Update on lattice study of $T_{cc}^{+}(3875)$ using three-body formalism

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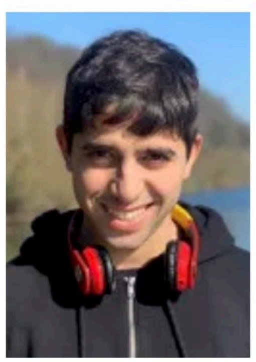
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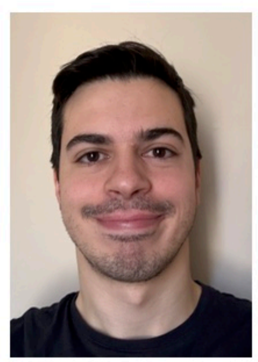
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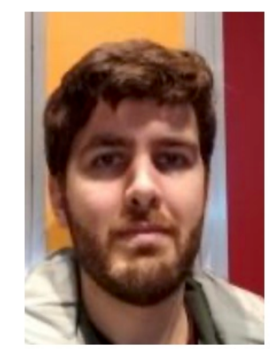
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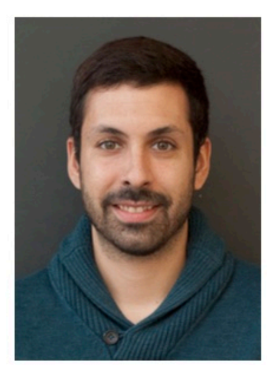
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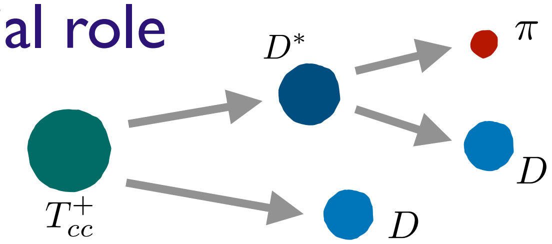
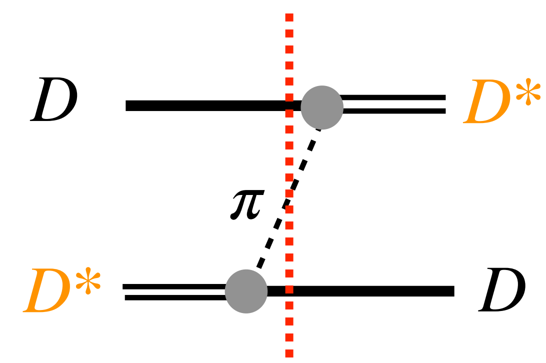
Introduction

- T_{cc}^{+} is an exotic resonance ($cc\bar{u}\bar{d}$) accessible to LQCD (no annihilation contractions)
  - Structure is unclear: DD^* molecule, diquark-antidiquark, compact tetraquark?
- Three-body dynamics plays a crucial role



  - In nature, decay is to $DD\pi$
  - For heavier-than-physical light quarks, thresholds are interchanged
    - T_{cc}^{+} can be studied in DD^* system
    - However, three-body effects enter through left-hand cut
    - States relevant for T_{cc}^{+} cannot be analyzed using standard Lüscher method

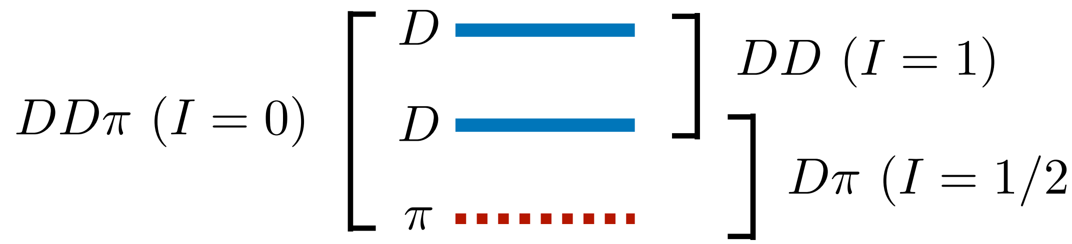


- Need a method that accounts for left-hand cut
  - We use the RFT three-particle formalism of [Hansen, Romero-López, SRS, 2024]
  - While perhaps more cumbersome than two-body alternatives, it remains valid for physical quark masses, and accounts for nearby $DD\pi$ states

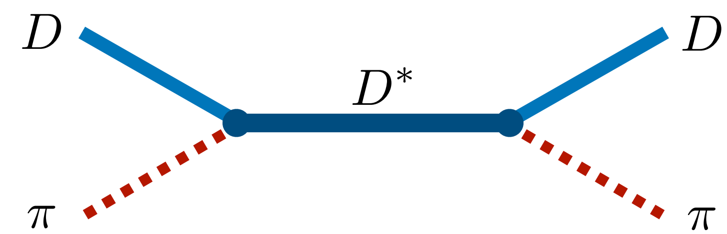
Overview of formalism

- Two 2-particle sub-channels to consider,

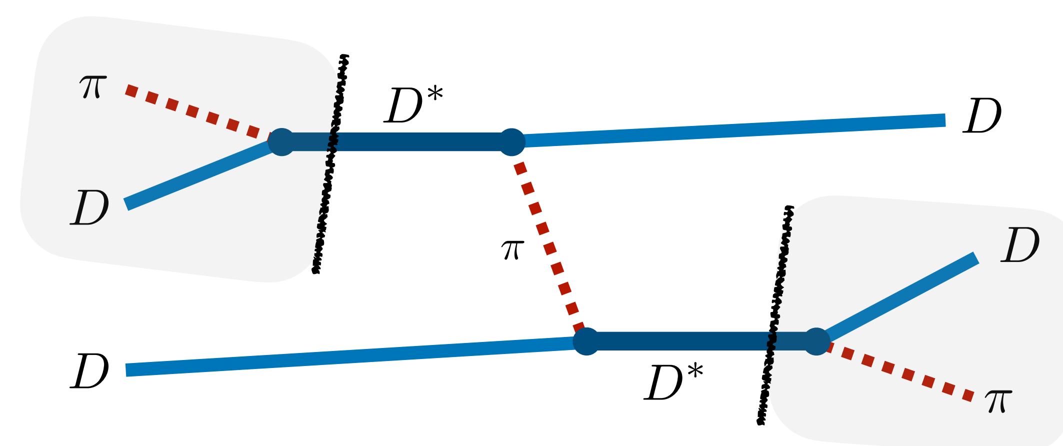
  $DD\ (I = 1),\quad D\pi\ (I = 1/2)$



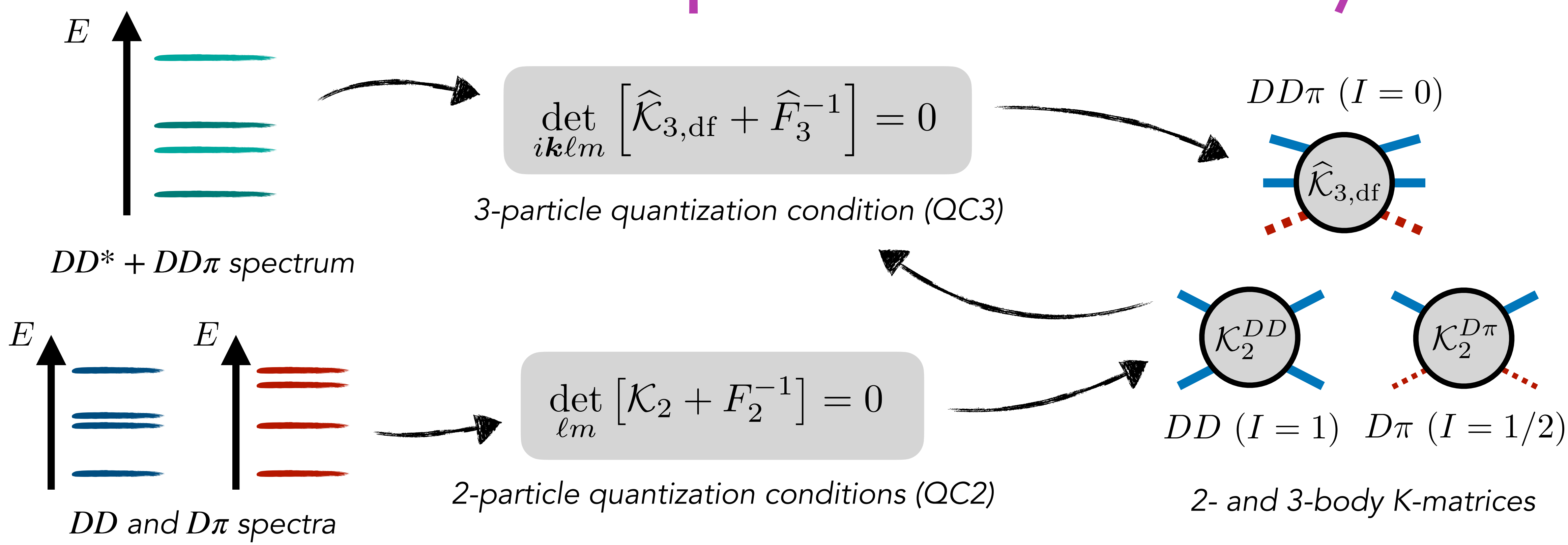
- D^* included as a **pole in the p-wave $D\pi$ amplitude** (i.e. a bound state)



- For stable D^*, DD^* amplitude accessed by **LSZ-reducing $DD\pi$ amplitude**



Workflow: spectrum analysis



Simultaneous fits of finite-volume spectra of two-body subsystems and three-body main system allow us to constrain infinite-volume K-matrices…

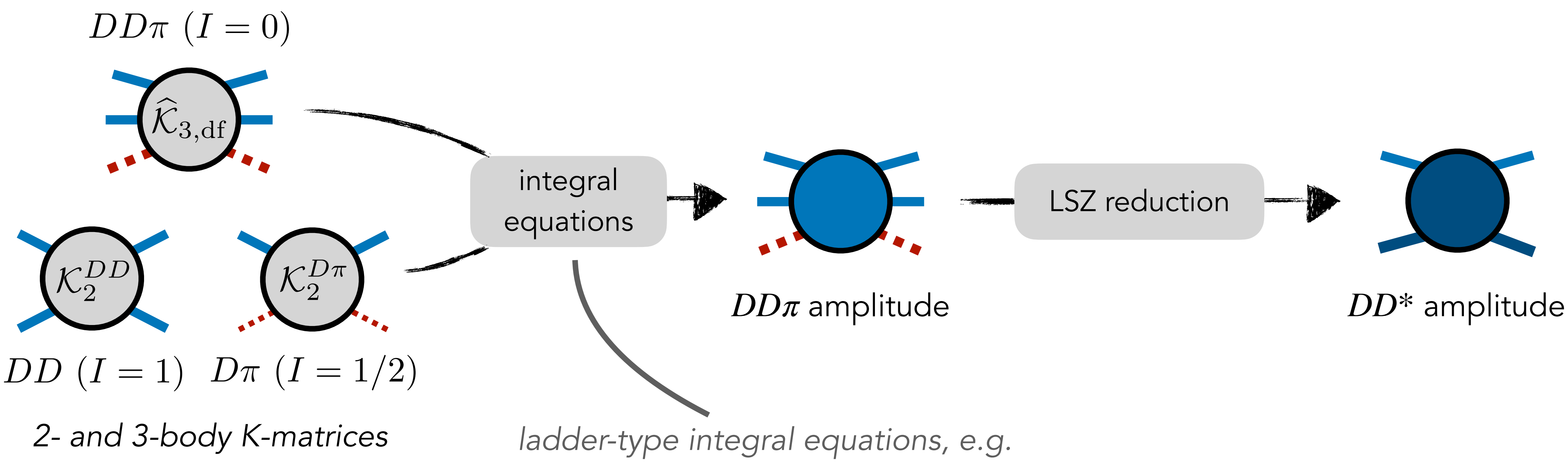
$$\det_{\boldsymbol{ik}\ell m}\left[\widehat{\mathcal{K}}_{3,\mathrm{df}} + \widehat{F}_3^{-1}\right] = 0 \qquad \widehat{F}_3 = \frac{\widehat{F}}{3} - \widehat{F}\frac{1}{1+\widehat{\mathcal{M}}_{2,L}\widehat{G}}\widehat{\mathcal{M}}_{2,L}\widehat{F}$$

$$\widehat{\mathcal{M}}_{2,L} = \frac{1}{\widehat{\mathcal{K}}_{2,L}^{-1} + \widehat{F}} \qquad \widehat{\mathcal{K}}_{2,L} = \begin{pmatrix} \mathcal{K}_{2,L}^{D\pi} & 0 \\ 0 & \frac{1}{2}\mathcal{K}_{2,L}^{DD} \end{pmatrix}$$

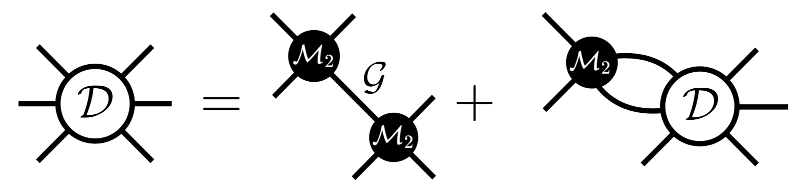
QC3 involves matrices in large index space:

- i — *spectator flavor*
- $\boldsymbol{k}$ — *spectator 3-momentum*
- ℓm — *2-pt subsystem angular momentum*

Workflow: integral equations



$$\boldsymbol{\mathcal{D}} = -\boldsymbol{\mathcal{M}}_2\,\boldsymbol{\mathcal{G}}\,\boldsymbol{\mathcal{M}}_2 - \boldsymbol{\mathcal{M}}_2\,\boldsymbol{\mathcal{G}}\,\boldsymbol{\mathcal{D}}$$



… we feed the K-matrices through **integral equations for $DD\pi$ amplitude**, **LSZ-reduce** and **analytically continue** to obtain DD^* amplitude and find T_{cc}^+ pole

Simulation details

- Use two CLS ensembles with same pion mass but different volume

	L/a	T/a	$m_\pi L$	m_π[MeV]	a[fm]	M_D[MeV]	$(M_{D^*} - M_D)$[MeV]	Configs
X252	36	128	3.4	290	0.063	1880	130	175
X253	40	128	3.7	290	0.063	1880	130	301
Physical values:				135		1870	140-145	

- Use frames (0,0,0), (0,0,1), (1,1,0), (1,1,1) and irreps coupling dominantly to $J^P = 1^+,\ 0^-$

- Use distillation, with multiple operators, and GEVP

  - Types of operator:

DD in $I = 1$: $DD\,,\ DD^*\,,\ D^*D^*\,,\ D_0^*D_0^*$ (D_0^* is a scalar, $D_0^*(2300)$)

$D\pi$ in $I = 1/2$: $D\pi\,,\ D^*\,,\ D^*\pi\,,\ D_0^*$

$DD\pi$ / DD^* in $I = 0$: bilocal/trilocal operators $DD^*\,,\ DD\pi\,,\ D^*D^*\,,\ D_0^*D\,,\ D_0^*D^*$

local tetraquark operators e.g. $T^i_{DD^*}(t,\boldsymbol{x}) = \left(\bar{u}\gamma^i c\,\bar{d}\gamma^5 c - \bar{d}\gamma^i c\,\bar{u}\gamma^5 c\right)(t,\boldsymbol{x})$

All results are PRELIMINARY

$D\pi$ subchannel

- Simultaneous fit to 16 X252 and 16 X253 levels with s- and p-wave ERE2 forms
  - $D\eta$ & $D^*\pi$ channels not included in QC2, but lie well above levels kept in fit

$$q \cot \delta_s = \frac{r_{0,s}}{2}(q^2 - q_{0,s}^2)$$

$$q^3 \cot \delta_p = \frac{r_{0,p}}{2}(q^2 - q_{0,p}^2)$$

HQEFT $DD\pi$ coupling g and D^* mass used as p-wave fit parameters

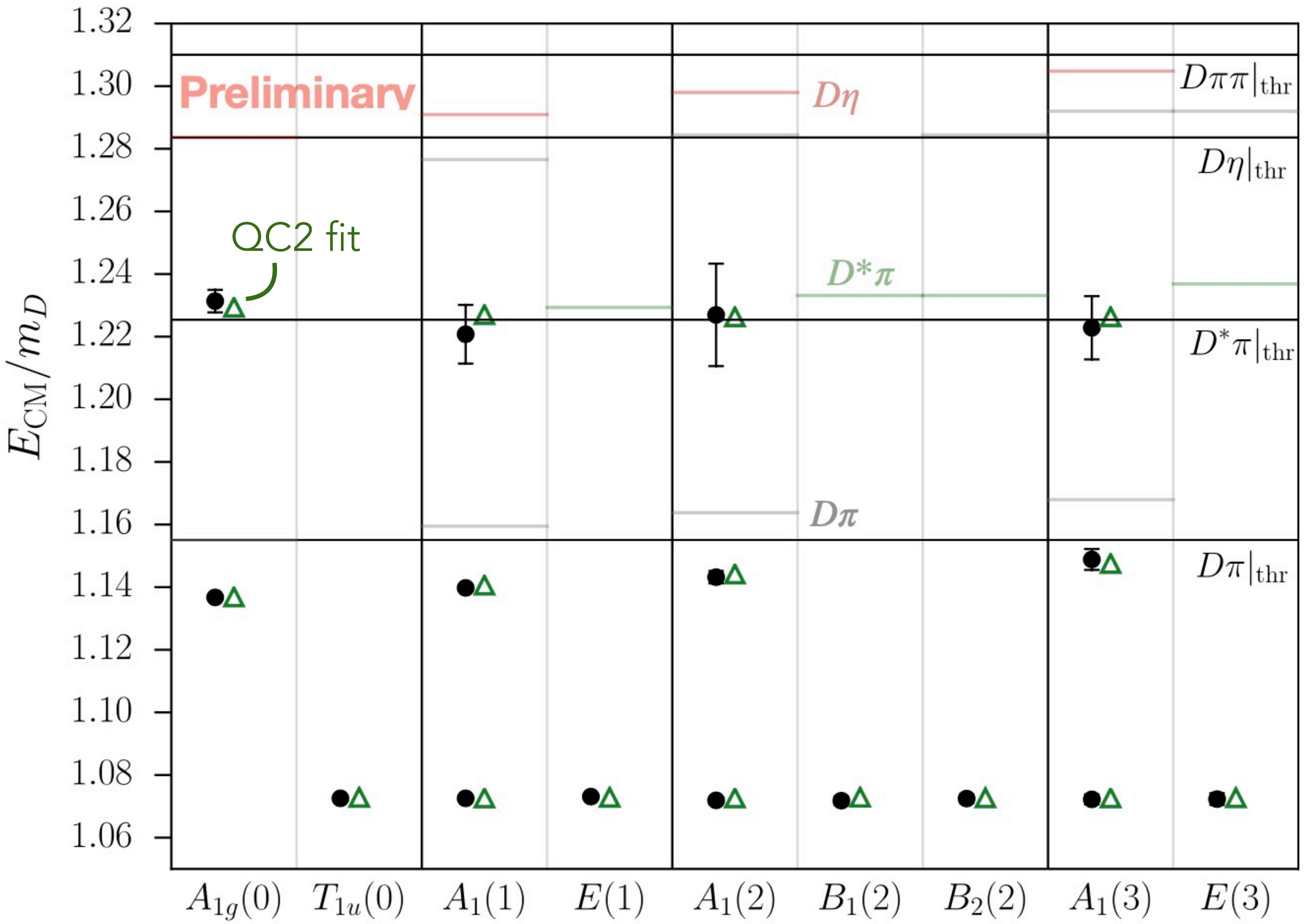
Constrained $g = 0.55 \pm 0.1$ based on experimental value (0.57) and lattice results

$$q_{0,s}^2 = 0.0084(14),\ r_{0,s} = -8.42(72),$$

$$M_{D^*} = 1.07195(56),\ g = 0.525(94)$$

Good fit: $\chi^2 = 23.15/29$ dof

E.g. X253 $\quad m_\pi L = 3.745$



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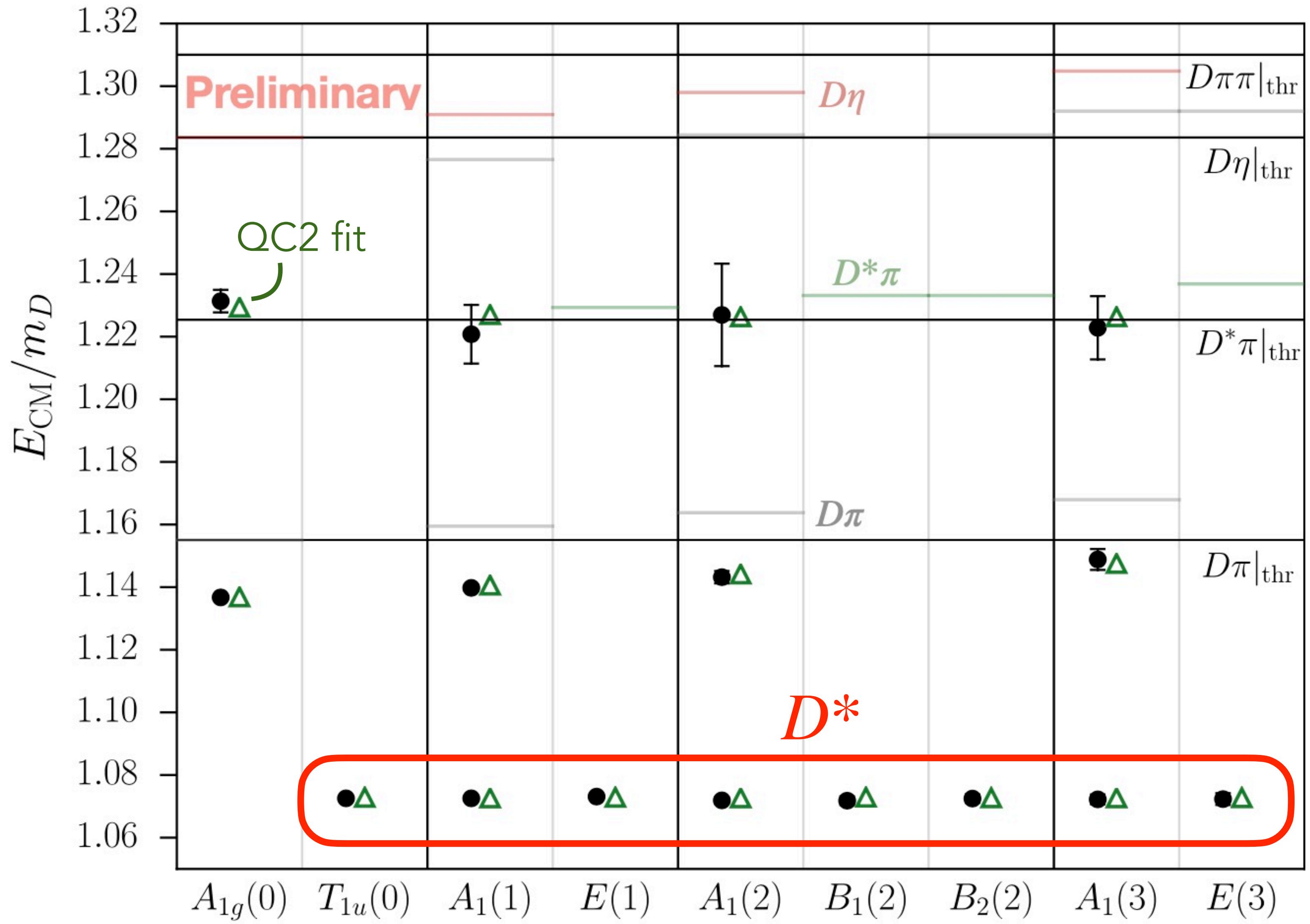

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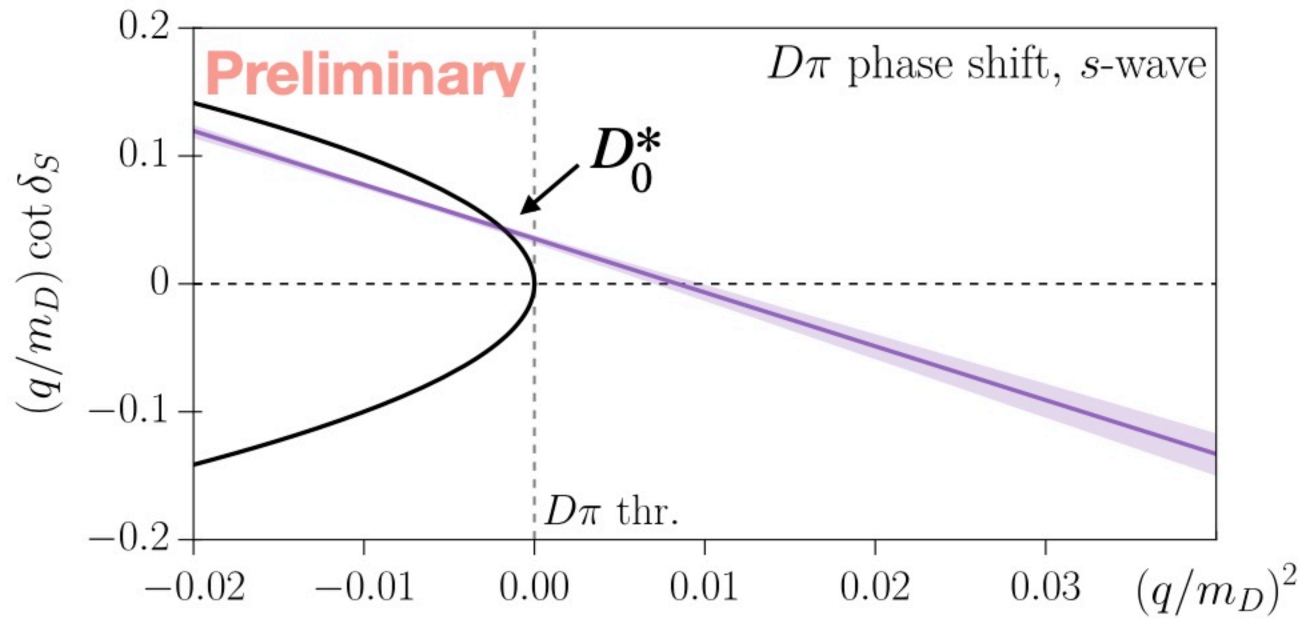

$$q^3\cot\delta_p = \frac{r_{0,p}}{2}(q^2 - q_{0,p}^2)$$

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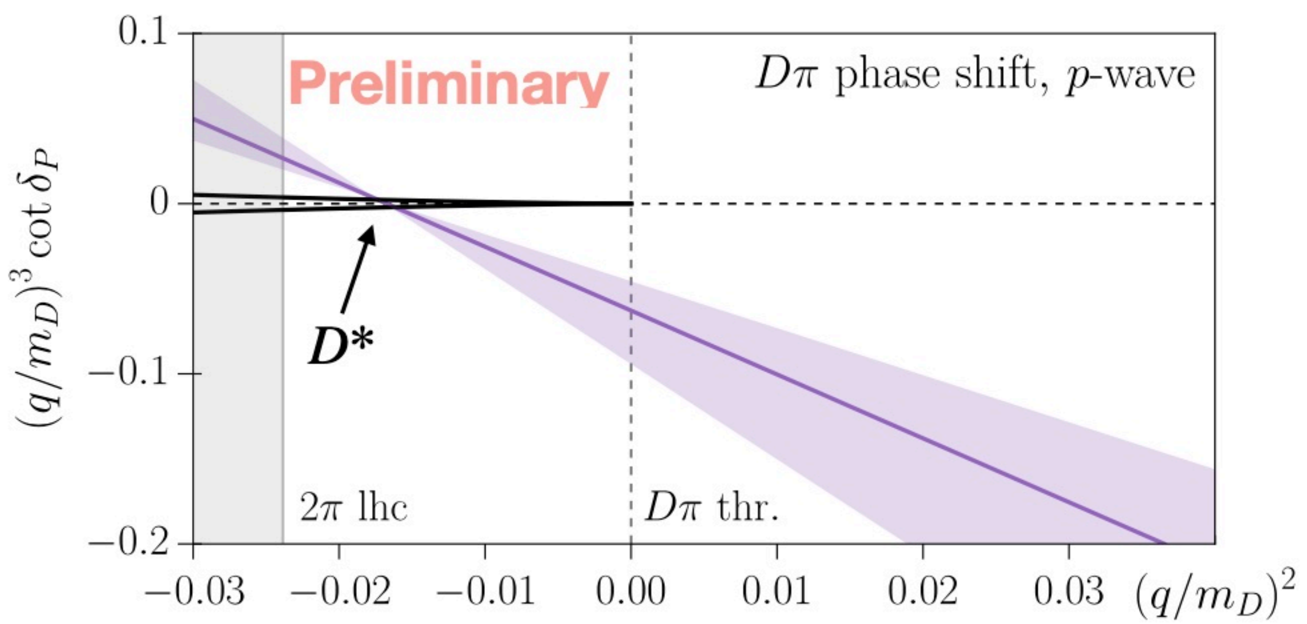
Largely determined by constraint rather than data

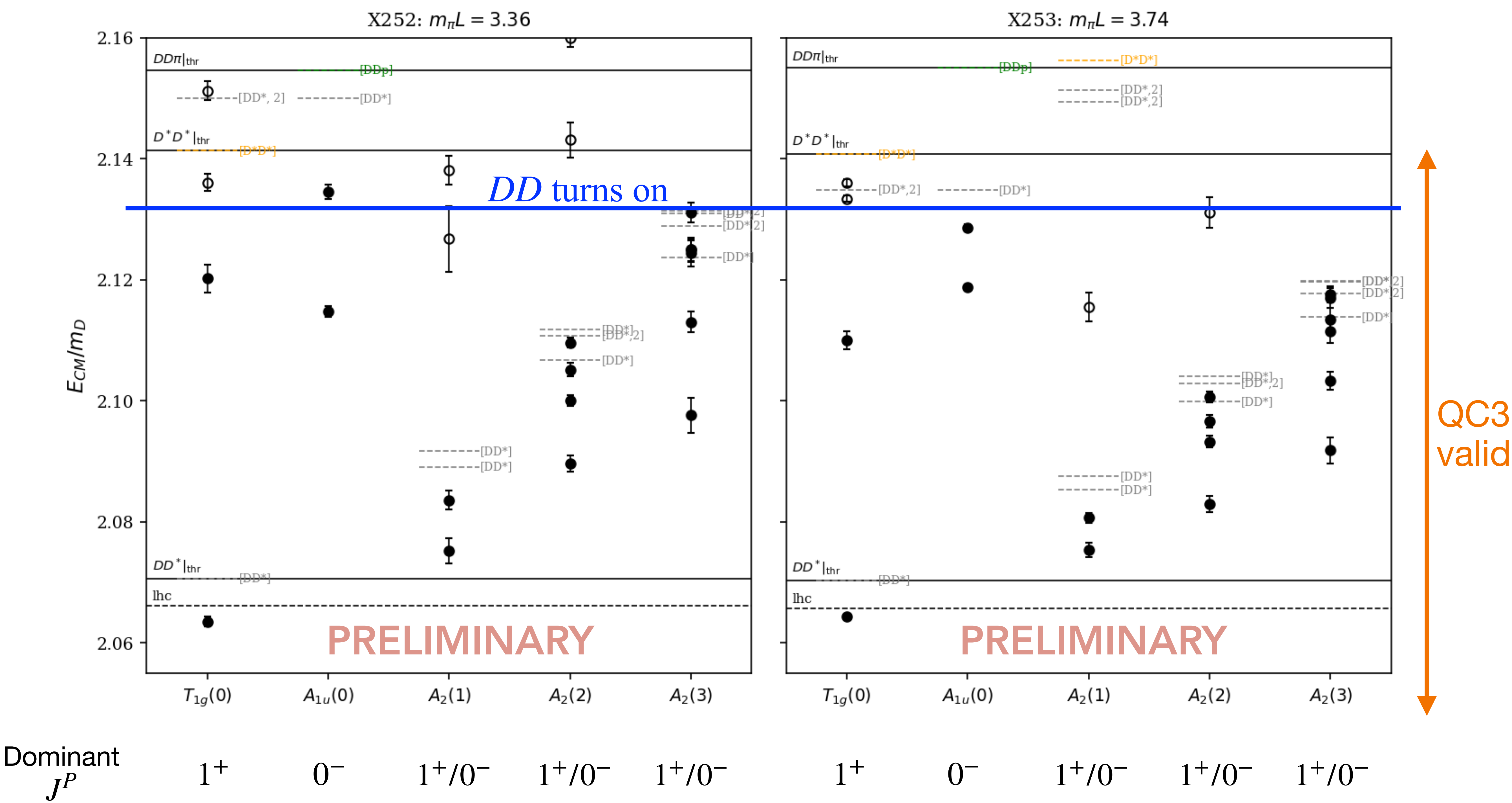


$I = 0$ $DD\pi$ channel

- Mixes with D^*D^*, $DD\pi\pi$, $DD_s\overline{K}$, and, in some moving irreps DD
  - QC3 only valid if such mixing is absent
  - Requires $E_{\rm CM} < 2M_{D^*} < 2M_D + M_\pi$, so all levels in fit are below $DD\pi$ threshold
  - In rest frame use T_{1g} (s/d-wave DD^*—T_{cc}^+ channel) and A_{1u} (p-wave)—no mixing with DD
  - In moving frames use A_2 irreps—all waves contribute, mixing with DD possible, but requires large relative angular momentum, and free DD levels far above $DD\pi$ threshold
- QC3 remains valid below u-channel DD^* one-pion-exchange left-hand cut at $E_{\rm LHC}^{(\pi)} = \sqrt{2M_D^2 + 2M_{D^*}^2 - M_\pi^2}$
  - QC3 fails below u-channel two-pion-exchange LHC at $E_{\rm LHC}^{(\pi\pi,u)} = \sqrt{2M_D^2 + 2M_{D^*}^2 - 4M_\pi^2}$

$DD\pi$ spectra

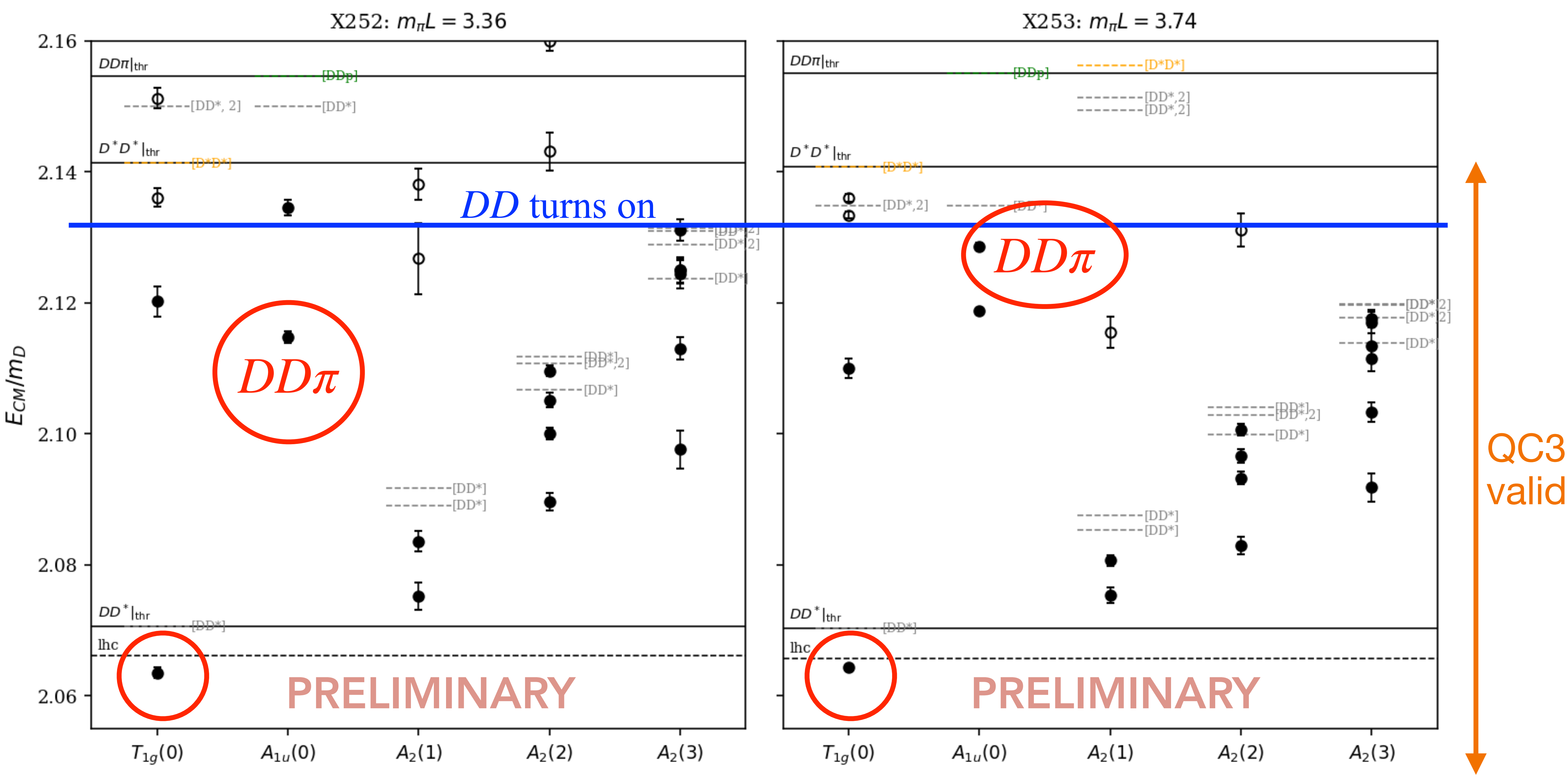
Keep16+16 levels; in some fits add 2+2 higher $A_{1u}(0)$ levels



$DD\pi$ spectra

Keep16+16 levels; in some fits add 2+2 higher $A_{1u}(0)$ levels

3-particle effects crucial



Dominant J^P	1^+	0^-	$1^+/0^-$	$1^+/0^-$	$1^+/0^-$	1^+	0^-	$1^+/0^-$	$1^+/0^-$	$1^+/0^-$

$DD\pi + D\pi + DD$ fit

- Tried many combinations of levels and parameters
  - Best global fit so far fits 84 levels (18 $DD\pi$, 16 $D\pi$, 8 DD on each ensemble)
  - 12 parameters (5 for $DD\pi$, 4 for $D\pi$, 3 DD), constraint $g = 0.55 \pm 0.1 \Rightarrow$ 73 dof

$$\chi^2 = 223.6,\ \chi^2_{\rm ref} = 3.06,\ (\text{Bayes} = 1.63)$$

$$\chi^2_{DD,X252} = 7.7,\ \chi^2_{D\pi,X252} = 28.6,\ \chi^2_{DD\pi,X252} = 33.9$$

$$\chi^2_{DD,X253} = 6.0,\ \chi^2_{D\pi,X253} = 77.3,\ \chi^2_{DD\pi,X253} = 41.4$$

DD ERE3: $B_0 = -0.435(11)\,,\quad B_1 = -3.99(73)\,,\quad B_2 = -20.9(8.8)\,,$

$D\pi$ s- & p-wave ERE2:

$$q^2_{0,s} = 0.0073(12)\,,\quad r_{0,s} = -8.76(70)\,,$$
$$M_{D^*} = 1.06739(24)\,,\quad g = 0.422(9)\,,$$

$\mathcal{K}_{\rm df,3}$:

$$c_{\rm iso,0} = -1.35(19)\times 10^5\,,\quad c_{\rm iso,1} = 2.16(47)\times 10^6\,,\quad c_B = 9.3(1.2)\times 10^4\,,$$
$$c_E = 1.49(21)\times 10^5\,,\quad c'_E = 2.8(3.2)\times 10^5\,.$$

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Poor fit!

$$\chi^2_{DD,X252} = 7.7,\ \chi^2_{D\pi,X252} = 28.6,\ \chi^2_{DD\pi,X252} = 33.9$$

$$\chi^2_{DD,X253} = 6.0,\ \chi^2_{D\pi,X253} = 77.3,\ \chi^2_{DD\pi,X253} = 41.4$$

Worst subsector

DD ERE3: $B_0 = -0.435(11)\,,\quad B_1 = -3.99(73)\,,\quad B_2 = -20.9(8.8)\,,$

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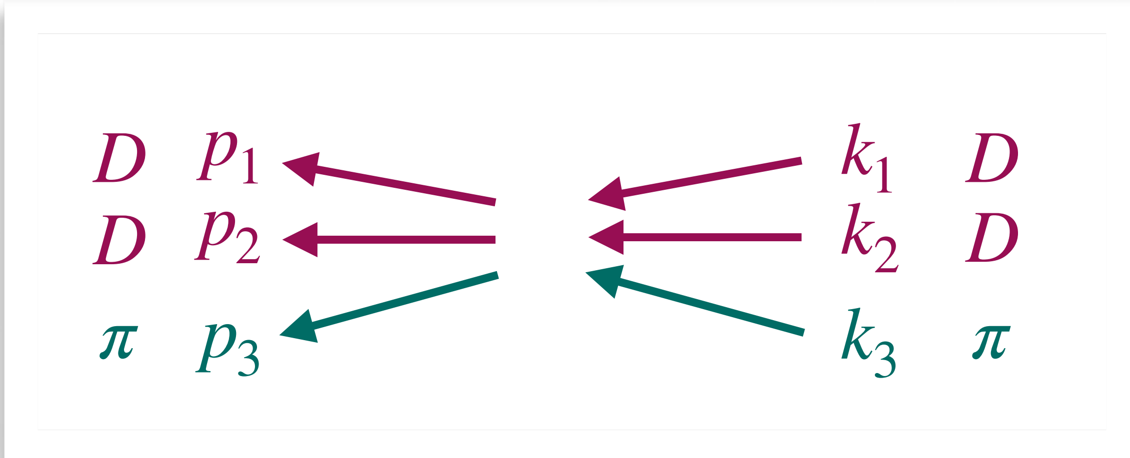
M_{D^*} much lower than in $D\pi$ fit

$\mathcal{K}_{\rm df,3}$: $c_{\rm iso,0} = -1.35(19)\times 10^5\,,\quad c_{\rm iso,1} = 2.16(47)\times 10^6\,,\quad c_B = 9.3(1.2)\times 10^4\,,$ $c_E = 1.49(21)\times 10^5\,,\quad c'_E = 2.8(3.2)\times 10^5\,.$

$\mathcal{K}_{\text{df},3}$ threshold expansion



$$\Delta = \frac{s - s_{\text{th}}}{s_{\text{th}}}, \quad s_{\text{th}} = (2M_D + M_\pi)^2,$$

$$t_{ij} = \frac{(p_i - k_j)^2 - (M_i - M_j)^2}{s_{\text{th}}}$$

$$\Delta_i = \frac{(k_j + k_k)^2 - (M_j + M_k)^2}{s_{\text{th}}}, \quad \Delta_i' = \frac{(p_j + p_k)^2 - (M_j + M_k)^2}{s_{\text{th}}},$$

For our data: $(-0.01) \lesssim \Delta \lesssim 0.15$

$\mathcal{O}(\Delta^0)$: includes dimer s waves

$$\mathcal{K}_{\text{df},3} = c_{\text{iso},0} +$$

$\mathcal{O}(\Delta^1)$: includes s & p waves

$$\underbrace{c_{\text{iso},1}\Delta + c_B(\Delta_3 + \Delta_3') + c_E t_{33}}+$$

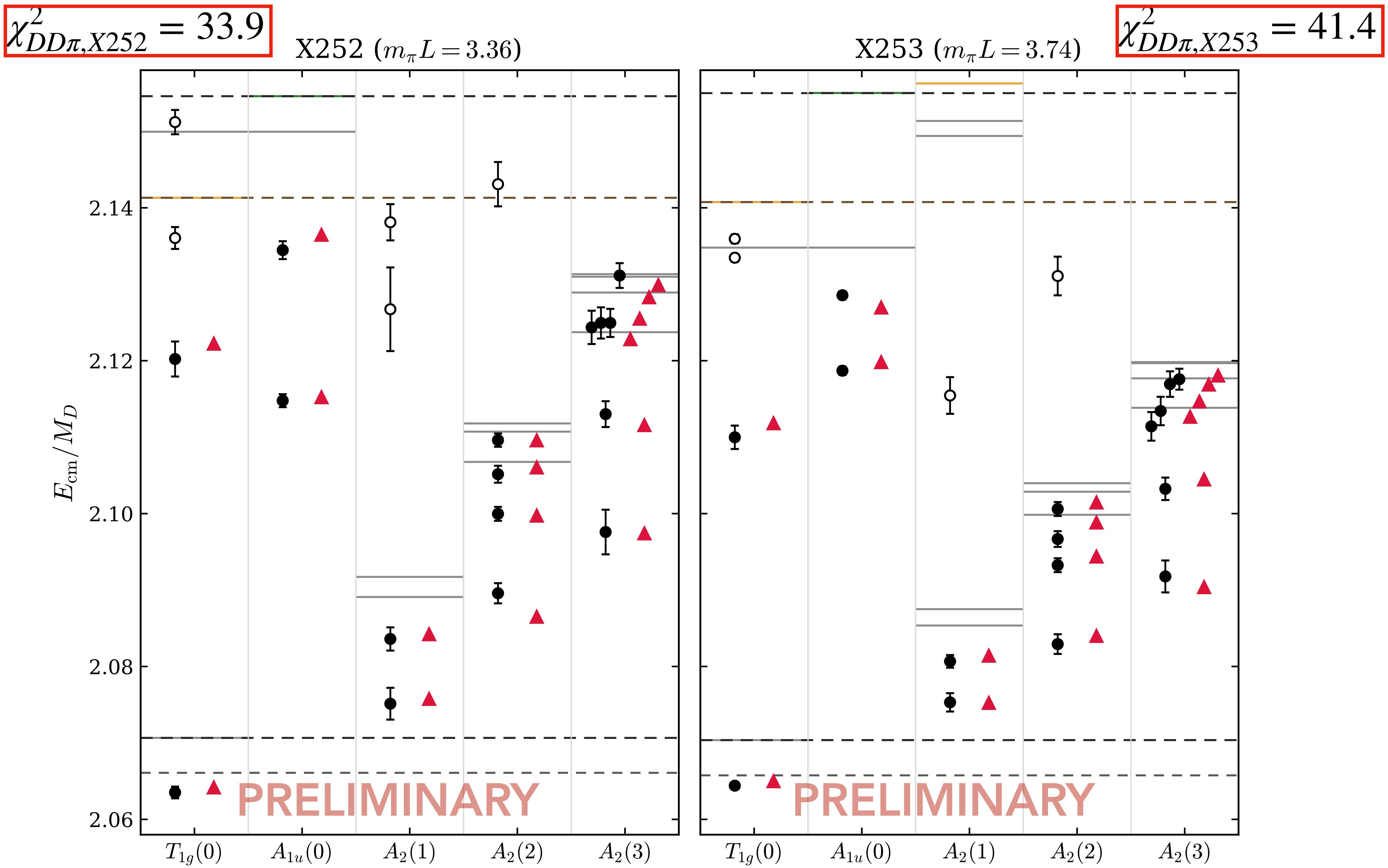
$$\underbrace{c_{\text{iso},2}\Delta^2 + c_B'\Delta(\Delta_3 + \Delta_3') + c_E'\Delta t_{33} + c_{E^2}(t_{33})^2 + \sum_{i=5}^{11} c_i \mathcal{O}_i^{(2)}} + \cdots$$

$\mathcal{O}(\Delta^2)$: includes s, p & d waves

- At $\mathcal{O}(\Delta^1)$, only c_E term couples to DD^* in T_{cc}^+ channel ($T_{1g}(0)$ irrep)
    - We fit with all $\mathcal{O}(\Delta^0)$ and $\mathcal{O}(\Delta^1)$ terms + c_E' term

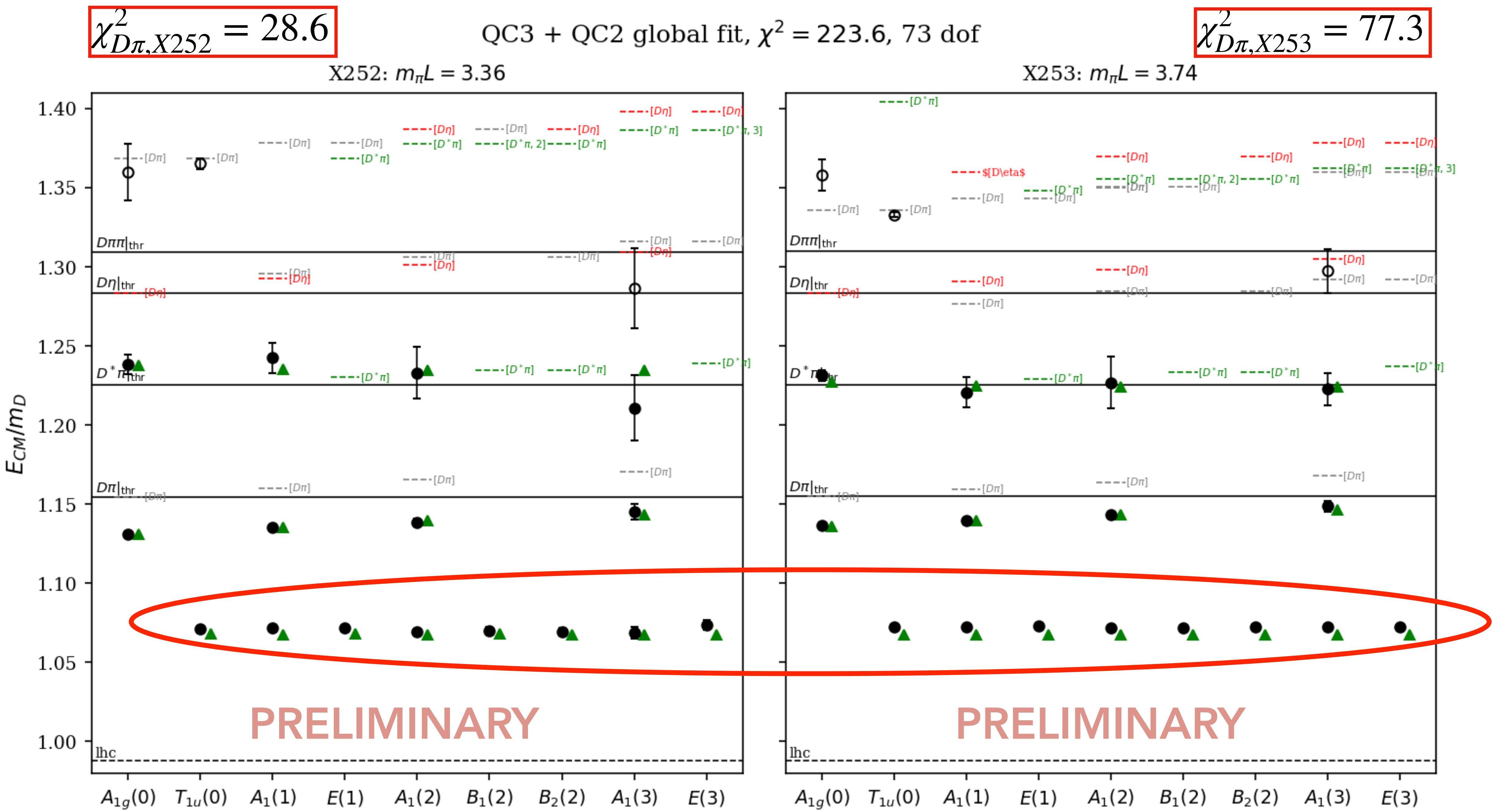
$DD\pi + D\pi + DD$ fit

- Fit result for $DD\pi/DD^*$ levels: reproduces levels below u-channel LHC reasonably well



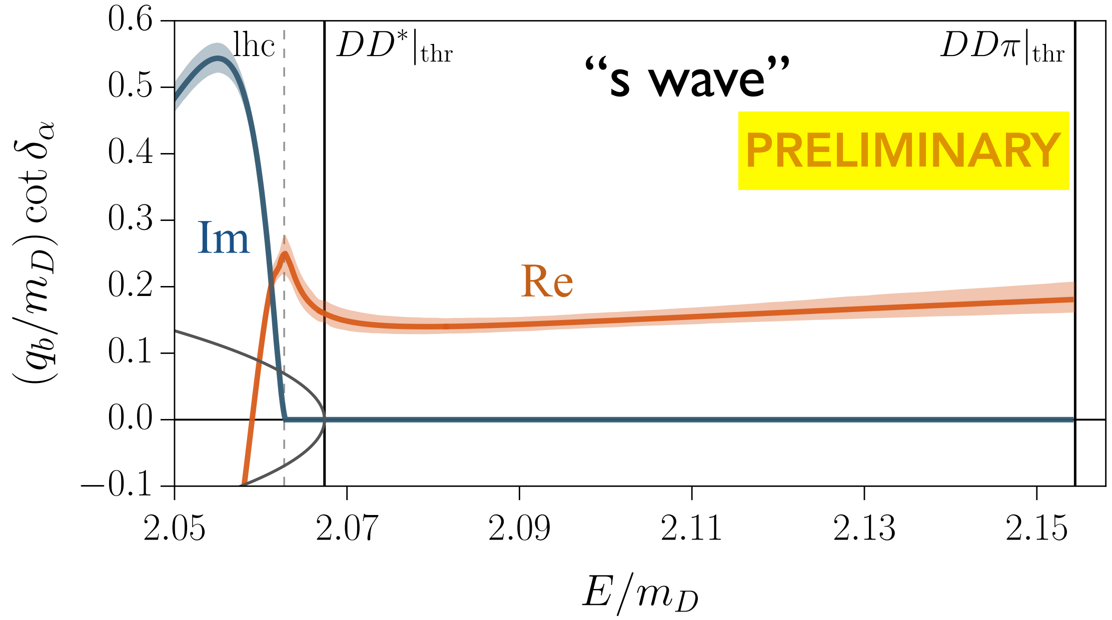
$DD\pi + D\pi + DD$ fit

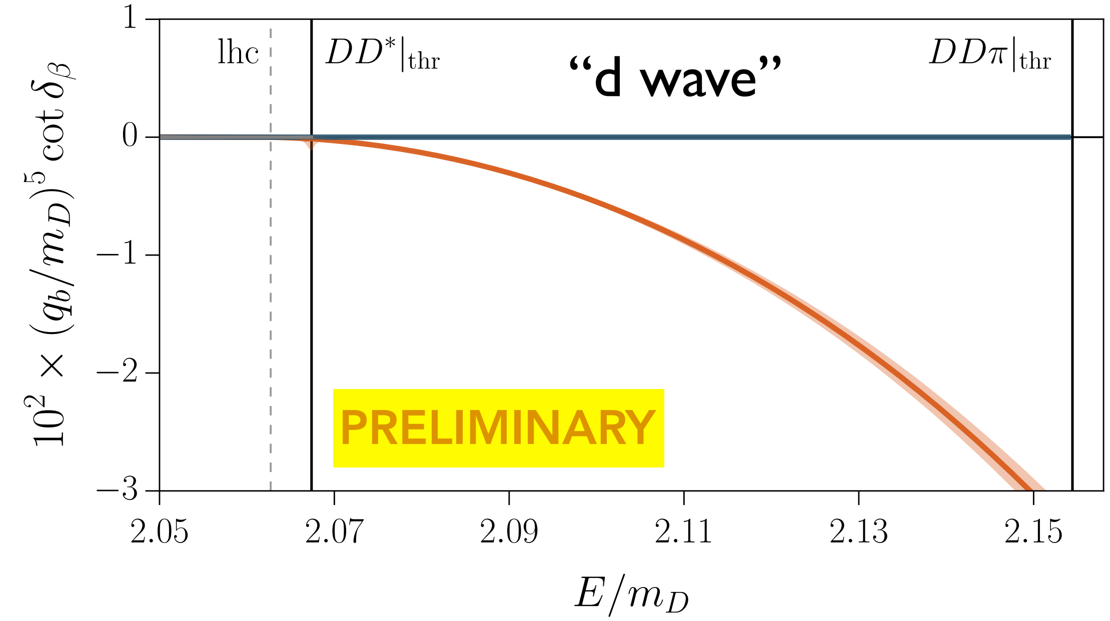
- Fit result for $D\pi$ levels: significantly undershoots the D^* states

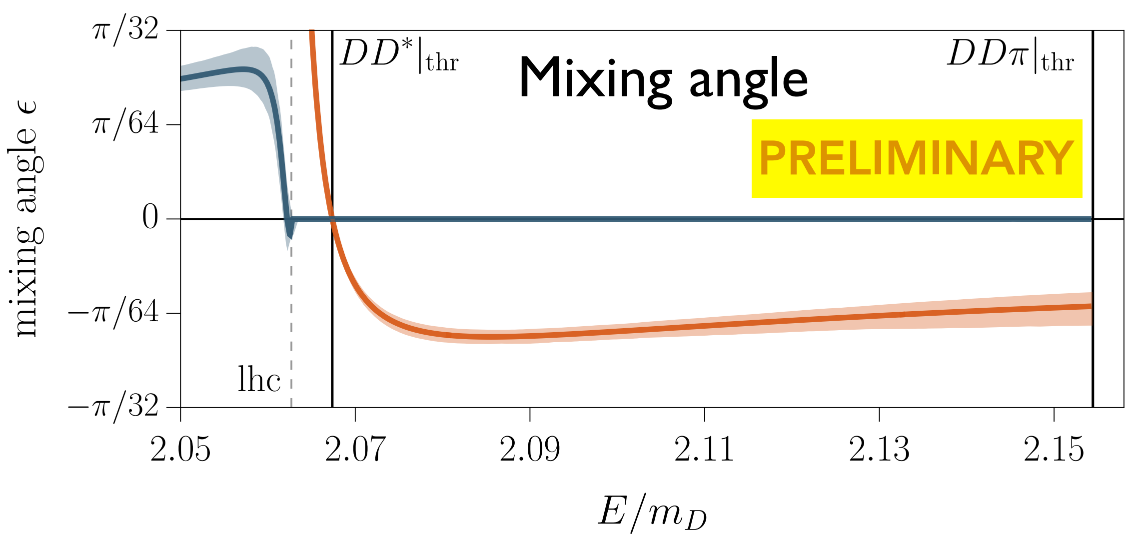


DD^* amplitude

- Given $\mathscr{K}_2$ and $\mathscr{K}_{\mathrm{df},3}$, can solve integral equations to obtain DD^* amplitude using LSZ
  - Using results from (imperfect) global fit as an example





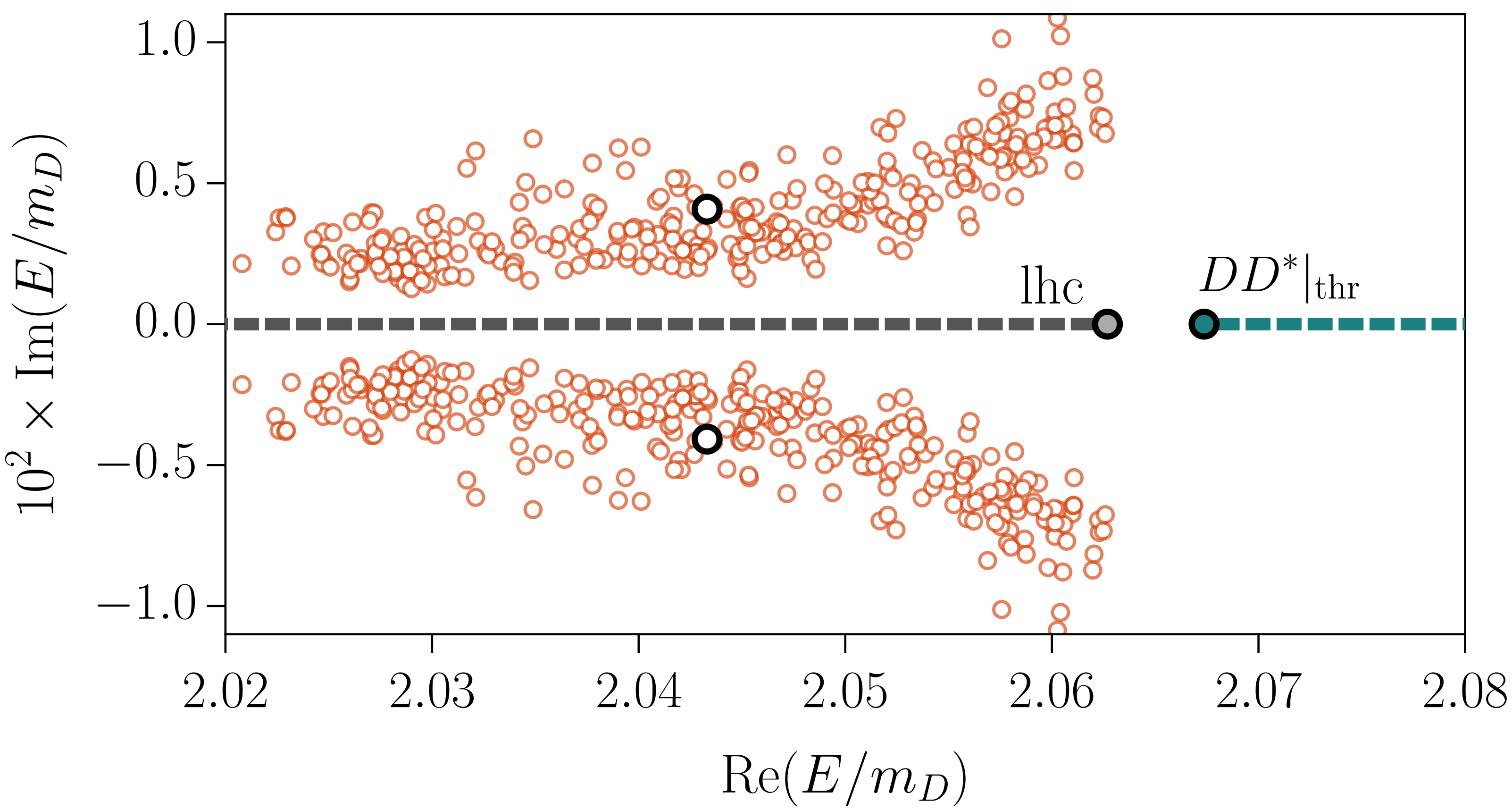


- Both s & d waves combine with spin of D^* to give $J^P = 1^+$
- Use Blatt-Biedenharn parametrization of K matrix

$$(8\pi E)\left[\mathcal{K}^{-1}_{DD^*}\right] = \begin{pmatrix} \cos(\epsilon) & -\sin(\epsilon) \\ \sin(\epsilon) & \cos(\epsilon) \end{pmatrix} \begin{pmatrix} q_b\cot(\delta_\alpha) & 0 \\ 0 & q_b\cot(\delta_\beta) \end{pmatrix} \begin{pmatrix} \cos(\epsilon) & \sin(\epsilon) \\ -\sin(\epsilon) & \cos(\epsilon) \end{pmatrix}$$

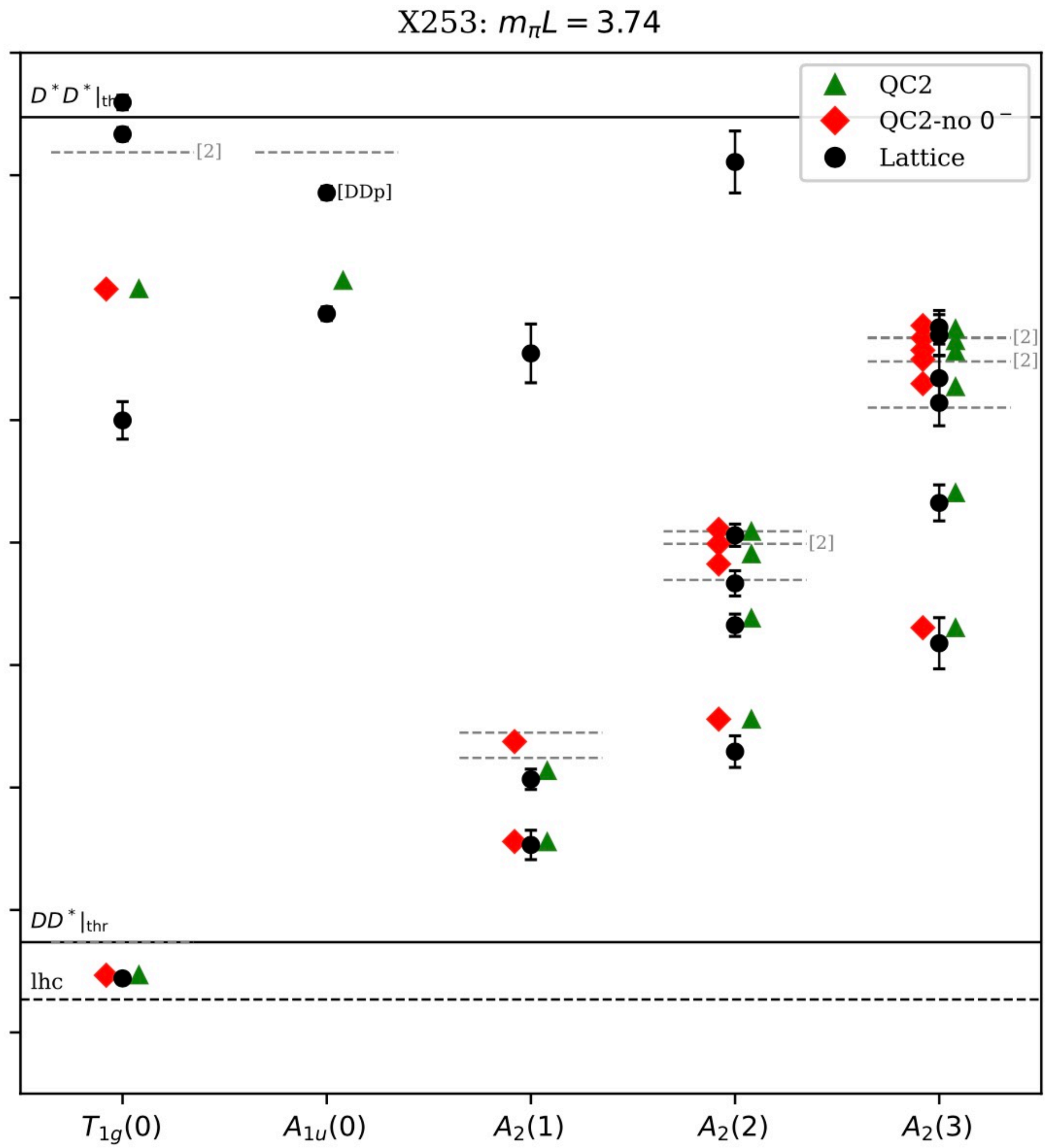
DD^* amplitude

- T_{cc}^+ appears as subthreshold resonance pole
    - Use 300 samples drawn from correlation matrix for K-matrix parameters from global fit



QC2 predictions

- Using the DD^* interactions from the integral equations (including also p waves), can solve the QC2
    - Should (roughly) reproduce the levels that are unaffected by 3-particle effects
        - Mismatch in M_{D^*} between global fit and direct extraction lifts lowest T_{1g} state above lhc



- Agreement is mediocre
- Need higher DD^* waves to match the moving A_2 levels

Conclusions & Outlook

- We are undertaking a comprehensive study of the T_{cc}^{+} channel including three-body effects
    - Inclusion of multiple moving frames and two volumes provides more constraints on the DD^* interaction
    - Good fits to two-particle subsystems, but combined fits are challenging
    - Likely need to include more terms in $\mathscr{K}_{\mathrm{df},3}$, so as to have more general DD^* interactions
        - Machinery to include all $11\ \mathcal{O}(\Delta^2)$ operators is in place
- Integral equation machinery in place awaiting successful FV fits
- Future plans
    - Calculations at lower M_π (where 3-body effects will be even larger)
    - Analyze data with other approaches (modified QC2 [Baiao Raposo & Hansen], plane wave approach, asymmetric QC3, …)
    - Include D^*D^* in formalism (required 4-body formalism)

Thanks!
Any questions?

Backup slides

$DD(I=1)$ subchannel

- Fit 16 levels on X252 + X253
- An s-wave Lüscher analysis on DD levels only gives a good fit

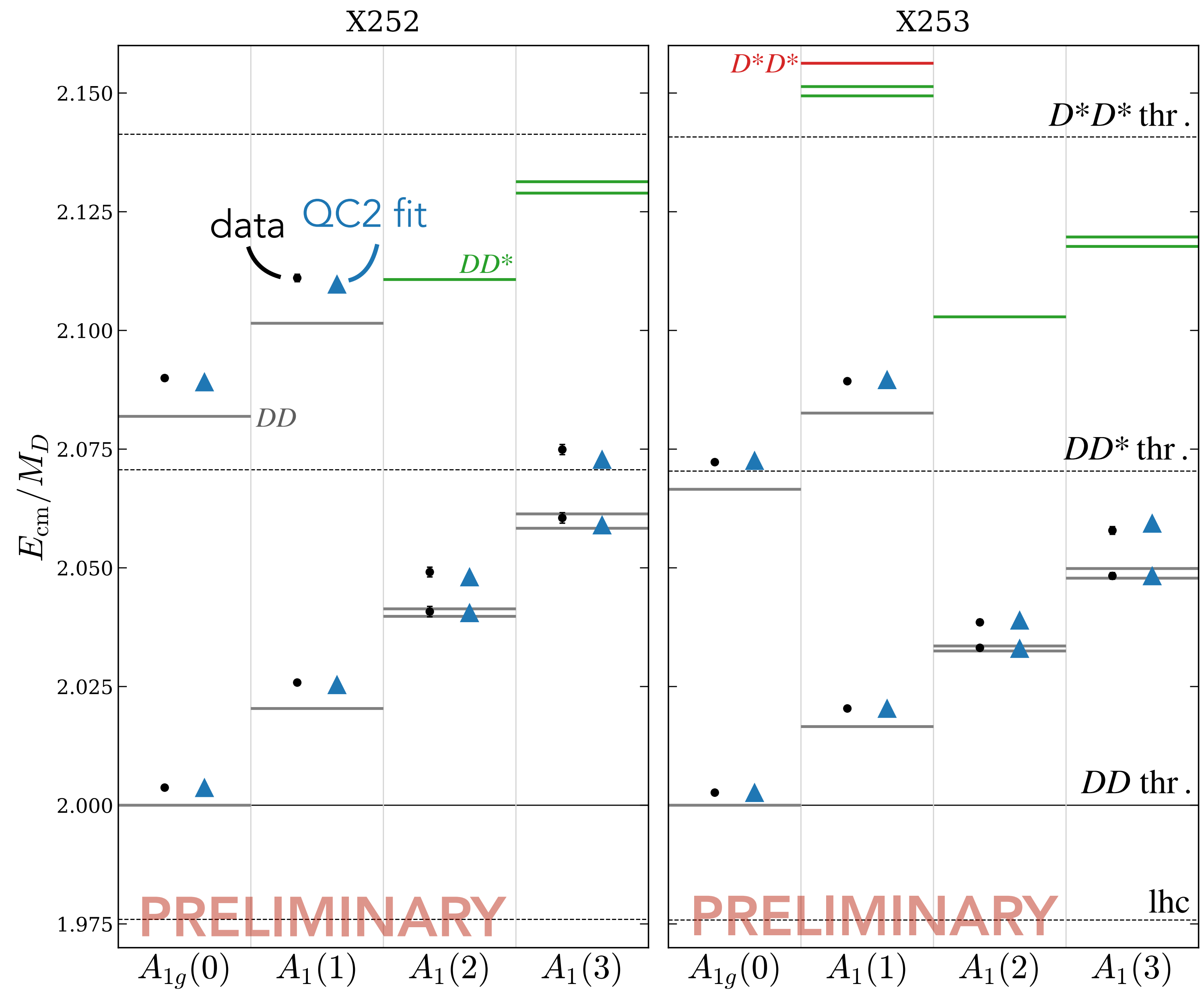
$$\frac{q}{M_D}\cot\delta_{DD,s} = C_1 + C_2\left(\frac{q}{M_D}\right)^2 + C_3\left(\frac{q}{M_D}\right)^4$$

$$C_1 = -0.437(12),\ C_2 = -4.48(85),\ C_3 = -14.2(9.8)$$

$$\chi^2/\mathrm{dof} = 13.02/13$$

- As expected, weakly repulsive interaction

$$a_0 = 2.29(6)/M_D\,,\quad r_0 = 8.9(1.7)M_D$$



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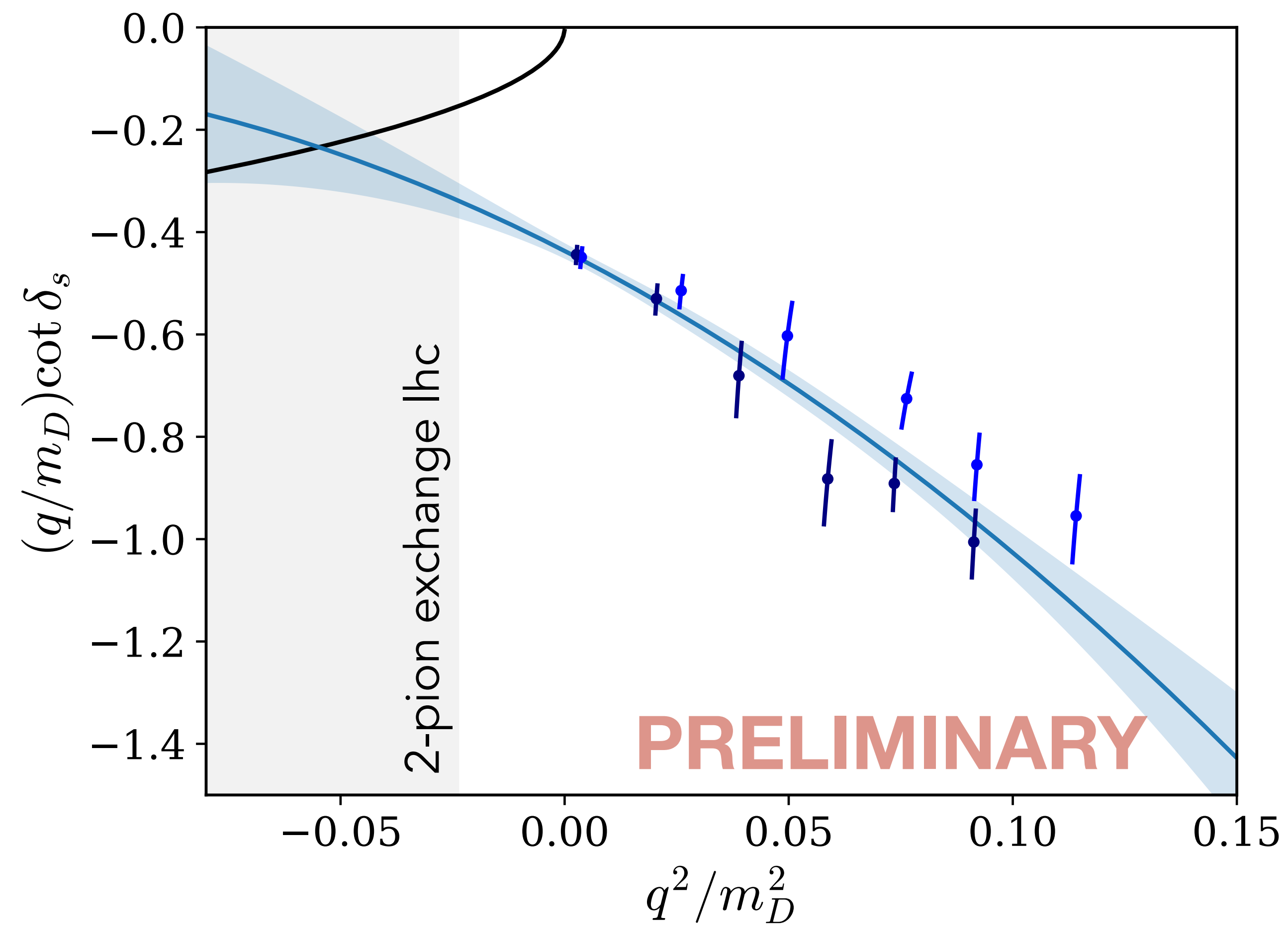
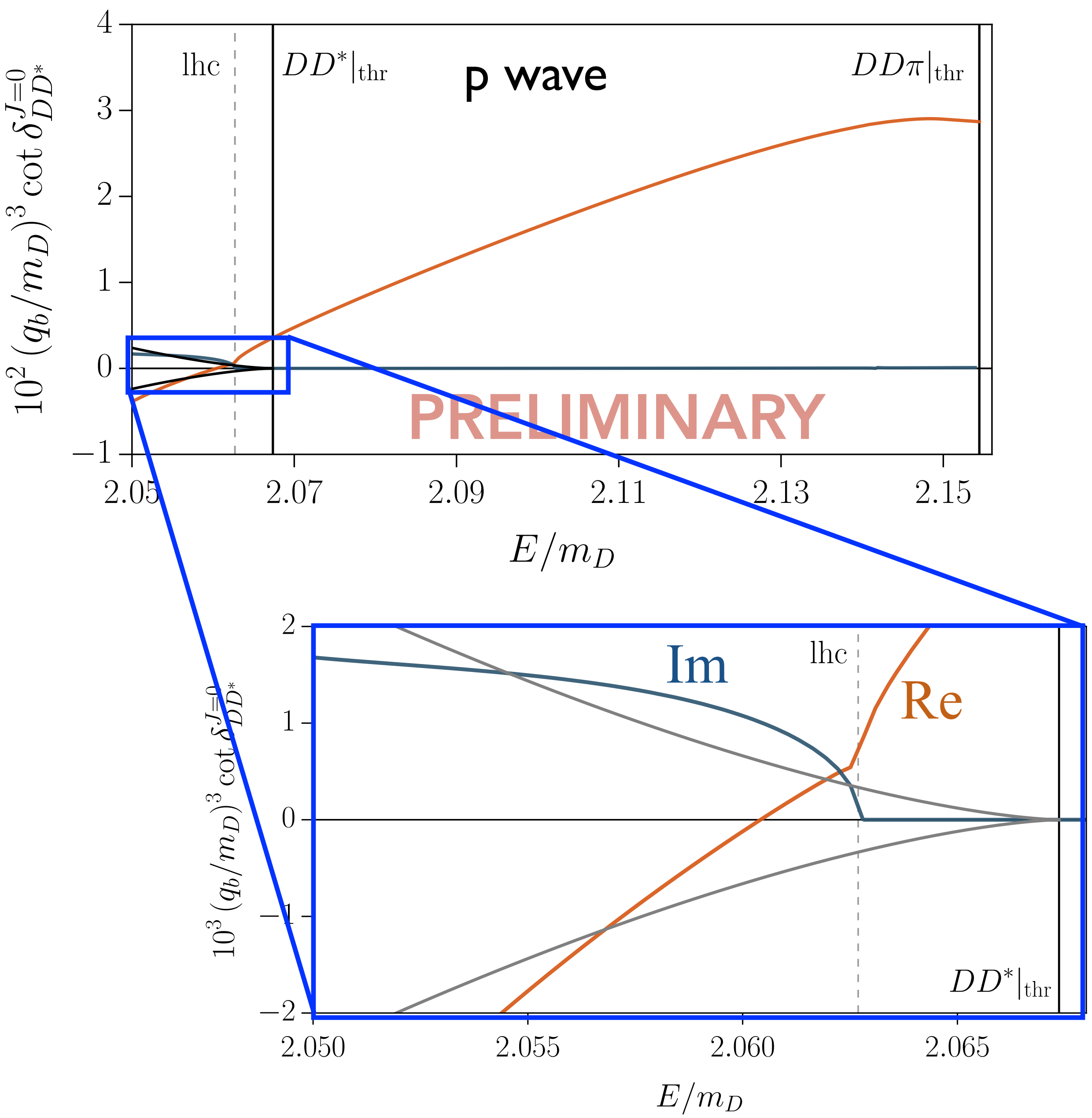


DD^* amplitude: p wave



Full $DD^*/DD\pi$ spectrum