through difference between $q^2 = 0$ and $q^2 = -m_\pi^2$

$$\partial_\mu A_\mu^3 = \hat{P}^3 + \hat{P}^0 - \pi K_0 \alpha F_{\mu\nu}^{em} \tilde{F}_{\mu\nu}^{em}$$

$$W(-q^2) = \hat{\mathcal{F}}^{(3)}(-q^2) + \hat{\mathcal{F}}^{(0)}(-q^2) - K_0$$

- At $q^2 = 0$, $W(0)$ contains a factor q^2

remaining part is regular when $m_l \neq 0 \Rightarrow W(0) = 0$

$$K_0 = \underbrace{\hat{\mathcal{F}}^{(3)}(0) + \hat{\mathcal{F}}^{(0)}(0)}_{\text{evaluated on lattice}} \Rightarrow \Gamma_{\pi^0\gamma\gamma}^{ABJ} = \frac{\pi m_\pi^3 \alpha^2 K_0^2}{4F_\pi^2}$$

exactly known evaluated on lattice

- At $q^2 = -m_\pi^2$, $W, \hat{\mathcal{F}}^{(3)}, \hat{\mathcal{F}}^{(0)}$ contain a π^0 pole.

For the pion pole contribution

$$K = \lim_{q^2+m_\pi^2 \to 0} \frac{q^2 + m_\pi^2}{m_\pi^2} \left(\hat{\mathcal{F}}^{(3)}(-q^2) + \hat{\mathcal{F}}^{(0)}(-q^2)\right) \Rightarrow \Gamma_{\pi^0\gamma\gamma} = \frac{\pi m_\pi^3 \alpha^2 K^2}{4F_\pi^2}$$

One can use PVV correlator at $q^2 = 0$ as a new reference observable and focus on:

$$\Delta K = K - K_0$$

The correction can be computed within the framework of 3-point function!

Decomposition of quark mass correction

➢ Determine correction of pion decay width

$$\Delta K = K - K_0 = \Delta K_{IS} + \Delta K_{IB} \Rightarrow \Delta\Gamma_{\pi^0\to\gamma\gamma} \approx \frac{2\Delta K}{K_0}\Gamma_{\pi^0\to\gamma\gamma}$$

$$\Delta K_{IS} = \underbrace{[\lim_{q^2+m_\pi^2\to 0}\frac{q^2+m_\pi^2}{m_\pi^2}\hat{\mathcal{F}}^{(3)}(-q^2) - \hat{\mathcal{F}}^{(3)}(0)]_{\delta m=0}}_{\propto m_l}$$

$$\Delta K_{IB} = \underbrace{-\delta m\, \mathcal{F}^{(0)}(0)(|_{\delta m=0}}_{\propto \delta m} + \mathcal{O}(m_l\delta m) + +\mathcal{O}(\delta m^2)$$

➢ E&M corrections to K: In ChPT, it arise at $O(e^2p^4)$ and estimated as a ~0.3% effect

Ananthanarayan & Moussallam, JHEP 2002

Since $K_0 = \hat{\mathcal{F}}^{(3)}(0) + \hat{\mathcal{F}}^{(0)}(0)$ holds with QED turned on, we demonstrate E&M effects are suppressed as $O(\alpha\, m_l)$ or $O(\alpha\, \delta m)$

Lattice Ensembles

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Ensembles	$a[\mathrm{GeV}^{-1}]$	L/a	T/a	L/fm	$m_\pi[\mathrm{MeV}]$	$m_\pi L$	# of confs
24D	1.023	24	64	4.7	142.8(2)	3.3	92
32Dfine	1.378	32	64	4.7	142.9(4)	4.7	37

Generated by RBC/UKQCD(RBC et al., 2016)

physical pion mass
Domain wall fermion + Iwasaki gauge action(+DSDR)
Two ensembles at different lattice spacing for extrapolation

IS contributions

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$$\Delta K_{IS} = \left[\lim_{q^2+m_\pi^2\to 0} \frac{q^2+m_\pi^2}{m_\pi^2}\hat{\mathcal{F}}^{(3)}(-q^2) - \hat{\mathcal{F}}^{(3)}(0)\right]_{\delta m=0}$$

contain only π-state contribution

contain all-state contributions with same quantum number as π

- Three-point correlator in Euclidean space

$$\hat{H}^{(a)}(t_z,x) = \epsilon_{\mu\nu\alpha 0}x_\alpha \int d^3\vec{z}\,\left\langle T\left[J_\mu\left(\frac{x}{2}\right)J_\nu\left(-\frac{x}{2}\right)\hat{P}^{(a)}(\vec{z},t_z)\right]\right\rangle$$

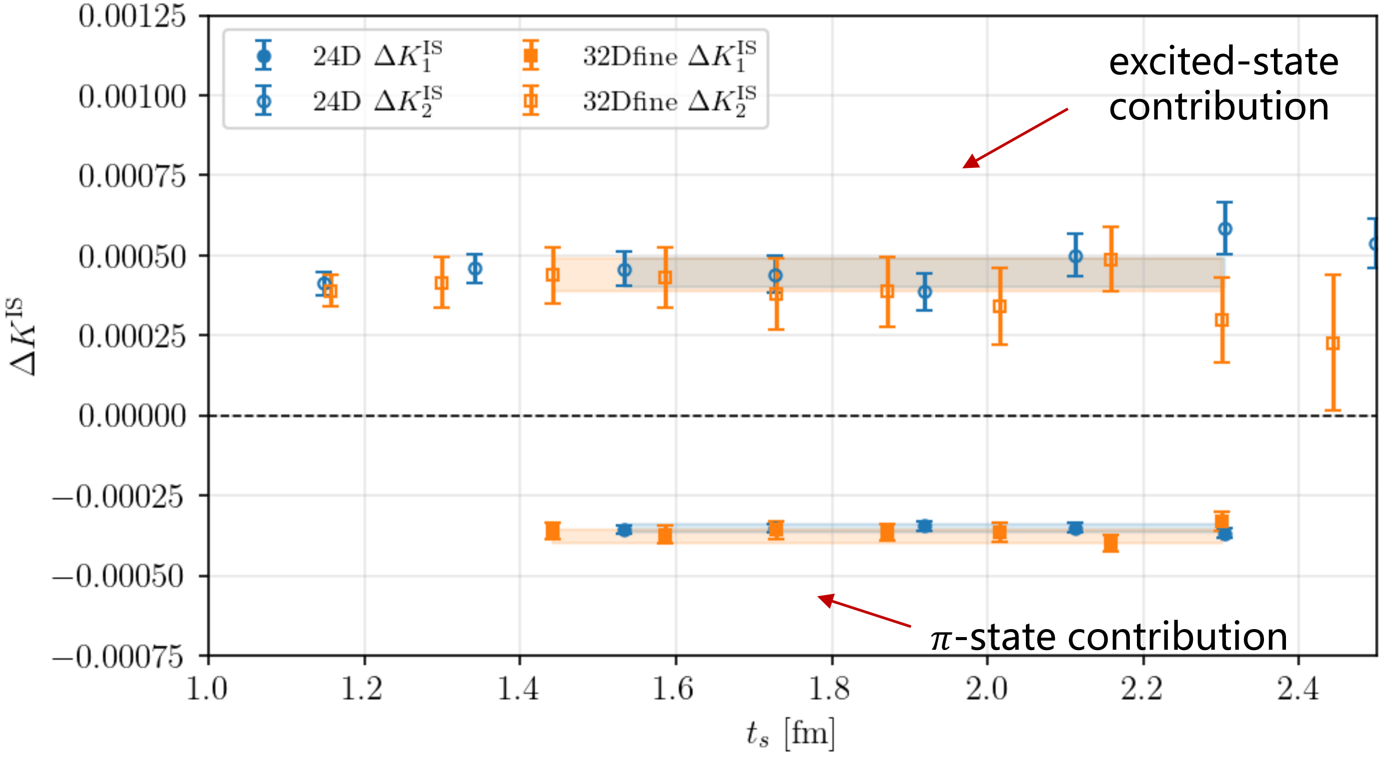
- Pion pole can be subtracted by defining

$$\widetilde{H}^{(3)}(t_z,-t_s,x) = \hat{H}^{(3)}(t_z,x) - e^{m_\pi(t_z+t_s)}\hat{H}^{(3)}(-t_s,x)$$

- The IS contribution is

$$\Delta K^{\mathrm{IS}} = \frac{2e^{m_\pi t_s}}{m_\pi^2}\int d^4x \left(\frac{j_1\left(\frac{m_\pi}{2}|\mathbf{x}|\right)}{m_\pi|\mathbf{x}|} - \frac{1}{6}\right)\hat{H}^{(3)}(-t_s;x)\Big|_{\delta m=0} + \int d^4x \int_{-t_s}^{0} dt_z\,\frac{t_z}{3}\widetilde{H}^{(3)}(t_z,-t_s;x)\Big|_{\delta m=0}$$

Significant cancellation observed



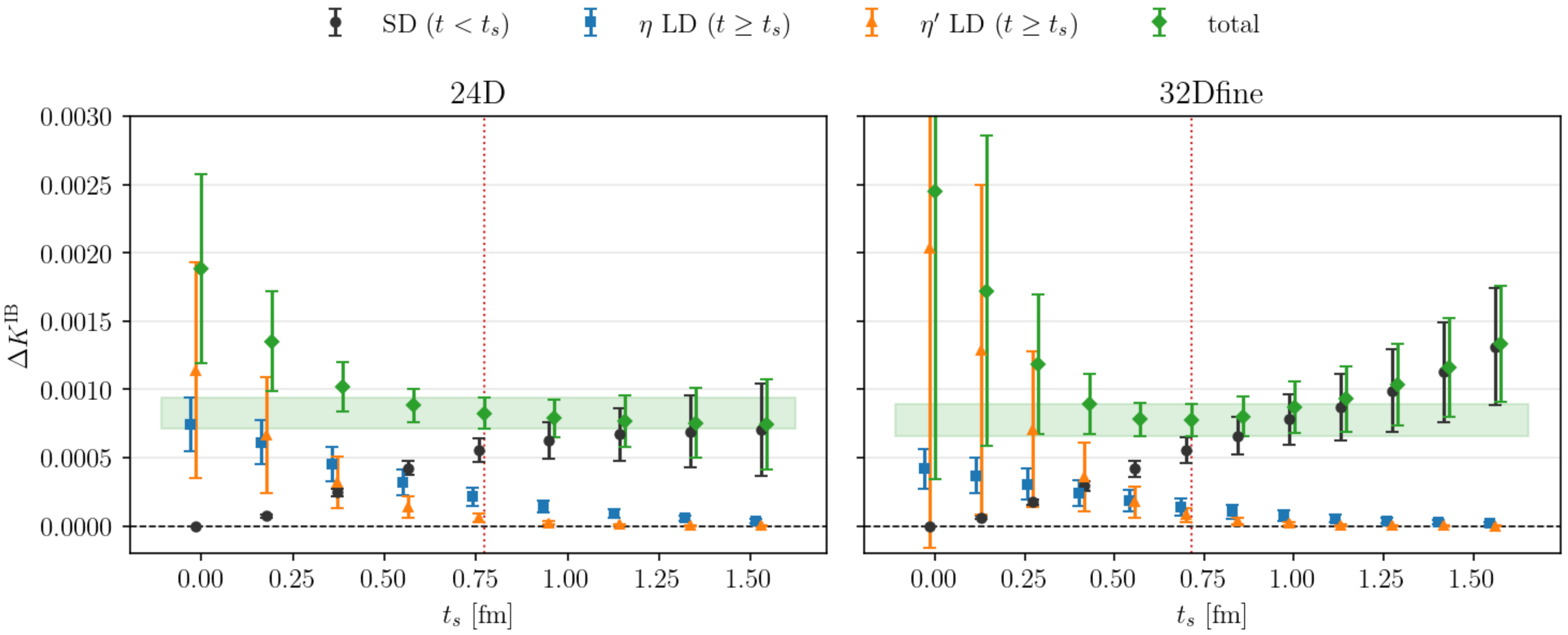
IB contributions

$$\Delta K_{IB} \approx -\hat{\mathcal{F}}^{(0)}(0) \approx -\delta m\, \mathcal{F}^{(0)}(0)\Big|_{\delta m=0}$$

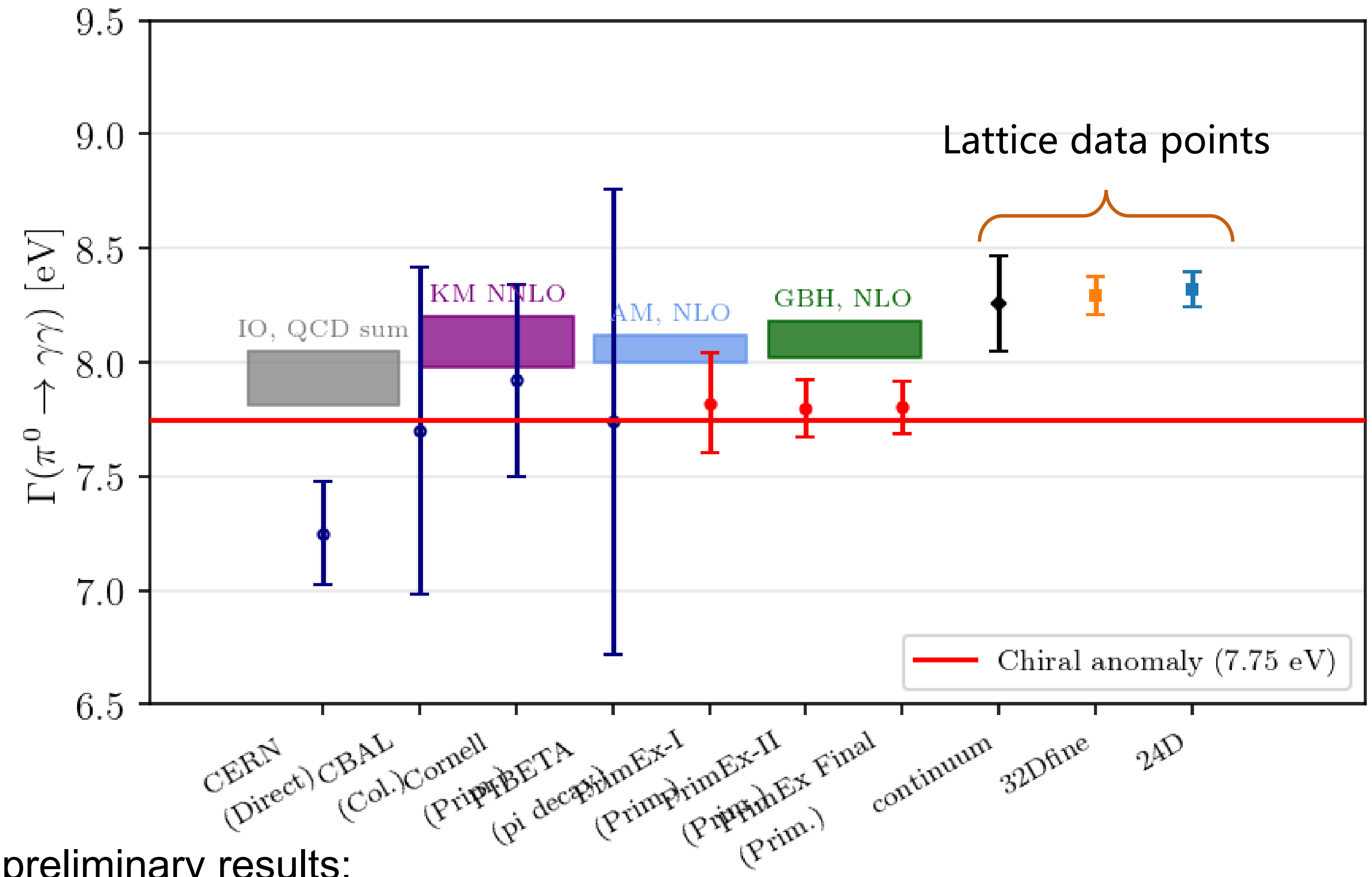
Operator is given by $P^0 = \frac{1}{2}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)$, indicating $\pi - \eta - \eta'$ mixing with isospin breaking

LD is dominated by η and η' state

Total = SD + η LD + η' LD



Preliminary lattice results



We obtain the preliminary results:

- IS contribution is only 10~20% of IB contribution, consistent with ChPT
- ~3% uncertainty after continuum extrapolation
- more compatiable with ChPT

Summary

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1
Precision motivation

$\pi^0 \to \gamma\gamma$ is entering a regime where experiment, ChPT, and lattice QCD can be compared at the percent level.

2
Strategy

Rather than relying on a direct-width calculation, we use the exactly known ABJ anomaly as a reference.

3
Reformulation

The anomalous Ward identity turns the physical-mass correction into a three-point-function problem.

4
Preliminary result

The correction is dominated by the isospin-breaking π^0-η-η' mixing contribution; the IS part is relatively small. Preliminary results are more consistent with ChPT prediction.