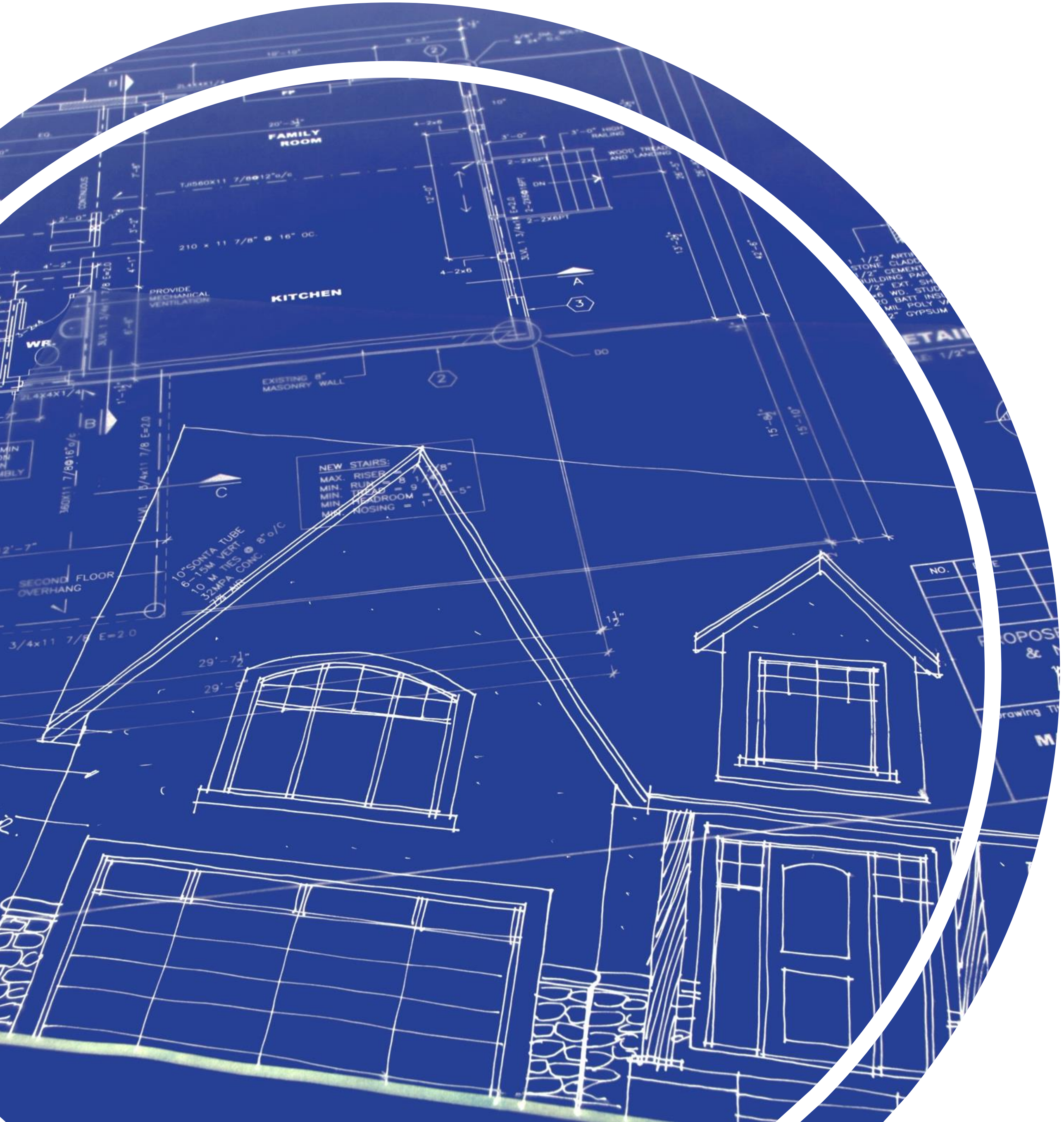
LATTICE CALCULATION OF GENERALIZED PARTON DISTRIBUTIONS WITH LARGE LOG RESUMMATION AT NONZERO SKEWNESS.

Jack Holligan[1], Huey-Wen Lin[2], Rui Zhang[3].

7/27/2026. 3:00pm

arXiv:2607.xxxxx

[1]Fudan University.
[2]Michigan State University.
[3]Massachusetts Institute of Technology.

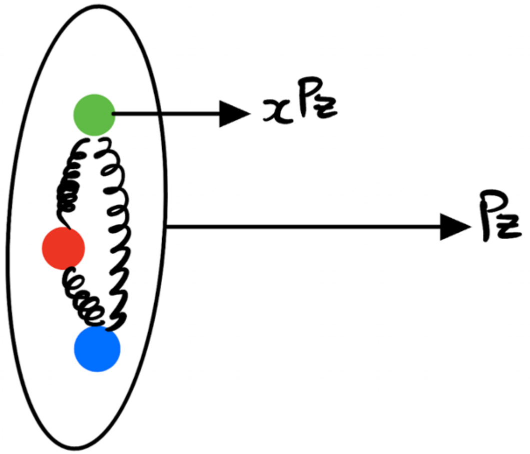


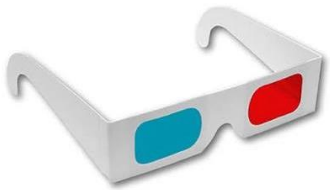
OUTLINE

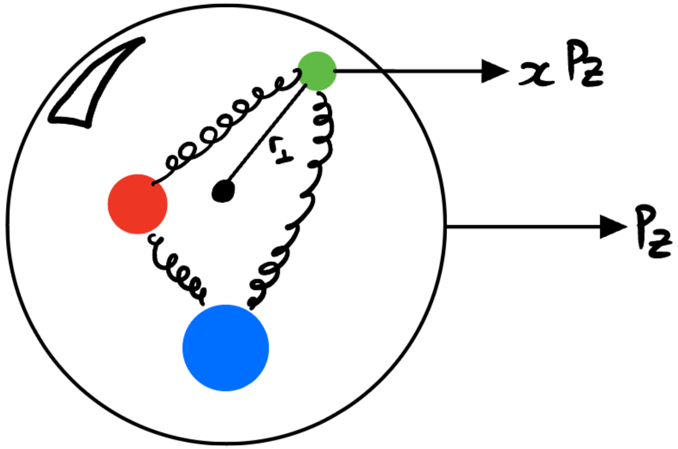
- Motivation; why study GPDs?
- Short summary of Large-Momentum Effective Theory (LaMET).
- Threshold Resummation (TR).
- Results.
- Conclusion.

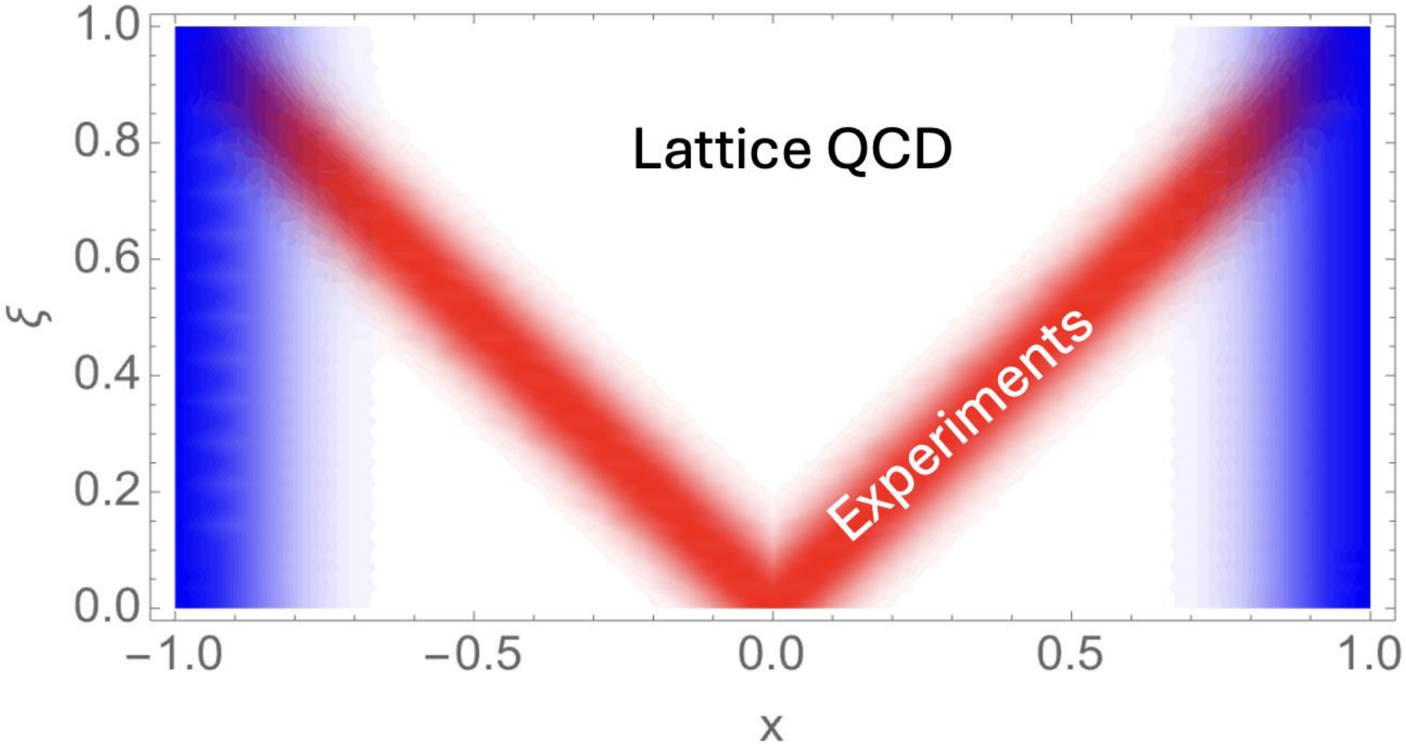
MOTIVATION

- Generalized Parton Distributions (GPDs) paint a 3D picture of the nucleon.
- Describe the distribution of longitudinal momentum in transverse impact-parameter space.
- x-dependent GPDs at skewness ξ are divided into two regions: DGLAP ($|x| > \xi$) and ERBL ($|x| < \xi$).







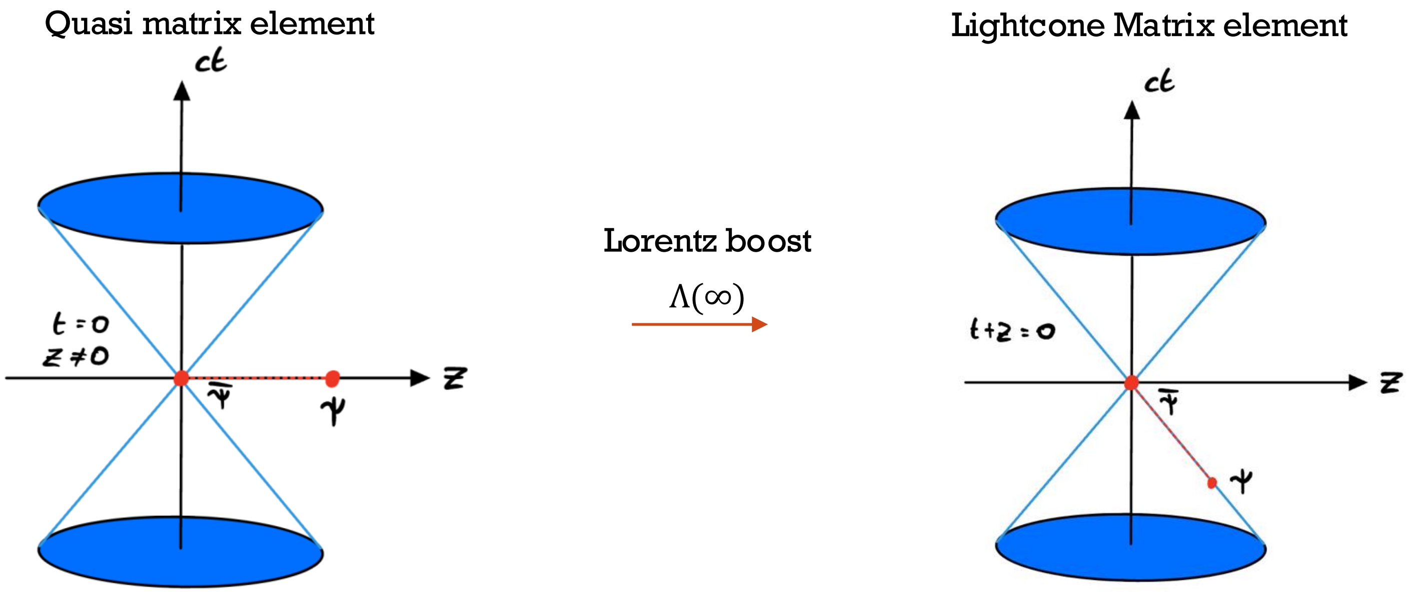


[1]Ji, *Phys.Rev.D* 55 (1997) 7114-7125
[2]Kriestan et. al. *Phys.Rev.D* 101 (2020) 5, 054021

MOTIVATION

- GPDs can be studied experimentally through the processes of deeply-virtual Compton scattering (DVCS)[1] and deeply-virtual meson production (DVMP)[2].
- Experiments have limited access to GPD data but can be supplemented by lattice calculations.

LAMET[1]



- Lightcone quantities are inaccessible on the Euclidean lattice due to their real-time dependence.
- Large-momentum effective theory (LaMET) studies equal-time, spatially separated correlators boosted to large-momentum.

[1] Ji, *Phys.Rev.Lett.* 110 (2013) 262002

THE LAMET CALCULATION

Hybrid Renormalization[1]

Fourier transform

$h^B(z, Q^2, \xi)$ → $h^R(z, Q^2, \xi)$ → $h^R(z \to \infty, Q^2, \xi)$ → $\tilde{F}(x, Q^2, \xi)$ → $F(x, Q^2, \xi)$

Extrapolation[2,3]

Lightcone matching

$$F(x, Q^2, \xi) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \mathcal{C}^{-1}\left(x, y, \xi, \frac{P_z}{\mu}\right) \tilde{F}(y, Q^2, \xi)$$

[1]Yao, et. al., *JHEP* 11 (2023) 021
[2]Ji, Liu, Su, *arXiv:2601.12189*
[3]Chen, et. al., *Phys.Rev.D* 113 (2026) 1, 1

THE LAMET CALCULATION

Hybrid Renormalization[1]

Fourier transform

$h^B(z, Q^2, \xi)$ → $h^R(z, Q^2, \xi)$ → $h^R(z \to \infty, Q^2, \xi)$ → $\tilde{F}(x, Q^2, \xi)$ → $F(x, Q^2, \xi)$

Extrapolation[2,3]

Lightcone matching

$$F(x, Q^2, \xi) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \mathcal{C}^{-1}\left(x, y, \xi, \frac{P_z}{\mu}\right) \tilde{F}(y, Q^2, \xi)$$

This is where the threshold resummation is applied

[1]Yao, et. al., *JHEP* 11 (2023) 021
[2]Ji, Liu, Su, *arXiv:2601.12189*
[3]Chen, et. al., *Phys.Rev.D* 113 (2026) 1, 1

MULTISCALE NATURE

- The relevant pieces of the one-loop matching kernel are[1]

$$\mathcal{C}\left(x,y,\xi,\frac{P_z}{\mu}\right)$$
$$\ni \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{|\xi+x|}{2\xi(\xi+y)}+\frac{|\xi+x|}{(\xi+y)(y-x)}\right)\left(\ln\left(\frac{4(\xi+x)^2P_z^2}{\mu^2}\right)-1\right)\right.$$
$$+\left(\frac{|\xi-x|}{2\xi(\xi-y)}+\frac{|\xi-x|}{(\xi-y)(x-y)}\right)\left(\ln\left(\frac{4(\xi-x)^2P_z^2}{\mu^2}\right)-1\right)$$
$$\left.+\left(\left(\frac{\xi+x}{\xi+y}+\frac{\xi-x}{\xi-y}\right)\frac{1}{|x-y|}-\frac{|x-y|}{\xi^2-y^2}\right)\left(\ln\left(\frac{4(x-y)^2P_z^2}{\mu^2}\right)-1\right)\right]_+^{[-\infty,\infty]}$$

"Momentum logs"

"Threshold logs"

The plus-prescription regulates the singularity at $x=y$: $[f(x,y)]_+^D = f(x,y) - \delta(x-y)\int_D dw\, f(w,y)$

Real Virtual

[1]Yao, Ji, Zhang, *JHEP* 11 (2023) 021

MOMENTUM LOGS

- The momentum logs are prevented from diverging by their prefactor $|\xi \pm x|$. The real part is simple[1].

$$\lim_{x \to \mp\xi} \left(\frac{|\xi \pm x|}{2\xi(\xi \pm y)} \pm \frac{|\xi \pm x|}{(\xi \pm y)(y - x)} \right) \left(\ln\left(\frac{4(\xi \pm x)^2 P_z^2}{\mu^2} \right) - 1 \right) = 0$$

- The virtual term requires more work but does vanish provided the substitution $x = \mp\xi$ is evaluated *before* the integral.

[1] ***JH***, Lin, Zhang, Zhao, *JHEP* 207 (2025) 241

THRESHOLD LOGS

- However, the whole situation changes in the threshold limit $x \to y$:

$$
\begin{aligned}
&\mathcal{C}\left(x \to y, \xi, \frac{P_z}{\mu}\right) \\
&\ni \frac{\alpha_s C_F}{4\pi}\left[\frac{\mathrm{sgn}(\xi+x)}{(y-x)}\left(\ln\left(\frac{4(\xi+x)^2 P_z^2}{\mu^2}\right)-1\right)+\frac{\mathrm{sgn}(\xi-x)}{(y-x)}\left(\ln\left(\frac{4(\xi-x)^2 P_z^2}{\mu^2}\right)-1\right)\right. \\
&\left.+\frac{2}{|x-y|}\left(\ln\left(\frac{4(x-y)^2 P_z^2}{\mu^2}\right)-1\right)\right]_{+}^{[-\infty,\infty]}
\end{aligned}
$$

The suppression factor disappears in the threshold limit[1]!

[1] ***JH***, Lin, Zhang, Zhao, *JHEP* 207 (2025) 241

THRESHOLD LOGS IN DEPTH

- Consider the matching process carried out over the narrow window $[x-\eta, x+\eta]$

$$\Delta\mathrm{F}(\epsilon = x - \xi, \eta) = \int_{x-\eta}^{x+\eta} dy \frac{|\xi - x| \ln^n|\xi - x|}{(\xi - y)(x - y)} - \int_{x-\eta}^{x+\eta} dz \frac{|\xi - z| \ln^n|\xi - z|}{(\xi - x)(z - y)}$$

- Isolating the piece nearest the threshold $\eta' \gtrsim m\epsilon$, we find.

$$\int_{m\epsilon}^{\eta} d\eta' \frac{\ln^n|\eta' + \epsilon|^2 + \ln^n|\eta' - \epsilon|^2}{\eta'} \sim -\frac{\ln^{n+1}(m\epsilon)^2}{n+1}$$

- This confirms that a genuine large logarithm survives – only one of several terms scaling this way - which justifies our resummation in a narrow window around $x = \xi$.

FACTORIZATION

- In the threshold limit, the matching kernel factorizes into a Sudakov factor, H, and a jet function, J[1,2].

$$\mathcal{C}\left(x \to y, \xi, \frac{P_z}{\mu}\right) = J(|x-y|P_z) \otimes H\big((x+\xi)P_z\big) \otimes H\big((x-\xi)P_z\big).$$

with

- $H\left(L_z^{\pm} = \ln\left(\frac{4x^2P_z^2}{\mu^2}\right) \mp i\pi\mathrm{sgn}(zxP_z), \mu\right) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\frac{1}{2}\left(L_z^{\pm}\right)^2 + L_z^{\pm} - 2 - \frac{5\pi^2}{12}\right)$
- $J\left(l_z = \ln\left(\frac{z^2\mu^2e^{2\gamma_E}}{4}\right), \mu\right) = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{2}l_z^2 + l_z + \frac{\pi^2}{12} + 2\right)$
- Thus, the resummed matching kernel is

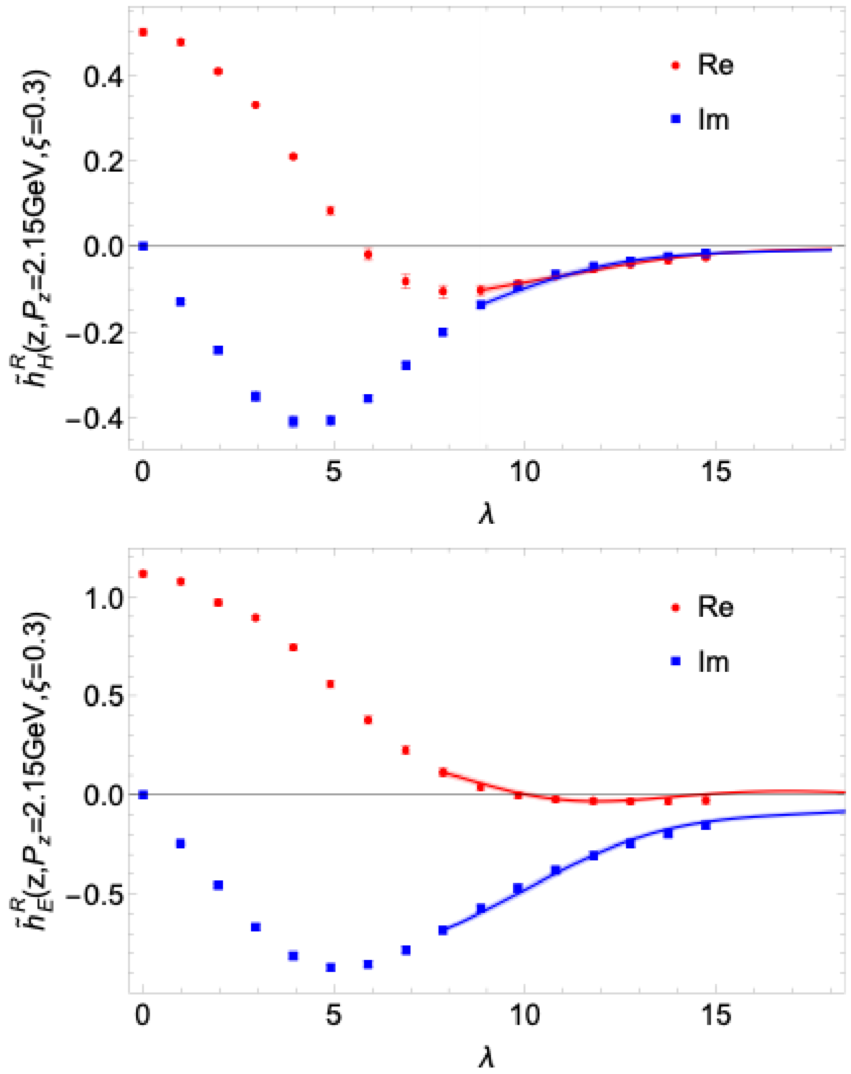
$$\mathcal{C}_{TR} = (H \otimes J)_{TR} \otimes (H \otimes J)_{NLO}^{-1} \otimes \mathcal{C}_{NLO}$$

[1] *Ji, Liu, Su, JHEP* 08 (2023) 037
[2] *Ji, Liu, Su, Zhang, JHEP* 03 (2025) 045

LATTICE SETUP

- Lattice configurations from MILC[1,2,3].
- $N_f = 2 + 1 + 1$ clover valence fermions.
- Pion mass $m_\pi \approx 130$ MeV.
- Lattice spacing $a = 0.09$ fm. Box width $L = 64a = 5.76$ fm.
- 1960 configurations × 256 measurements per configuration = 501,760 measurements.
- Skewness values $\xi = 0.1$ and $\xi = 0.3$ with momentum transfers $Q^2 = 0.23$ and $0.43\ GeV^2$, respectively.

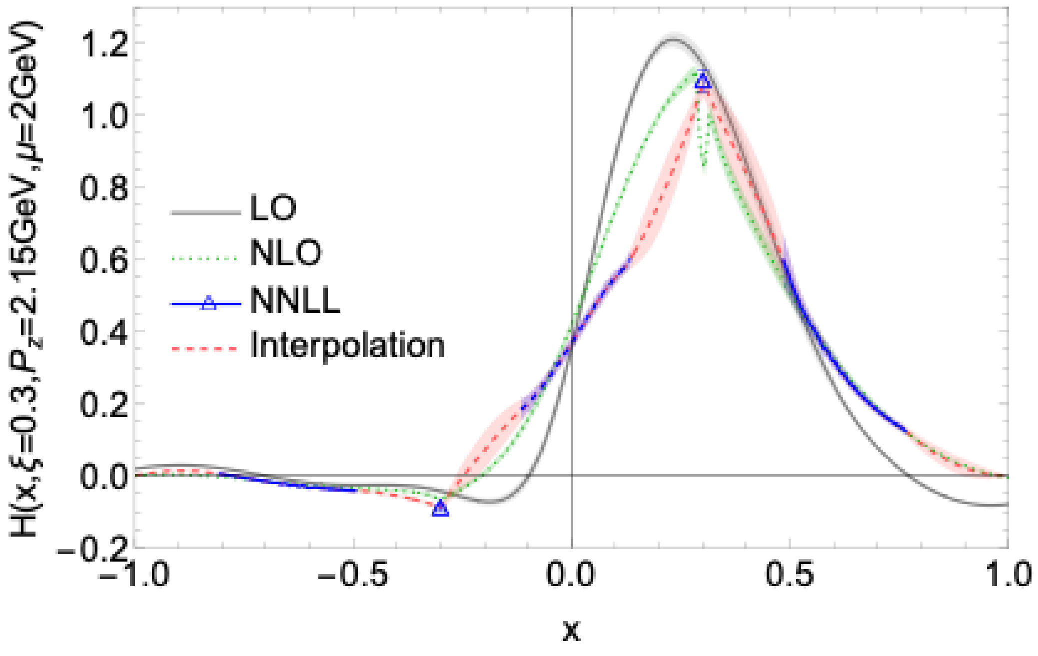
[1]MILC. *Phys.Rev.D* 82 (2010) 074501
[2]MILC. *Phys.Rev.D* 87 (2013) 5, 054505
[3]MILC. *Phys.Rev.D* 93 (2016) 9, 094510

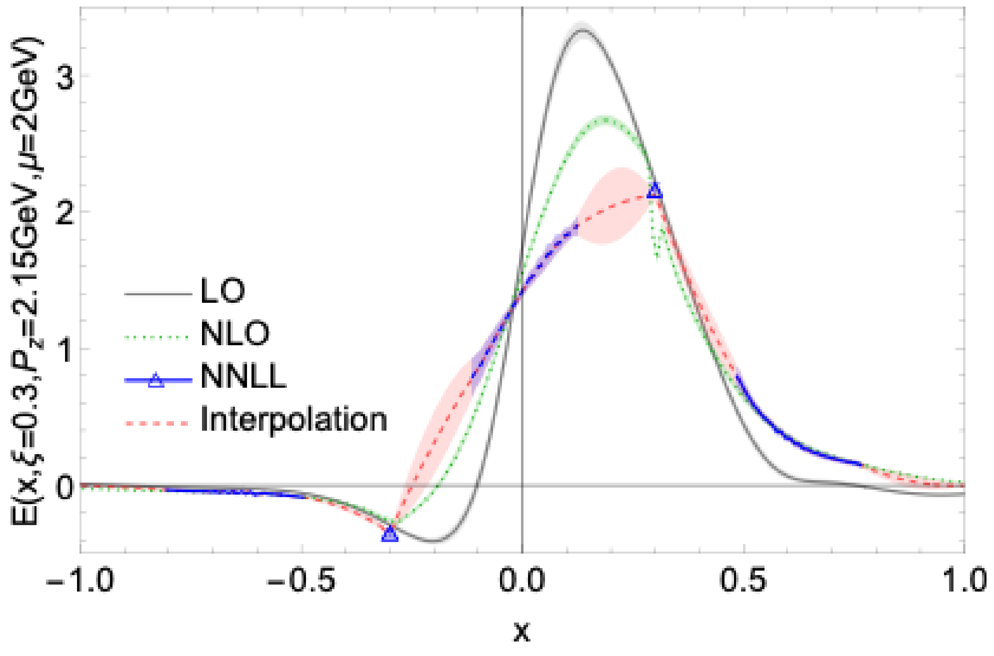


MATRIX ELEMENTS TO QGPD

- Matrix elements are renormalized in the hybrid-ratio scheme[1] with large-distance extrapolation.
- Fourier transforming converts from coordinate- to momentum-space.
- Figure: Renormalized matrix elements for $\xi = 0.3$, $Q^2 = 0.43\ \text{GeV}^2$ as a function of $\lambda = zP_z$. Top (bottom) is H (E) function.
- Renormalization and matching for real (red) and imaginary (blue) parts at $(NLO + LRR) \times RGR$[2,3].

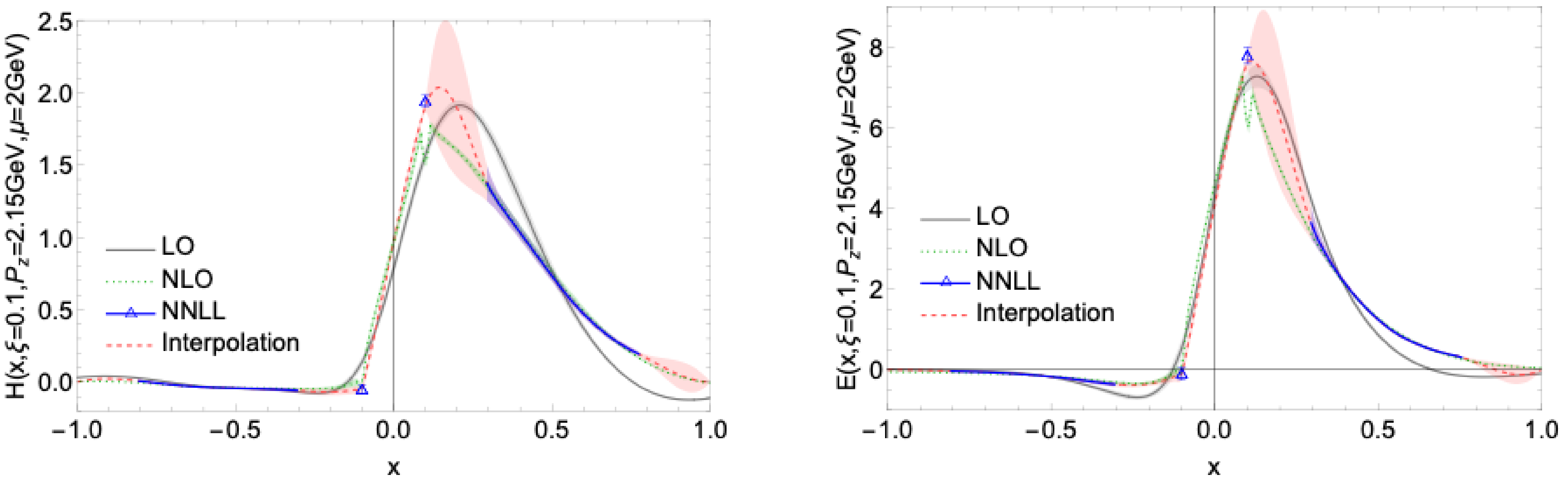
[1]Ji, et. al., *Nucl.Phys.B* 964 (2021) 115311
[2]*Zhang*, ***JH***, Ji, Su, *Phys.Lett.B* 844 (2023) 138081
[3]Su, **JH**, Ji, Yao, Zhang, *Nucl.Phys.B* 991 (2023) 116201





RESULTS. $\xi = 0.3$

- x-dependent (quasi-)GPDs at $\xi = 0.3$. Left (right) is H (E).
- Quasi-GPD (black), fixed order matching (green), threshold resummation (blue), simple interpolation where LaMET breaks down (red).
- Mild effect in DGLAP region ($|x| > \xi$), significant effect in ERBL region ($|x| < \xi$).



RESULTS. $\xi = 0.1$

- x-dependent (quasi-)GPDs at $\xi = 0.1$. Left (right) is H (E).
- Quasi-GPD (black), fixed order matching (green), threshold resummation (blue) , simple interpolation where LaMET breaks down (red).
- Mild effect in DGLAP region ($|x| > \xi$), significant effect in ERBL region ($|x| < \xi$). Effects reduced compared to $\xi = 0.3$.

CONCLUSION

- We have made the first application of threshold resummation to real x-dependent GPD data computed on the Euclidean lattice.
- Revisited the log structure near $x = \pm\xi$: the log vanishes pointwise at threshold, but a genuine logarithm survives once convolved against a smooth GPD — confirming resummation is needed in a narrow threshold window.
- Small effects in DGLAP regions. Significant effect in ERBL region.
- Effect is even greater with increasing ξ.

BACKUP SLIDES

- $H(p,\mu,\pm) = |H|(p,\mu)e^{S(\mu_h,\mu)-a_H(\mu_h,\mu)}e^{iA_\pm(\mu_h)\mp\frac{i\pi}{2}a_\Gamma(\mu_h,\mu)}\left(\frac{\mu_h}{2p}\right)^{a_\Gamma(\mu_h,\mu)}$
- $J(\Delta,\mu) = e^{[-2S(\mu_i,\mu)+a_J(\mu_i,\mu)]}\tilde{J}_z(l_z = -2\partial_\eta, \alpha_s)\left[\frac{\sin\left(\frac{\eta\pi}{2}\right)}{|\Delta|}\left(\frac{2|\Delta|}{\mu_i}\right)^\eta\right]_* \frac{\Gamma(1-\eta)e^{-\eta\gamma_E}}{\pi}\Big|_{\eta=2a_\Gamma(\mu_i,\mu)}$
- Resummed Sudakov factor:

$$H(p,\mu,\pm) = |H|(p,\mu)e^{S(\mu_h,\mu)-a_H(\mu_h,\mu)}e^{iA_\pm(\mu_h)\mp\frac{i\pi}{2}a_\Gamma(\mu_h,\mu)}\left(\frac{\mu_h}{2p}\right)^{a_\Gamma(\mu_h,\mu)}$$

- Resummed jet function:

$$J(\Delta,\mu) = e^{[-2S(\mu_i,\mu)+a_J(\mu_i,\mu)]}\tilde{J}_z(l_z = -2\partial_\eta, \alpha_s)\left[\frac{\sin\left(\frac{\eta\pi}{2}\right)}{|\Delta|}\left(\frac{2|\Delta|}{\mu_i}\right)^\eta\right]_* \frac{\Gamma(1-\eta)e^{-\eta\gamma_E}}{\pi}\Big|_{\eta=2a_\Gamma(\mu_i,\mu)}$$