

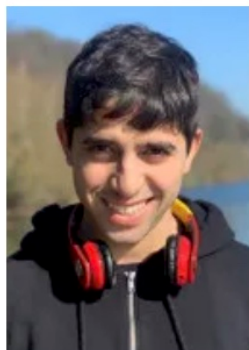
Update on lattice study of $T_{cc}^+(3875)$ using three-body formalism



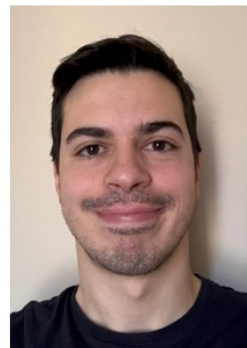
Steve Sharpe
University of Washington



In collaboration with:



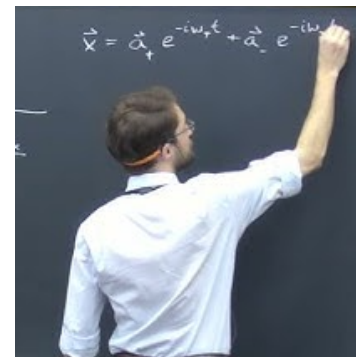
Herzallah Alharazin (Bochum)



André Baião-Raposo (Bochum)



John Bulava (Bochum)



Sebastian Dawid (Indiana)



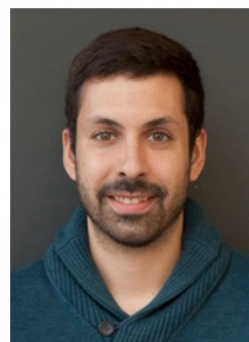
Jeremy Green (DESY)



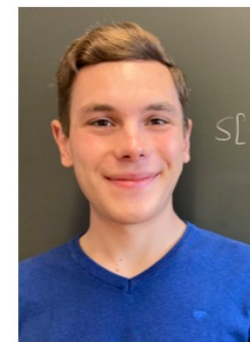
Colin Morningstar (CMU)



Davide Laudicina (Bochum)



Fernando Romero-López (U. Bern)



Miguel Salg (U. Bern)

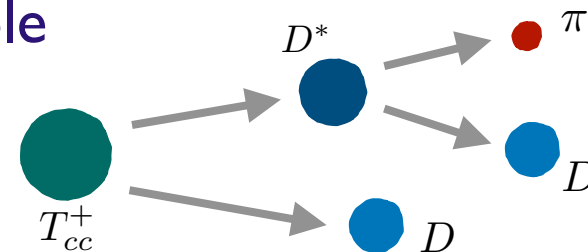


Andres Stump (Humboldt U.)

Introduction

- T_{cc}^+ is an exotic resonance ($cc\bar{u}\bar{d}$) accessible to LQCD (no annihilation contractions)
 - Structure is unclear: DD^* molecule, diquark-antidiquark, compact tetraquark?

- Three-body dynamics plays a crucial role



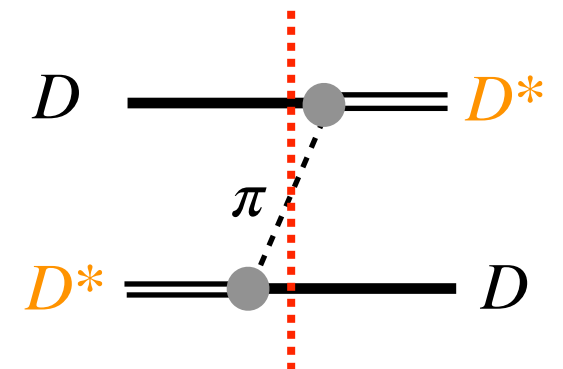
- In nature, decay is to $DD\pi$

- For heavier-than-physical light quarks, thresholds are interchanged

- T_{cc}^+ can be studied in DD^* system

- However, three-body effects enter through left-hand cut

- States relevant for T_{cc}^+ cannot be analyzed using standard Lüscher method



- Need a method that accounts for left-hand cut

- We use the RFT three-particle formalism of [Hansen, Romero-López, SRS, 2024]
- While perhaps more cumbersome than two-body alternatives, it remains valid for physical quark masses, and accounts for nearby $DD\pi$ states

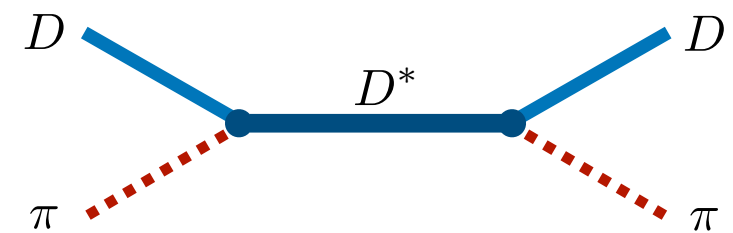
Overview of formalism

- Two 2-particle sub-channels to consider,

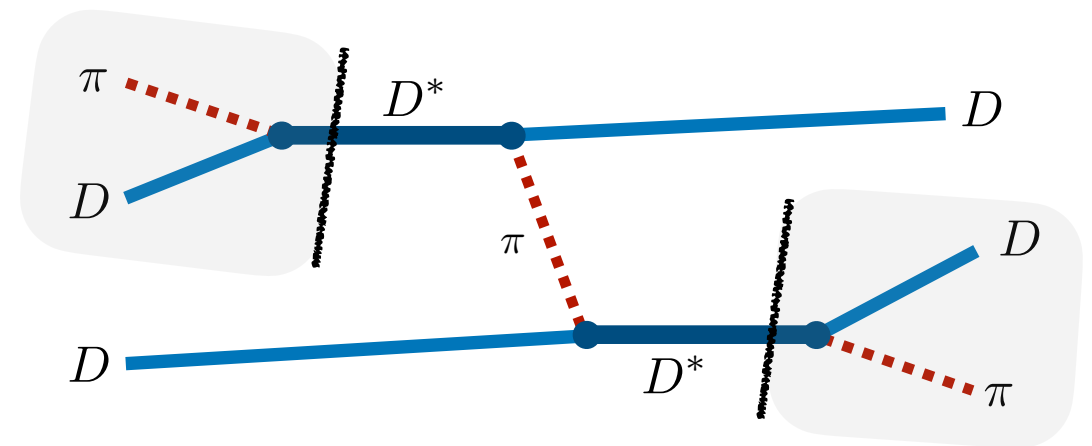
DD ($I = 1$), $D\pi$ ($I = 1/2$)

$$DD\pi \ (I = 0) \left[\begin{array}{c} D \text{ ————— } \\ D \text{ ————— } \\ \pi \text{ } \end{array} \right] \left[\begin{array}{l} DD \ (I = 1) \\ D\pi \ (I = 1/2) \end{array} \right]$$

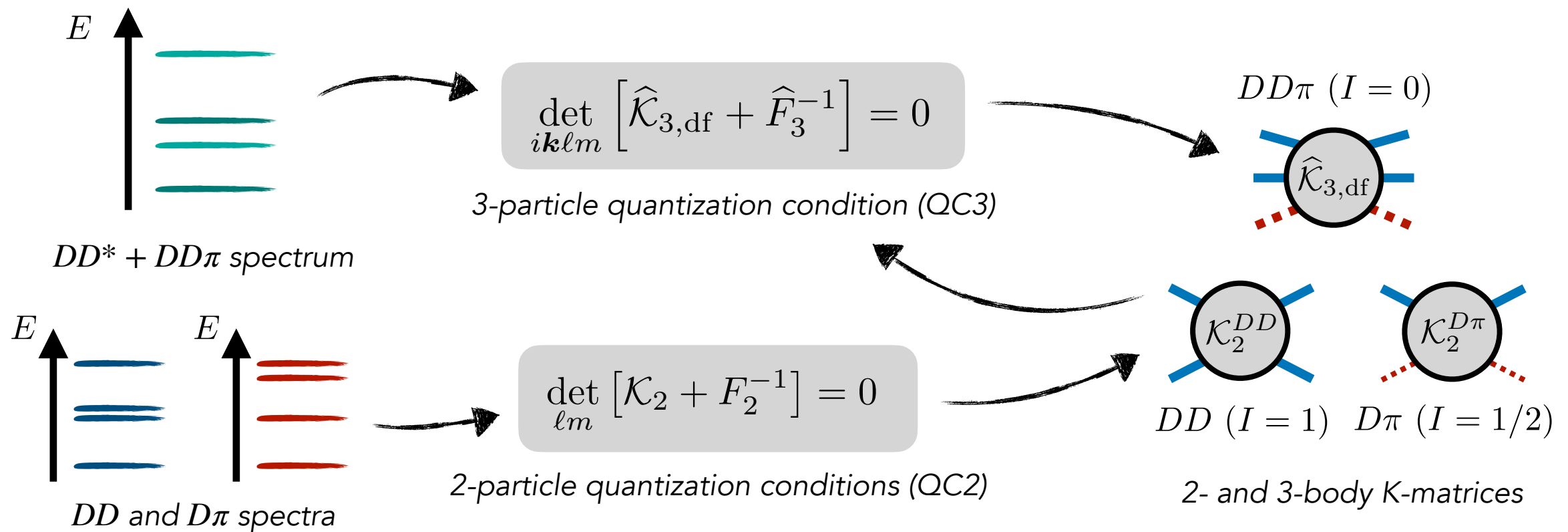
- D^* included as a **pole in the p -wave $D\pi$ amplitude** (as a bound state)



- For stable D^* , DD^* amplitude accessed by **LSZ-reducing $DD\pi$ amplitude**



Workflow: spectrum analysis



Simultaneous fits of finite-volume spectra of two-body subsystems and three-body main system allow us to constrain infinite-volume K-matrices...

$$\det_{i\mathbf{k}\ell m} [\hat{\mathcal{K}}_{3,\text{df}} + \hat{F}_3^{-1}] = 0 \quad \hat{F}_3 = \frac{\hat{F}}{3} - \hat{F} \frac{1}{1 + \hat{\mathcal{M}}_{2,L} \hat{G}} \hat{\mathcal{M}}_{2,L} \hat{F}$$

$$\hat{\mathcal{M}}_{2,L} = \frac{1}{\hat{\mathcal{K}}_{2,L}^{-1} + \hat{F}} \quad \hat{\mathcal{K}}_{2,L} = \begin{pmatrix} \mathcal{K}_{2,L}^{D\pi} & 0 \\ 0 & \frac{1}{2} \mathcal{K}_{2,L}^{DD} \end{pmatrix}$$

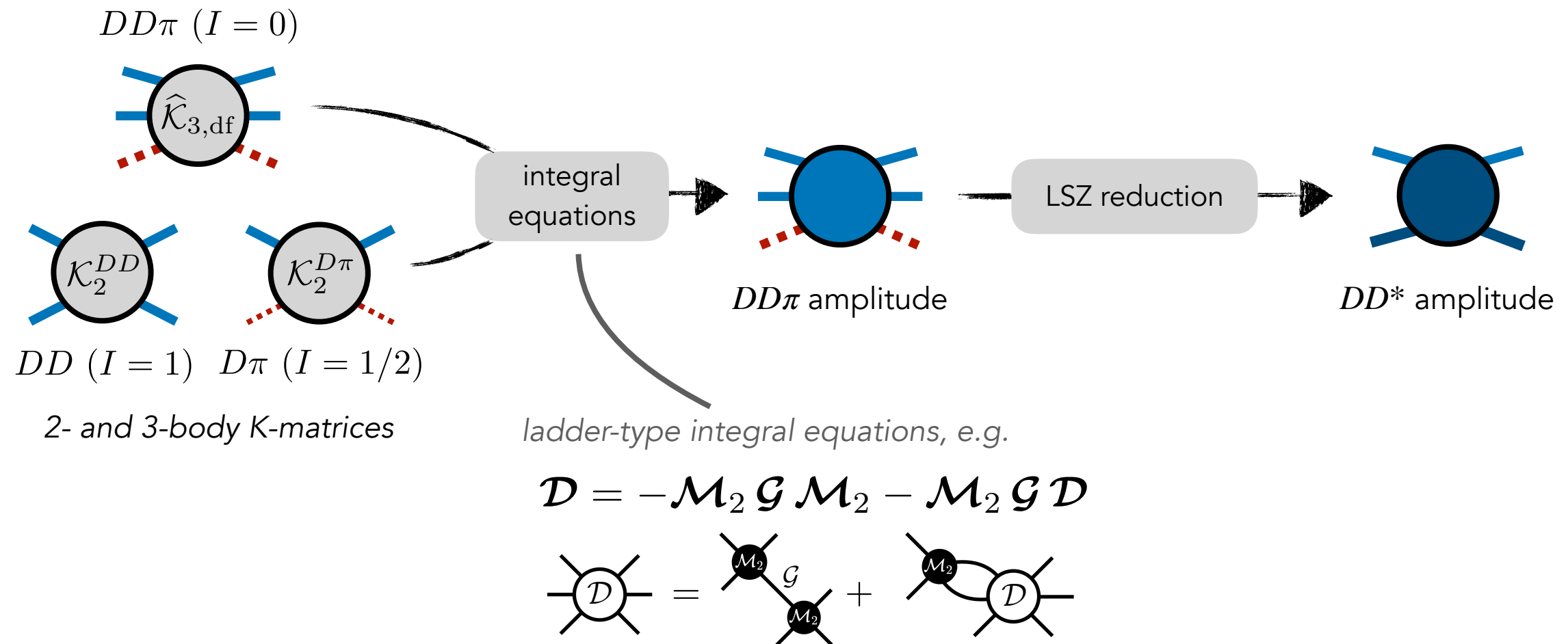
QC3 involves matrices in large index space:

i — spectator flavor

$\mathbf{k}$ — spectator 3-momentum

ℓm — 2-pt subsystem angular momentum

Workflow: integral equations



... we feed the K-matrices through **integral equations for $DD\pi$ amplitude**, **LSZ-reduce** and **analytically continue** to obtain DD^* amplitude and find T_{cc}^+ pole

Simulation details

- Use two CLS ensembles with same pion mass but different volume

	L/a	T/a	$m_\pi L$	$m_\pi [\text{MeV}]$	$a [\text{fm}]$	$M_D [\text{MeV}]$	$(M_{D^*} - M_D) [\text{MeV}]$	Configs
X252	36	128	3.4	290	0.063	1880	130	175
X253	40	128	3.7	290	0.063	1880	130	301

Physical values:

135

1870

140-145

- Use frames (0,0,0), (0,0,1), (1,1,0), (1,1,1) and irreps coupling dominantly to $J^P = 1^+, 0^-$
- Use stochastic LapH method, with multiple operators, and GEVP

- Types of operator:

DD in $I = 1$: $DD, DD^*, D^*D^*, D_0^*D_0^*$ (D_0^* is a scalar, $D_0^*(2300)$)

$D\pi$ in $I = 1/2$: $D\pi, D^*, D^*\pi, D_0^*$

$DD\pi / DD^*$ in $I = 0$: bilocal/trilocal operators $DD^*, DD\pi, D^*D^*, D_0^*D, D_0^*D^*$

local tetraquark operators e.g. $T_{DD^*}^i(t, \mathbf{x}) = (\bar{u}\gamma^i c \bar{d}\gamma^5 c - \bar{d}\gamma^i c \bar{u}\gamma^5 c)(t, \mathbf{x})$

All results are PRELIMINARY

$D\pi$ subchannel

- Simultaneous fit to 16 X252 and 16 X253 levels with s- and p-wave ERE2 forms
- $D\eta$ & $D^*\pi$ channels not included in QC2, but lie well above levels kept in fit

Good fit: $\chi^2 = 23.15 / 29$ dof

E.g. X253 $m_\pi L = 3.745$

$$q \cot \delta_s = \frac{r_{0,s}}{2}(q^2 - q_{0,s}^2)$$

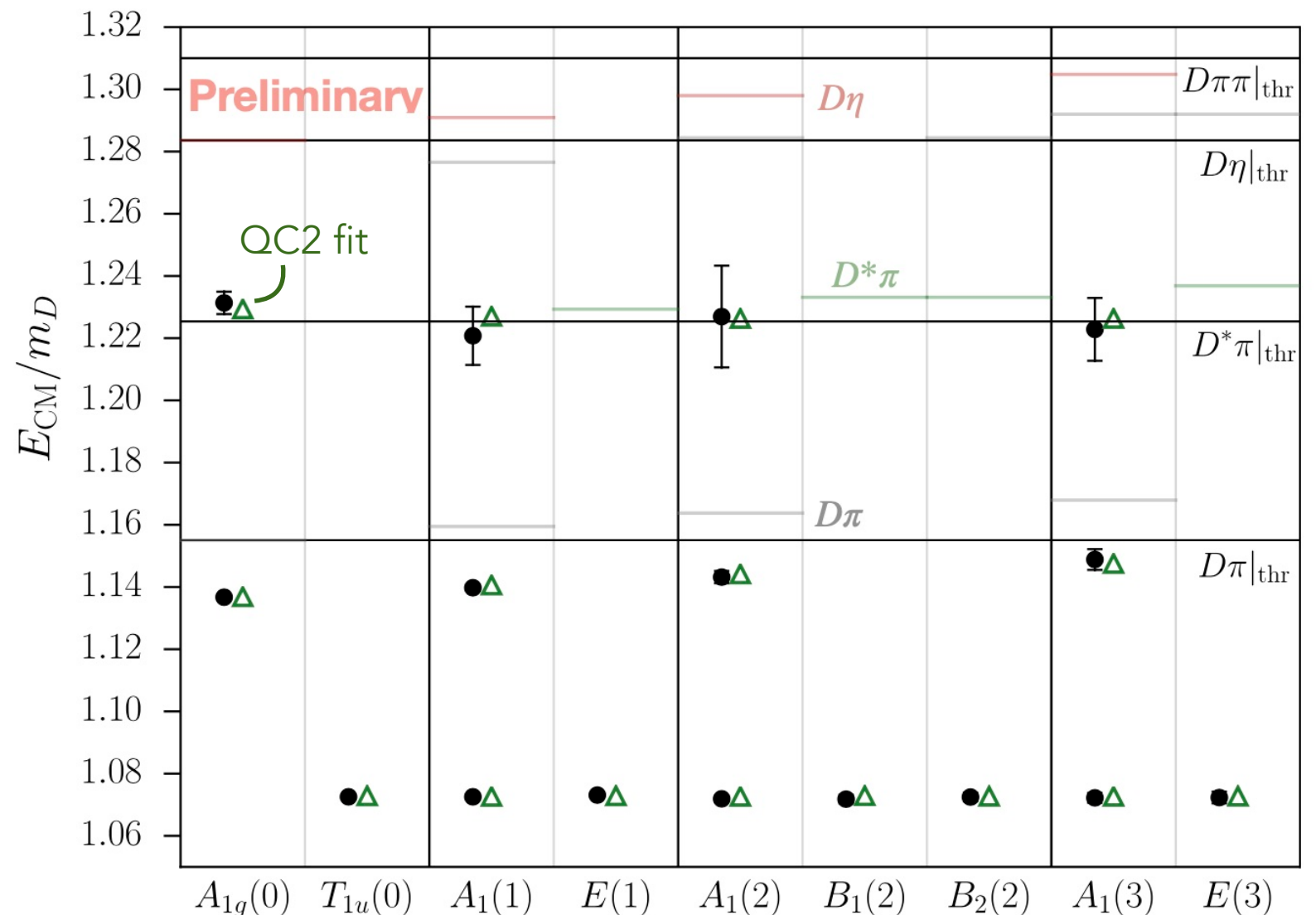
$$q \cot \delta_p = \frac{r_{0,p}}{2}(q^2 - q_{0,p}^2)$$

HQEFT $DD\pi$ coupling g and D^* mass used
as p -wave fit parameters

Constrained $g = 0.55 \pm 0.1$ based on
experimental value (0.57) and lattice results

$$q_{0,s}^2 = 0.0084(14), r_{0,s} = -8.42(72),$$

$$M_{D^*} = 1.07195(56), g = 0.525(94)$$



$D\pi$ subchannel

- Simultaneous fit to 16 X252 and 16 X253 levels with s- and p-wave ERE2 forms
- $D\eta$ & $D^*\pi$ channels not included in QC2, but lie well above levels kept in fit

Good fit: $\chi^2 = 23.15 / 29$ dof

E.g. X253 $m_\pi L = 3.745$

$$q \cot \delta_s = \frac{r_{0,s}}{2}(q^2 - q_{0,s}^2)$$

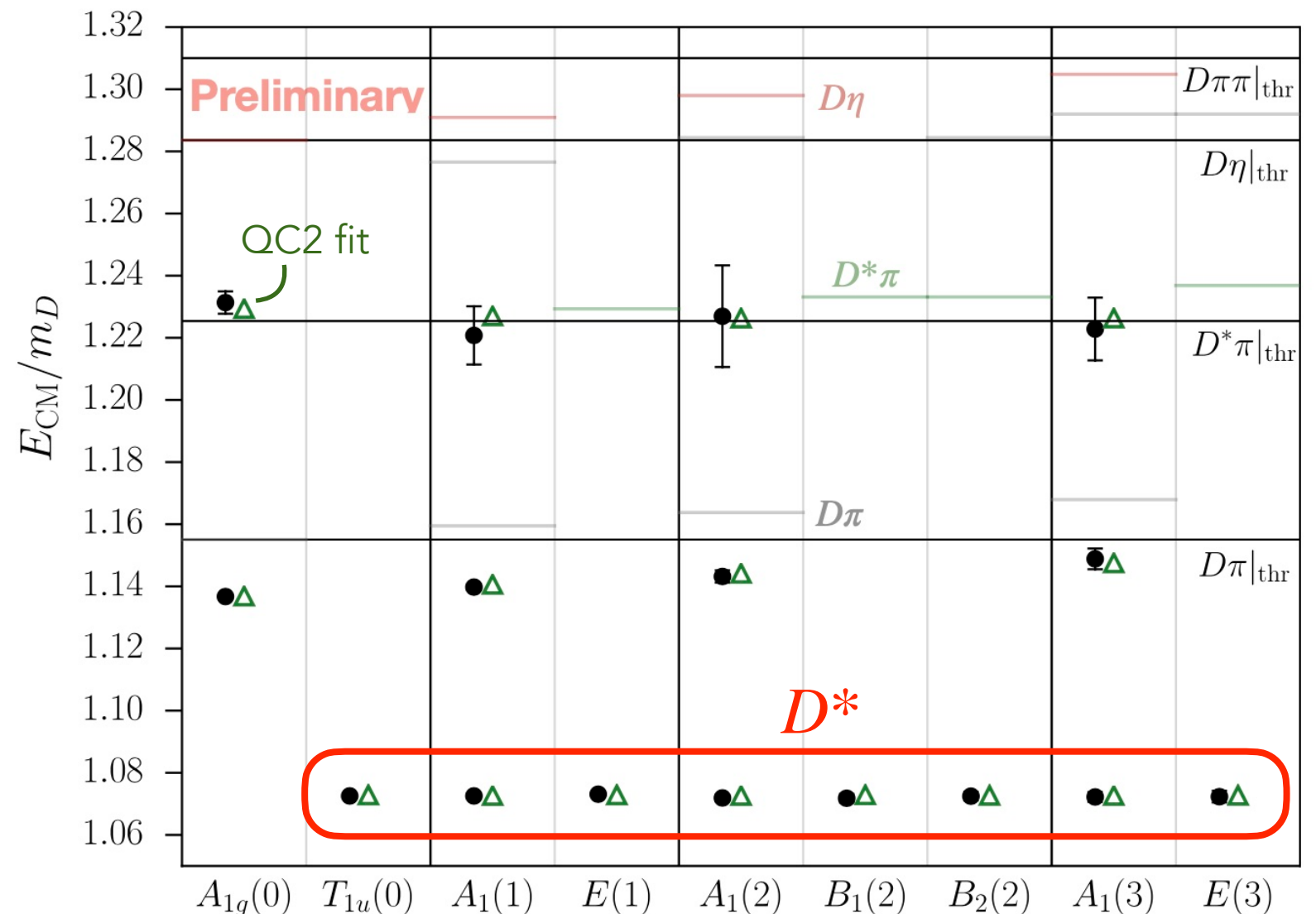
$$q \cot \delta_p = \frac{r_{0,p}}{2}(q^2 - q_{0,p}^2)$$

HQEFT $DD\pi$ coupling g and D^* mass used
as p -wave fit parameters

Constrained $g = 0.55 \pm 0.1$ based on
experimental value (0.57) and lattice results

$$q_{0,s}^2 = 0.0084(14), r_{0,s} = -8.42(72),$$

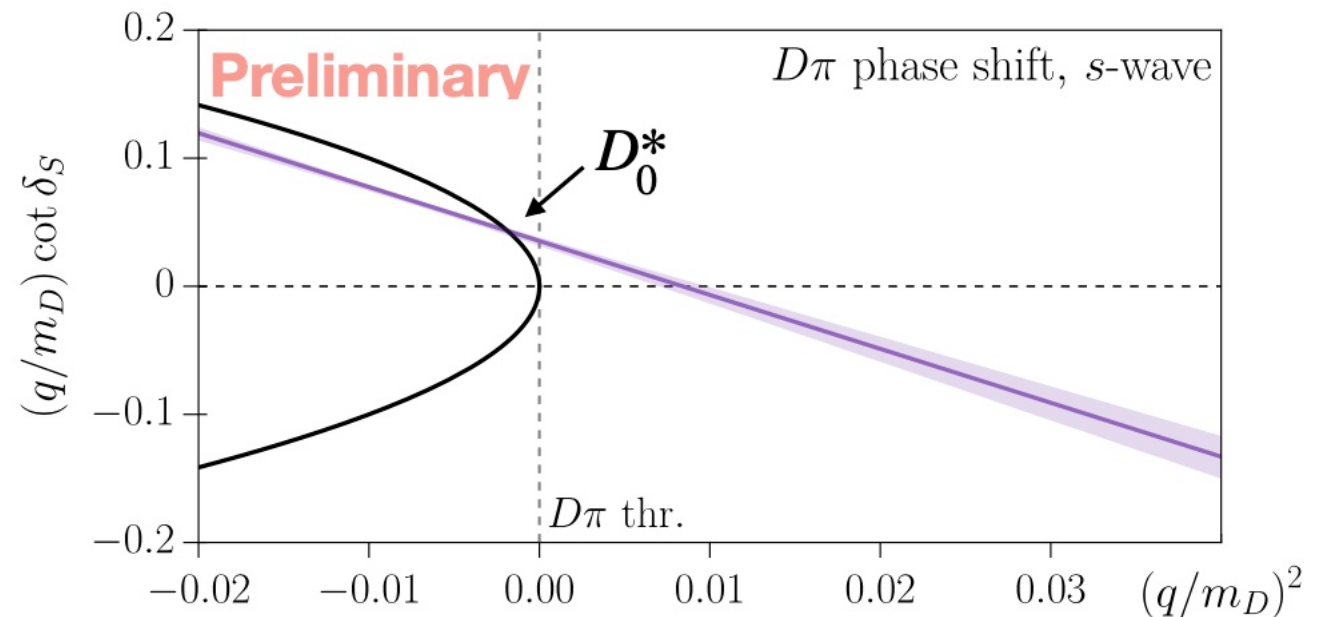
$$M_{D^*} = 1.07195(56), g = 0.525(94)$$



$D\pi$ subchannel

$$q \cot \delta_s = \frac{r_{0,s}}{2}(q^2 - q_{0,s}^2)$$

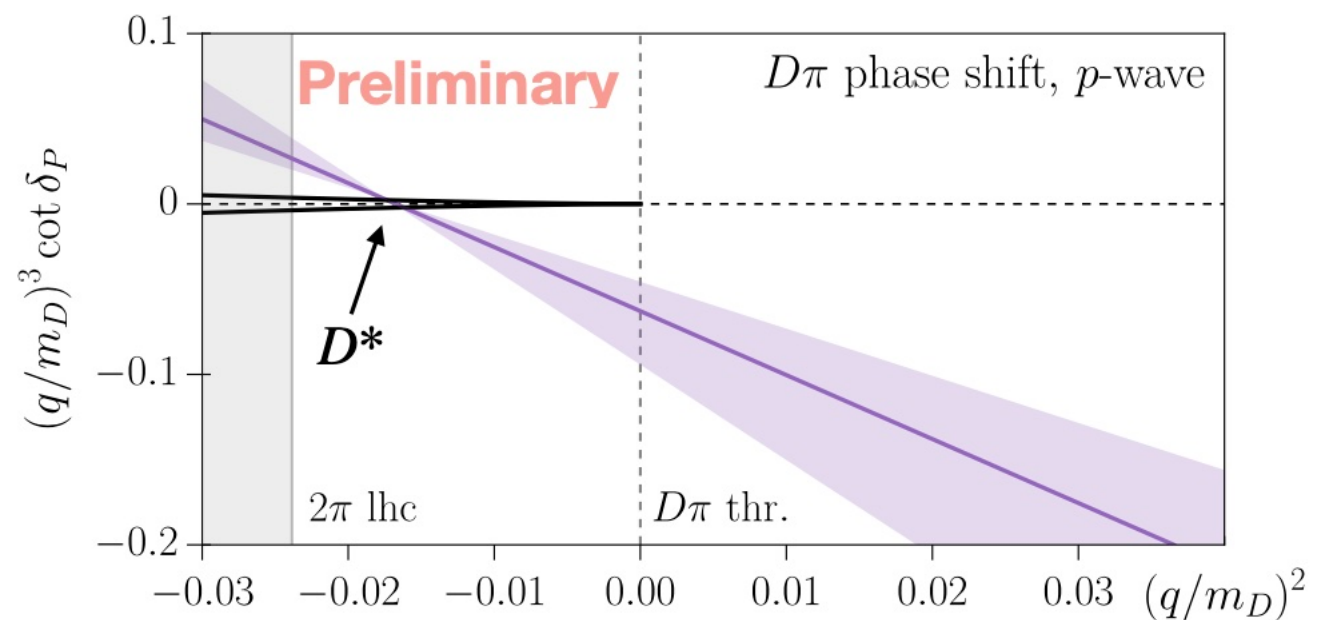
$$q_{0,s}^2 = 0.0084(14), \quad r_{0,s} = -8.42(72),$$



$$q \cot \delta_p = \frac{r_{0,p}}{2}(q^2 - q_{0,p}^2)$$

$$M_{D^*} = 1.07195(56), \quad g = 0.525(94)$$

Largely determined by constraint rather than data

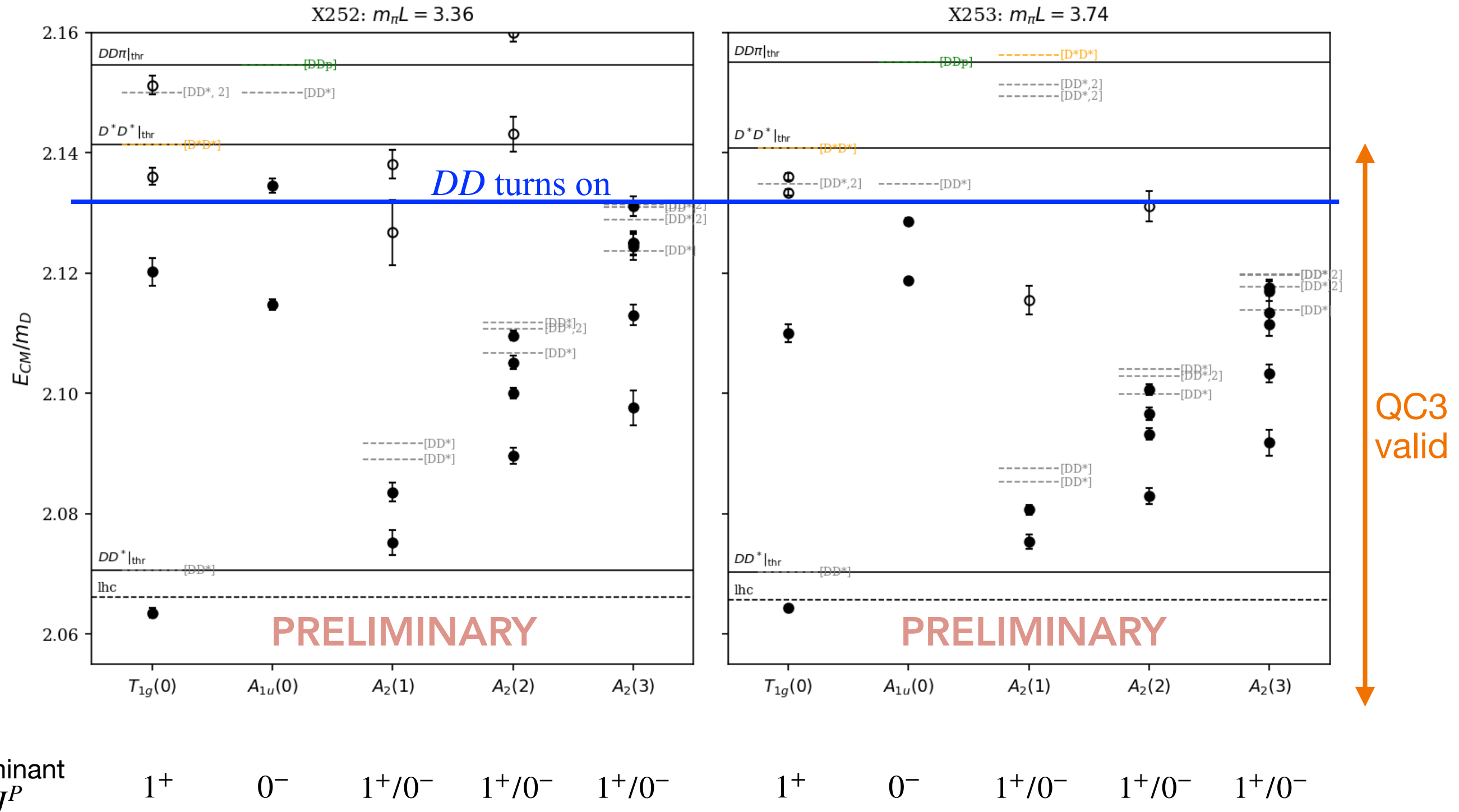


$I = 0$ $DD\pi$ channel

- Mixes with D^*D^* , $DD\pi\pi$, $DD_s\bar{K}$, and, in some moving irreps DD
 - QC3 only valid if such mixing is absent
 - Requires $E_{\text{CM}} < 2M_{D^*} < 2M_D + M_\pi$, so all levels in fit are below $DD\pi$ threshold
 - In rest frame use T_{1g} (s/d-wave DD^* — T_{cc}^+ channel) and A_{1u} (p-wave)—no mixing with DD
 - In moving frames use A_2 irreps—all waves contribute, mixing with DD possible, but requires large relative angular momentum, and free DD levels far above $DD\pi$ threshold
- QC3 remains valid below u-channel DD^* one-pion-exchange left-hand cut at
$$E_{\text{LHC}}^{(\pi)} = \sqrt{2M_D^2 + 2M_{D^*}^2 - M_\pi^2}$$
- QC3 fails below u-channel two-pion-exchange LHC at $E_{\text{LHC}}^{(\pi\pi,u)} = \sqrt{2M_D^2 + 2M_{D^*}^2 - 4M_\pi^2}$

$DD\pi$ spectra

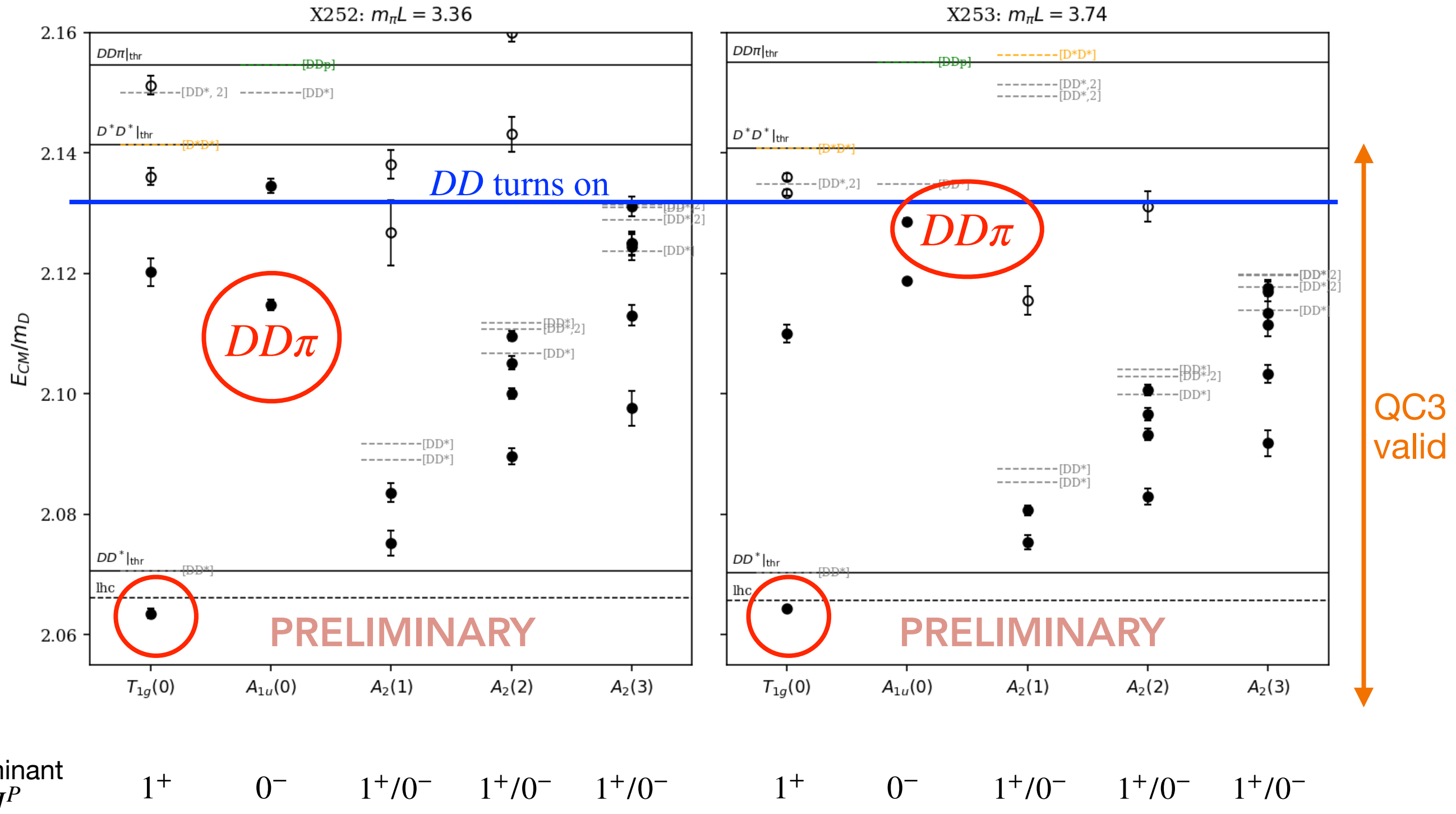
Keep 16+16 levels; in some fits add 2+2 higher $A_{1u}(0)$ levels



$DD\pi$ spectra

3-particle effects crucial

Keep 16+16 levels; in some fits add 2+2 higher $A_{1u}(0)$ levels



$DD\pi + D\pi + DD$ fit

- Tried many combinations of levels and parameters
 - Best global fit so far fits 84 levels (18 $DD\pi$, 16 $D\pi$, 8 DD on each ensemble)
 - 12 parameters (5 for $DD\pi$, 4 for $D\pi$, 3 DD), constraint $g = 0.55 \pm 0.1 \Rightarrow 73$ dof

$$\chi^2 = 223.6, \chi_{\text{ref}}^2 = 3.06, (\text{Bayes} = 1.63)$$

$$\chi_{DD,X252}^2 = 7.7, \chi_{D\pi,X252}^2 = 28.6, \chi_{DD\pi,X252}^2 = 33.9$$

$$\chi_{DD,X253}^2 = 6.0, \chi_{D\pi,X253}^2 = 77.3, \chi_{DD\pi,X253}^2 = 41.4$$

$$DD \text{ ERE3: } B_0 = -0.435(11), \quad B_1 = -3.99(73), \quad B_2 = -20.9(8.8),$$

$$q_{0,s}^2 = 0.0073(12), \quad r_{0,s} = -8.76(70),$$

$D\pi$ s- & p-wave ERE2:

$$M_{D^*} = 1.06739(24), \quad g = 0.422(9),$$

$\mathcal{K}_{\text{df},3}$:

$$c_{\text{iso},0} = -1.35(19) \times 10^5, \quad c_{\text{iso},1} = 2.16(47) \times 10^6, \quad c_B = 9.3(1.2) \times 10^4,$$

$$c_E = 1.49(21) \times 10^5, \quad c'_E = 2.8(3.2) \times 10^5.$$

$DD\pi + D\pi + DD$ fit

- Tried many combinations of levels and parameters
 - Best global fit so far fits 84 levels (18 $DD\pi$, 16 $D\pi$, 8 DD on each ensemble)
 - 12 parameters (5 for $DD\pi$, 4 for $D\pi$, 3 DD), constraint $g = 0.55 \pm 0.1 \Rightarrow 73$ dof

$$\chi^2 = 223.6, \chi_{\text{ref}}^2 = 3.06, (\text{Bayes} = 1.63)$$

Poor fit!

$$\chi_{DD,X252}^2 = 7.7, \chi_{D\pi,X252}^2 = 28.6, \chi_{DD\pi,X252}^2 = 33.9$$

$$\chi_{DD,X253}^2 = 6.0, \chi_{D\pi,X253}^2 = 77.3, \chi_{DD\pi,X253}^2 = 41.4$$

Worst subsector

$$DD \text{ ERE3: } B_0 = -0.435(11), \quad B_1 = -3.99(73), \quad B_2 = -20.9(8.8),$$

$D\pi$ s- & p-wave ERE2:

$$q_{0,s}^2 = 0.0073(12), \quad r_{0,s} = -8.76(70),$$

$$M_{D^*} = 1.06739(24), \quad g = 0.422(9),$$

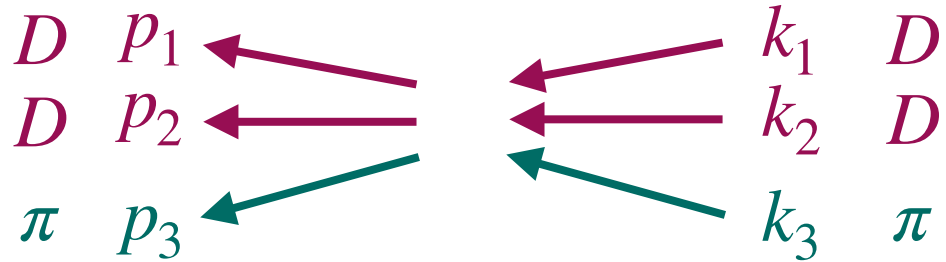
M_{D^*} much lower than in $D\pi$ fit

$\mathcal{K}_{\text{df},3}$:

$$c_{\text{iso},0} = -1.35(19) \times 10^5, \quad c_{\text{iso},1} = 2.16(47) \times 10^6, \quad c_B = 9.3(1.2) \times 10^4,$$

$$c_E = 1.49(21) \times 10^5, \quad c'_E = 2.8(3.2) \times 10^5.$$

$\mathcal{K}_{\text{df},3}$ threshold expansion



$$\Delta = \frac{s - s_{\text{th}}}{s_{\text{th}}}, \quad s_{\text{th}} = (2M_D + M_\pi)^2,$$

$$t_{ij} = \frac{(p_i - k_j)^2 - (M_i - M_j)^2}{s_{\text{th}}}$$

$$\Delta_i = \frac{(k_j + k_k)^2 - (M_j + M_k)^2}{s_{\text{th}}}, \quad \Delta'_i = \frac{(p_j + p_k)^2 - (M_j + M_k)^2}{s_{\text{th}}},$$

For our data: $(-0.01) \lesssim \Delta \lesssim 0.15$

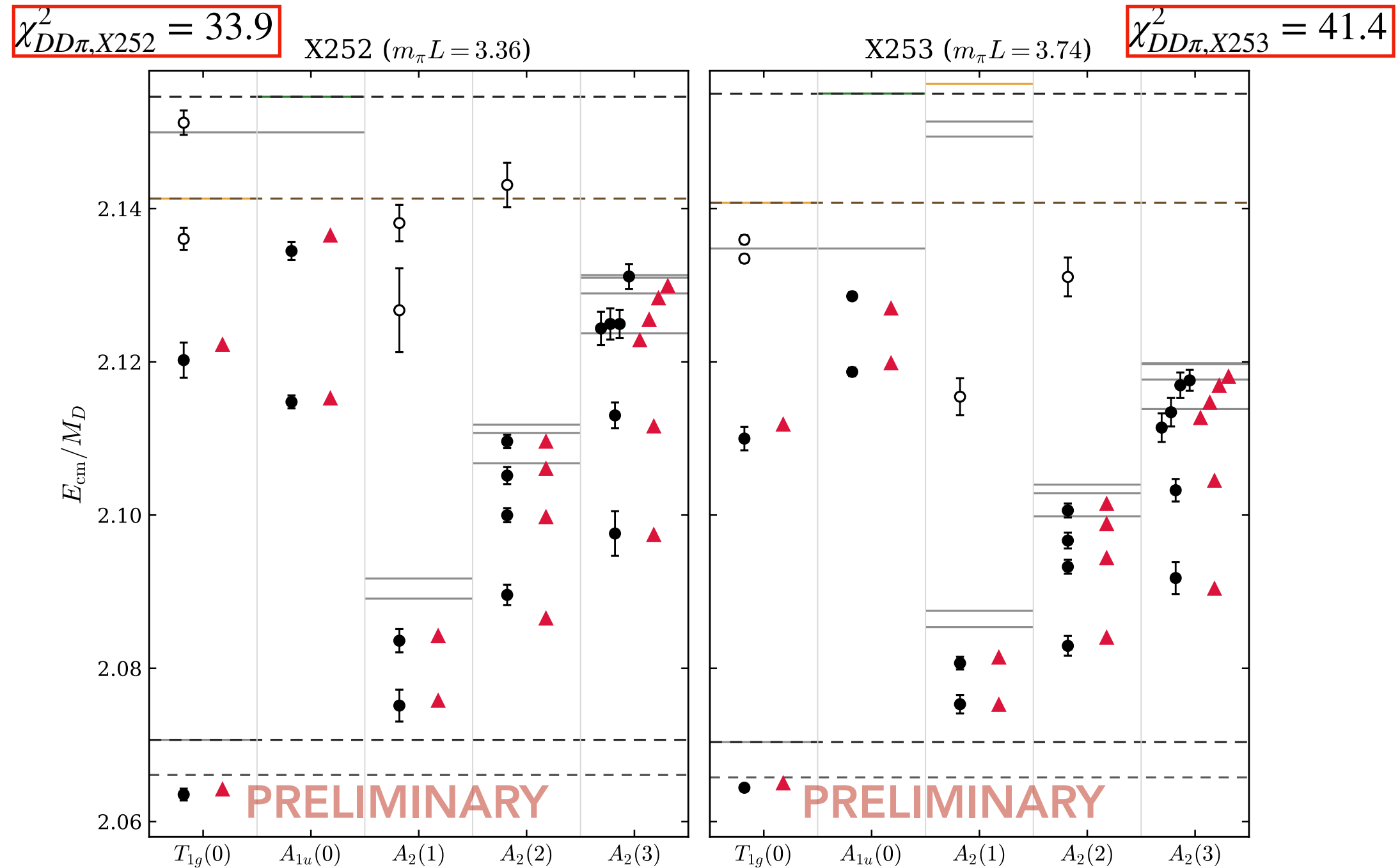
$$\begin{aligned} \mathcal{K}_{\text{df},3} = & c_{\text{iso},0} + \underbrace{c_{\text{iso},1}\Delta + c_B(\Delta_3 + \Delta'_3) + c_E t_{33}}_{\mathcal{O}(\Delta^0): \text{ includes dimer s waves}} + \underbrace{c_{\text{iso},2}\Delta^2 + c'_B\Delta(\Delta_3 + \Delta'_3) + c'_E\Delta t_{33} + c_{E^2}(t_{33})^2 + \sum_{i=5}^{11} c_i \mathcal{O}_i^{(2)} + \dots}_{\mathcal{O}(\Delta^2): \text{ includes s, p \& d waves}} \end{aligned}$$

$\mathcal{O}(\Delta^1): \text{ includes s \& p waves}$

- At $\mathcal{O}(\Delta^1)$, only c_E term couples to DD^* in T_{cc}^+ channel ($T_{1g}(0)$ irrep)
- We fit with all $\mathcal{O}(\Delta^0)$ and $\mathcal{O}(\Delta^1)$ terms + c'_E term

$DD\pi + D\pi + DD$ fit

- Fit result for $DD\pi/DD^*$ levels: reproduces levels below u-channel LHC reasonably well



$DD\pi + D\pi + DD$ fit

- Fit result for $D\pi$ levels: significantly undershoots the D^* states

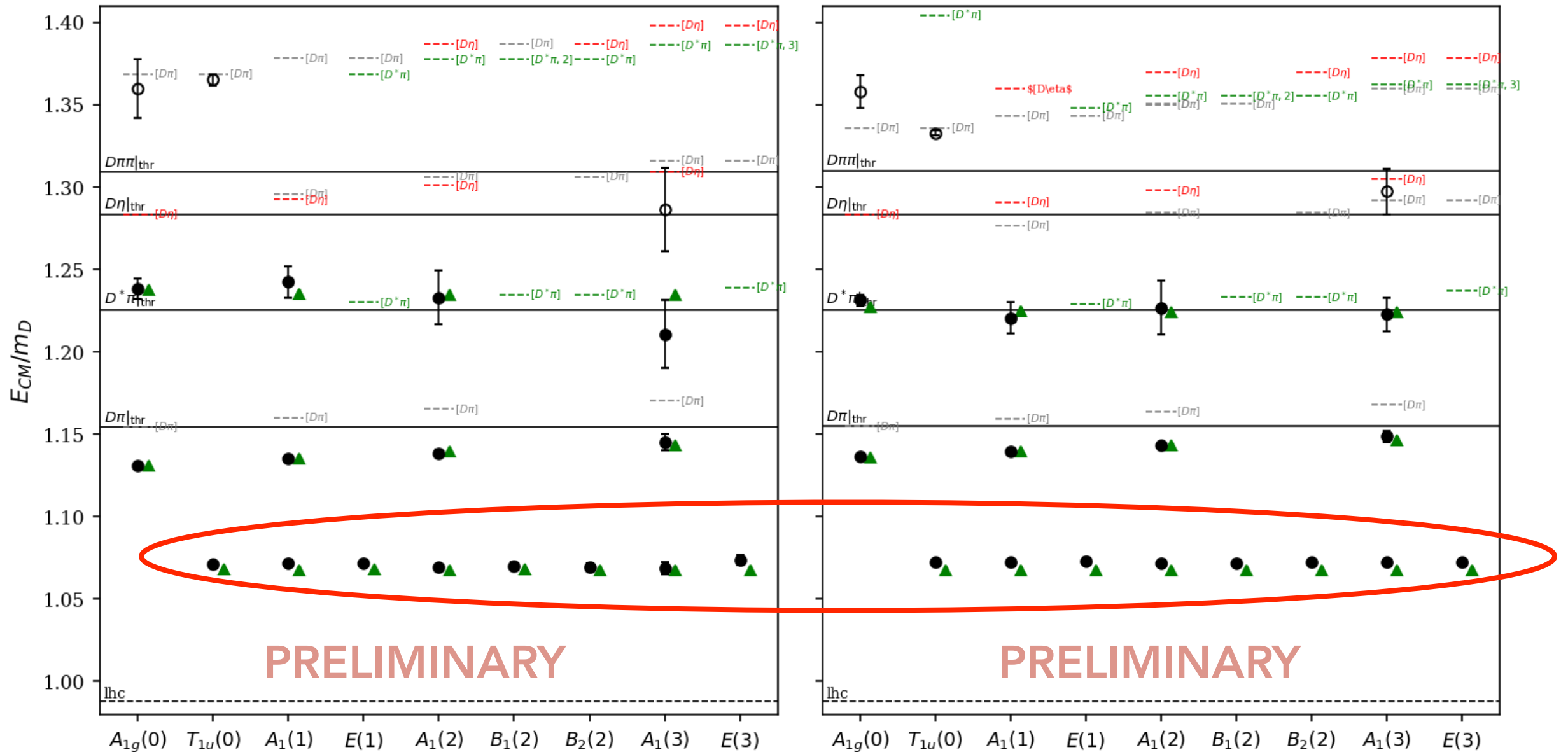
$$\chi^2_{D\pi, X252} = 28.6$$

QC3 + QC2 global fit, $\chi^2 = 223.6$, 73 dof

$$\chi^2_{D\pi, X253} = 77.3$$

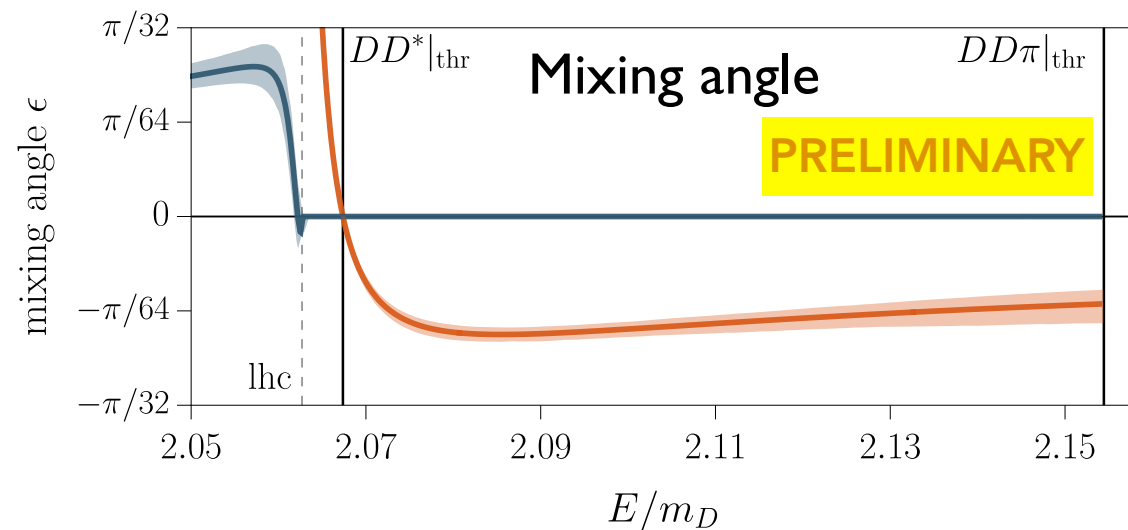
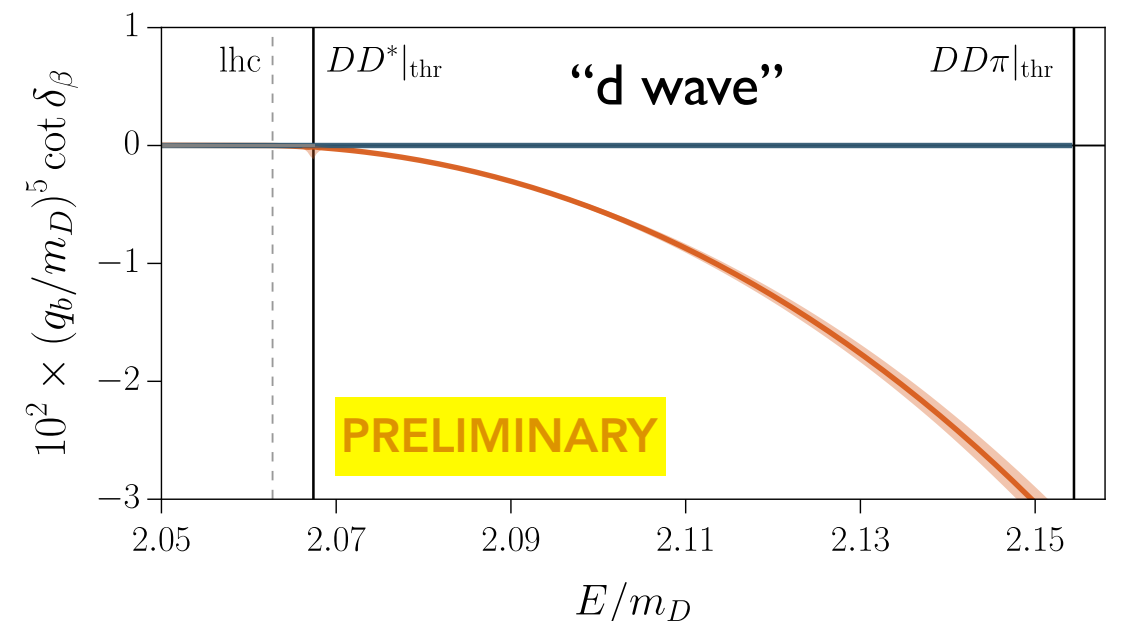
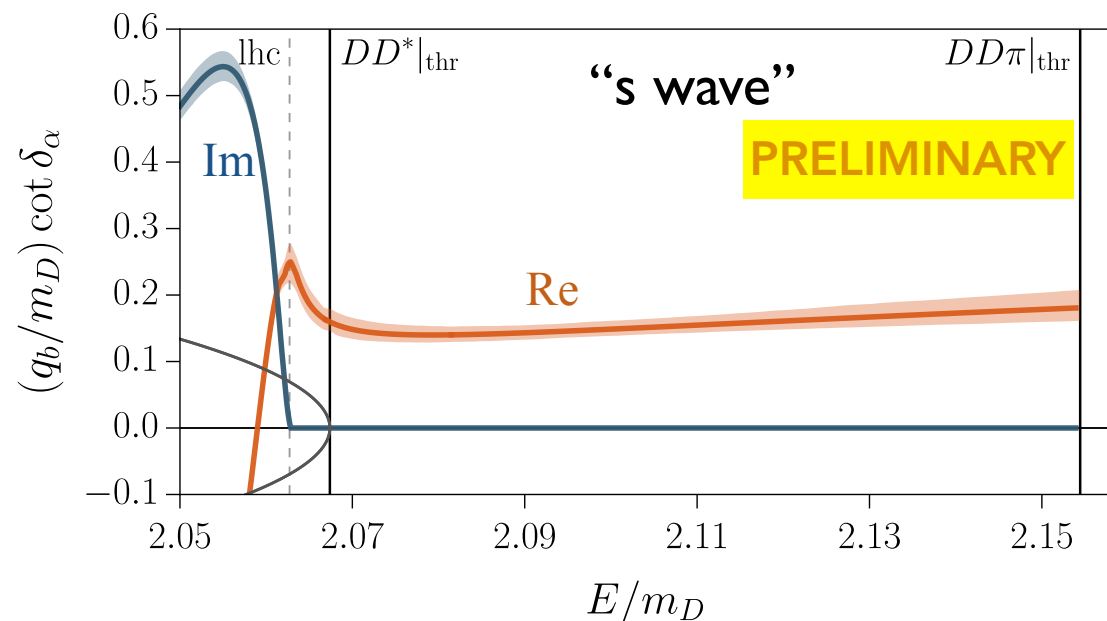
X252: $m_\pi L = 3.36$

X253: $m_\pi L = 3.74$



DD^* amplitude

- Given $\mathcal{K}_2$ and $\mathcal{K}_{\text{df},3}$, can solve integral equations to obtain DD^* amplitude using LSZ
- Using results from (imperfect) global fit as an example

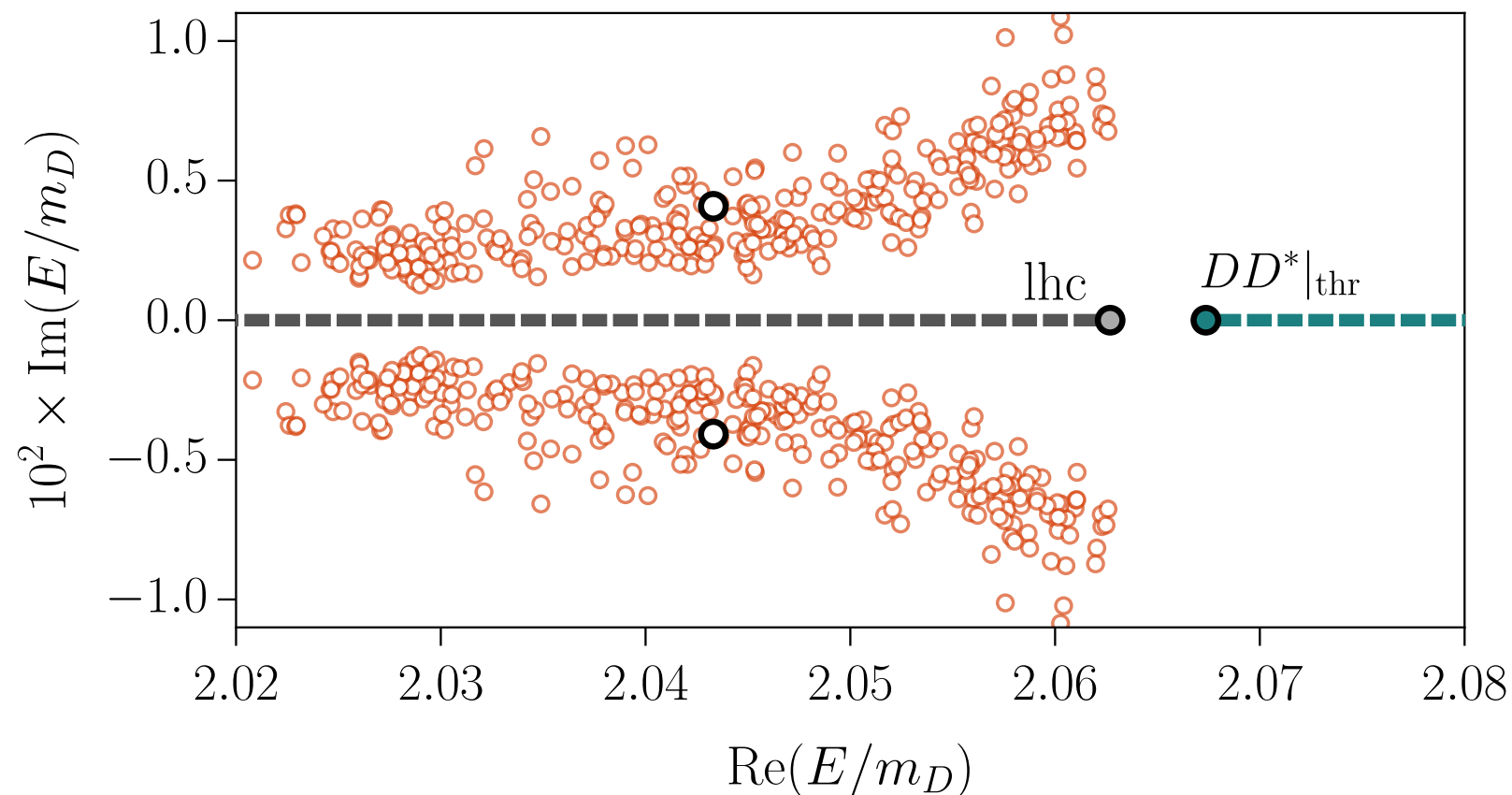


- Both s & d waves combine with spin of D^* to give $J^P = 1^+$
- Use Blatt-Biedenbarn parametrization of K matrix

$$(8\pi E) [\mathcal{K}_{DD^*}^{-1}] = \begin{pmatrix} \cos(\epsilon) & -\sin(\epsilon) \\ \sin(\epsilon) & \cos(\epsilon) \end{pmatrix} \begin{pmatrix} q_b \cot(\delta_\alpha) & 0 \\ 0 & q_b \cot(\delta_\beta) \end{pmatrix} \begin{pmatrix} \cos(\epsilon) & \sin(\epsilon) \\ -\sin(\epsilon) & \cos(\epsilon) \end{pmatrix}$$

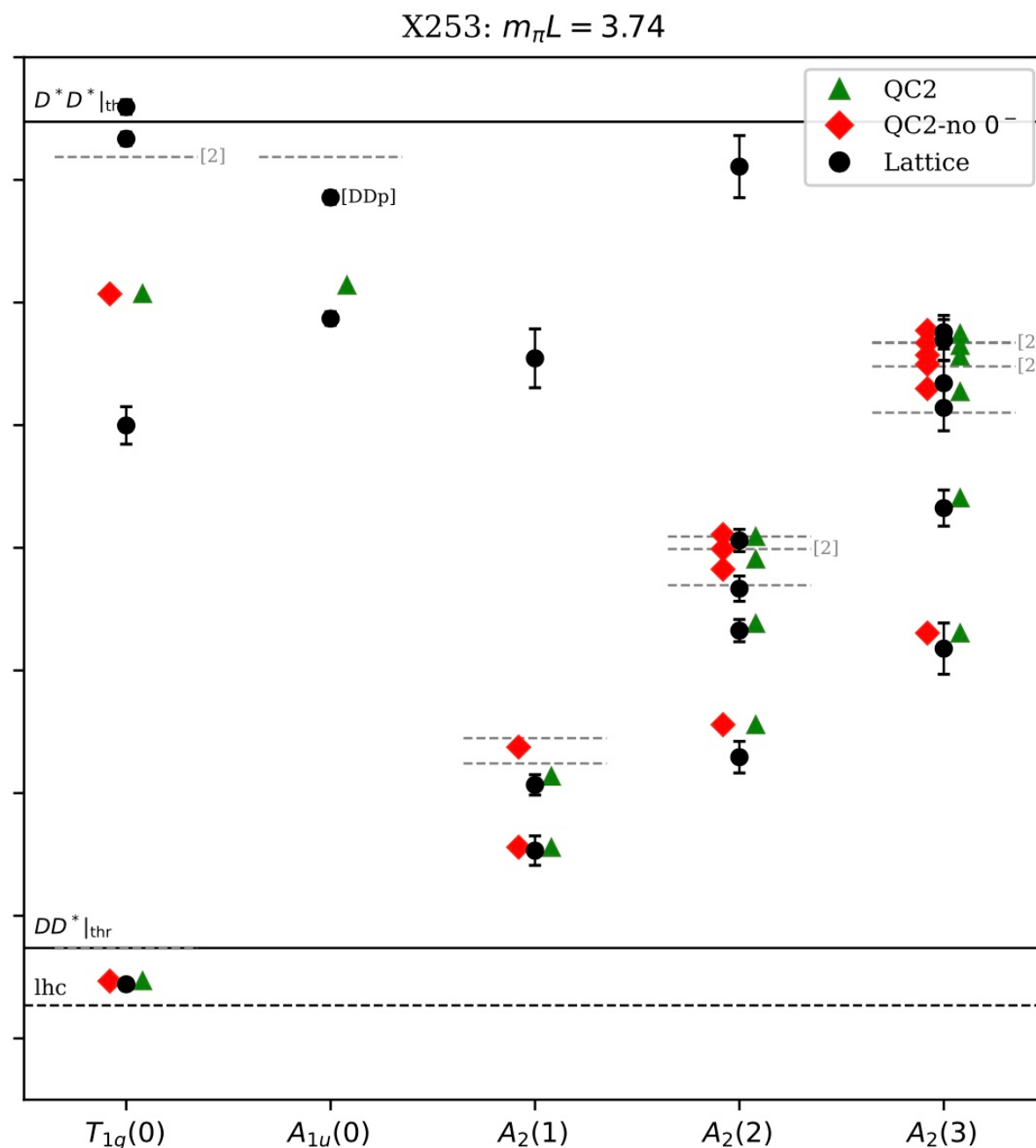
DD^* amplitude

- T_{cc}^+ appears as subthreshold resonance pole
 - Use 300 samples drawn from correlation matrix for K-matrix parameters from global fit



QC2 predictions

- Using the DD^* interactions from the integral equations (including also p waves), can solve the QC2
 - Should (roughly) reproduce the levels that are unaffected by 3-particle effects
 - Mismatch in M_{D^*} between global fit and direct extraction lifts lowest T_{1g} state above lhc



- Agreement is mediocre
- Need higher DD^* waves to match the moving A_2 levels

Conclusions & Outlook

- We are undertaking a comprehensive study of the T_{cc}^+ channel including three-body effects
 - Inclusion of multiple moving frames and two volumes provides more constraints on the DD^* interaction
 - Good fits to two-particle subsystems, but combined fits are challenging
 - Likely need to include more terms in $\mathcal{K}_{\text{df},3}$, so as to have more general DD^* interactions
 - Machinery to include all 11 $\mathcal{O}(\Delta^2)$ operators is in place
- Integral equation machinery in place awaiting successful FV fits
- Future plans
 - Calculations at lower M_π (where 3-body effects will be even larger)
 - Analyze data with other approaches (modified QC2 [Baiao Raposo & Hansen], plane wave approach, asymmetric QC3, ...)
 - Include D^*D^* in formalism (required 4-body formalism)

Thanks!
Any questions?

Backup slides

$DD(I = 1)$ subchannel

- Fit 16 levels on X252 + X253
- An s -wave Lüscher analysis on DD levels only gives a good fit

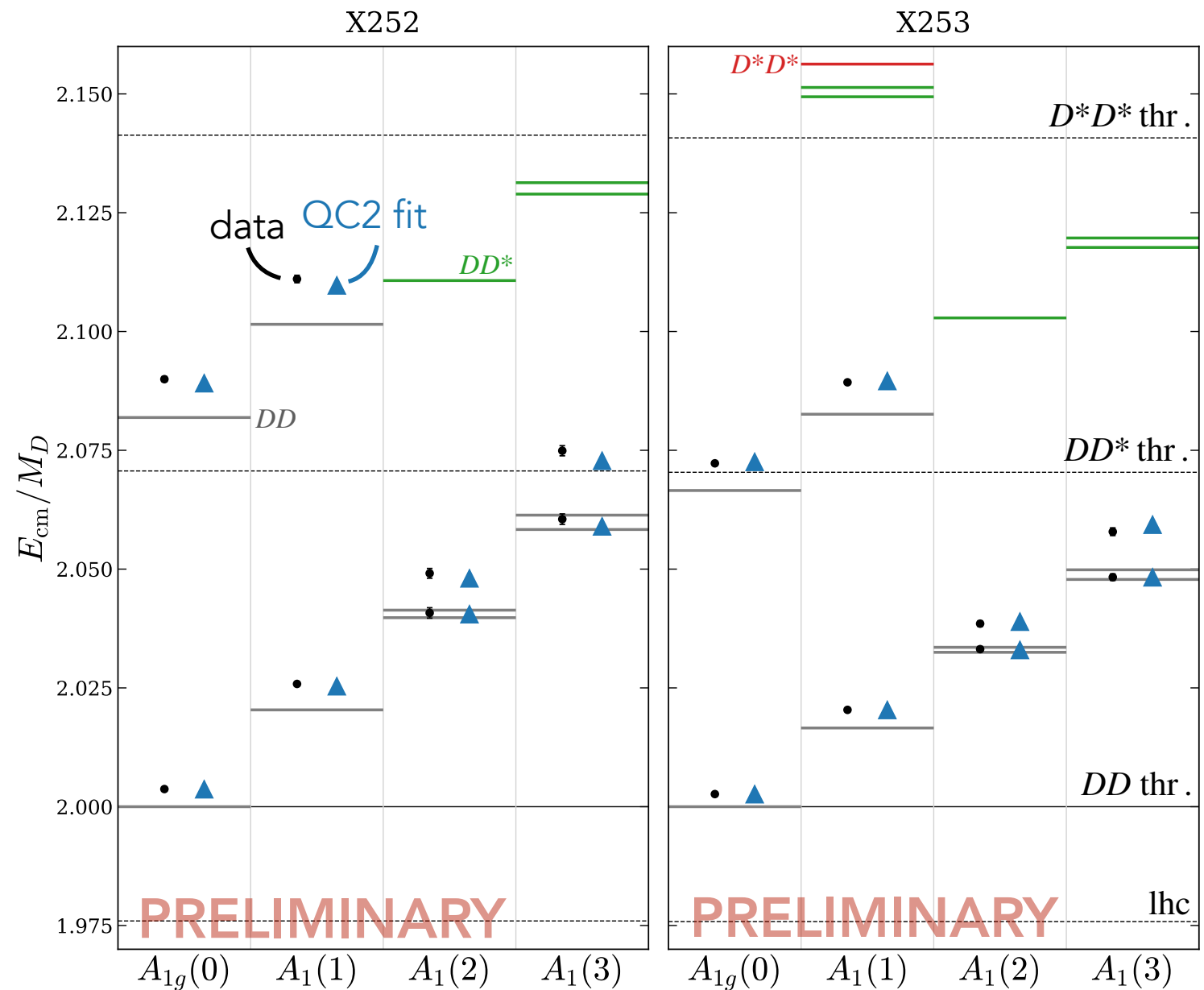
$$\frac{q}{M_D} \cot \delta_{DD,s} = C_1 + C_2 \left(\frac{q}{M_D} \right)^2 + C_3 \left(\frac{q}{M_D} \right)^4$$

$$C_1 = -0.437(12), \quad C_2 = -4.48(85), \quad C_3 = -14.2(9.8)$$

$$\chi^2/\text{dof} = 13.02/13$$

- As expected, weakly repulsive interaction

$$a_0 = 2.29(6)/M_D, \quad r_0 = 8.9(1.7)M_D$$



$DD(I = 1)$ subchannel

- Fit 16 levels on X252 + X253
- An s -wave Lüscher analysis on DD levels only gives a good fit

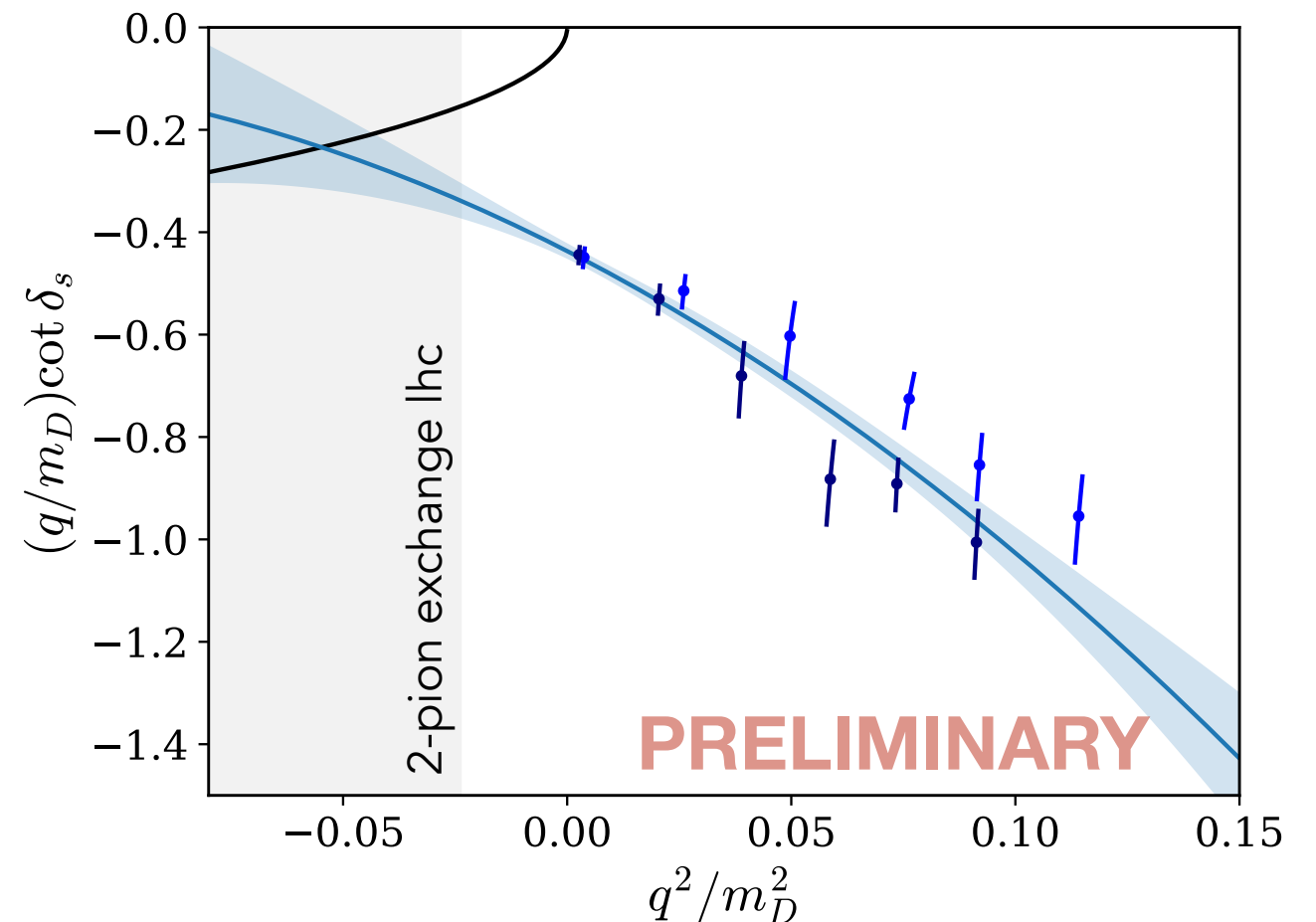
$$\frac{q}{M_D} \cot \delta_{DD,s} = C_1 + C_2 \left(\frac{q}{M_D} \right)^2 + C_3 \left(\frac{q}{M_D} \right)^4$$

$$C_1 = -0.437(12), \quad C_2 = -4.48(85), \quad C_3 = -14.2(9.8)$$

$$\chi^2/\text{dof} = 13.02/13$$

- As expected, weakly repulsive interaction

$$a_0 = 2.29(6)/M_D, \quad r_0 = 8.9(1.7)M_D$$



DD^* amplitude: p wave

