Hybrid-Quarkonium Mixing & Hybrid Spin-Dependent Potentials Using Lattice Gauge Theory

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In collaboration with P. Pütz, A. Vairo, and M. Wagner

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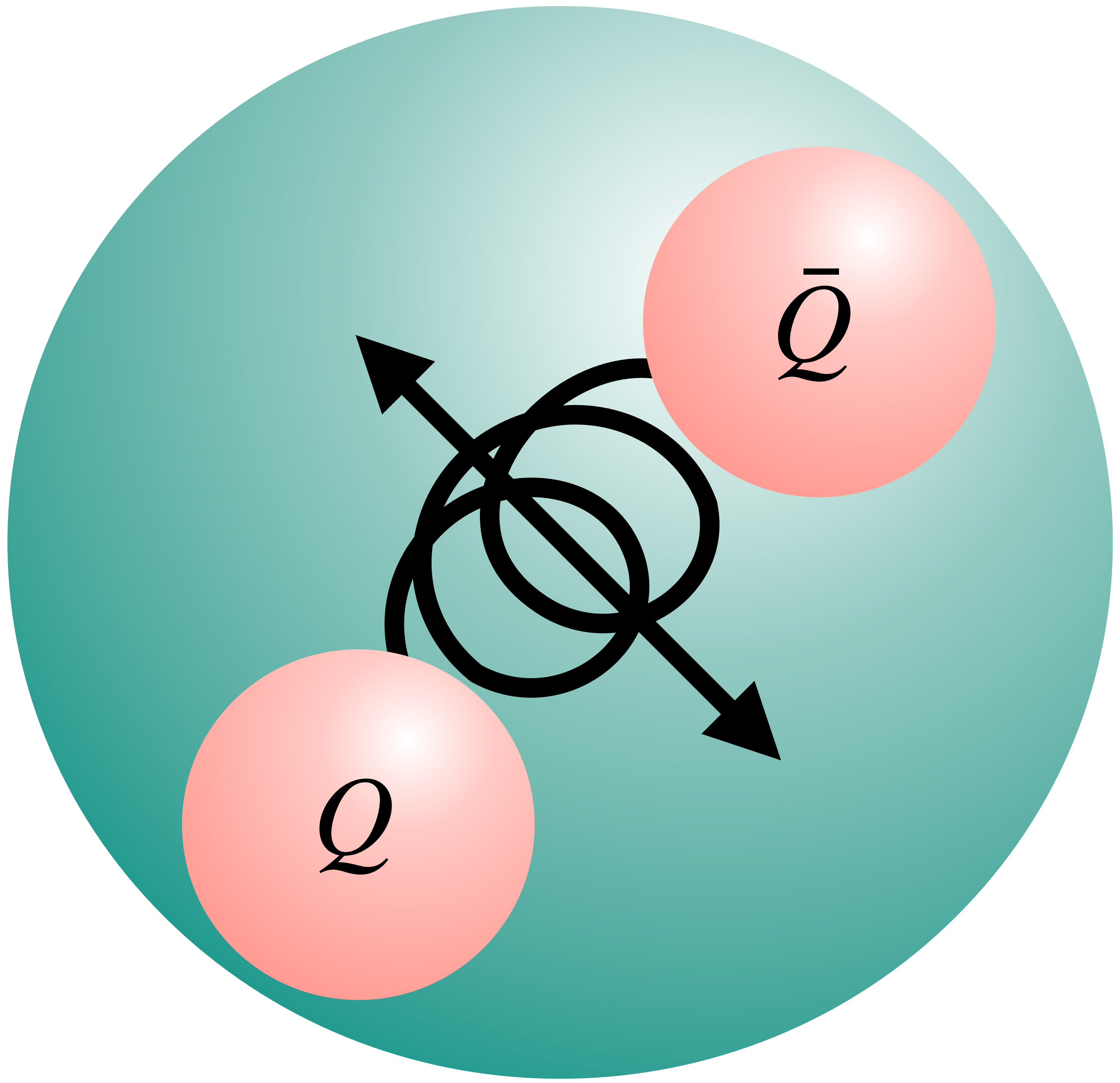
Heavy hybrid mesons



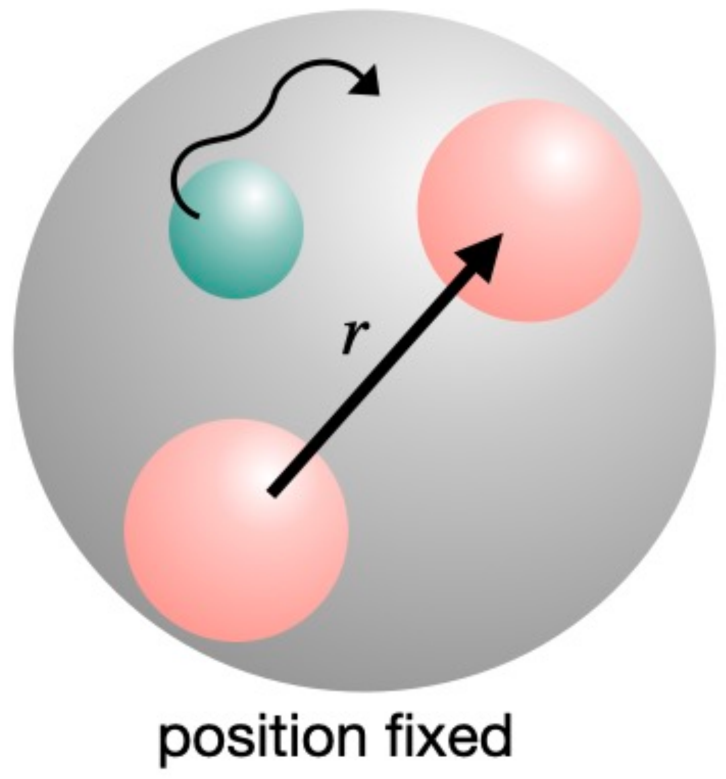
- Heavy $Q\bar{Q}$ pair coupled to an excited gluonic field
- Hybrids can mix with ordinary quarkonium $\rightarrow$ even the ground-state $\eta_c(1S)$ may contain $\sim 10\,\%$ hybrid component
- Guiding experimental searches requires precise predictions of hybrid spectra, spin splittings, and hybrid admixtures

Born–Oppenheimer (BO) approach for heavy hybrids

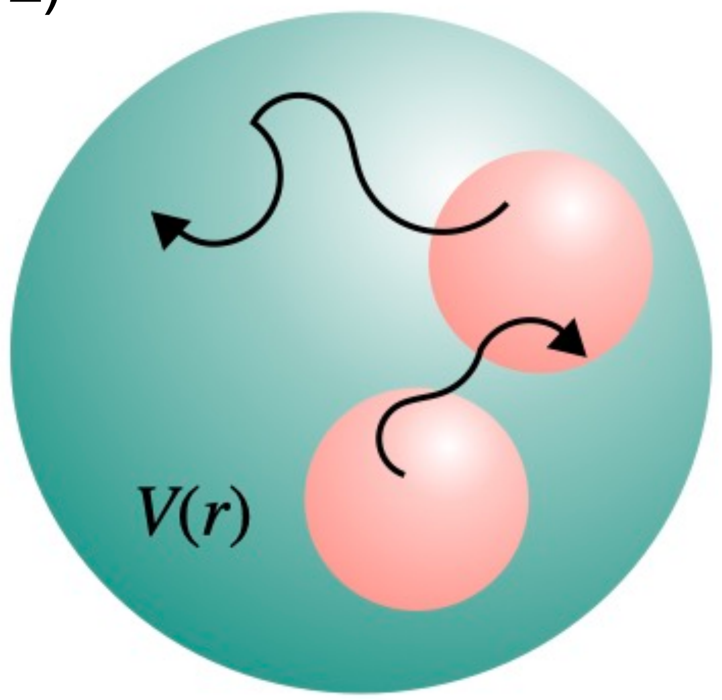
Two-step description

BO effective field theory (BOEFT): expand in $1/m_Q$

1)



2)



1) Pure $SU(3)$ lattice gauge theory (static quarks)

↓

Nonperturbative hybrid potentials

↓

2) BOEFT + coupled Schrödinger equations

↓

Hybrid spectrum

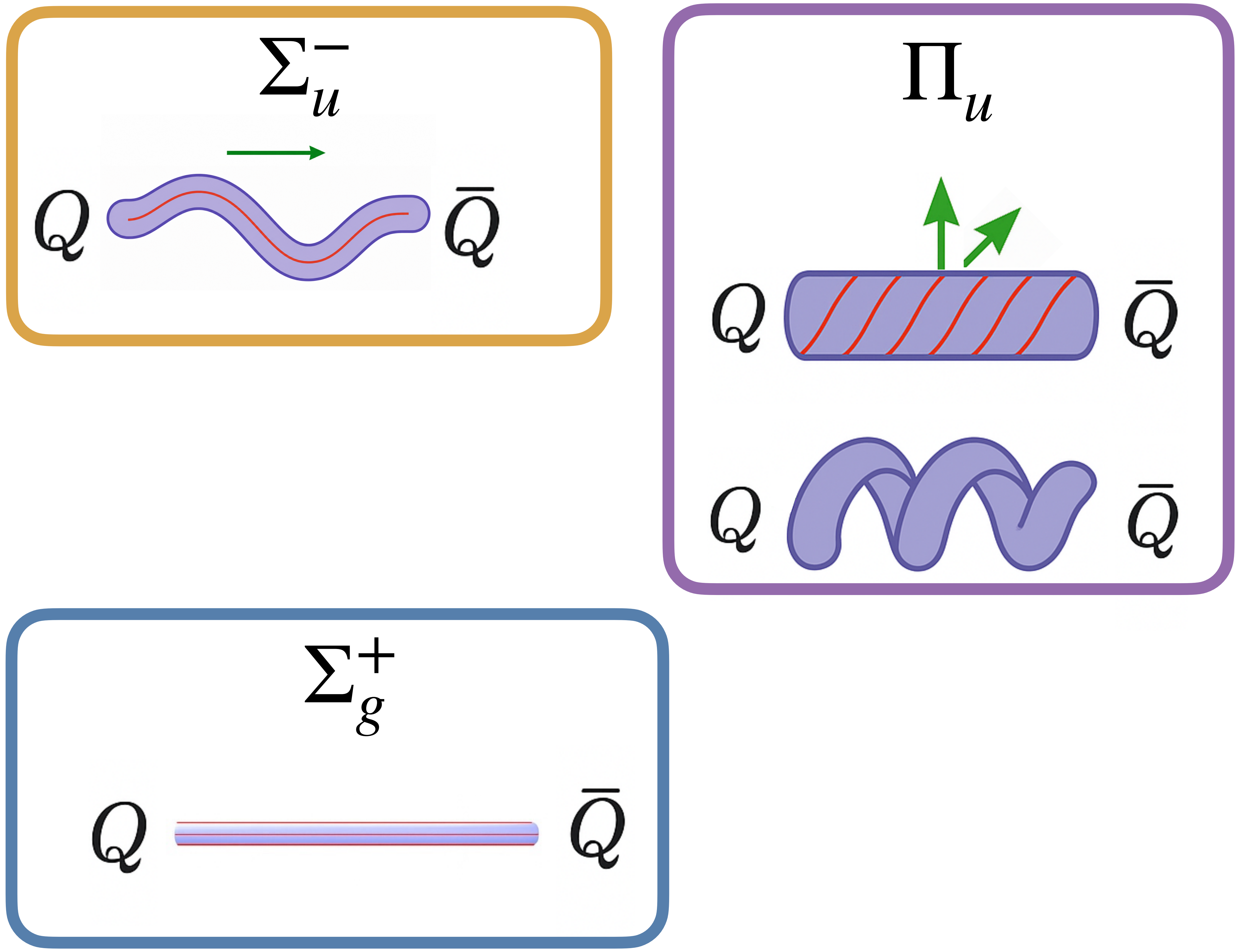
<u>Leading order (LO)</u>	<u>Next-to-leading order (NLO)</u>
Static $\Sigma_g^+, \Sigma_u^-, \Pi_u$	Mixing $V_{\Pi_u}^{\text{mix}}(r)$, $V_{\Sigma_u^-}^{\text{mix}}(r)$
	Spin $V_{11}^{sa}(r)$, $V_{10}^{sb}(r)$

Visualizing heavy hybrids

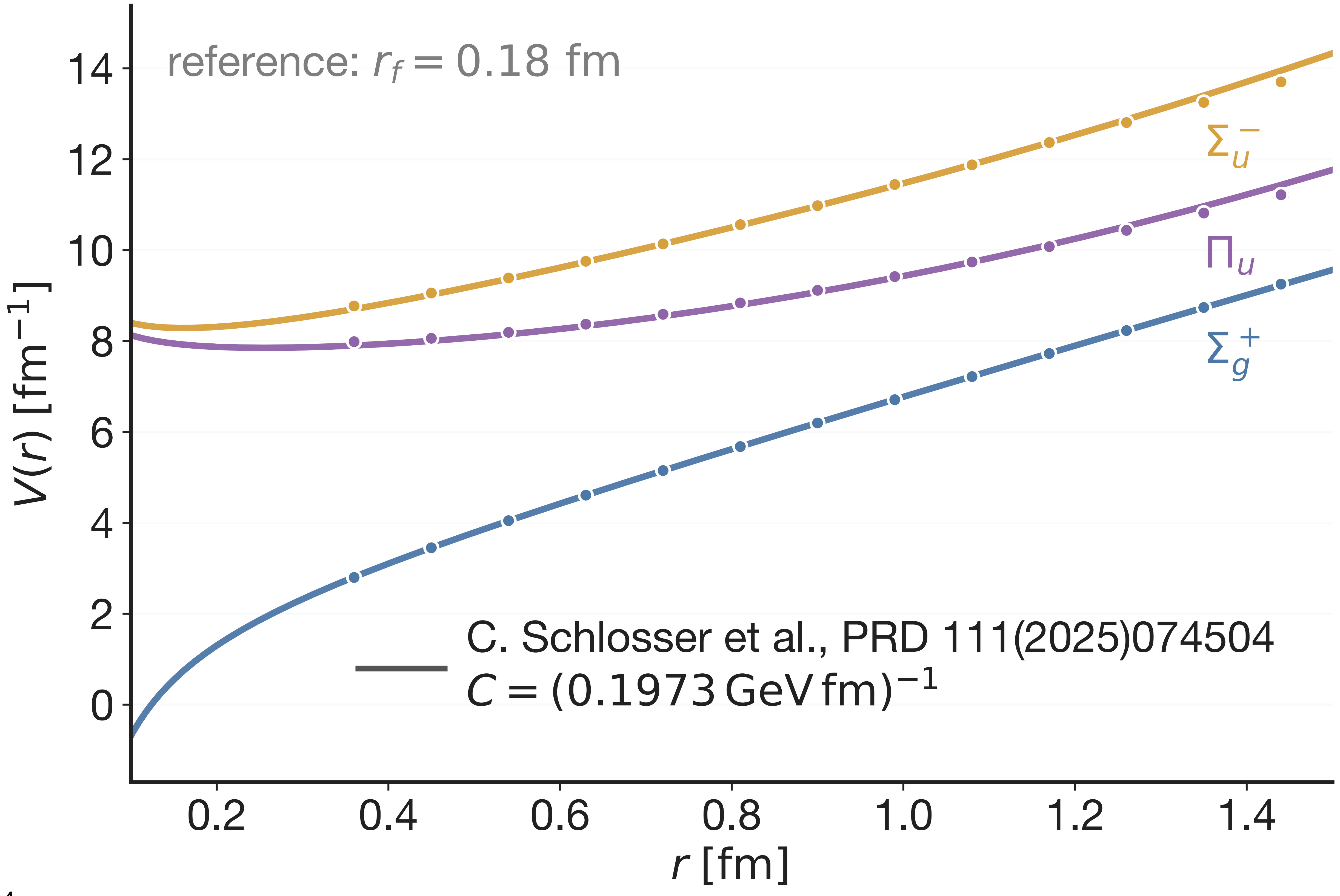
Lowest gluonic excitation gluon with $J^{PC} = 1^{+-}$

$\Sigma \equiv \Lambda = 0$
$\Pi \equiv \Lambda = 1$
g, u : even/odd under CP
$\pm$: even/odd under reflection

Flux tubes



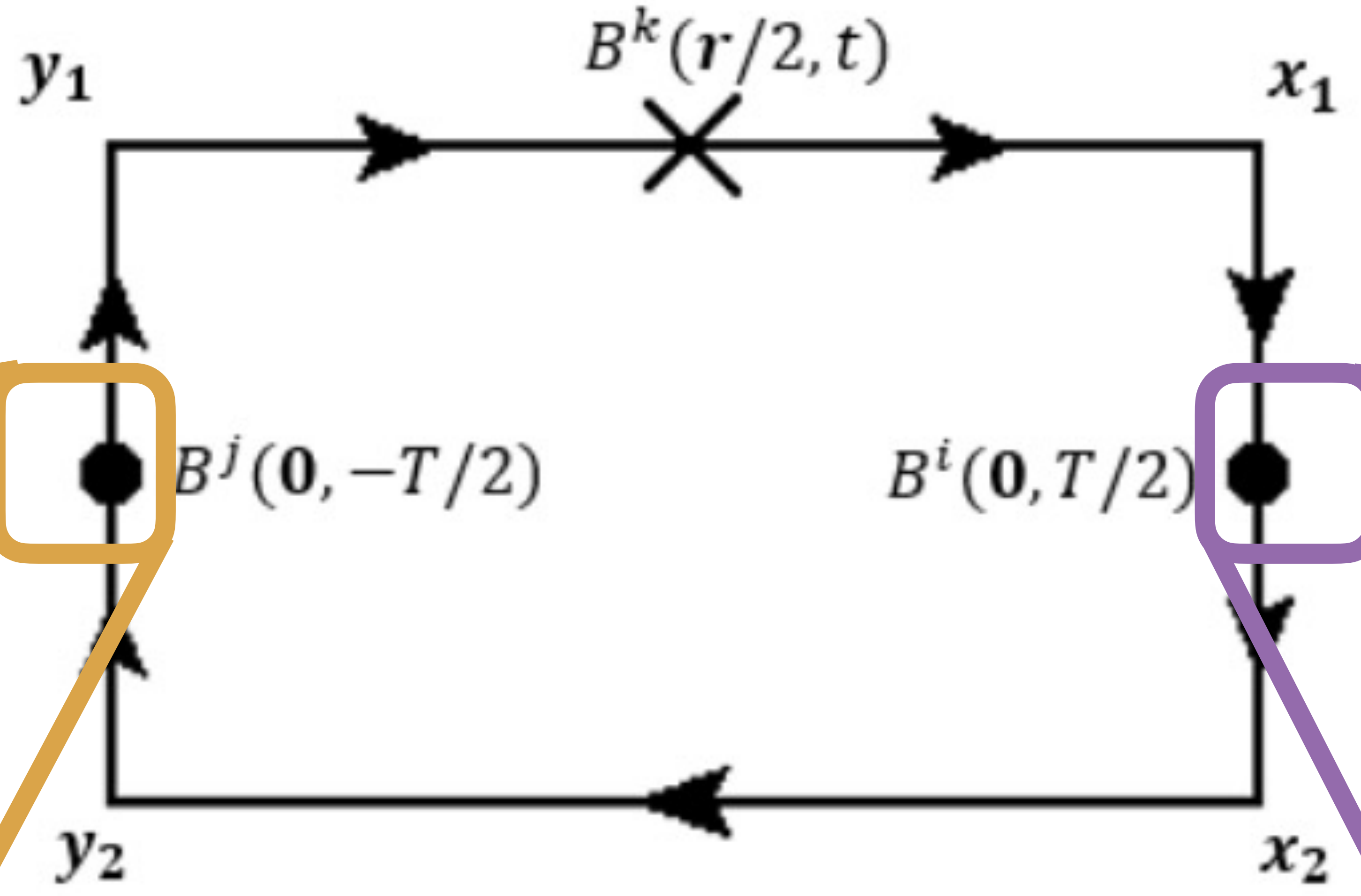
Lowest static BO potentials

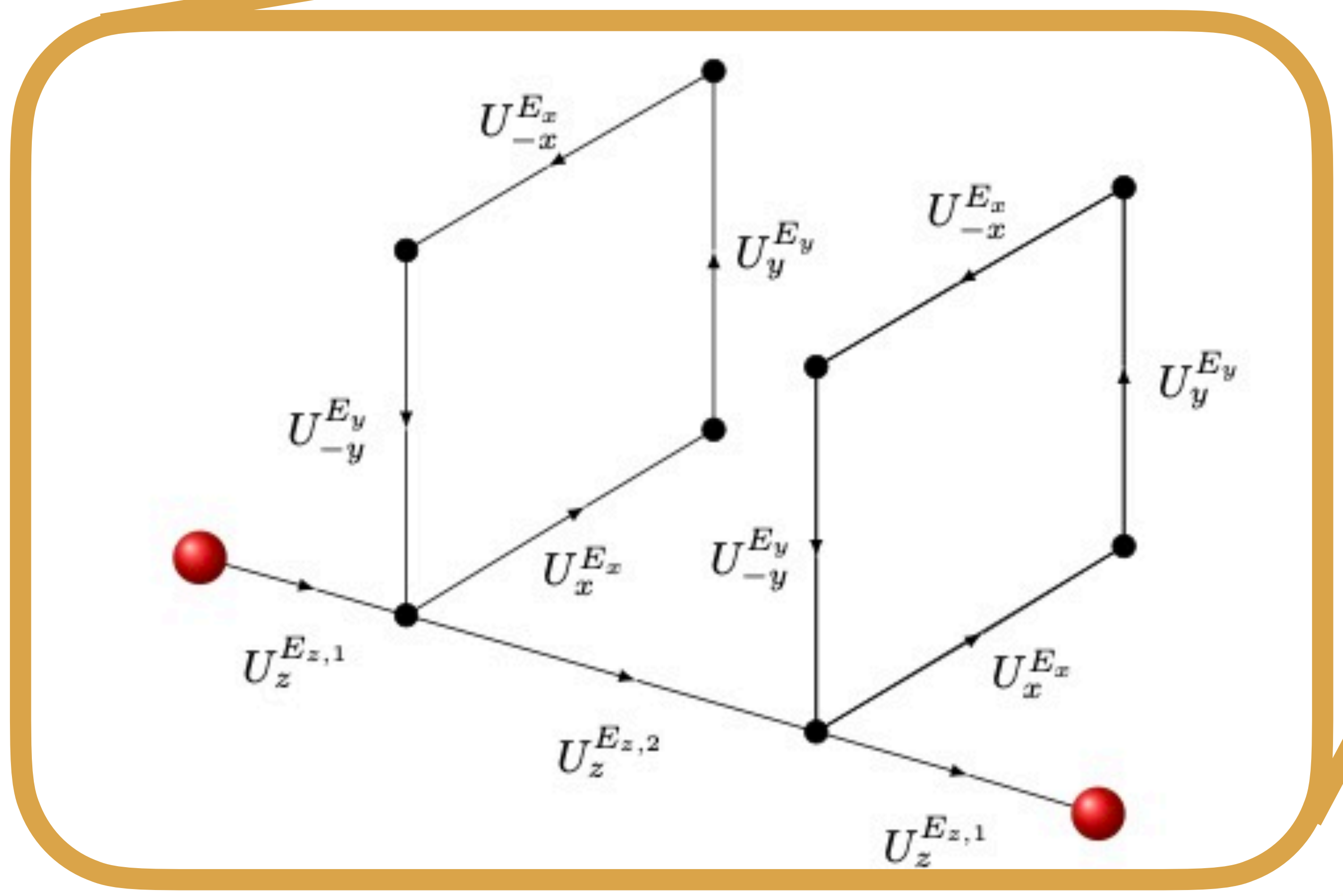


Hybrid Operators for the NLO Potentials

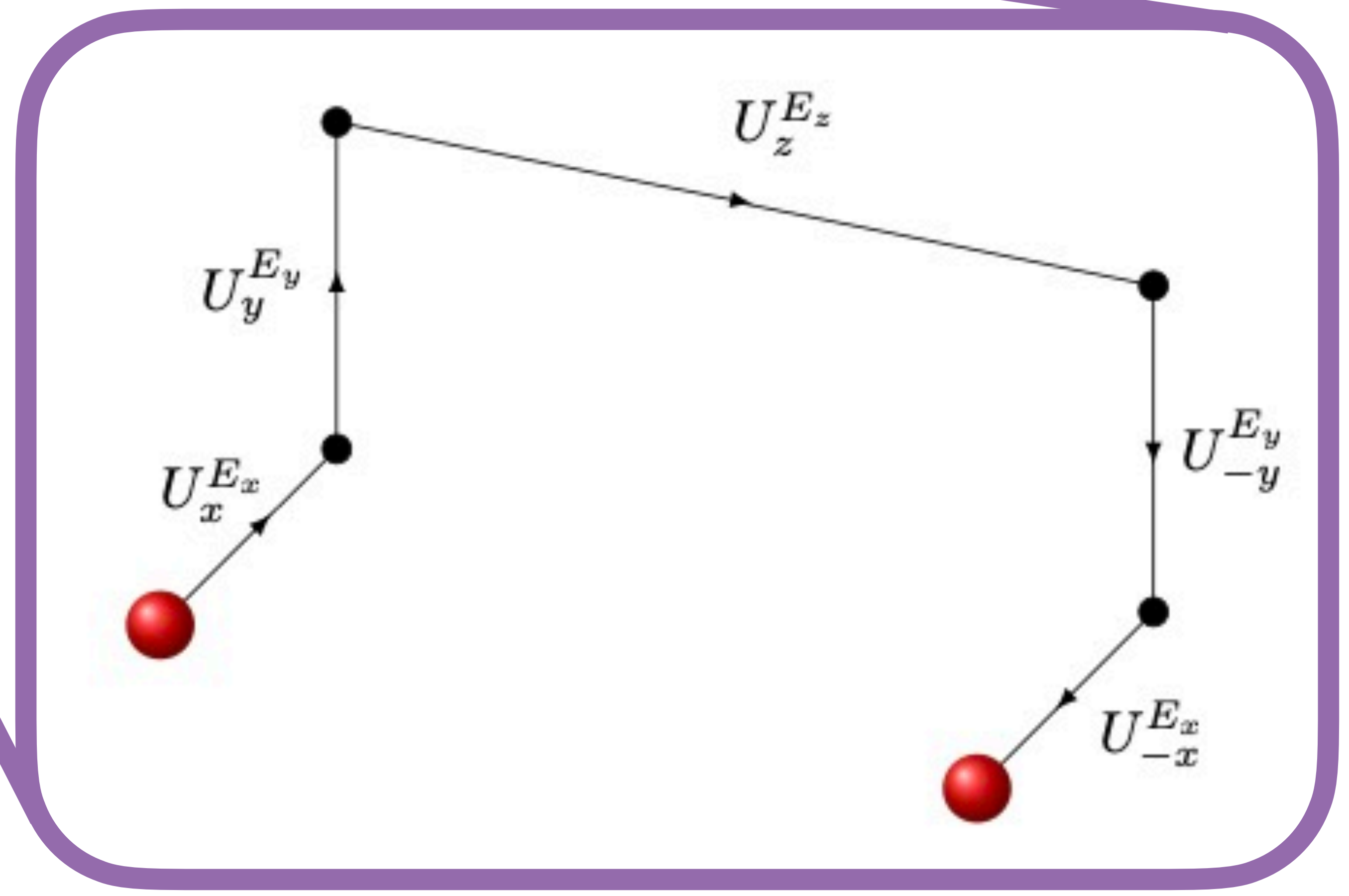
Spin-dependent

$$V_{10}^{sb}(r) = igc_F \left\langle 0,\Sigma_u^- \middle| B_y(-r/2) \middle| 0,\Pi_u^+ \right\rangle$$

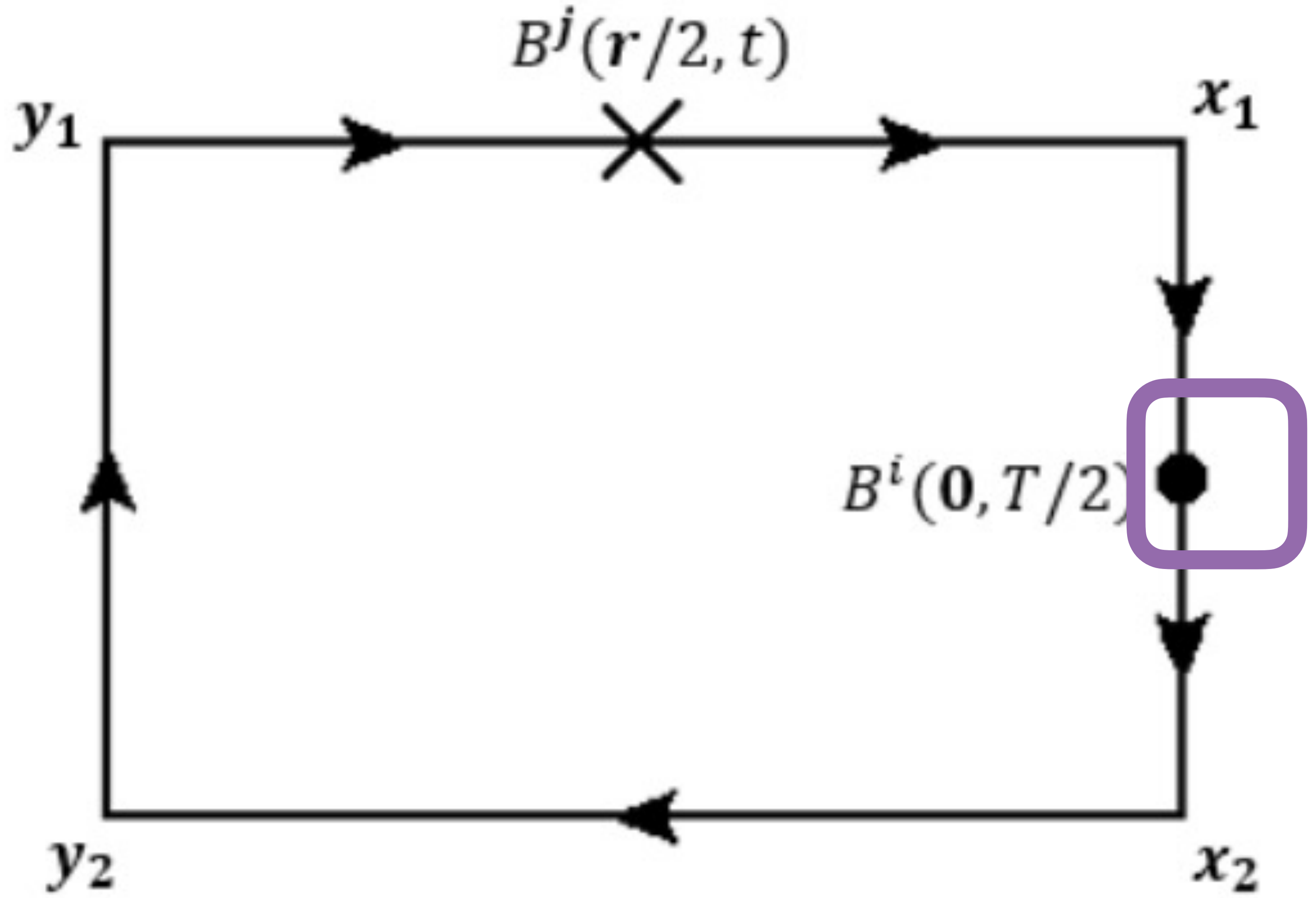




Optimized creation operator for Σ_u^-



Optimized creation operator for Π_u^+



Mixing

$$V_{\Pi_u}^{\text{mix}}(r) = \frac{igc_F}{2m_Q} \left\langle 0,\Sigma_g^+ \middle| B_x(-r/2) \middle| 0,\Pi_u^+ \right\rangle$$

[J. Soto and J. Tarrus Castella Phys. Rev. D 102 no. 1, (2020) 014012]
[R. Oncala and J. Soto, Phys. Rev. D96 no. 1, (2017) 014004]
[C. Schlosser and M. Wagner, Phys. Rev. D 111 (2025) no. 7, 074504]

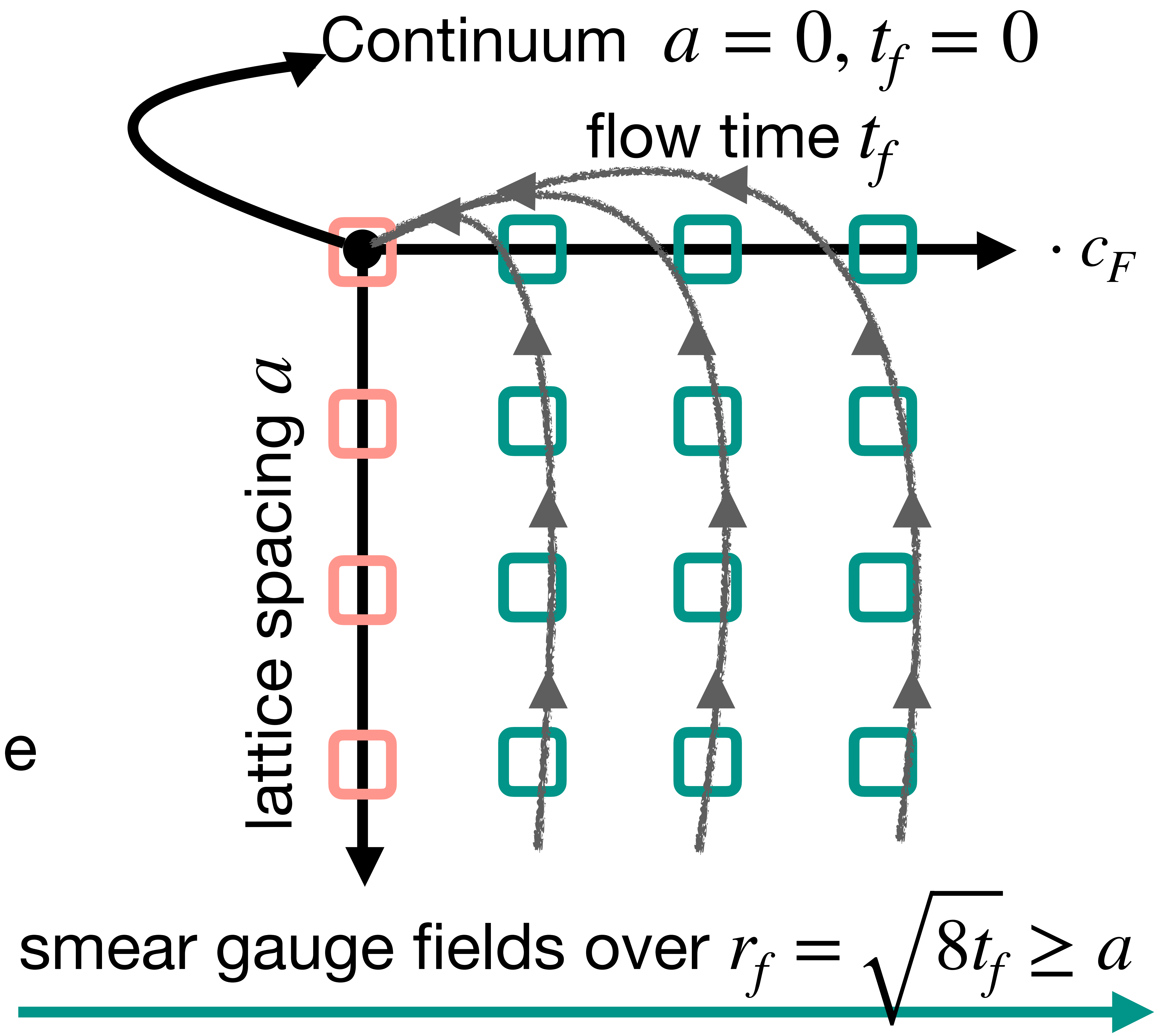
Computing hybrid potentials on the lattice

- Hybrid NLO operators dominated by UV noise → severe statistical and discretization errors

- We need gradient flow to suppress UV fluctuations

- Gradient flow → matching scheme

→ Several million CPU hours on the Goethe NHR cluster



[M. Eichberg and M. Wagner, PoS LATTICE2024 (2024) 117]

From gauge fields to hybrid potentials

Heat bath algorithm to generate gauge configurations

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Gradient flow + APE smear

↓

Generalized Wilson loops with chromomagnetic field insertions

$R^{B_k}_{\Lambda^\epsilon_\eta \Lambda'_\eta}(t; r, T)$

↓

Plateau fits

↓

Potentials

Repeat for several ensembles →

Continuum extrapolation

↓

Matching/renormalization

↓

Gradient flow extrapolation

numerically costly

[J. Soto and J. Tarrus Castella Phys. Rev. D 102 no. 1, (2020) 014012]
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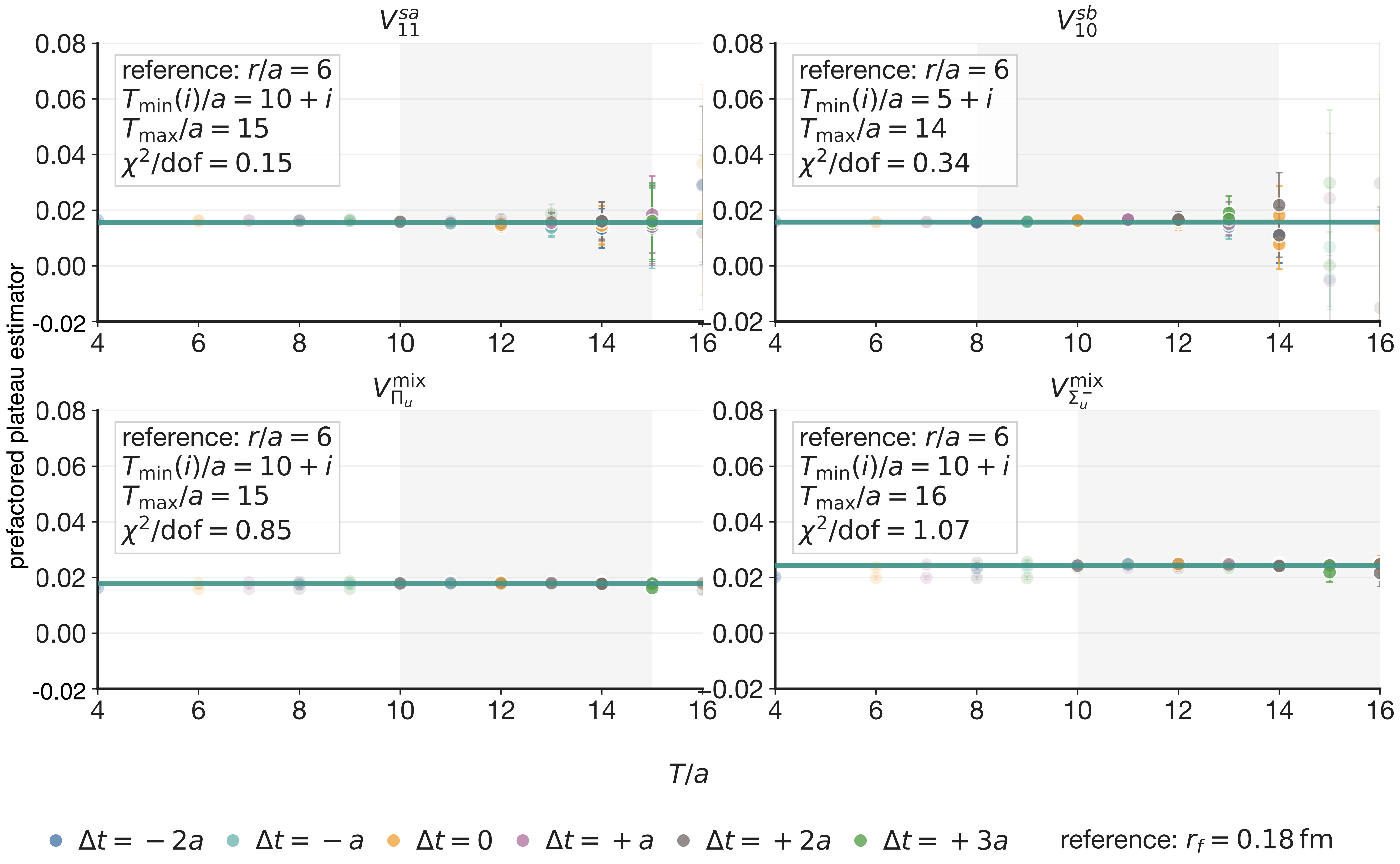
[M. Eichberg and M. Wagner, PoS LATTICE2024 (2024) 117]
[C. Schlosser and M. Wagner, Phys. Rev. D 111 (2025) no. 7, 074504]
[ETM Collaboration, K. Jansen, C. Michael, A. Shindler, and M. Wagner, JHEP 12 (2008) 058]

Numerical setup

Pure SU(3) gauge theory (Wilson plaquette action, CL2QCD)

Ensemble	β	a	Volume	Configurations	Start
A	6.020	0.09 fm	$(4.32\,\mathrm{fm})^4$	4×1000	Cold
B	6.091	0.08 fm	$(4.32\,\mathrm{fm})^4$	2×500	Cold
C	6.285	0.06 fm	$(4.32\,\mathrm{fm})^4$	2×500	Cold

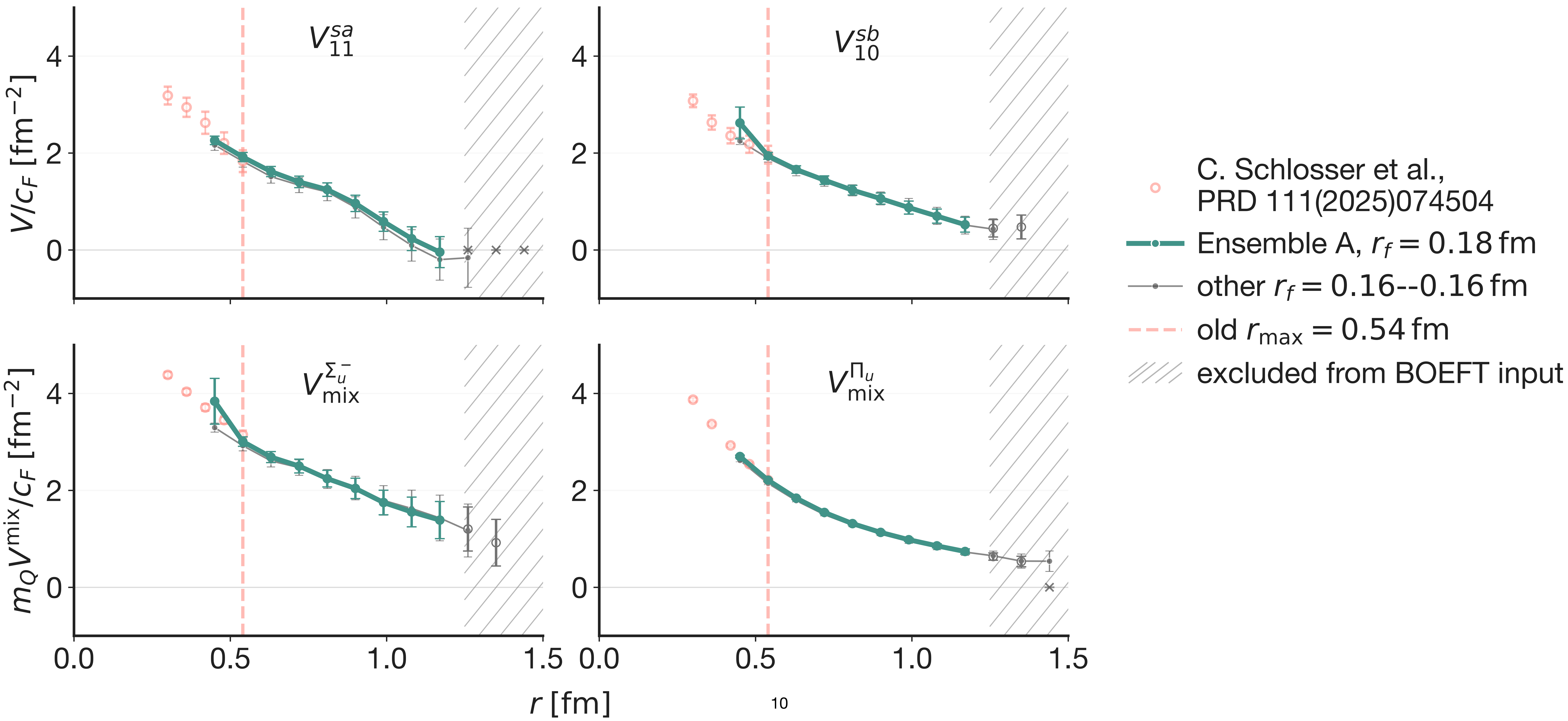
From Wilson-loop ratios to NLO hybrid potentials



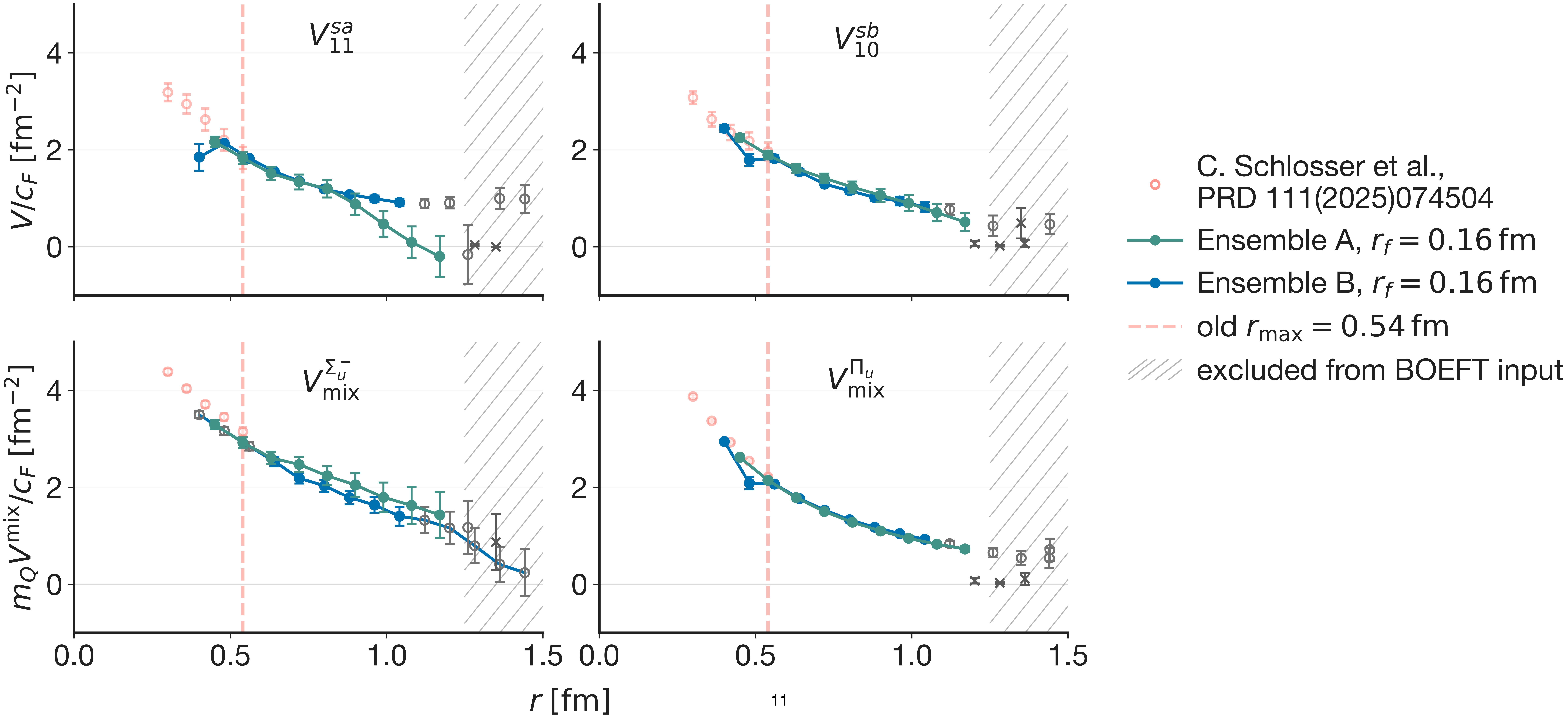
$$R^{B_k}_{\Lambda_\eta^\epsilon \Lambda_\eta'}(t; r, T) = \frac{W^{B_k}_{\Lambda_\eta' \Lambda_\eta'}(t; r, T)}{\sqrt{W_{\Lambda_\eta^\epsilon}(r, T) W_{\Lambda_\eta'}(r, T)}} \times \sqrt{\frac{W_{\Lambda_\eta'}(r, T/2 - t) W_{\Lambda_\eta'}(r, T/2 + t)}{W_{\Lambda_\eta^\epsilon}(r, T/2 - t) W_{\Lambda_\eta'}(r, T/2 + t)}} \xrightarrow{T \to \infty} \left\langle 0, \Lambda_\eta^\epsilon \middle| B_k(-r/2) \middle| 0, \Lambda_\eta^{\epsilon'} \right\rangle (r) \propto V_{NLO}$$

with BO quantum numbers Λ_η^ϵ, spatial, temporal extent of the Wilson-loop r, T and temporal displacement of B_k insertion from midpoint $\Delta t \equiv t - \dfrac{T}{2}$

Long-distance NLO hybrid potentials (A)

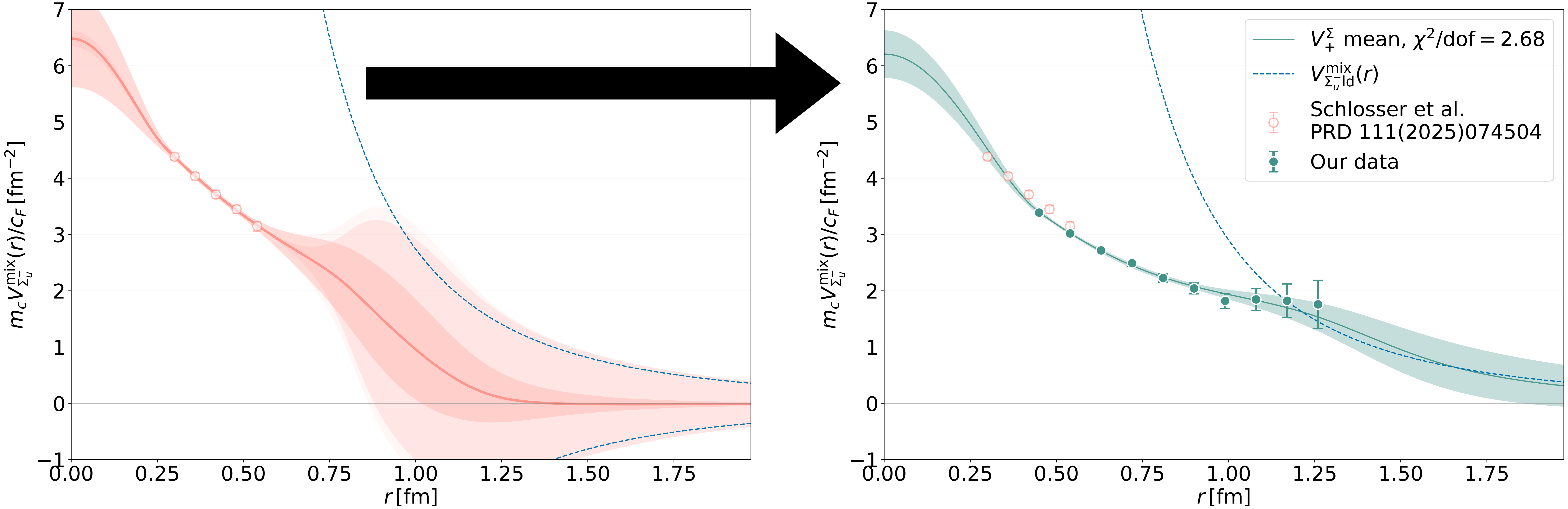


Long-distance NLO hybrid potentials (A+B)



Physics implication (BOEFT)

New lattice data extends to $r \approx 1.25 fm$



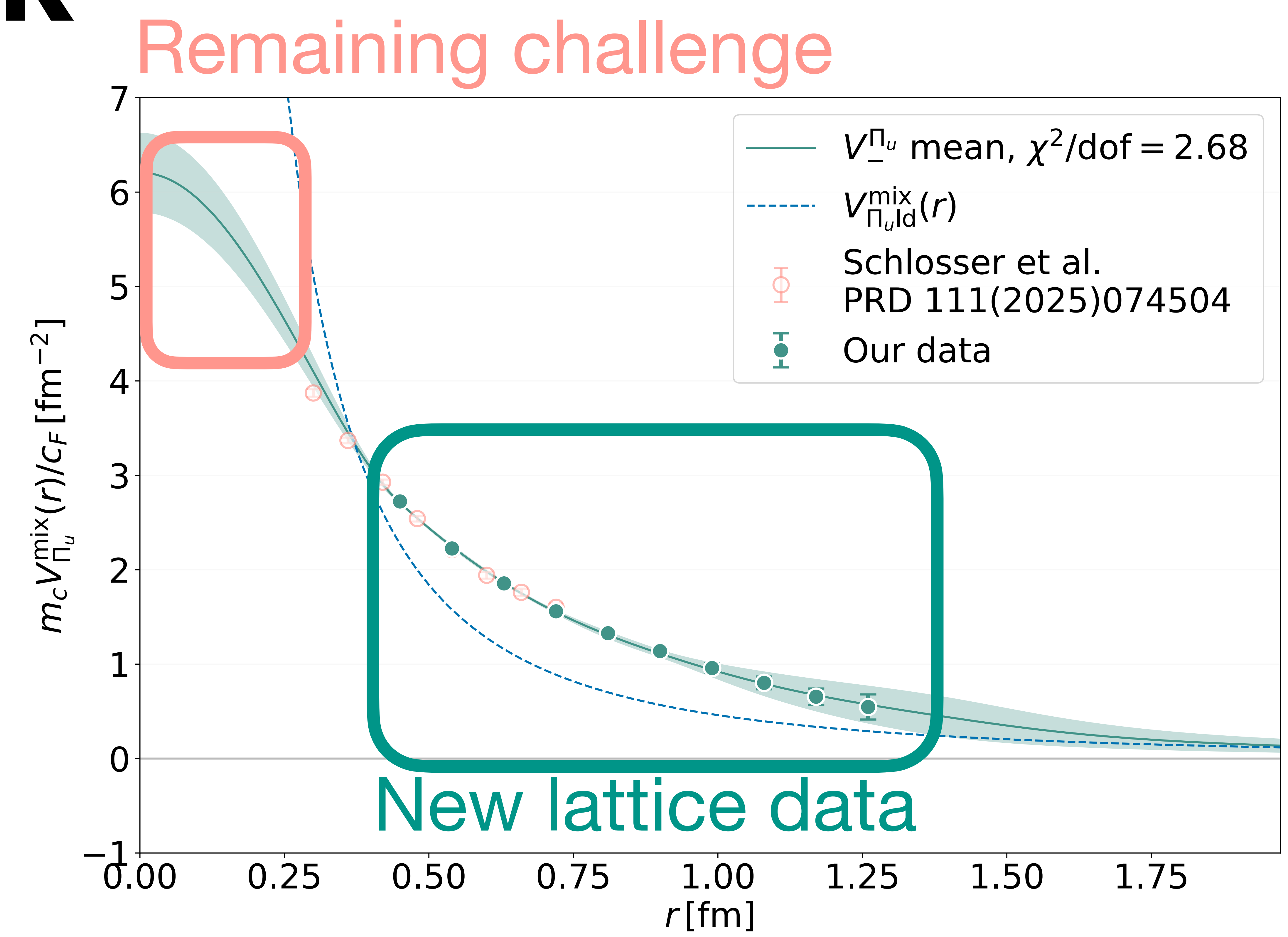
Compact shape leads to prediction of $\eta_c(1S) \sim 10\,\%$ hybrid admixture

→ Improve potentials at short distances

[Talk: P. Pütz, Confinement 2026]

Conclusion and Outlook

- First long-distance lattice determination of all four NLO hybrid potentials up to $r \approx 1.25$ fm
- Next: matching, continuum and gradient-flow extrapolations
- Precision mixing requires small-r lattice data



(1) Topological freezing

(2) Large and fine lattices

[Talk: P. Pütz, Confinement 2026]

Questions?

Lattice Computations

Gauge Configurations with gradient flow

- Flow the gauge link with flow time t_f
  $\rightarrow$ smear gauge fields over
  $r_f = \sqrt{8t_f} \geq a$

- Flowed equations for the gauge links

$$\dot{V}_{t_f}(x,\mu) = -\,g_0^2 \left(\partial_{x,\mu} S \left(V_{t_f} \right) \right) V_{t_f}(x,\mu)$$

$$V_{t_f}(x,\mu)\Big|_{t=0} = U(x,\mu)\,.$$

A. Significantly improves continuum limit:

- Smaller statistical errors
- Smaller discretization erros for $r, T \geq 2r_f$

B. Suppresses UV fluctuations

C. Renormalization scheme (matching coefficients c_F)

$$V^{(1)}_{\mathrm{phys}}(r, m_Q) = V^{(1)}(r, t_f, a)\, c_F^{\overline{\mathrm{MS}}}(m_Q, \mu)/(a^2 c_F(t_f, \mu))$$

NLO potentials of heavy hybrid mesons

$$V_{11}^{sa}(r) = igc_F \left\langle 0, \Pi_u^- \right| B_z(-r/2) \left| 0, \Pi_u^+ \right\rangle$$

$$V_{10}^{sb}(r) = igc_F \left\langle 0, \Sigma_u^- \right| B_y(-r/2) \left| 0, \Pi_u^+ \right\rangle$$

$$V_{\Pi_u}^{\text{mix}}(r) = \frac{igc_F}{2m_Q} \left\langle 0, \Sigma_g^+ \right| B_x(-r/2) \left| 0, \Pi_u^+ \right\rangle$$

$$V_{\Sigma_u^-}^{\text{mix}}(r) = \frac{igc_F}{2m_Q} \left\langle 0, \Sigma_g^+ \right| B_z(-r/2) \left| 0, \Sigma_u^- \right\rangle$$

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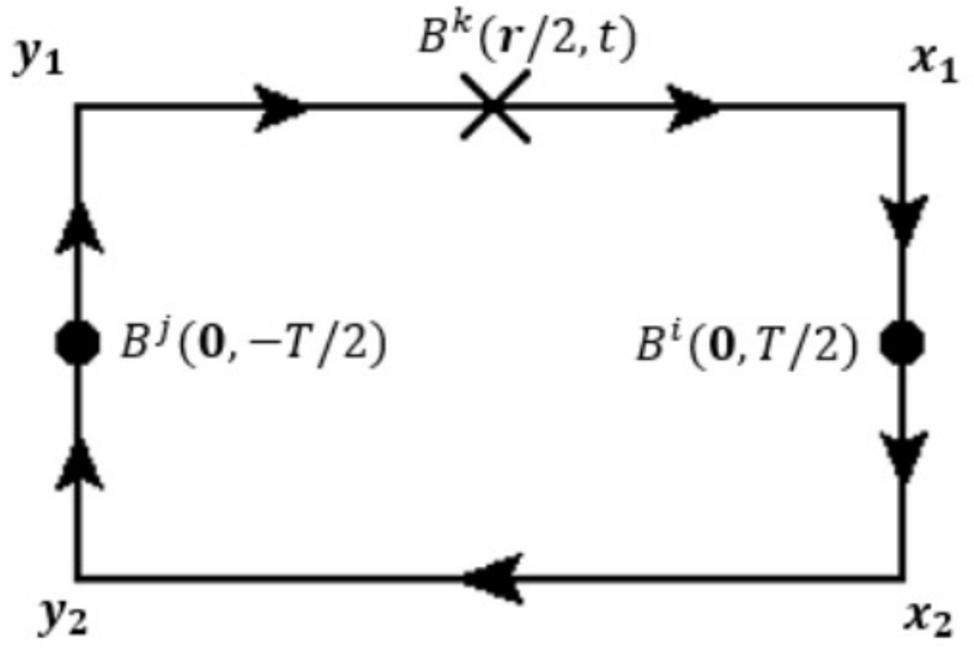

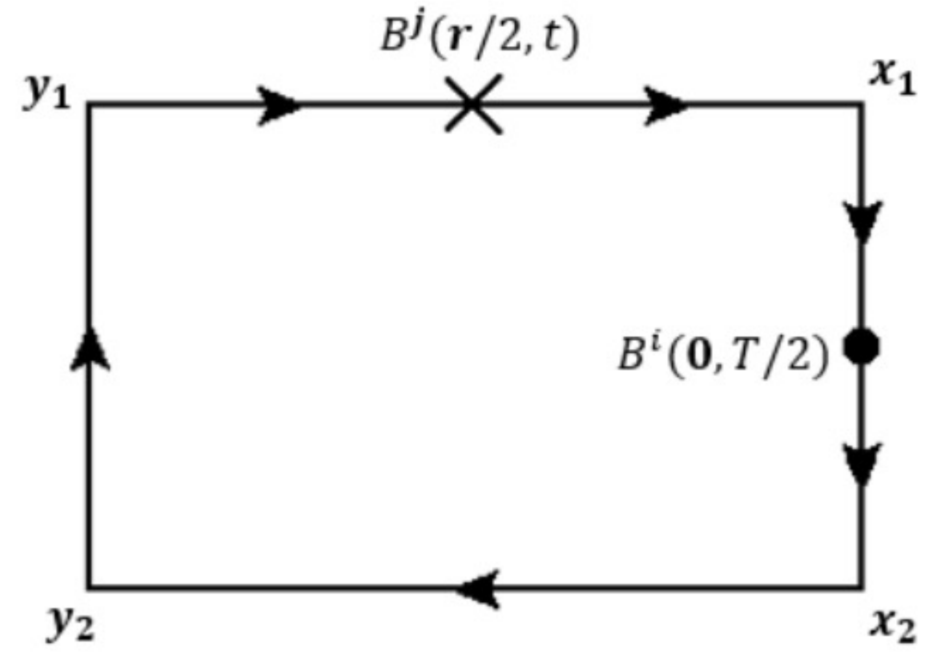

Why is the η_c mixing so large?

$V_{mix}(r)$ determines overlap integrals

↓

Compact $1S$ wave function has the largest overlap

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Largest hybrid admixture among bound charmonia: $\eta_c(1S) \sim 10\,\%$

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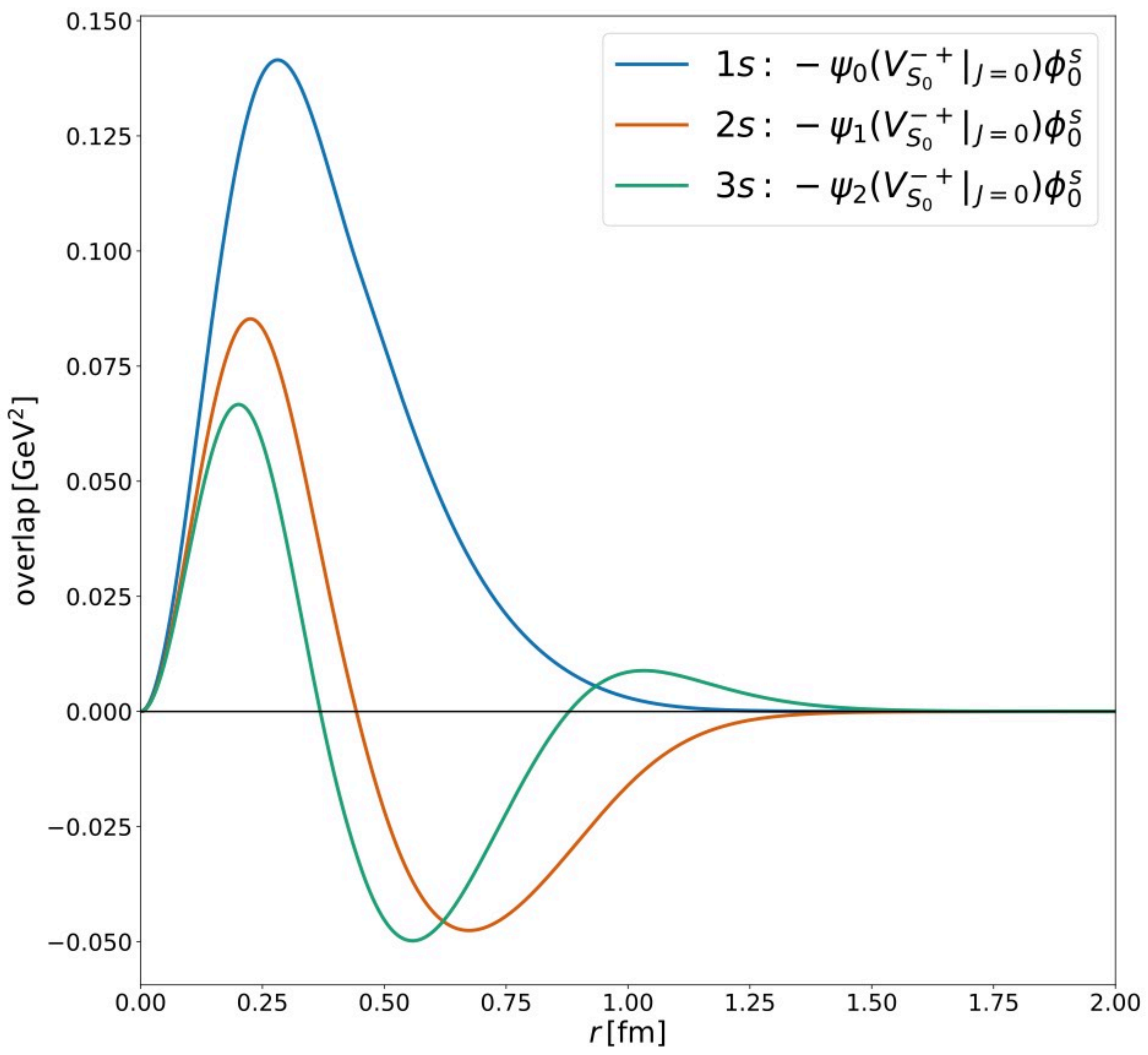
$\rightarrow$ Next step: precise short-distance lattice data



[Talk: P. Pütz, Confinement 2026]

Topological freezing

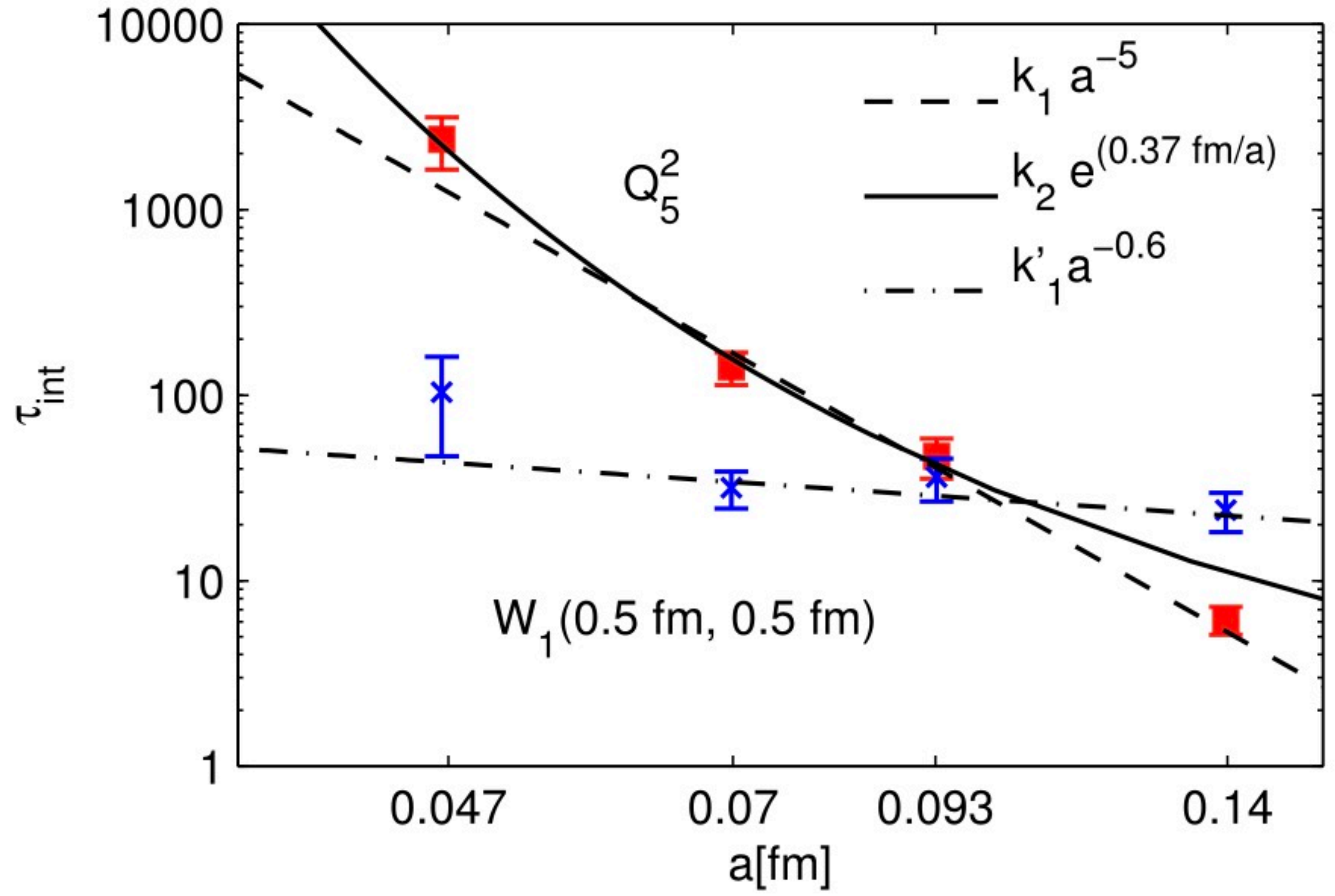


Fig. 4. Auto-correlation time of Q_5^2 and the (0.5 fm) × (0.5 fm) square Wilson loop as a function of the lattice spacing using the DD-HMC algorithm. For Q_5^2 the two curves are fits through the last three points of the two ansätze of Eq. (4.1). For the Wilson loop only the fit to the power law has been performed, through all points.

[S. Schaefer, R. Sommer and F. Virotta (ALPHA), Nucl. Phys. B 845, 93–119 (2011)]

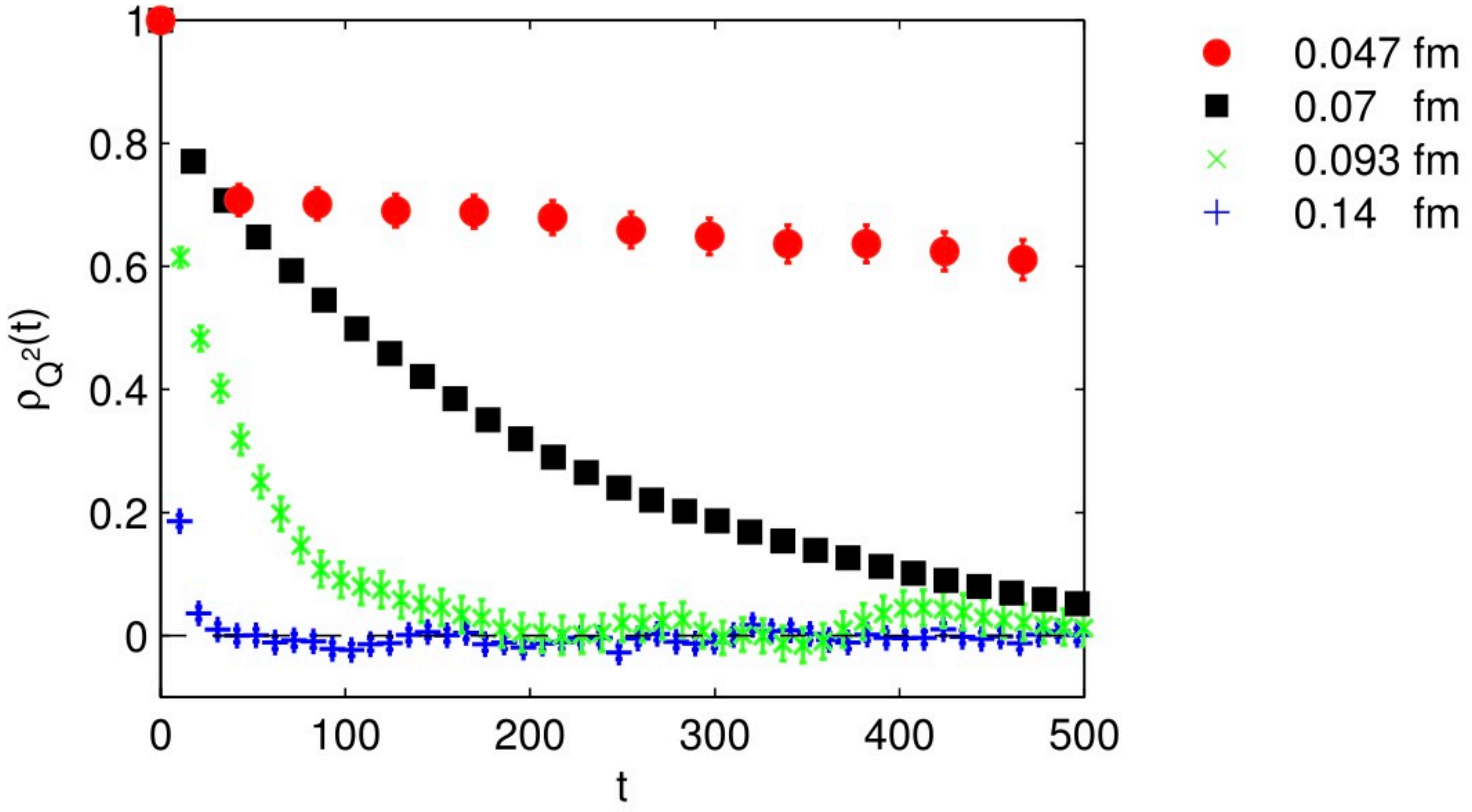


Fig. 2. Normalized auto-correlation function of Q_1^2 at various lattice spacings. The Monte Carlo time is given in molecular dynamics units multiplied by R.