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UNIVERSITY OF MARYLAND, COLLEGE PARK

# Non-static mesonic screening masses up to electroweak-scale temperatures

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# Outline

I Screening masses at high temperature

- Why screening masses? Why up to $T = 165~\mathrm{GeV}$?

II Computational strategy and lattice setup

- Lines of constant physics and ensembles
- Random wall sources
- Combined continuum limits

III Discussion of results

- Structure of the screening spectrum
- Temperature dependence and comparison with perturbation theory
- On the restoration of chiral and $\mathrm{U}(1)_A$ symmetry

IV Closing

# Screening spectrum of QCD

**Screening masses** probe the **spatial structure** of the Quark-Gluon Plasma (QGP)

[Detar & Kogut, 1987]

$$C^{(n)}_{O_\Gamma}(x_3) = \int_0^{L_0} \mathrm{d}x_0\; e^{i\omega_n x_0} \int \mathrm{d}x_1 \mathrm{d}x_2\; \langle O_\Gamma(x) O^\dagger_\Gamma(0)\rangle$$
$$\underset{x_3\to\infty}{\propto} \exp\Big\{-m^{(n)}_O x_3\Big\}\Big(1+\mathcal{O}\Big(\exp\Big\{-\Delta^{(n)}_m x_3\Big\}\Big)\Big)$$

Focus on mesonic flavor non-singlet channels in static and non-static Matsubara sectors:

$$\omega_n = \begin{cases} 0 & (n=0) \\ 2\pi T & (n=1) \\ 4\pi T & (n=2) \end{cases}, \qquad O_\Gamma = \bar\psi \Gamma \frac{\lambda^a}{2}\psi$$

| Label | $S^a$ | $P^a$ | $V^a_T$ | $A^a_T$ | $V^a_0$ | $A^a_0$ |
|---|---|---|---|---|---|---|
| $\Gamma$ | $\mathbb{1}$ | $\gamma_5$ | $\gamma_{1,2}$ | $\gamma_5\gamma_{1,2}$ | $\gamma_0$ | $\gamma_5\gamma_0$ |

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Probe **symmetry restoration** at high $T$:
→Talk by Jana Günther

- Non-abelian **chiral symmetry**: degeneracy of $V^a$ and $A^a$ channels
- $U(1)_A$ (topology suppression, "effective restoration"): degeneracy of $P^a$ and $S^a$
  →Talk by Antonio Smecca
- Larger emergent symmetries?

[Aoki et al. 2025; Rohrhofer et al. 2019]

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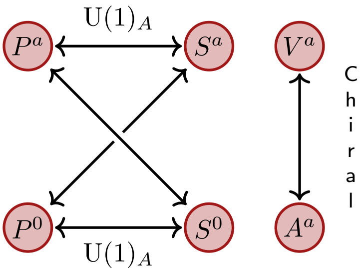


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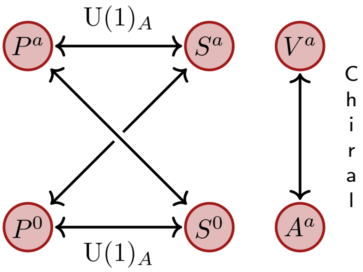


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# Lattice QCD up to the Electroweak scale

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$$L^{-1} \ll M_{\mathrm{Had}} \ll T \ll a^{-1}, \quad \textbf{unfeasible} \text{ for } T > 1\ \mathrm{GeV}$$

Solved with **non-perturbative finite-volume scheme** (SF,GF) + step scaling [Lüscher et al. 1992, ALPHA 2016-2018]

Fix the temperature imposing $\bar{g}^2(g_0, a\mu) = \bar{g}^2(\mu)$ with $a\mu \ll 1$, $\mu \sim T$

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Matched to **hadronic scheme** at low energy $\mu_{\mathrm{Had}}$ to determine overall physical scale [Bruno et al. 2017]

| Label | $T[\mathrm{GeV}]$ | $\bar{g}^2_{\mathrm{SF}}(\mu)$ |
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| $T_0$ | 165(6) | 1.01636 |
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First applications in $N_f = 3$ QCD:

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Sizeable higher order contributions, some of which are **non-perturbative**

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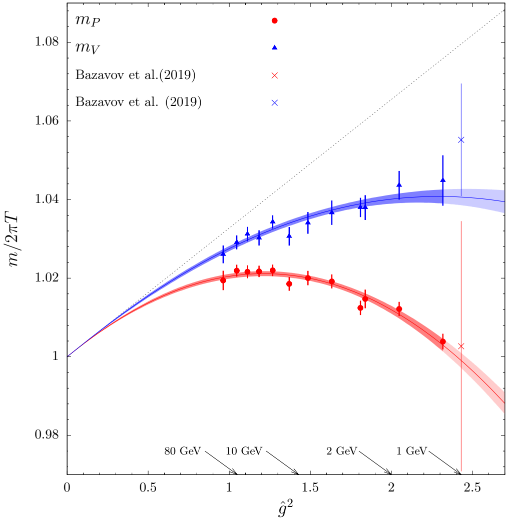


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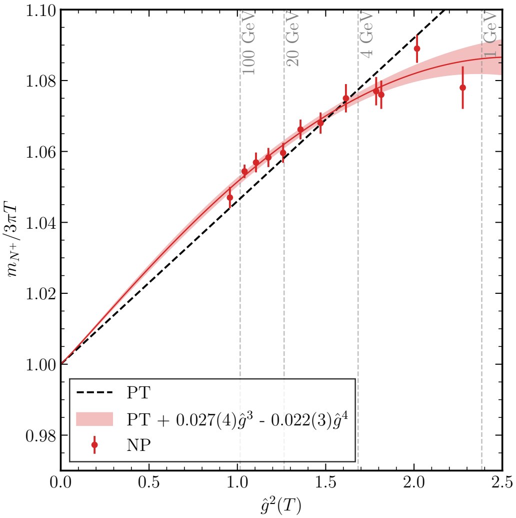


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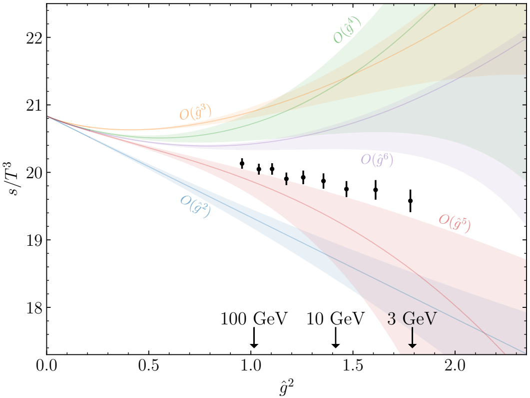


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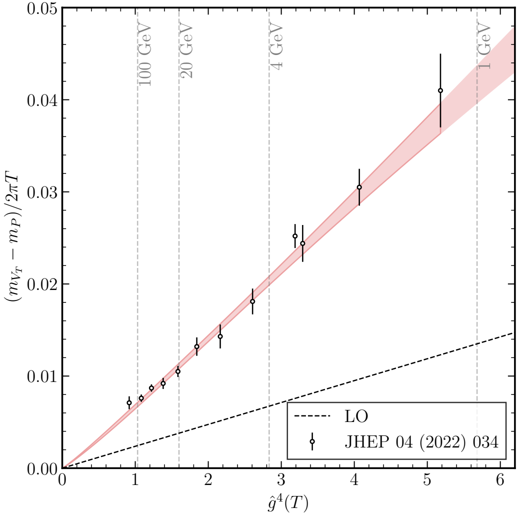


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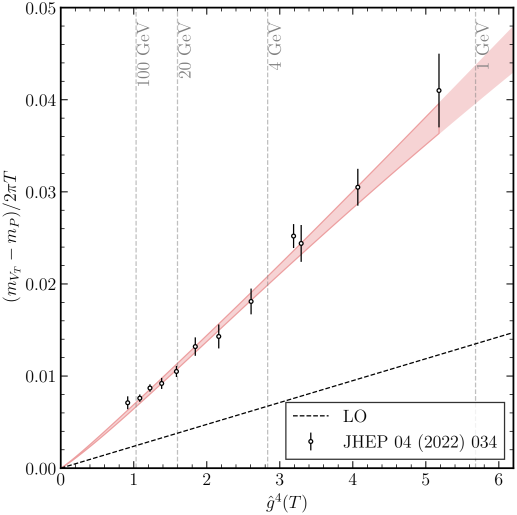


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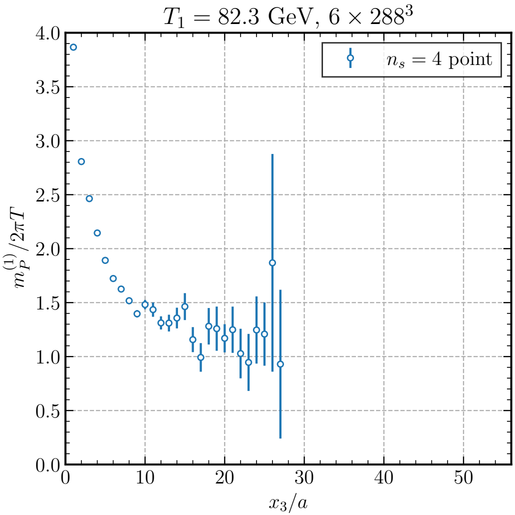
# The exponential Signal-to-Noise problem

$$C^{(n)}_{O_\Gamma}(x_3) = \sum_{x_0,x_1,x_2} e^{i\omega_n x_0} \left\langle O_\Gamma(x) O^\dagger_{O_\Gamma}(0) \right\rangle$$

- $n \geq 1$ sectors suffer from exponential depletion of the Signal-to-Noise ratio

$$\frac{C^{(n)}_{O_\Gamma}}{\sigma_{C^{(n)}_{O_\Gamma}}}(x_3) \overset{x_3\to\infty}{\sim} \exp\left\{-\left(m^{(n)}_{O_\Gamma} - m^{(0)}_P\right) x_3\right\}$$

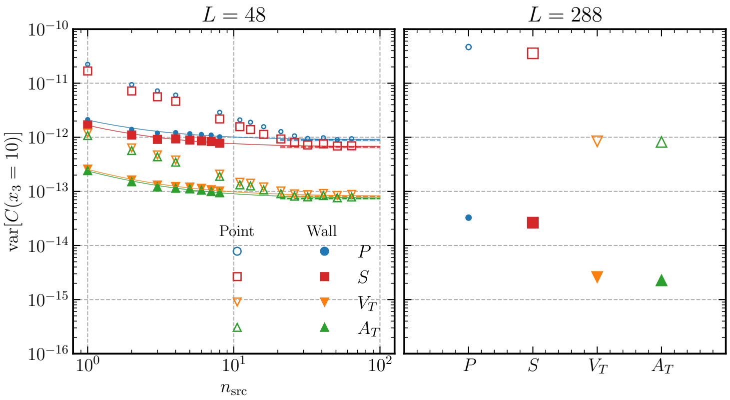
- Projection of point-source static correlators on desired $\omega_n$ is **not a feasible way** to extract the screening masses




▶ Employ and **algorithmic improvement:** estimator with same expectation value but **reduced variance**

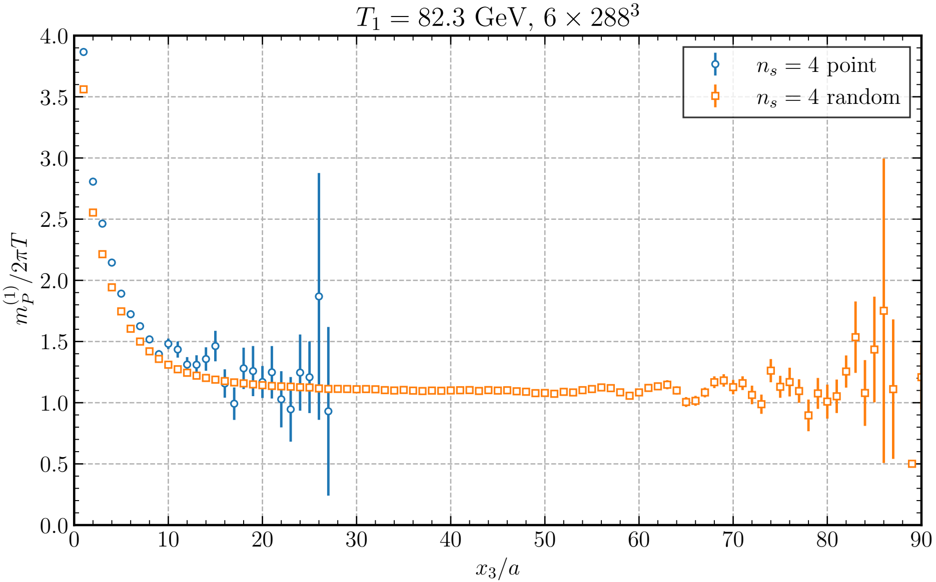
# Random sources

Implement $\mathrm{U}(1)$ noise, study variance on a fixed slice for different $n_{\mathrm{src}}$




- Expected behavior in $n_{\mathrm{src}}$ and $V$ **correctly reproduced**, significant gain even with a single random source
- Variance reduction works also for **vector channels**, differently than what observed at $T = 0$ [Giusti et al., 2019]

# Effective masses – $P$, $n = 1$



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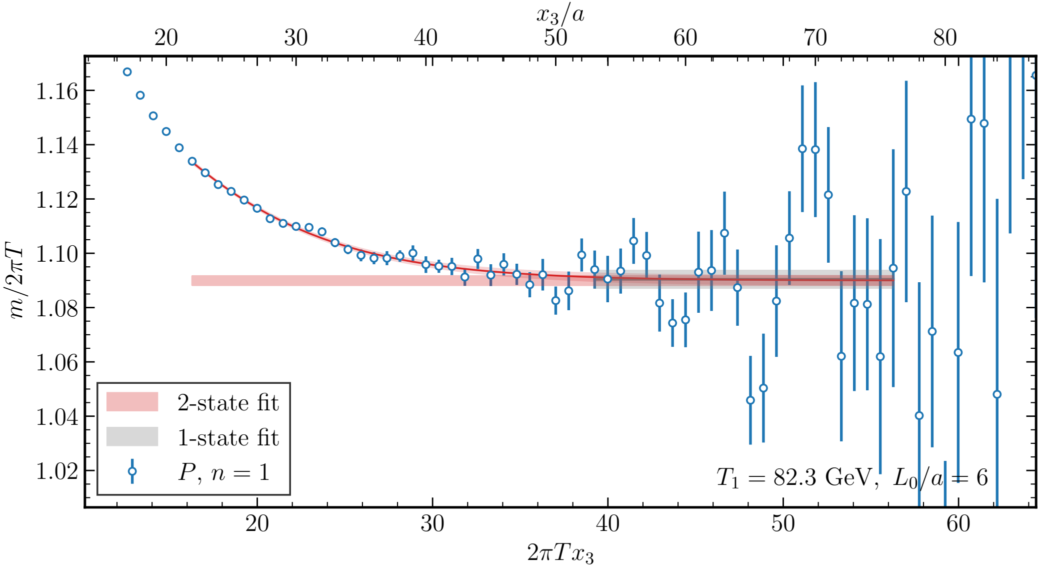
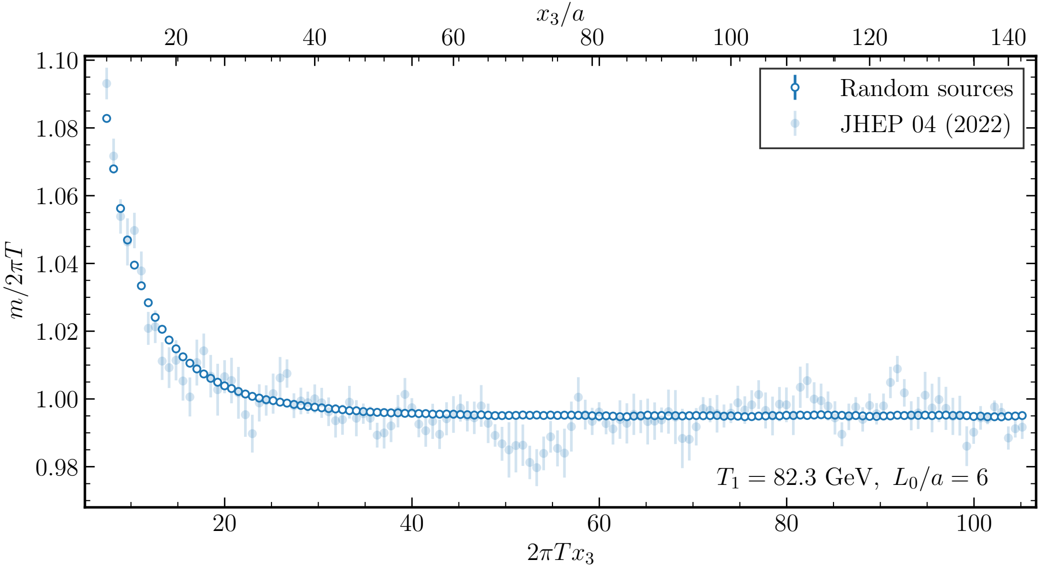


# Effective masses – $P,\ n = 0$



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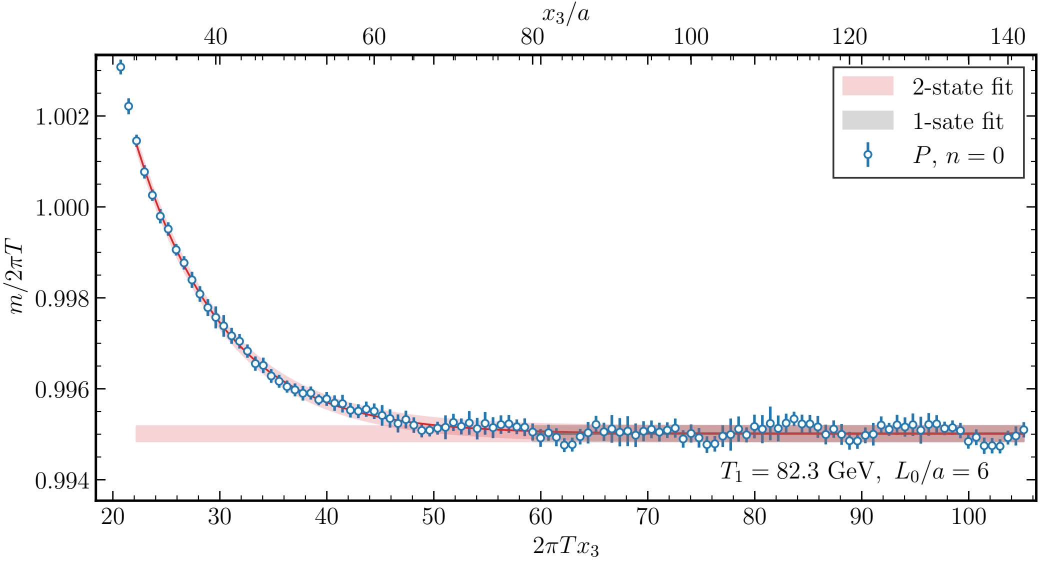

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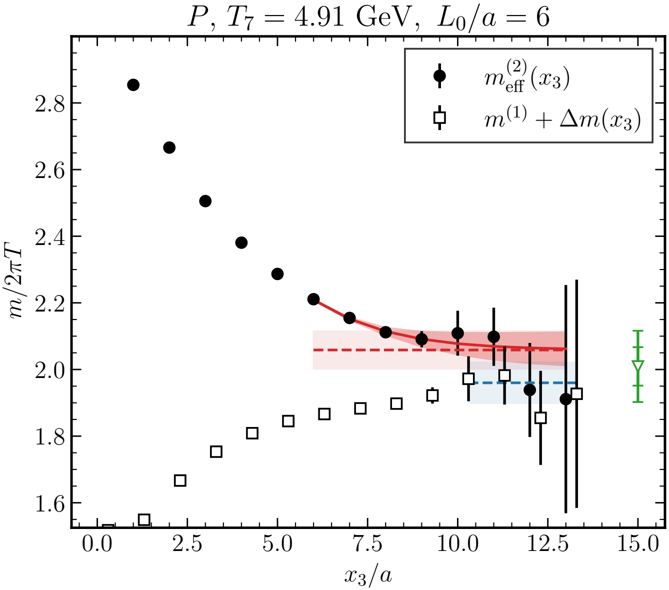
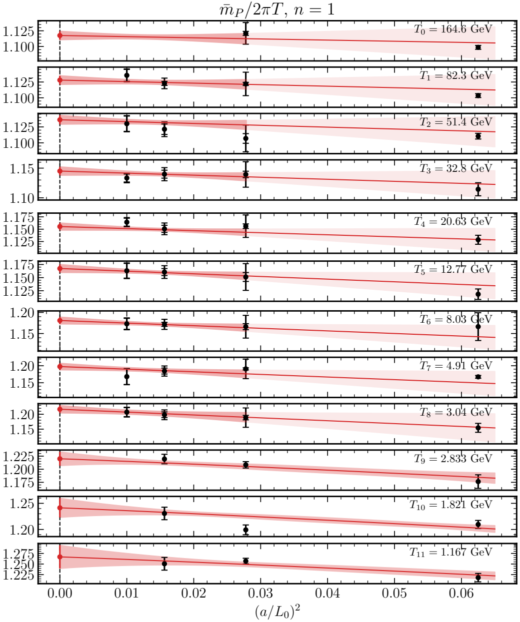


# Global continuum-temperature fit

- Simultaneous **continuum extrapolation** and **interpolation of continuum values** in $\hat{g}(T)$

$$\frac{m}{2\pi T}(a/L_0, T) = \left(1 + b_2\ \hat{g}^2(T) + b_3\ \hat{g}^3(T) + b_4\ \hat{g}^4(T)\right) + \left(\frac{a}{L_0}\right)^2 \left(d^X_{2,2}\ \hat{g}^2(T) + d^X_{2,3}\ \hat{g}^3(T)\right) + d^X_{3,2}\ \left(\frac{a}{L_0}\right)^3\ \hat{g}^2(T).$$

- Different discretization effects $d^X_{i,j}$ for high / low temperatures
- **Final fit**: $L_0/a > 4$ for $T_0 - T_8$, no $(a/L_0)^3$, full form for $T_9 - T_{11}$

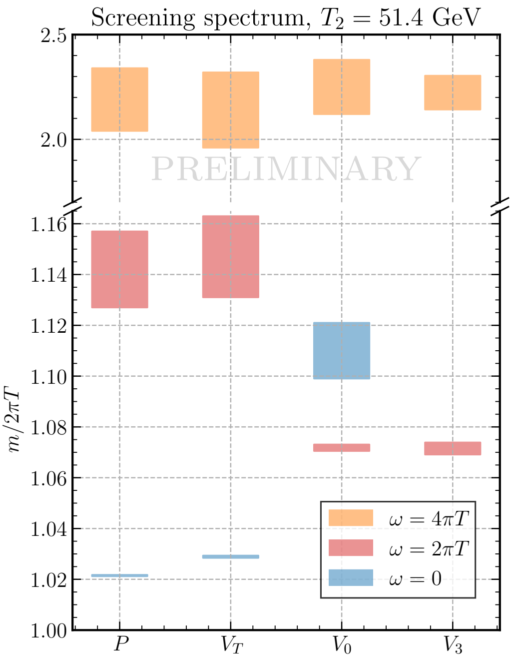


# Structure of the spectrum

- Continuum precision varies significantly across channels:
  - $<< 0.1\%$ for $P,\ V_T$ at $\omega = 0$ (StN$\sim$ **constant**)
  - $\lesssim 0.1\%$ for $V_0,\ V_3$ at $\omega = 2\pi T$
  - $\sim 0.5\%$ for $P, V_T$ at $\omega = 2\pi T$
  - $\sim 1\%$ for $V_0$ at $\omega = 0$
  - $\sim 5 - 10\%$ in the $\omega = 4\pi T$ sector

- **Vector current conservation**: $m_{V_0}^{(n)} \approx m_{V_3}^{(n)}$ for $n \neq 0$. $m_{V_3}^{(0)}$ not defined

- Spin-splitting in the $\omega = 2\pi T, 4\pi T$ sectors **compatible with 0** within errors



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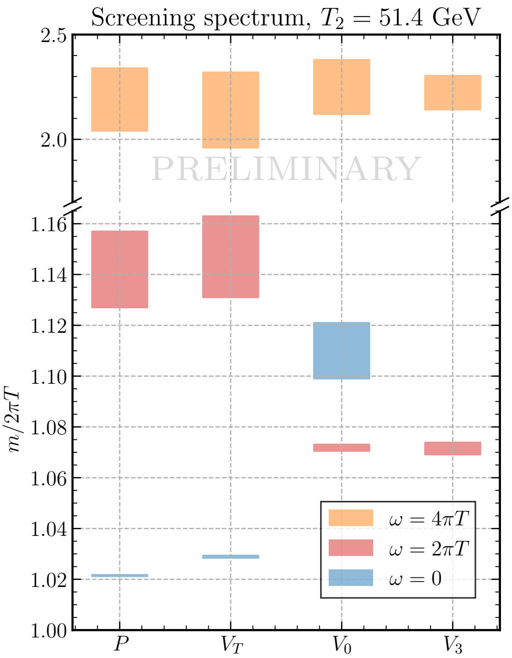

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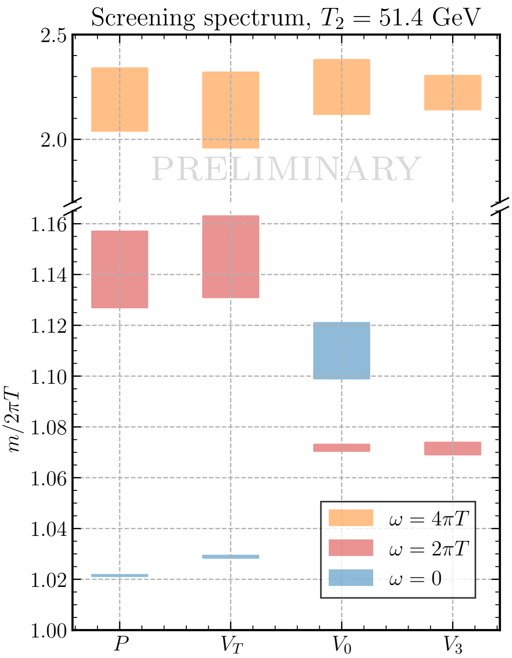

# Temperature dependence

- Precise parameterization for all $T \geq 1$ GeV
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- Quantitative agreement only at **asymptotically high temperatures**, much larger than the Electroweak scale
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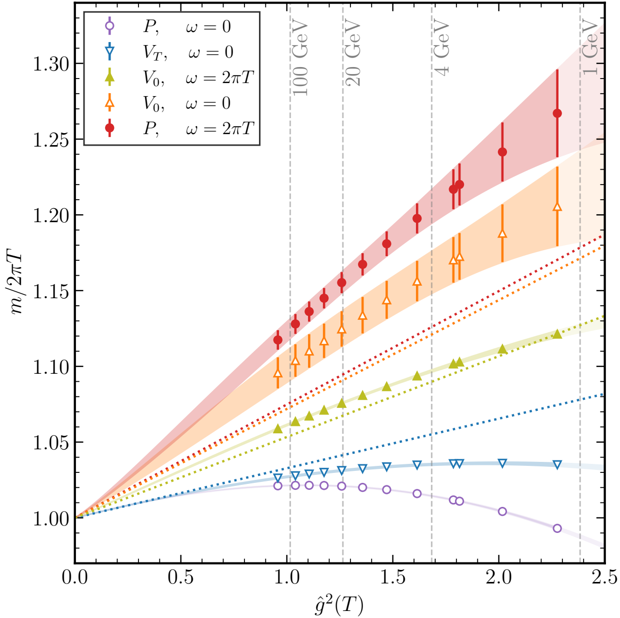

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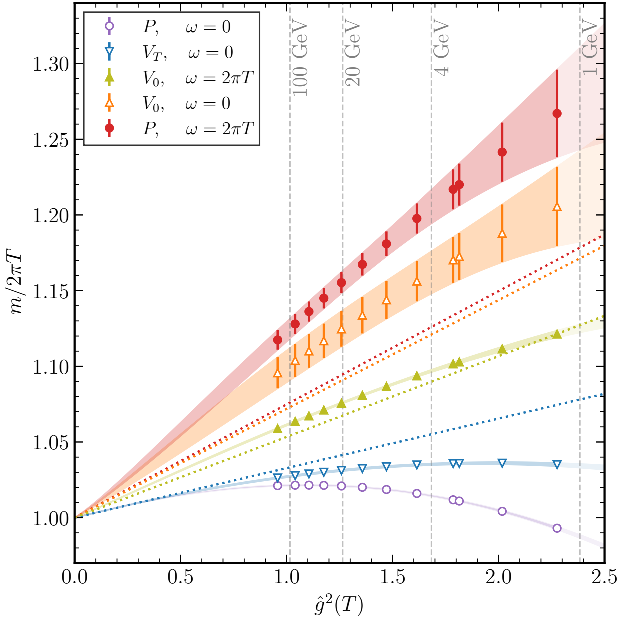

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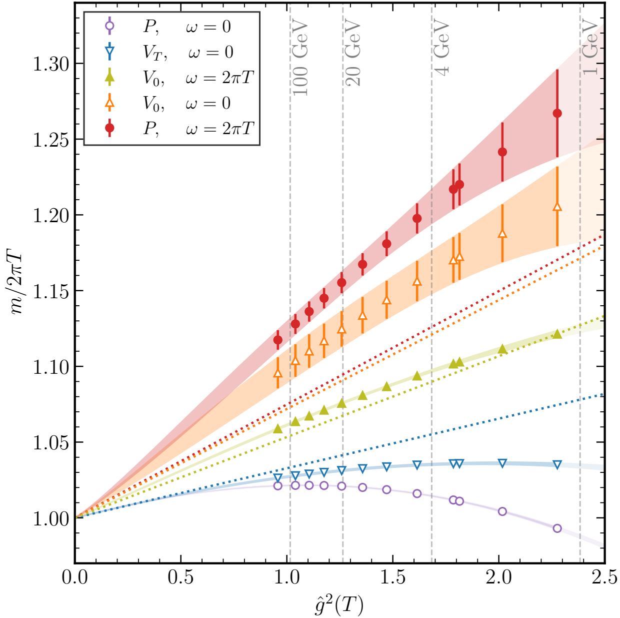

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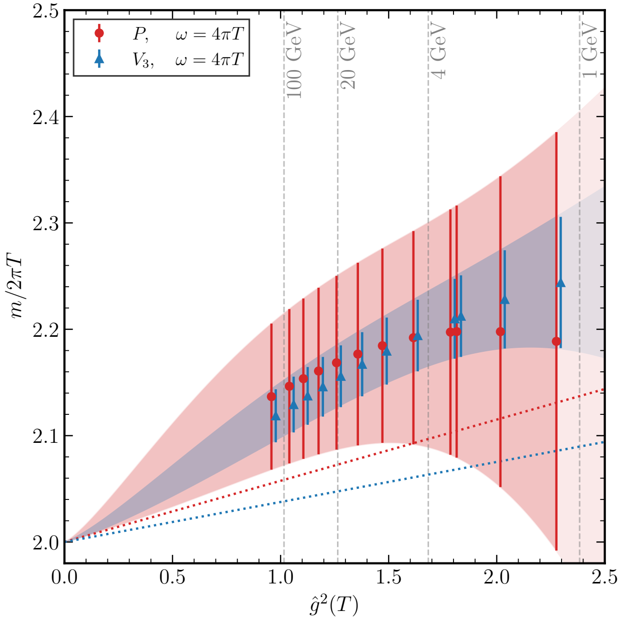


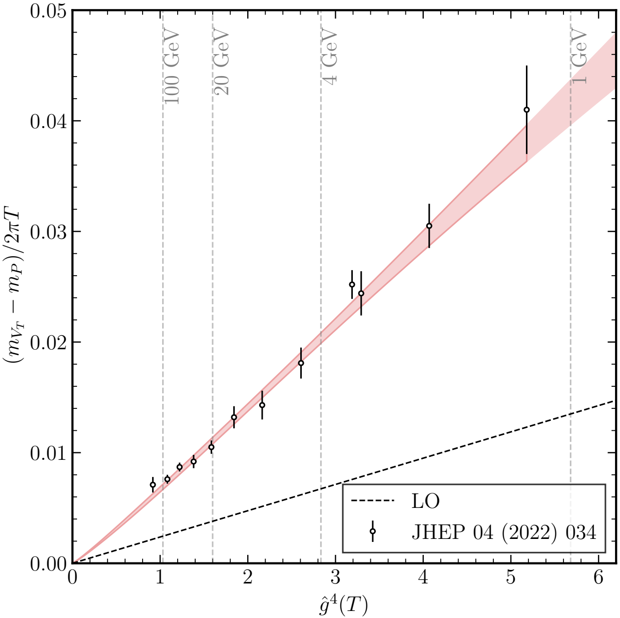
# Hyperfine splitting

- $\mathcal{O}(g^4)$ PT compared with point-source data: effective slope is $\sim 3\times$ larger than PT
[Cè, Giusti, Laudicina, Pepe, **PR** 2025]

- New data, up to $10\times$ more precise, **can not be fit** with a single $\mathcal{O}(g^4)$ term: combined fit to

$$\frac{m_{V_T} - m_P}{2\pi T} = 0.002\,376\,\hat{g}^4 + s_5\hat{g}^5 + s_6\hat{g}^6$$

- Dominated up to $T \sim 165$ GeV by (effective) **higher-order terms** (**non-perturbative** string term in the interquark potential contributes at $\mathcal{O}(g^5)$)

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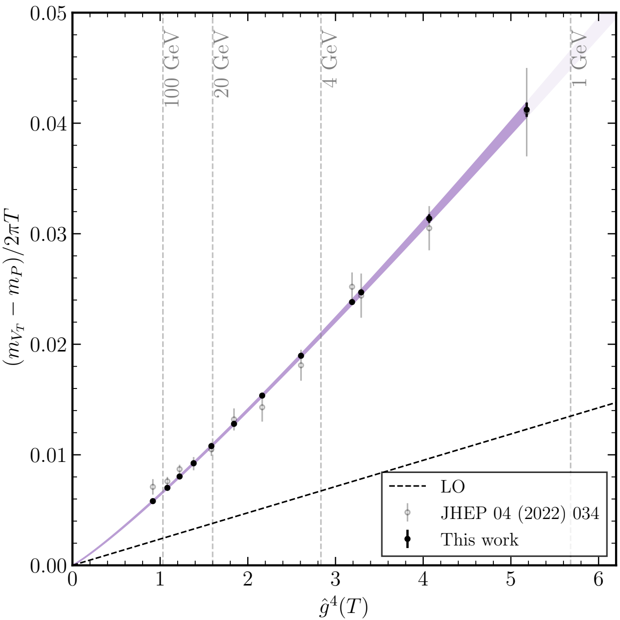

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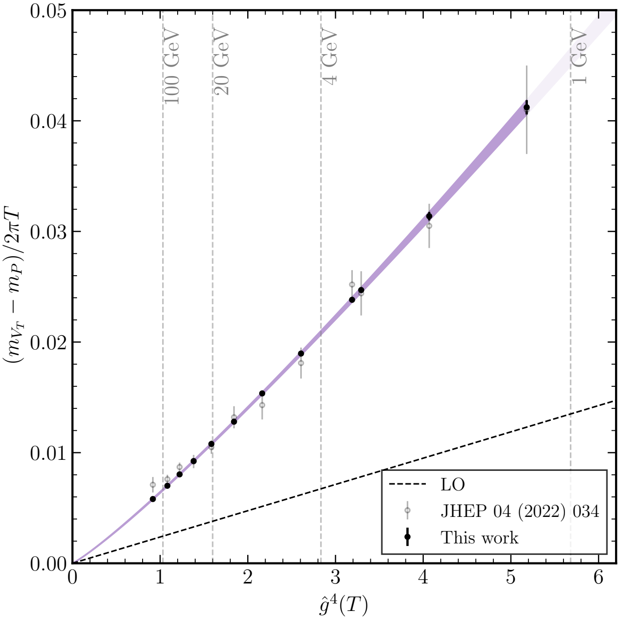
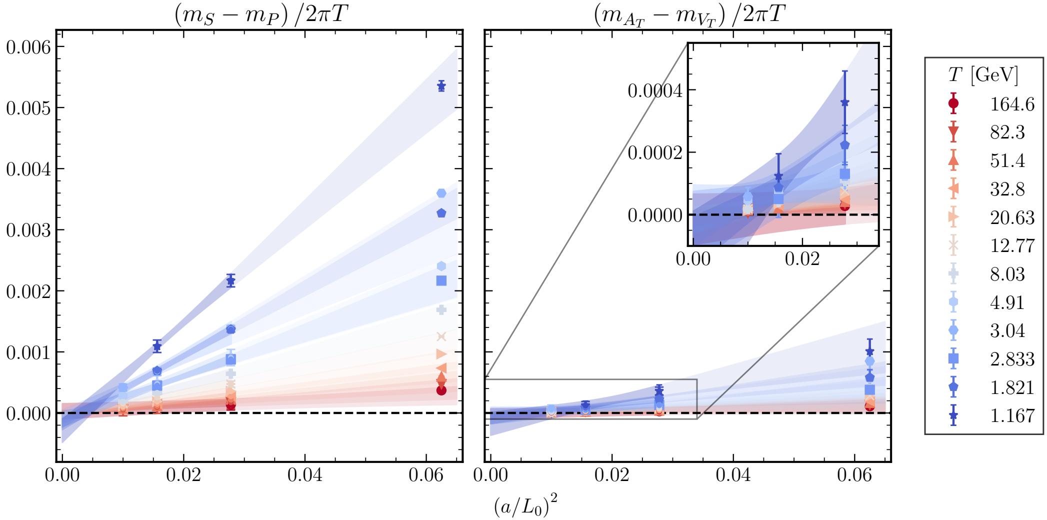


# Chiral and $U(1)_A$ symmetry

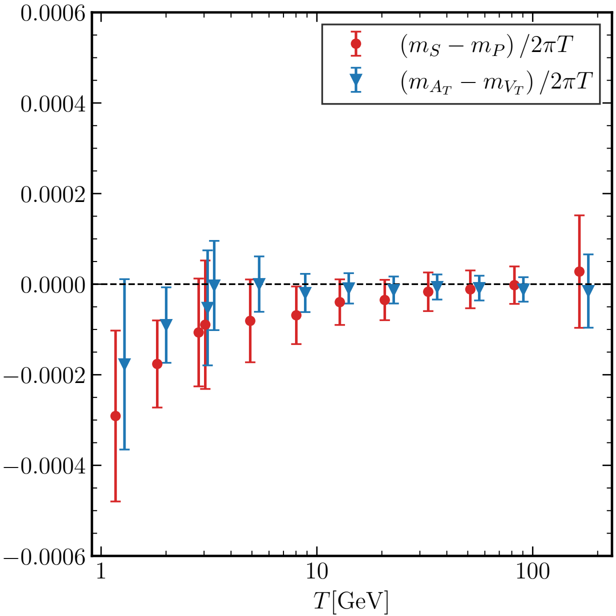



$(n = 1, 2 : m_S - m_P \sim m_A - m_V \sim 0$ at all $a$ and $T)$

# Chiral and $U(1)_A$ symmetry

- **Chiral symmetry** restored over whole temperature range

- Simulations with **fixed** $Q = 0$: confirm the **expected** $S/P$ degeneracy



# Conclusions

- Carried an extensive study of the QCD **screening spectrum** at high temperatures in several mesonic channels and different Matsubara sectors

- **First results** up to the **Electroweak scale** for non-static mesonic channels

- Perturbation theory is **useful** to interpret and parameterize the temperature-dependence of non-perturbative data. However, it **does not offer a quantitative description** over the whole temperature range and in most channels. Similar behavior observed in the QCD Equation of State [Bresciani et al. 2025]

- **Consolidated strategy** to obtain non-perturbative thermal QCD predictions up to the Electroweak scale. Our **high-precision** results:
  - constrain the **range of applicability** of the perturbative expansion of EFT;
  - indicate that the Quark-gluon plasma remains **non-perturbative** up to the Electroweak scale.

- **Continuum limit** and **high precision** crucial to draw conclusions about **strucure** of the spectrum and **symmetries** of the QGP.

Thank you for your attention!

# Backup

# Relevant energy scales

**Three relevant scales** are developed at high $T$

$E$

$\pi T$ (**Hard**) $\psi,\ A_\mu\ (\omega_n \neq 0)$

$gT$ (**Soft**) $A_0\ (w_n = 0)$

$g^2 T$ (**Ultra-soft**) $A_i\ (w_n = 0)$

Ultrasoft gluons cause **breakdown** of PT:

Observables computable only up to a **finte order** after which **infinite** $\#$ of diagrams are needed

E.g. QCD Equation of state computable **only up to $\mathcal{O}(g^6)$** [Kajante et al., 2003]

**Infrared problem** of non-Abelian Yang-Mills gas [Linde, 1980]

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# 3-Dimensional EFT [Braaten and Nieto, 1994]

At high temperature, **hierarchy** sets in: $\frac{g^2T}{\pi} \ll gT \ll \pi T$

QCD undergoes **dimensional reduction** to an Effective Field Theory of the lightest modes

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QCD

Integrate out **hard scale** (non-static modes)

EQCD – 3D YM $A_i$ + adjoint scalar $A_0$

Perturbative series in $g(T)$, starting at $g^2(T)$

MQCD – 3D YM $A_i$

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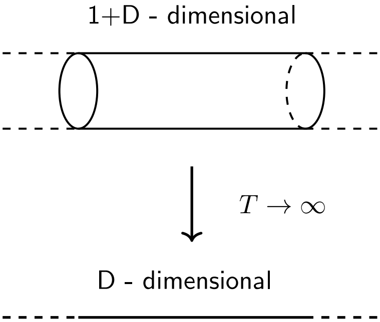
Series in $g^2(T)$, coefficients from lattice



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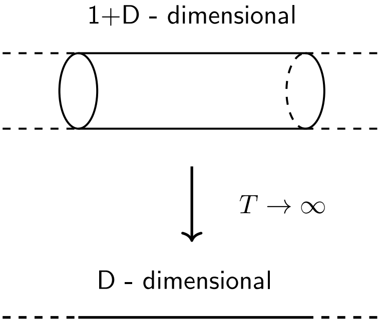

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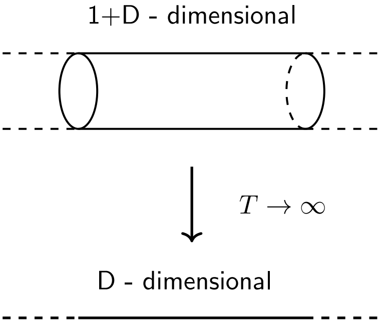

# 3-Dimensional EFT: $\mathrm{QCD}_3$

**Valence heavy quarks** for fermionic observables – NRQCD, perturbative [Brambilla et al. 2005]

- $$S_{\mathrm{NRQCD}} = i\int \mathrm{d}^3x \left\{ \bar{\chi}\left[M - g_{\mathrm{E}}A_0 + D_3 - \frac{1}{2\pi T}\left(D_\perp^2 + \frac{g_{\mathrm{E}}}{4i}[\sigma_j,\sigma_k]F_{jk}\right)\right]\chi + \bar{\phi}\left[M - g_{\mathrm{E}}A_0 - D_3 - \frac{1}{2\pi T}\left(D_\perp^2 + \frac{g_{\mathrm{E}}}{4i}[\sigma_j,\sigma_k]F_{jk}\right)\right]\phi \right\} + \ldots ,$$

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EFT **matched** to QCD in perturbation theory

[Laine and Vepsalainen 2004, Laine and Schroder 2005, Ghisoiu et al. 2015]

$$g_{\mathrm{E}}^2 = g^2T + \mathcal{O}(g^4T), \qquad m_{\mathrm{E}}^2 = g^2T^2\left(\frac{N_c}{3} + \frac{N_f}{6}\right) + \mathcal{O}(g^4T^2)$$

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- **Increased statistical precision** calls for a **detailed investigation**: computed in perturbation theory to interpret the non-perturbative data [Cè, Giusti, Laudicina, Pepe, PR - JHEP 05 (2025) 044]

At $\mathcal{O}(g^2)$, $m_O = 2\pi T\,(1 + 0.032\,740\ldots g^2)\,, \quad O = \{P, V_T\}$ [Laine and Vepsalainen, 2004]

First non-zero term: $\dfrac{\Delta m_{\mathrm{VP}}^{\mathrm{LO}}}{2\pi T} = 0.002\,376 \cdot g^4 \quad + \mathcal{O}(g^5)$

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# Hyperfine splitting: computation

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Comes from Schrödinger equation with spin-independent potential

$$\left[-\frac{\nabla_{\boldsymbol{r}}^2}{\pi T} + 2M + U_0(\boldsymbol{r}) - E_0\right]\psi_0(\boldsymbol{r}) = 0$$

$$U_0(\boldsymbol{r}) = g_{\mathrm{E}}^2 \frac{C_F}{2\pi}\left[\ln\left(\frac{m_{\mathrm{E}} r}{2}\right) + \gamma_{\mathrm{E}} - K_0(m_{\mathrm{E}} r)\right]$$

# Leading Spin-dependent potential

The first spin-dependent potential has the form [Cè, Giusti, Laudicina, Pepe, PR - JHEP 05 (2025) 044] [Bala et al. 2025]

$$U_{\mp}(\boldsymbol{r}) = \mp g_{\mathrm{E}}^2 \frac{C_F}{(2\pi T)^2} \delta(\boldsymbol{r}), \quad \mp \text{ for } P \text{ and } V_T$$

Temperature-suppressed: treat it as a **perturbation** to $U_0(\boldsymbol{r})$

$$E_{\mp} = E_0 + \int_{\mathbf{r}} U_{\mp}(\mathbf{r})|\psi_0(\mathbf{r})|^2 = E_0 \mp g_{\mathrm{E}}^2 \frac{C_F}{(2\pi T)^2} |\psi_0(\mathbf{0})|^2$$

Solved numerically in dimensionless units: $\hat{r} = m_{\mathrm{E}} r$, $\hat{\psi}_0(\hat{r}) = \psi_0(r)\sqrt{2\pi}/m_{\mathrm{E}}$

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# Wavefunction analysis

- A **non-perturbative string term** is expected to appear in the spin-independent potential: $\sigma r \sim \frac{g_{\rm E}^2}{m_{\rm E}}\hat{\sigma}\hat{r} \sim g^3\hat{\sigma}\hat{r}$

- This induces a $\mathcal{O}(g)$ correction in $|\psi_0(\mathbf{0})|^2$ and thus an $\mathcal{O}(g^5)$ term in the hyperfine splitting

- **Non-perturbative effects** are needed to go beyond LO

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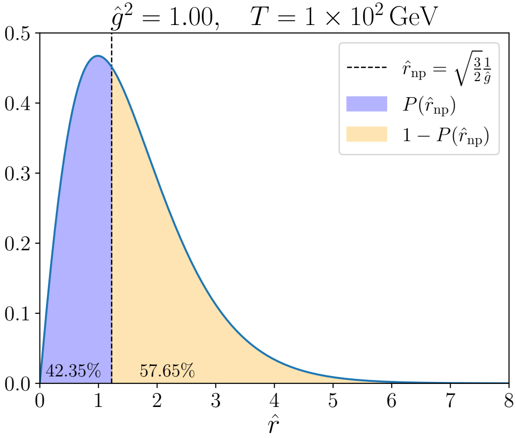


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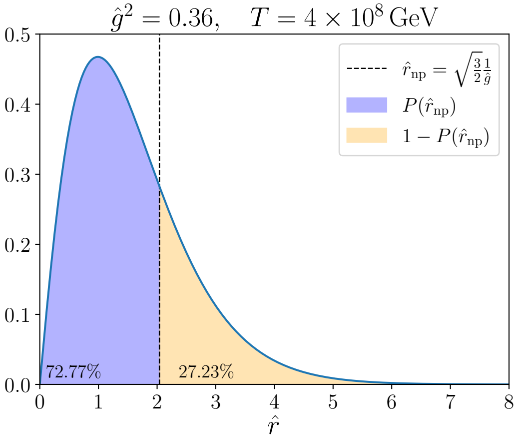


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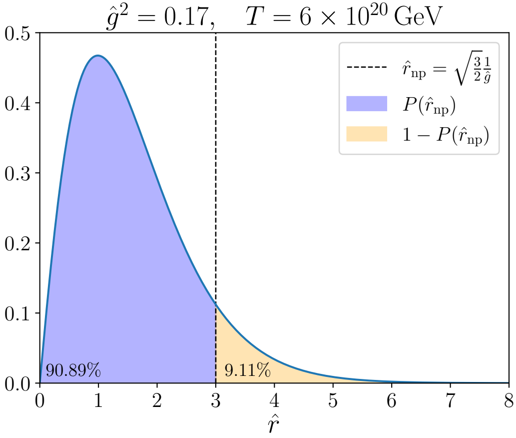


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I **Two-state fit**

- End point: relative error on correlator $\geq 12.5\%$
- Scan start point, look for **stability** and good $\chi^2$/d.o.f.

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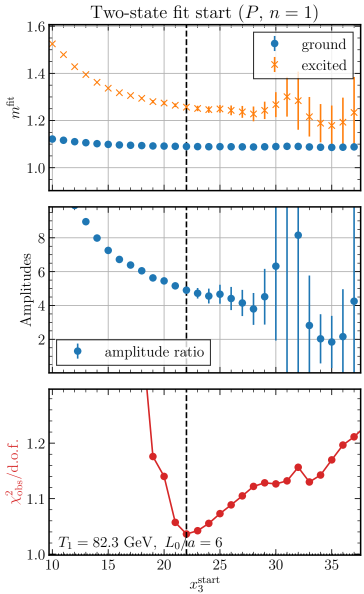

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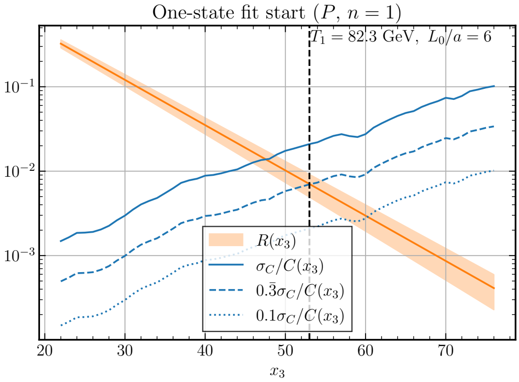

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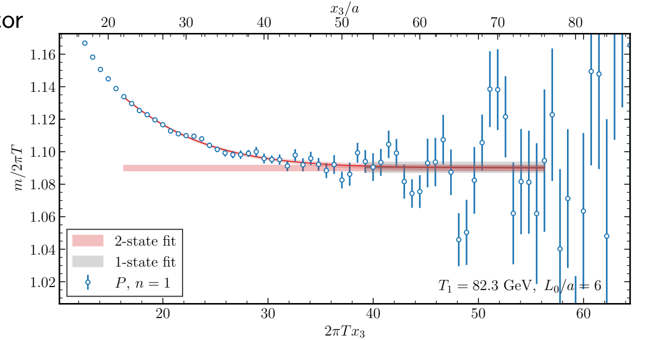

# Random sources

Exploiting translational invariance and our large lattices $V = L_0\, L^2/a^3 \sim 3 - 8 \times 10^5$:

$$\overline{C}(x_3 - y_3) \equiv \frac{a^3}{V} \sum_{\boldsymbol{y}} C(x_3, y) = C(x_3, y) \quad \Rightarrow \quad \sigma_{\overline{C}} \propto \frac{1}{\sqrt{V}} \sigma_C$$

With local point sources requires $V$ inversions, each inversion costing $\propto V$

$\mathcal{O}\left(V^2\right)$ for $\mathcal{O}(V)$ gain in variance!

**Random (wall) sources:** $\eta_i^a(y) = \delta_{y_3 \bar{y}_3} \eta_i(\boldsymbol{y})$, $i = 1 \ldots n_{\mathrm{src}}$, $a = 0 \ldots 11$

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- **Efficient way** to implement volume average **if** one can **suppress** noise variance with moderate number of sources $n_{\mathrm{src}} << V$

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