Baryon-baryon interactions in the S = -1 channel and inelastic contaminations in physical-point lattice QCD

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for HAL QCD Collaboration

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Background for S = -1 interaction

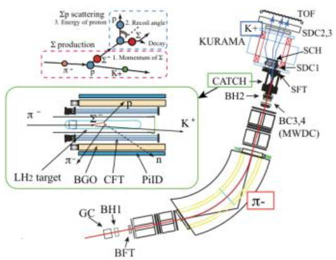
***S* = -1**: Systems of minimum number of strangeness
→ Most common/accessible non-nucleonic systems

Present study
YN (2-body, S = -1) w/ the HAL QCD method

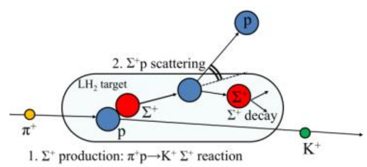
Examples

Scattering exp. Σp, Λp

Cross section、phase shift、cusp、etc.

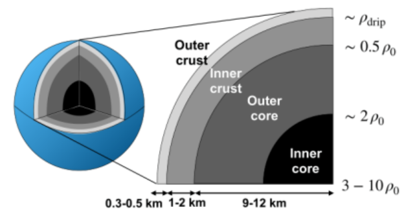


Miwa et al. [J-PARC E40] (2021)



Nanamura et al. [J-PARC E40] (2022)

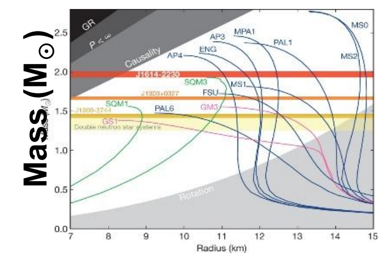
Neutron star EoS



Massive neutron stars

Hyperon puzzle

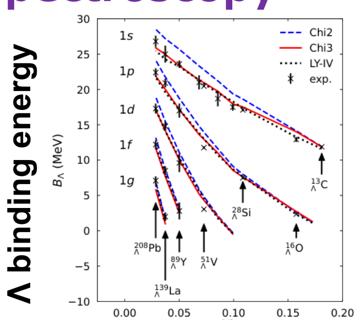
***M-R* relation**



Radius (km)

Demorest et al. (2010)

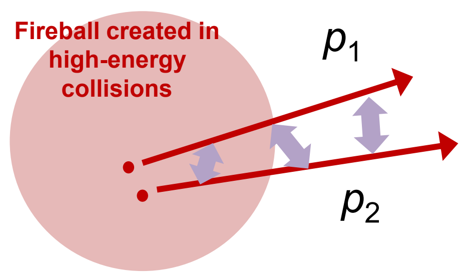
Hypernuclear spectroscopy



Jinno, KM, Nara, Ohnishi PRC 108 (2023)

High energy heavy-ion collisions

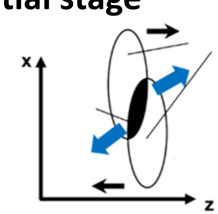
Femtoscopy



Single-particle potential

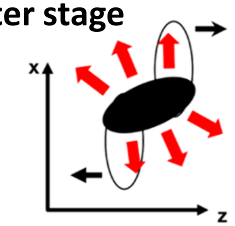
→ Collective flow in RQMD

Initial stage



Figures: Jinno (2023)

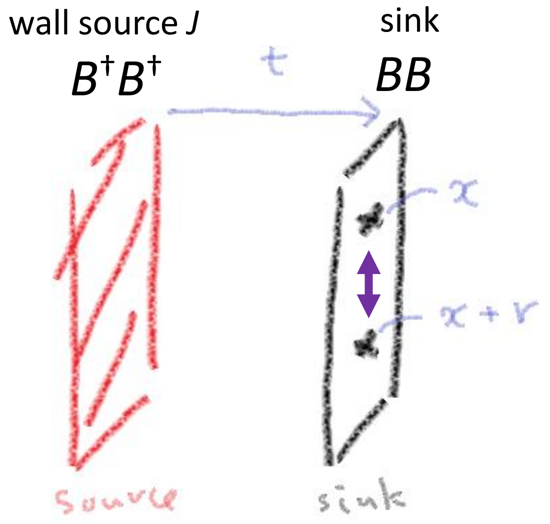
later stage



Nara, Jinno, KM, Ohnishi PRC (2023)

Time-dependent HAL QCD method

N. Ishii et al [HAL], PLB 712 (2012), 437



(1) ***R*(*r*, *t*):** measure the normalized 4pt functions

$$R^c{}_d(r,t) = \frac{\sum_x \langle B_{c_1}(x,t) B_{c_2}(x+r,t) | \mathcal{J}_d \rangle}{\exp[-(m_{c1}+m_{c2})t]}.$$

(2) Consider time-dependent Schroedinger eq. for ***R***

$$\left(\frac{1+3\delta_c^2}{8\mu_c} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_c} \right) R^c{}_d(\vec{r},t) = \sum_{c'} \int d^3r' \underline{U^c{}_{c'}(\vec{r},\vec{r}\,')} \Delta^c{}_{c'} R^{c'}{}_d(\vec{r}\,',t),$$

Relativistic correction

Potential

(3) Assume "$U(r, r') \sim \delta(r-r')$ ***V*(*r*)** + (derivative expansion)" to solve ***V*(*r*)**

~ neglected (for small *p*)

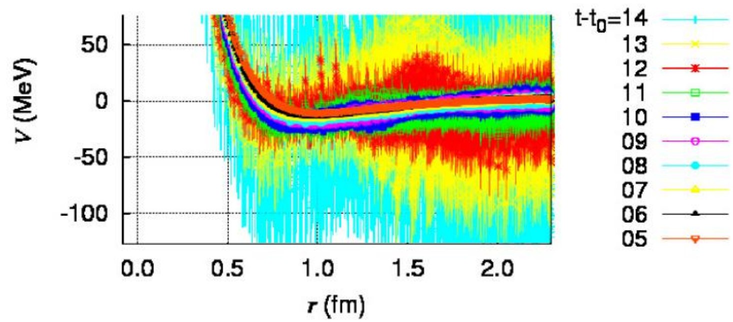
$$V^c{}_d = \left[\left(\frac{1+3\delta_c^2}{8\mu_c} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_c} \right) R^c{}_{c'} \right] (R^d{}_{c'})^{-1} / \Delta^c{}_d$$

Gauge configurations at physical point

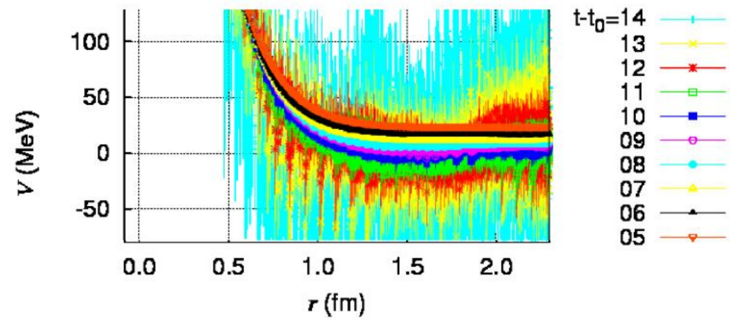
K-conf

- N_f = 2+1, *almost physical point* (m_π, m_K) = (146, 525) MeV on K supercomputer
- size 96^4, a = 0.085 fm, ～400 configurations
- existing S = -1 analysis: H. Nemura, AIP Conf. Proc. 2130 (2019) 1, 040005 [HYP2018], H. Nemura, PoS LATTICE2021 (2022) 272, etc.

$N\Sigma$(I=3/2) $^1S_0$



$N\Sigma$(I=3/2) $^3S_1$



F-conf (HAL-conf-2023) Aoyama et al, PRD110 (2024) 9, 094502.

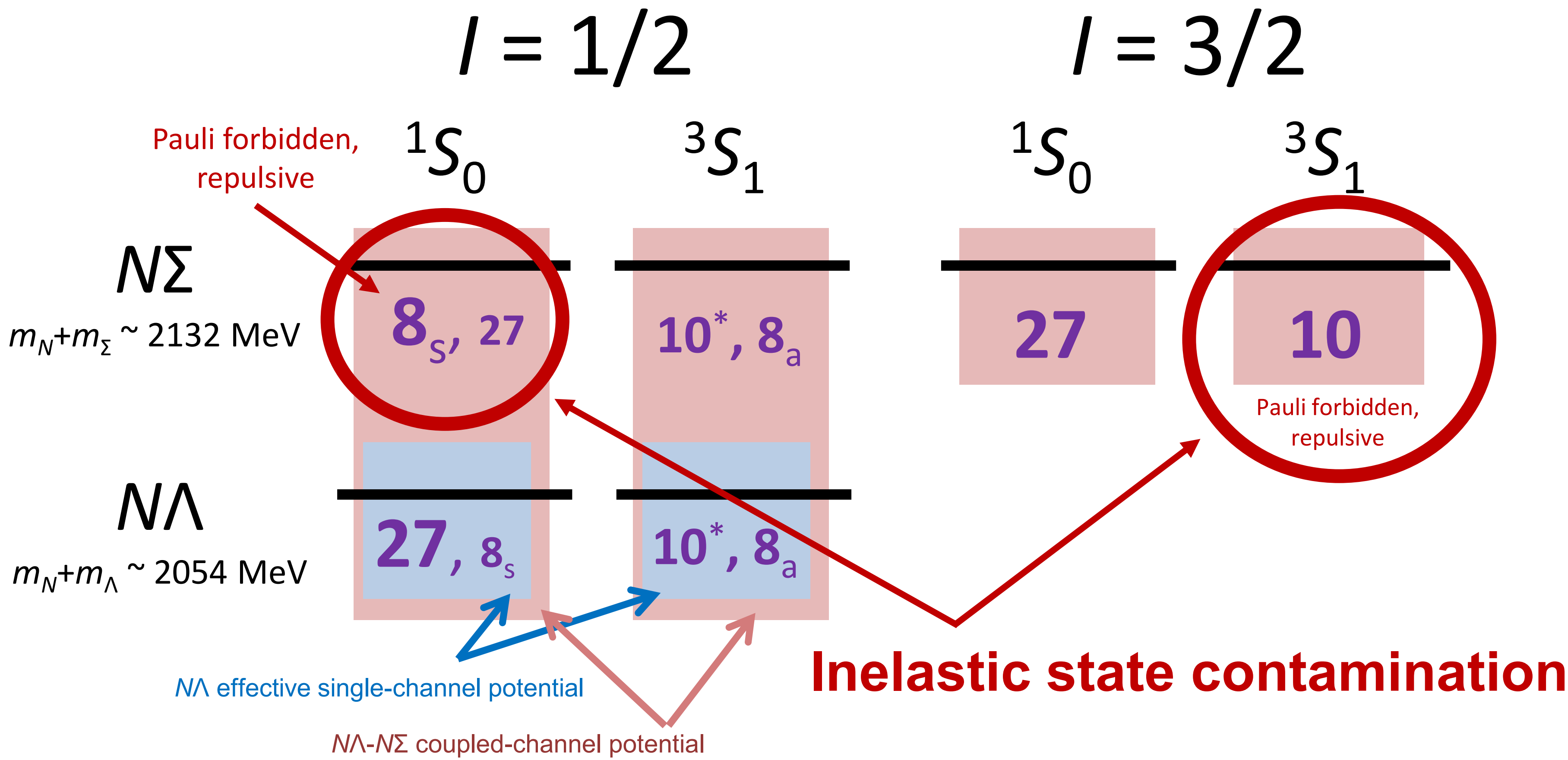
- N_f = 2+1, *physical point* (m_π, m_K) = (137, 502) MeV at Fugaku computer
- size 96^4, a = 0.084372(54) fm, 1600 configs

→ Potential analysis based on F-conf

S = -1 Baryon-Baryon (S-wave)

Note: $^3D_1$ & tensor forces are not considered **for now**

27, 10, 8_s etc.: SU(3)$_f$ rep.

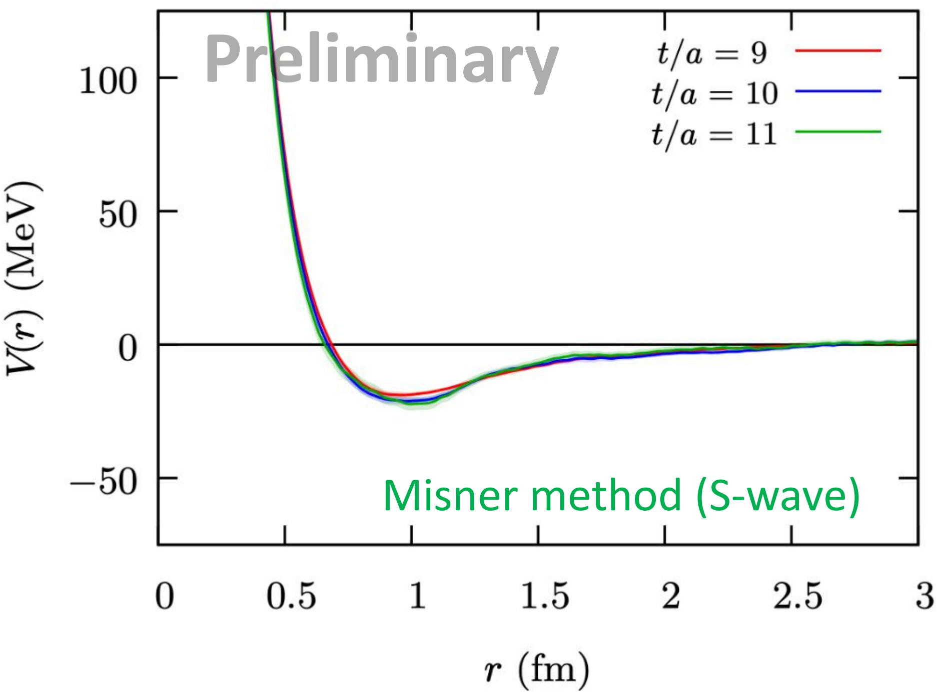


S = -1 has particularly bad S/N.
Removing inelastic contamination becomes important

RESULTS

Results: $N\Sigma$(I=3/2) potentials

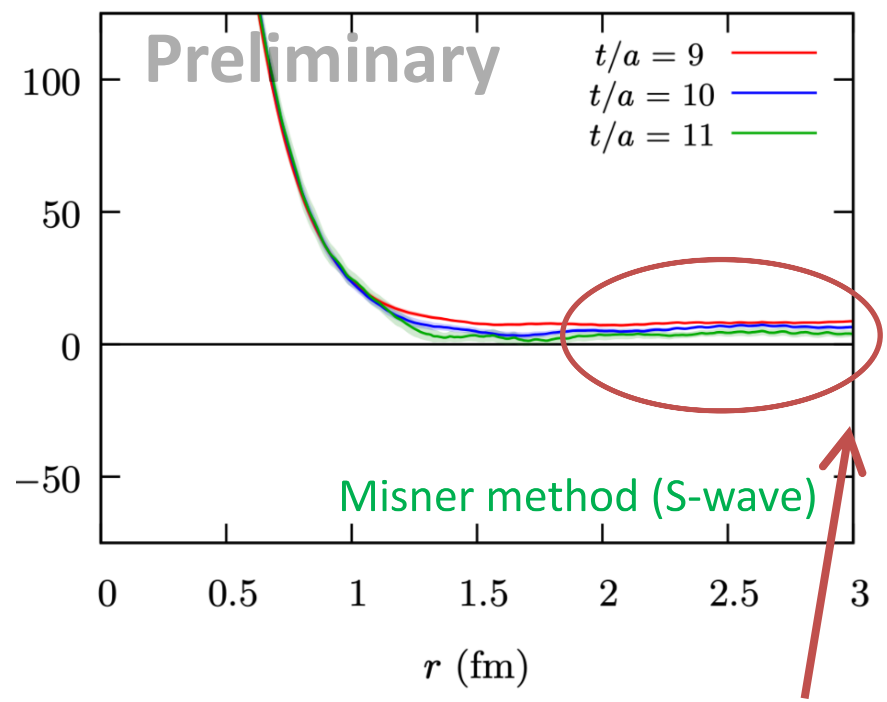
$^{1}S_{0}$



~ SU(3) **27**

~ NN(I=1)

$^{3}S_{1}$



~ SU(3) **10**

~ Pauli forbidden, repulsive

Constant-offset coming from inelastic contamination

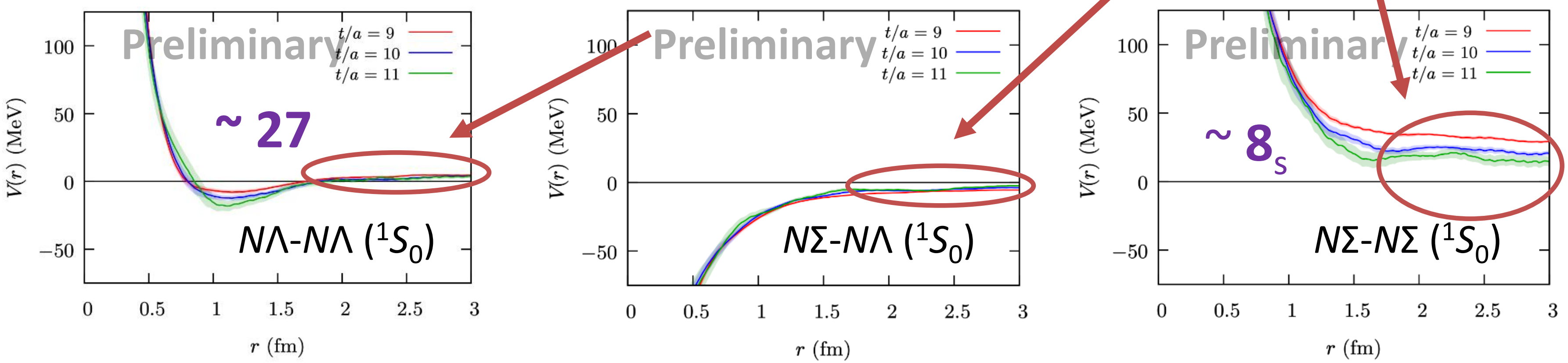
Misner method: Extract partial wave and average in [r-a, r+a]

Results: $N\Lambda$-$N\Sigma$ coupled-channel (I=1/2)

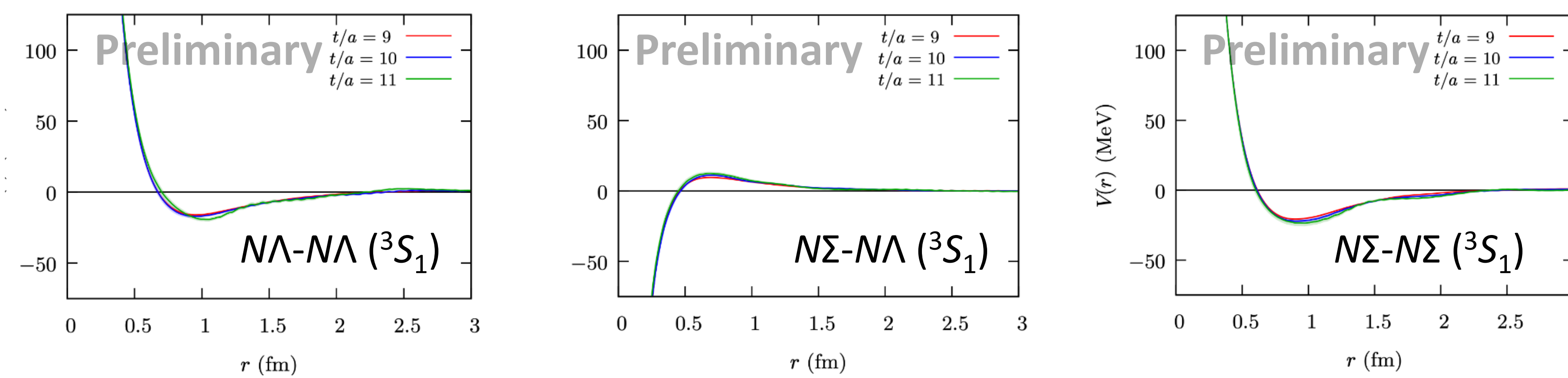
Other channelsalso suffer from

constant offsets

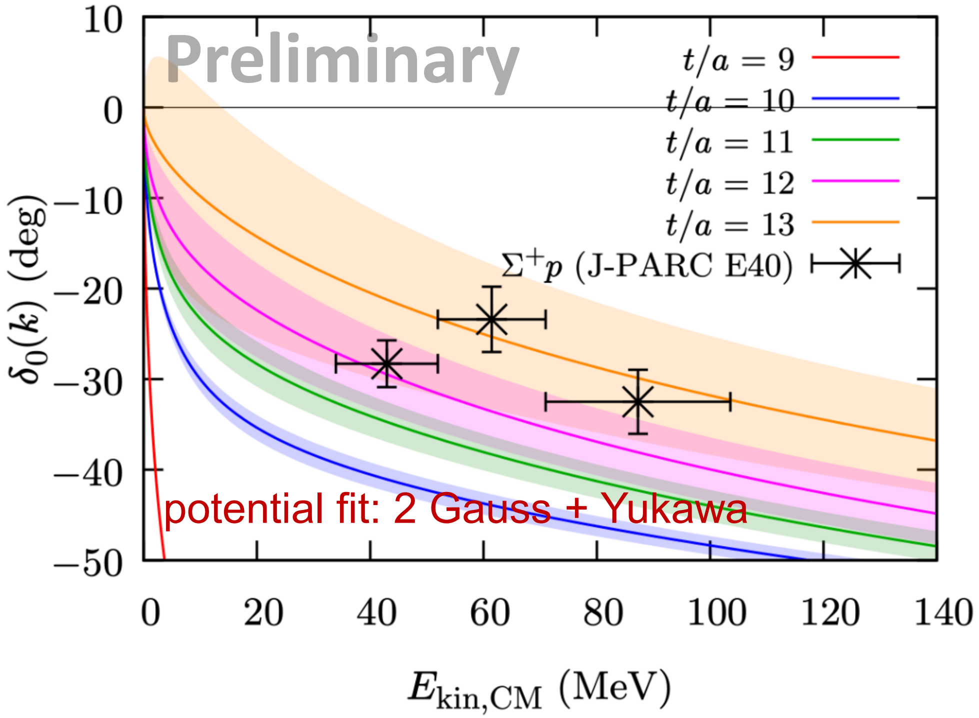
2x2 $N\Lambda$-$N\Sigma$ coupled-channel potential ($^1S_0$)



2x2 $N\Lambda$-$N\Sigma$ coupled-channel potential ($^3S_1$)

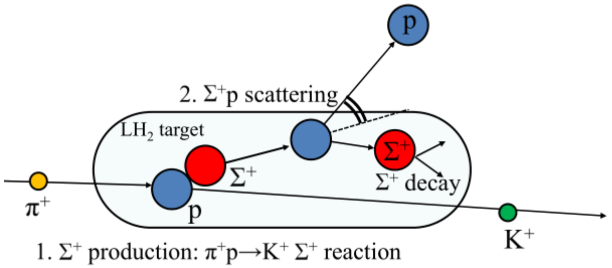


$N\Sigma$(I=3/2) $^3S_1$ phase shift



Experimental data:
Σ^+p elastic scattering [J-PARC E40]
to determine the size of repulsion

Nanamura et al. [J-PARC E40] PTEP 2022 (2022) 9, 093D01



$t/a \leqq 11$: large contamination from inelastic excited states

$t/a \geqq 12$: larger noise

Exponential extrapolation to $t/a \rightarrow \infty$

Contribution from inelastic state

Inelastic contributions enter the R correlator in the form of the sum of the exponentials exp(-bt)

$$R(r,t) = \underbrace{\sum_{k:\text{elastic}} A_k \psi_k(r) e^{-\Delta E_k t}}_{\text{Elastic part } (N\Sigma)} + \underbrace{\sum_i \psi_{\text{inel,i}}(r) e^{-\Delta E_{\text{inel,i}} t}}_{\text{Inelastic state contamination}}$$

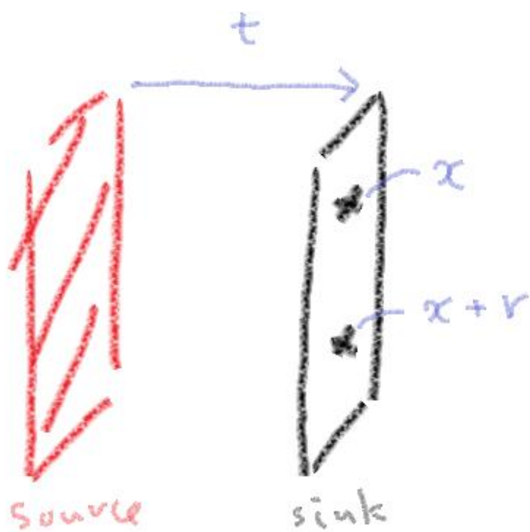
***N* + Σ**

***N*+*π* + Σ**

Δ + Σ

N* + Σ

etc…



***t* → ∞** → ***N* + Σ**

Too small t/a → Large inelastic contamination

Mass difference of inelastic state remains at large r → constant shift

Too large t/a → Signal (elastic states) becomes too small.

Large relative noise

Extrapolation of $V(r)$ to $t/a \rightarrow \infty$

Naively, **inelastic state contamination** enter
the R correlator in a sum of exponential forms exp(-bt):

$$R(r,t) = \underbrace{\sum_{k:\text{elastic}} \psi_k(r) e^{-\Delta E_k t}}_{\text{Elastic part}} + \underbrace{\psi_{\text{inel}}(r) e^{-\Delta E_{\text{inel}} t} + \cdots}_{\text{Inelastic contamination...}} .$$

Assumption for extracted potential: V = (1/R) (− H_0 − ∂/∂t) R

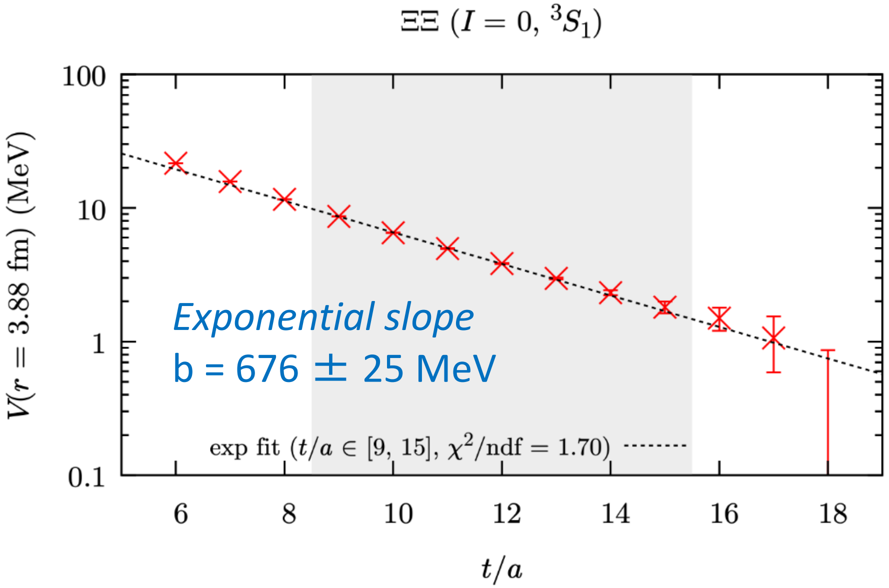
$$V(r,t) \simeq \underbrace{V_{\text{el}}(r)}_{\text{we want this}} + \underbrace{V_{\text{inel}}(r) e^{-bt}}_{\text{contamination (leading)}}$$

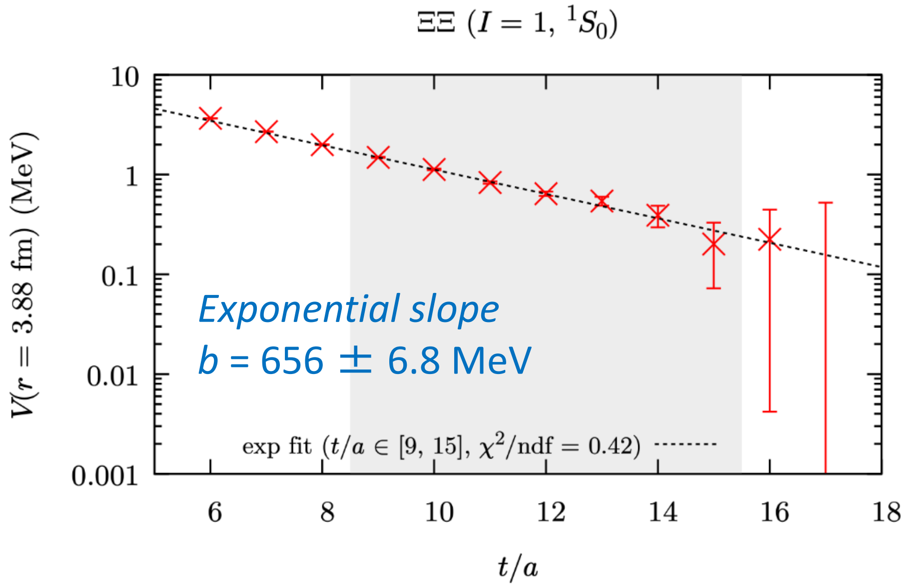
Steps to get *V*(*r*)

1. Determine ***b*** (> 0) at a **large *r*** of $V(r, t)$
2. Fit V_{el} and V_{inel} at each ***r***. → Pick only V_{el} (Extrapolation for $t/a \rightarrow \infty$)

Inelastic contamination in ΞΞ

Small noise in ΞΞ → We can check the contamination behavior for a wide t/a range



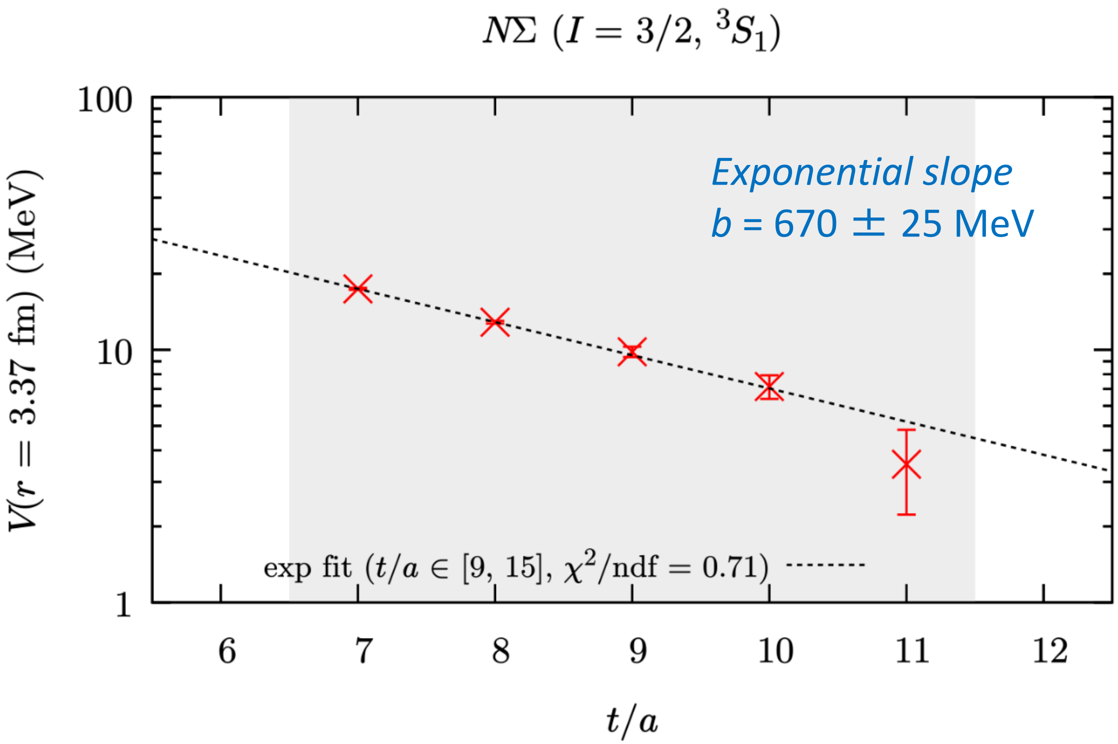


points: calculated potential values
line: exponential fit by $a \exp(-bt)$
shaded region: fitting range

Contamination is well described by a single exponential form
at large r (i.e., constant offsets)
in ΞΞ (where the contamination is small)

Inelastic contamination in $N\Sigma$ (I=3/2, $^3S_1$)

*N*Σ is noisier … We need to fit to fewer data points

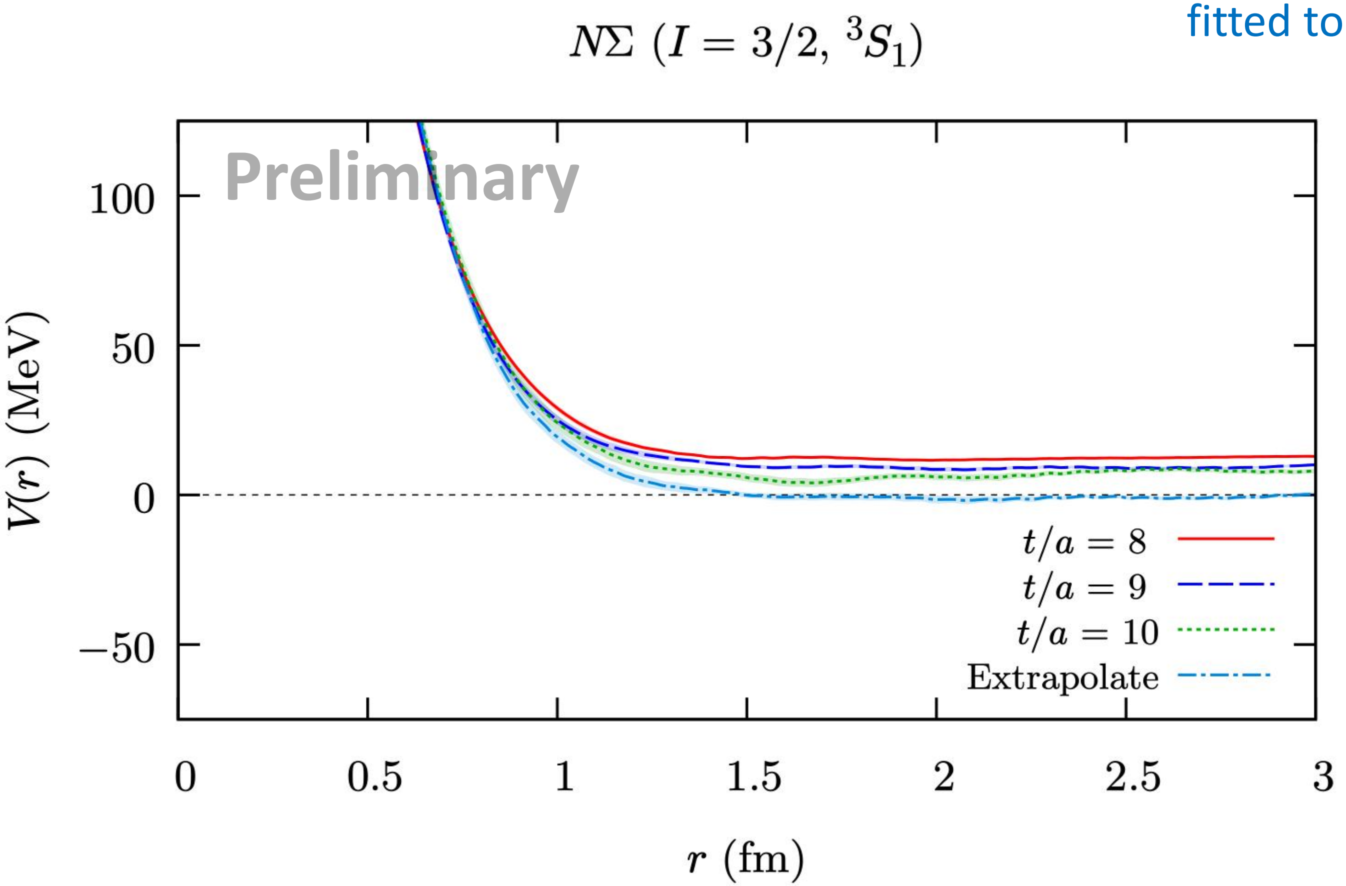


The constant offset of *N*Σ also behaves in a single exponential decay

Extrapolated potential $V_{N\Sigma}(r)$ ($I = 3/2$, ${}^3S_1$)

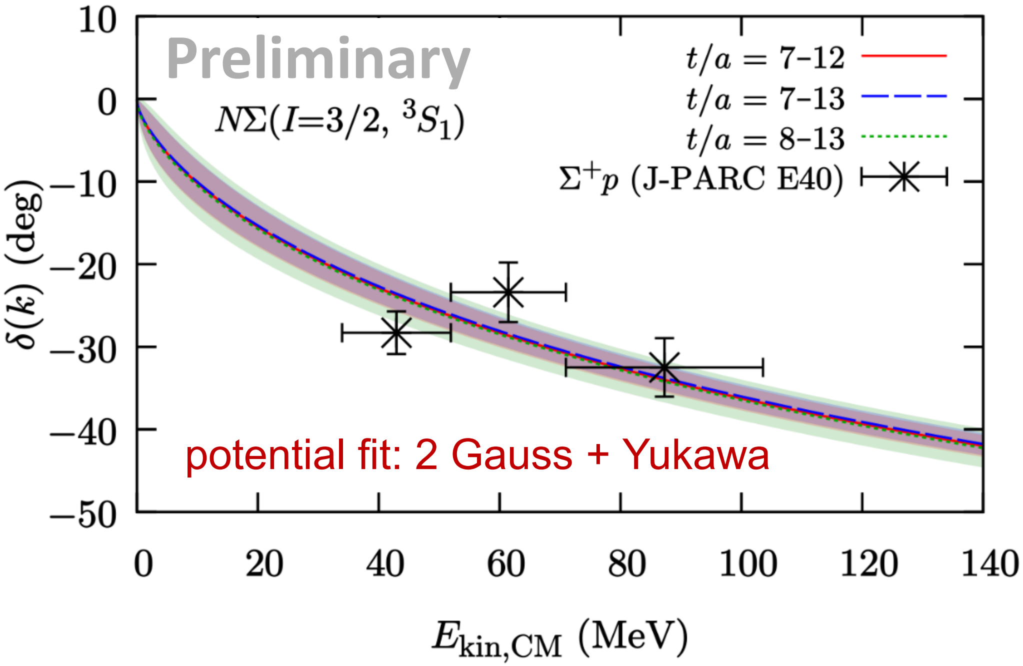
Result of the extrapolation to $t/a \to \infty$

$b = 0.2947(7) = 690$ MeV
fitted to r = 3-3.38 fm



Modification by the extrapolation is small at r < 1
This is not actually constant subtraction.

Phase shift with extrapolated potentials



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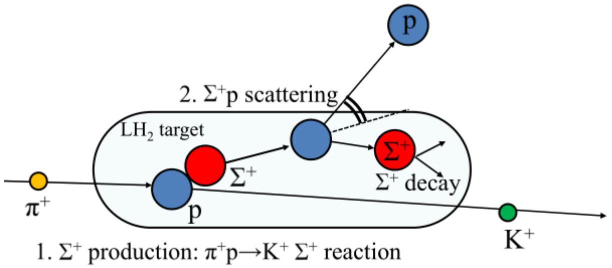


The phase shift seems stable for *slight change* of the time range to fit.

Note: The potential difference at short range (r < 1) does not affect δ_0 much.

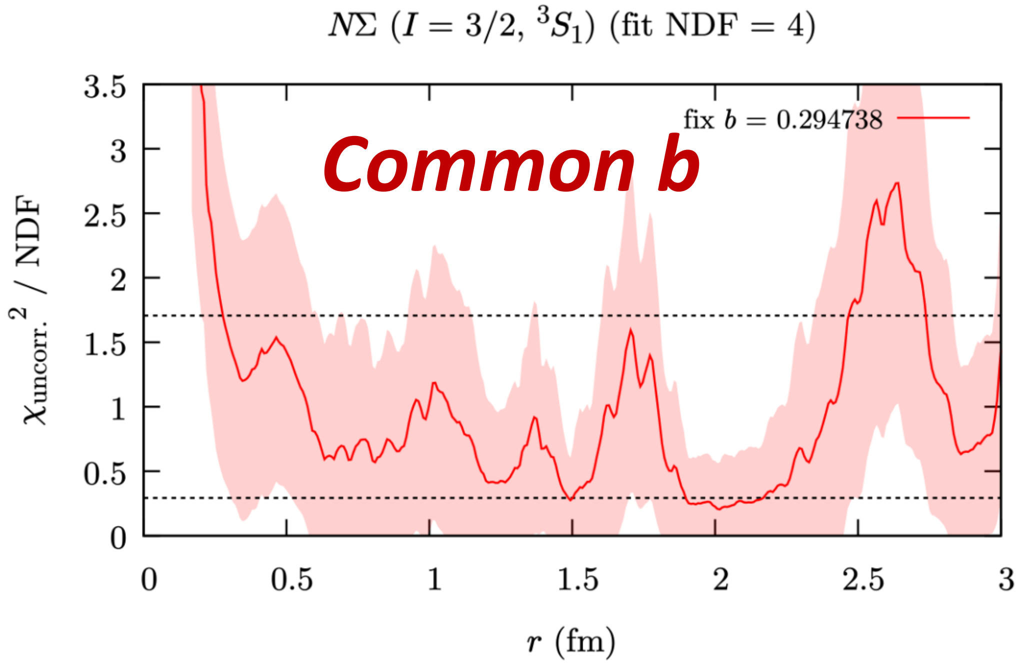
Discussion: χ^2 vs r

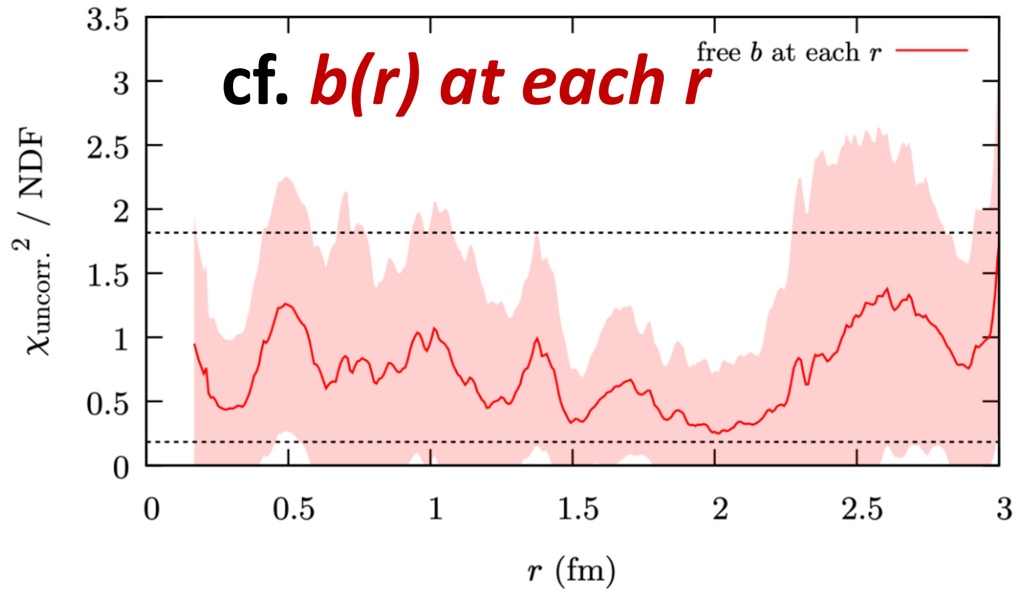
We determined b at large r.
Does it work for small r?

By default, b is determined at large r, and only V and V_{inel} are fitted for each r
$V(r,t) = V(r) + V_{inel}(r) \exp[-bt]$.

Another variation of the fitting form:
$b(r)$ is determined for each r:
$V(r,t) = V(r) + V_{offset}(r) \exp[-b(r)t]$.

Both cases seem to produce consistent results





Discussion: *b* ~ 700 MeV

b: The "mass" for the inelastic contribution

$V_{\mathrm{inel}}(r,t) \propto \exp(-bt)$,

naively associated with the gap for *the lowest inelastic state*.

b for large *r* is identified to be $0.286(11)/a = 670(25)$ MeV

for $N\Sigma(I=3/2,\ {}^3S_1)$.

Too large?

Possibilities:

- Cancellation in $R = (4\mathrm{pt})/(2\mathrm{pt})^2$
  (contaminations in denominator vs numerator)?
  → Unlikely? The gap doesn't change much even with $R = (4\mathrm{pt})/\exp[-(m_1+m_2)t]$
- "Single exponential" is a bad assumption?
  → There is the dependencies on the time range to fit
  We need to checks with multi exponential fitting (with ΞΞ).
- Smaller overlap with the source for lower inelastic states such as $N\pi$-Σ?
- etc.

Summary

- Analyses *ongoing* for **$N\Lambda$ & $N\Sigma$ potentials** at the physical pt. using HAL QCD method for **F-conf (HAL-conf-2023)**
- Repulsive channels, $N\Sigma(I=1/2)\ {}^1S_0$ & $N\Sigma(I=3/2)\ {}^3S_1$, receive constant offsets in the potential from inelastic contamination: (Large offset at **small** t/a) vs (Large noise at **large** t/a)
  - *Extrapolation* from smaller t/a is consistent (assuming $\Delta V \sim e^{-bt}$)
- Comparison
  - I=3/2 $\delta({}^3S_1)$ consistent with J-PARC E40 Σ^+p scattering?
  - I=3/2 $a({}^1S_0)$ and $a({}^3S_1)$ [~ ALICE femtoscopy] large uncertainty

Outlook

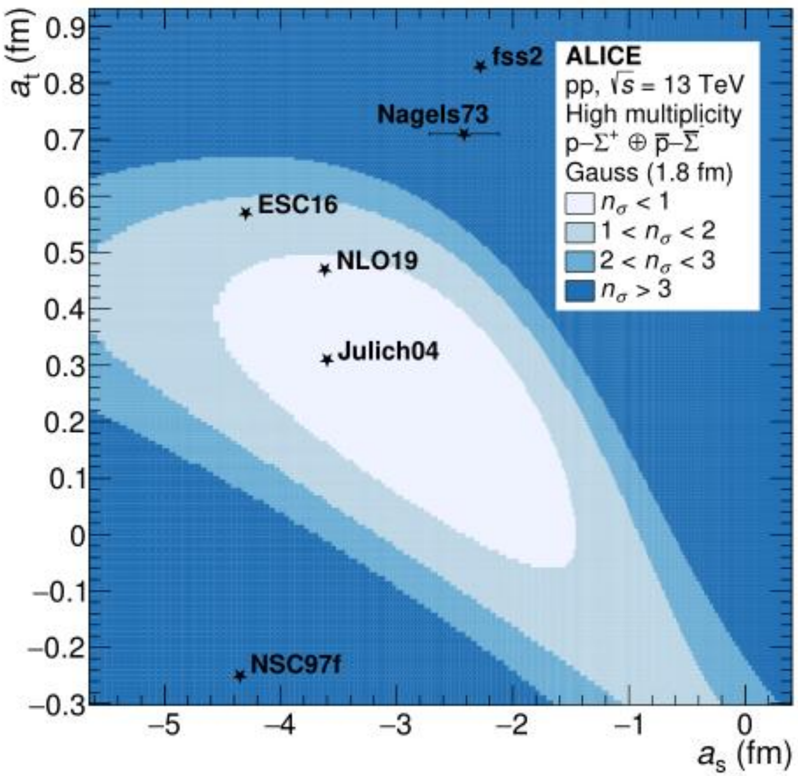
- Systematic validation of *extrapolation* method
- Detailed analysis on the fitting-form dependence
- ${}^3D_1$ and tensor forces

BACKUP

Femtoscopy of $p\Sigma^+$ (ALICE)

Legend (fitting function):
G = Gauss,
OPE = Yukawqa with m_π

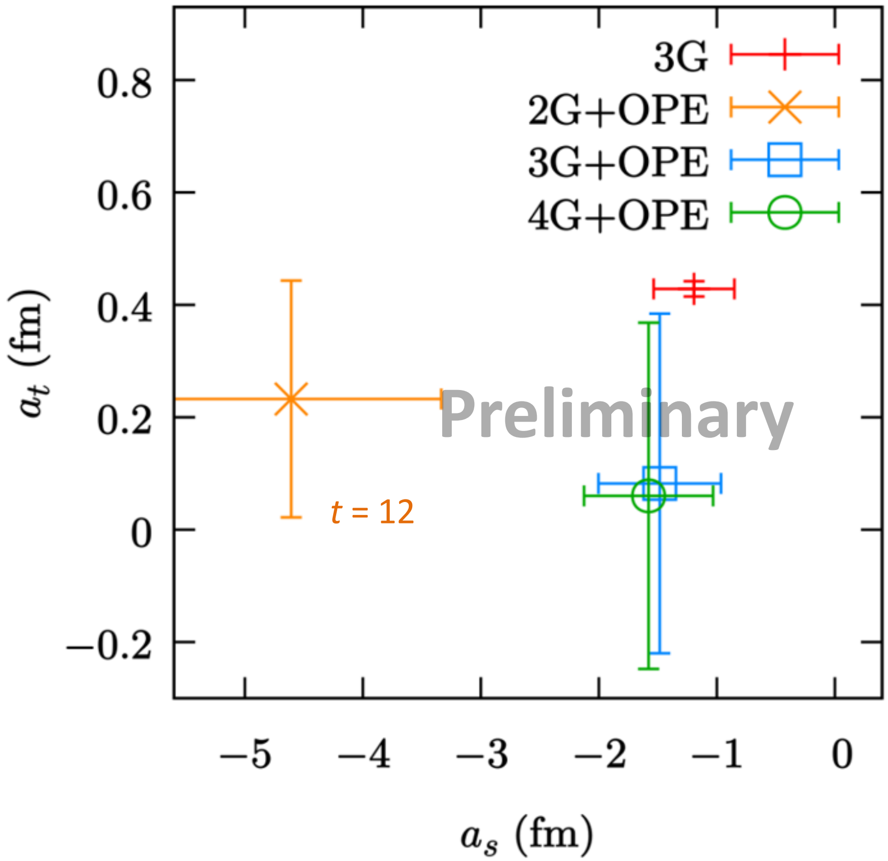
ALICE, arXiv:2510.14448v1 [nucl-ex] (2025)
(a_s, a_t) for Gaussian potential (1.8 fm)



$^3S_1$ scattering length →

$^1S_0$ scattering length →

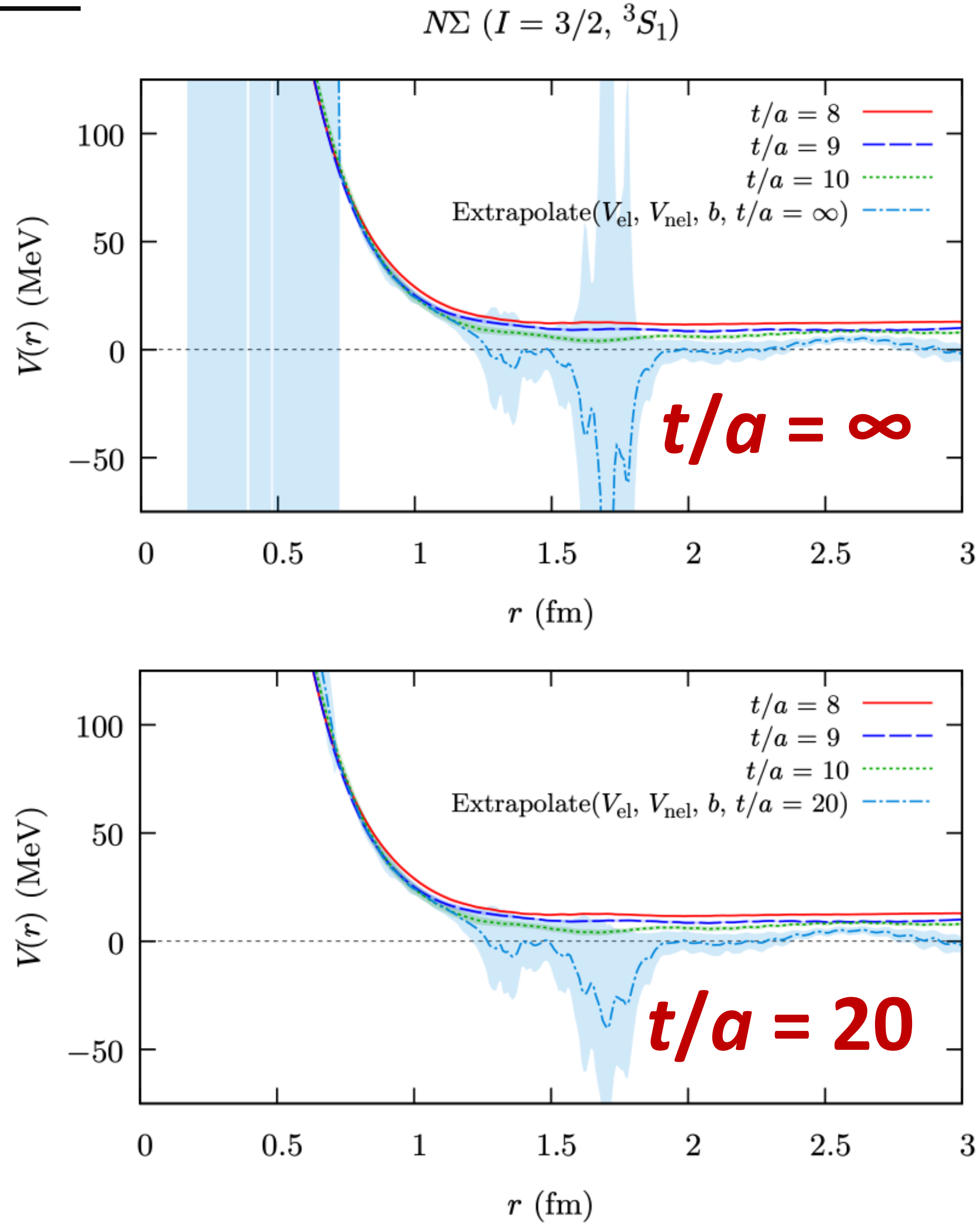
(Coulomb **Un**modified) Scattering length from HAL-conf-2023
in the same plotting range



Fitting function dependence & time dependence are large.

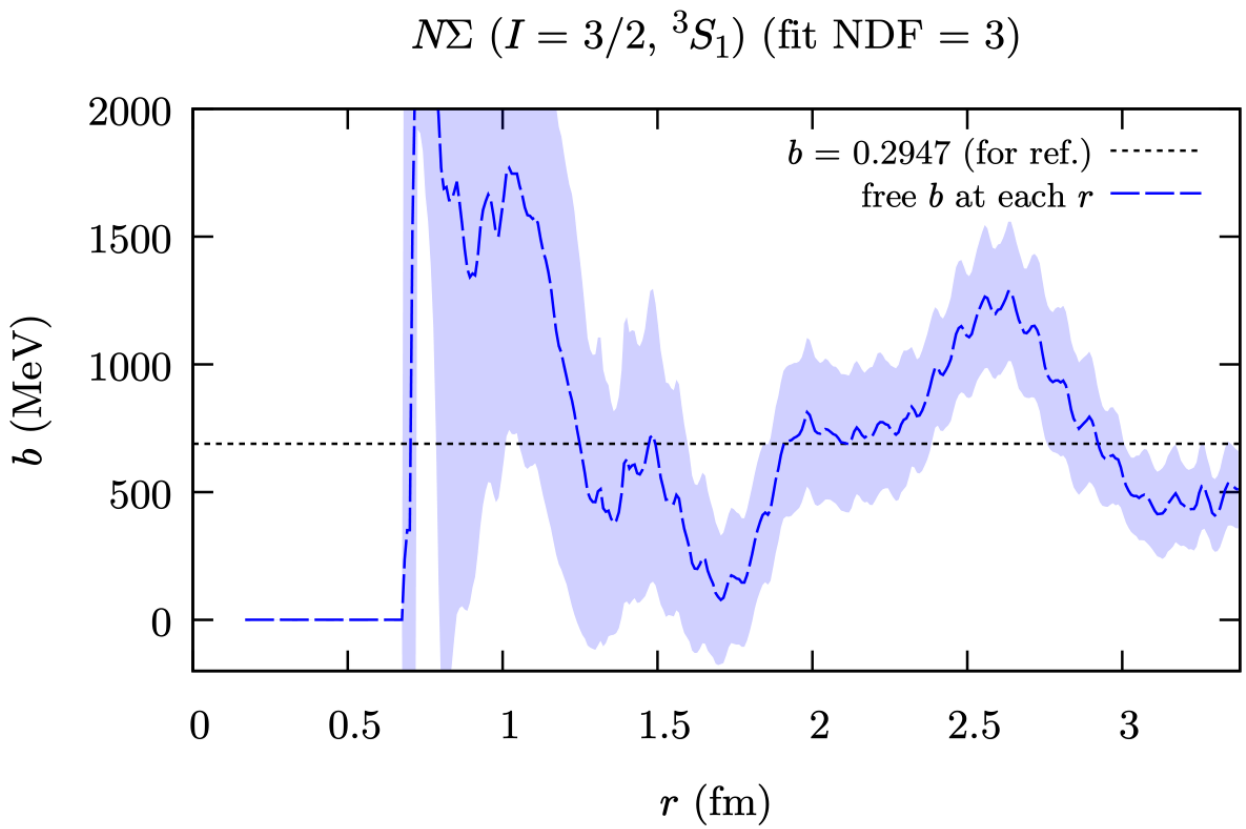
Extrapolation methods

When ***b*** is also varied for each **r**



Extrapolation methods

$b(r)$ is for fitting $V(r,t) = V_{el}(r) + V_{inel}(r) \exp[-b(r)t]$

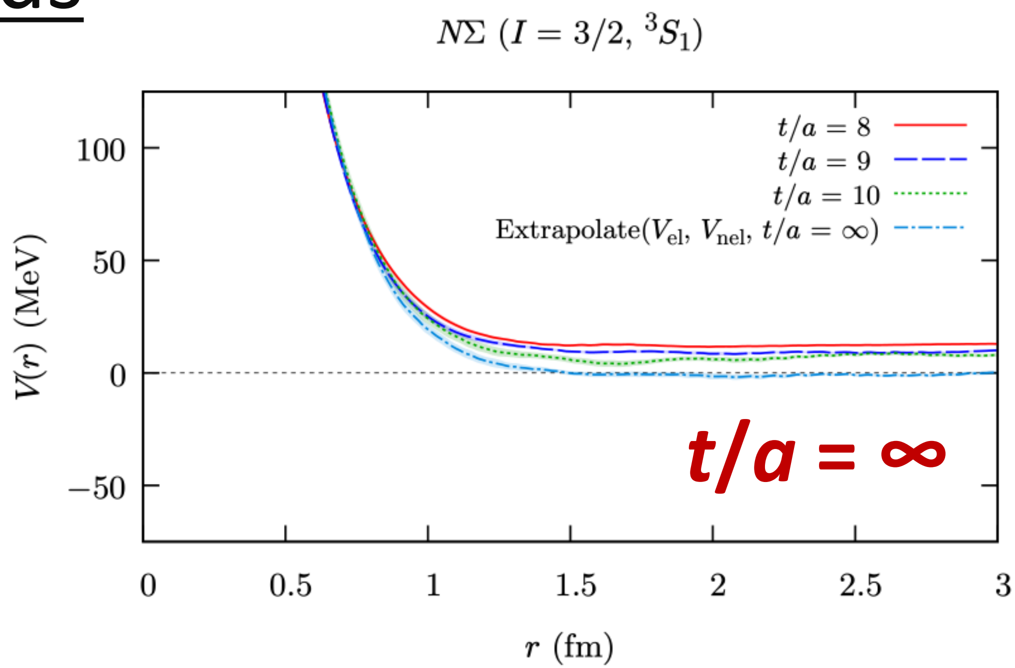


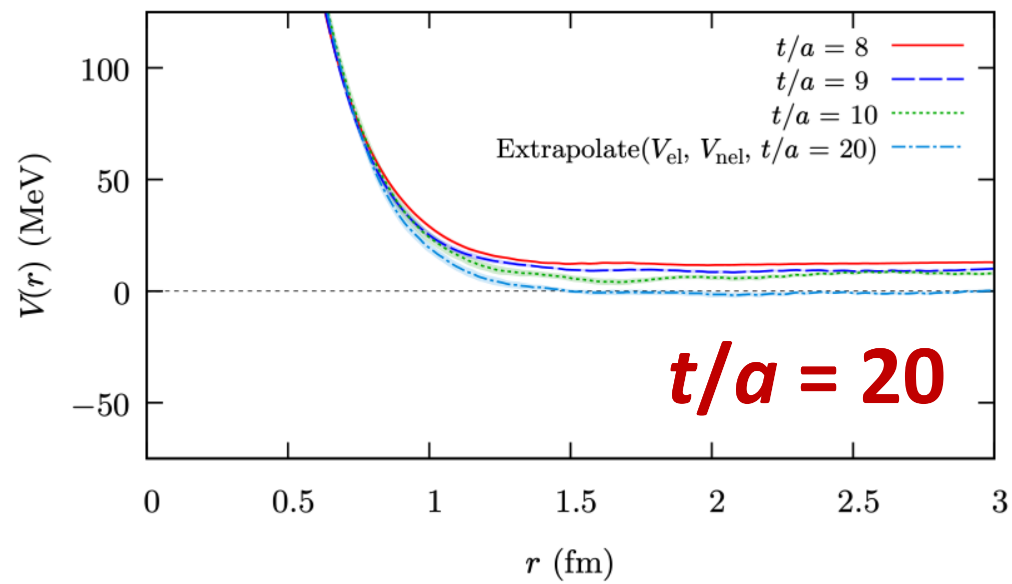
b tends to be large at the entire r region

Note: $b = 0.2947(7)$ in *the other extrapolation method*
was obtained for a different fitting form
$V(r,t) = V_{offset}(r) \exp[-b(r)t]$, so is systematically different.

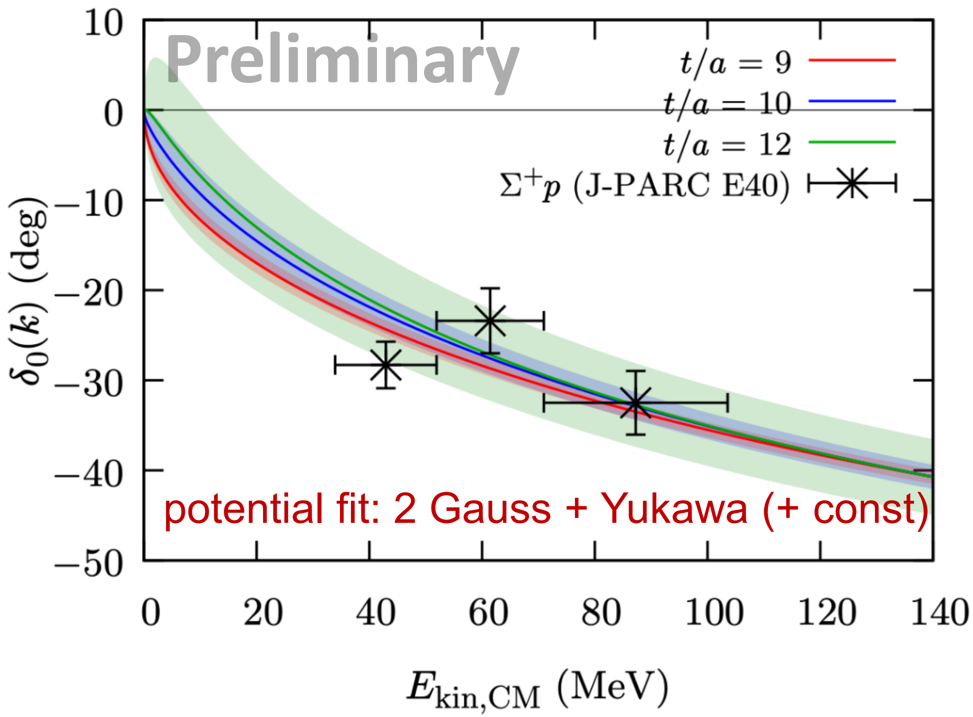
Extrapolation methods

When ***b* = 0.2947**
is determined at
r = 3.0 … 3.38
for V ∝ exp(-*bt*)





NΣ(I=3/2) $^3S_1$ phase shift



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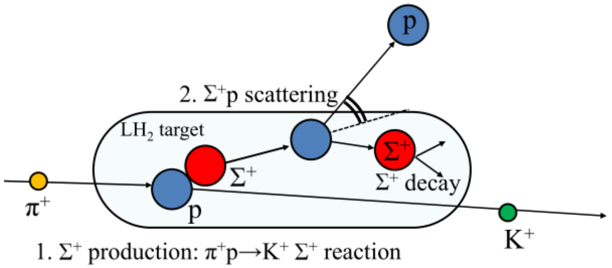
Nanamura et al. [J-PARC E40] PTEP 2022 (2022) 9, 093D01



With the constant offset of the potential subtracted in fitting, the result becomes **stable** for a larger *t/a* range.

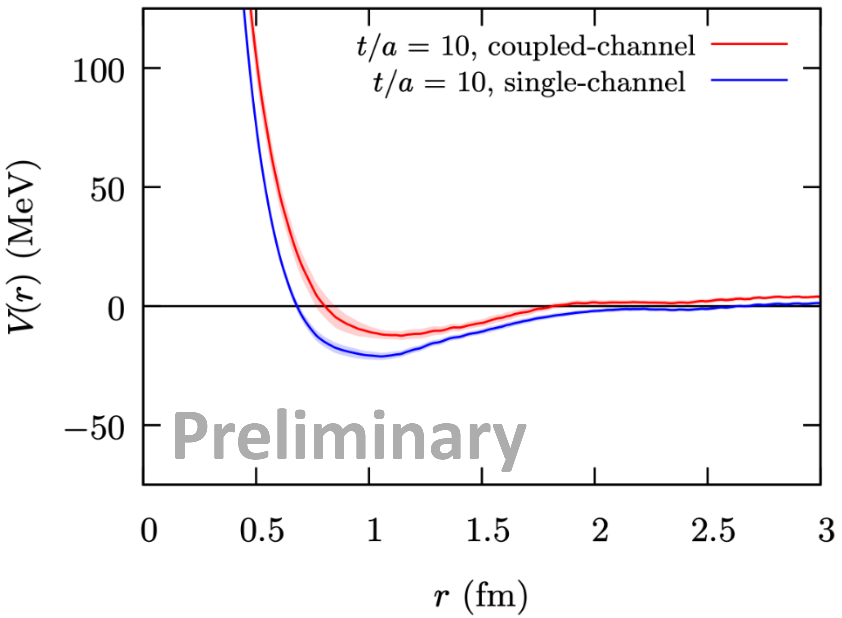
→ Consistent with experimental estimation of phase shifts (with $\delta < 0$ assumed)

NΛ(I=1/2) Effective single-channel pot.

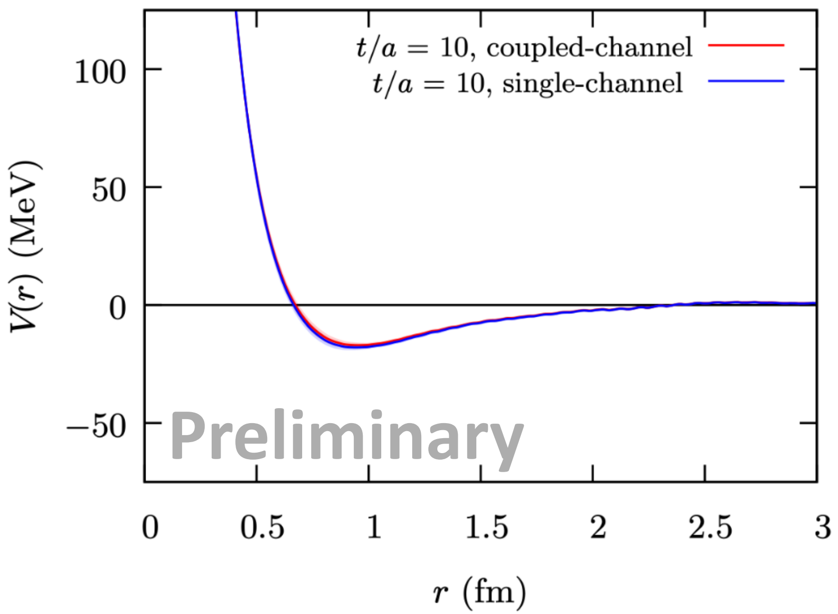
Analysis ongoing…

2x2 *NΛ-NΣ* coupled-channel potential
vs **NΛ effective single-channel potential**

$^1S_0$

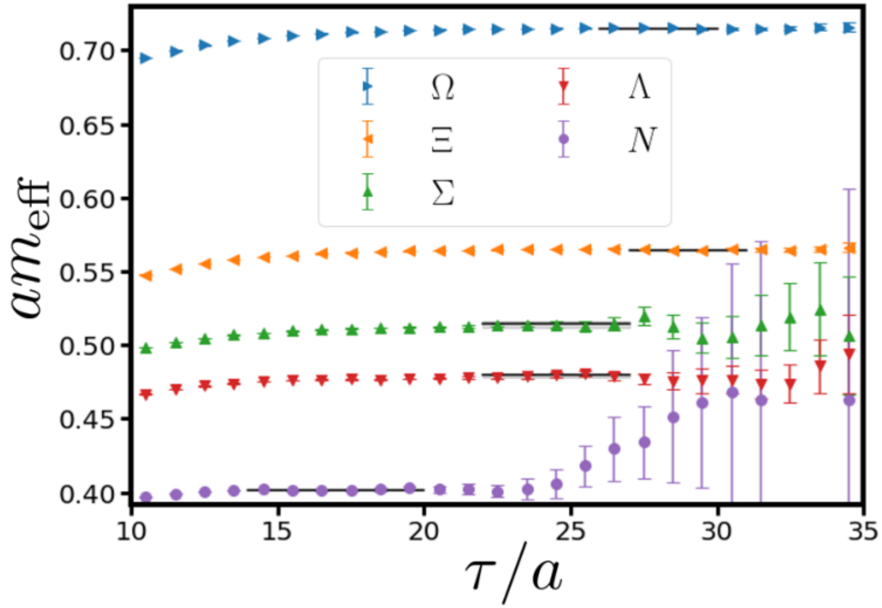


$^3S_1$



Baryon masses

Aoyama et al [HAL QCD], PRD110 (2024)



hadron	experiment [MeV]	lattice [MeV]
π	138.04	$137.1(0.3)(^{+0.0}_{-0.2})$
K	495.64	$501.8(0.3)(^{+0.0}_{-0.7})$
N	938.92	$939.7(1.8)(^{+0.2}_{-1.7})$
Λ	1115.68	$1121.4(3.6)(^{+0.5}_{-3.7})$
Σ	1193.15	$1202.5(5.6)(^{+0.3}_{-4.0})$
Ξ	1318.29	$1320.7(2.1)(^{+1.5}_{-1.7})$
Ω	1672.45	input

Baryon effective masses