$B_{(s)} \to D_{(s)} \ell \nu$ with HISQ

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Fermilab Lattice and MILC Collaborations

allHISQ Working Group

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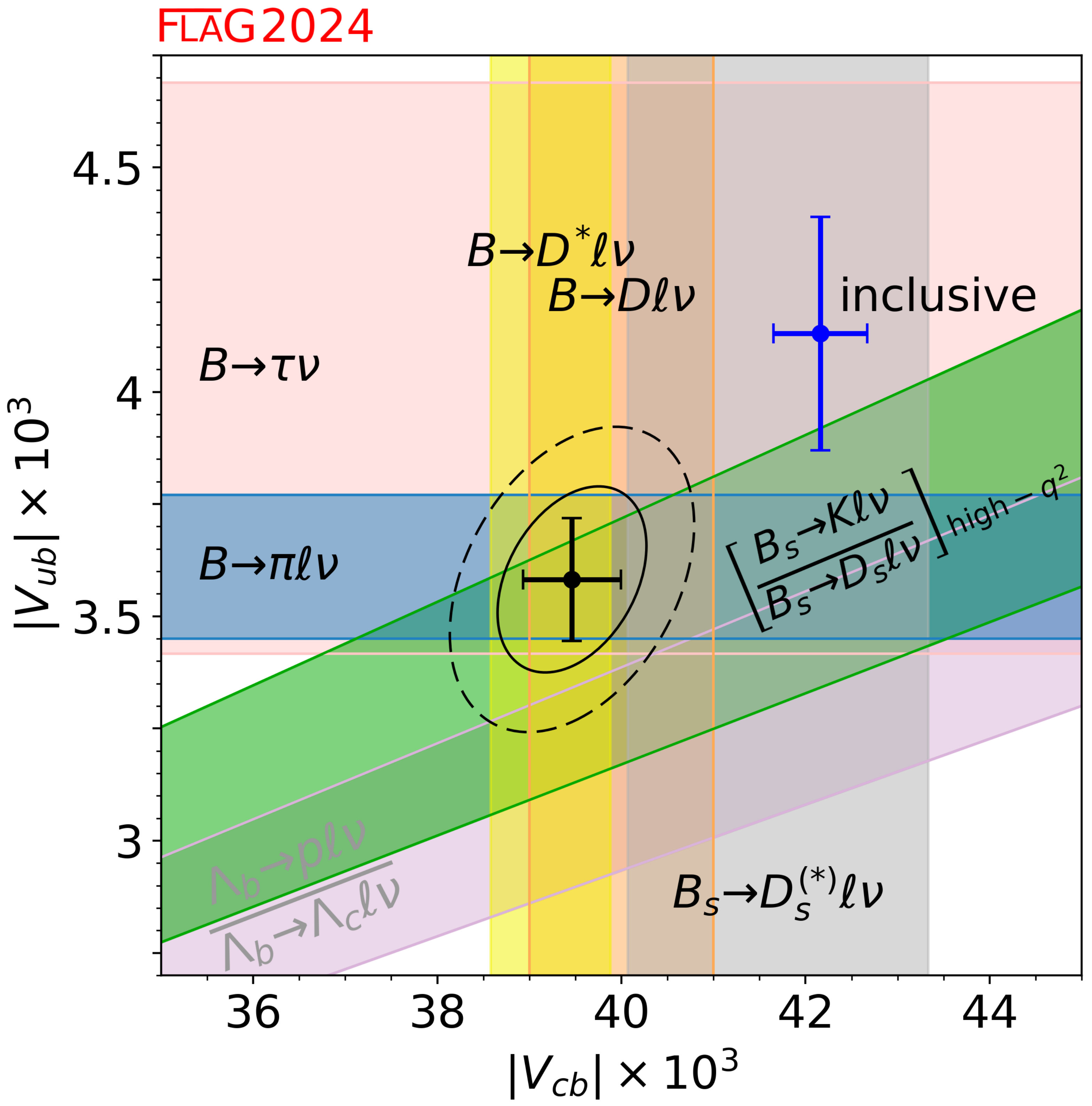
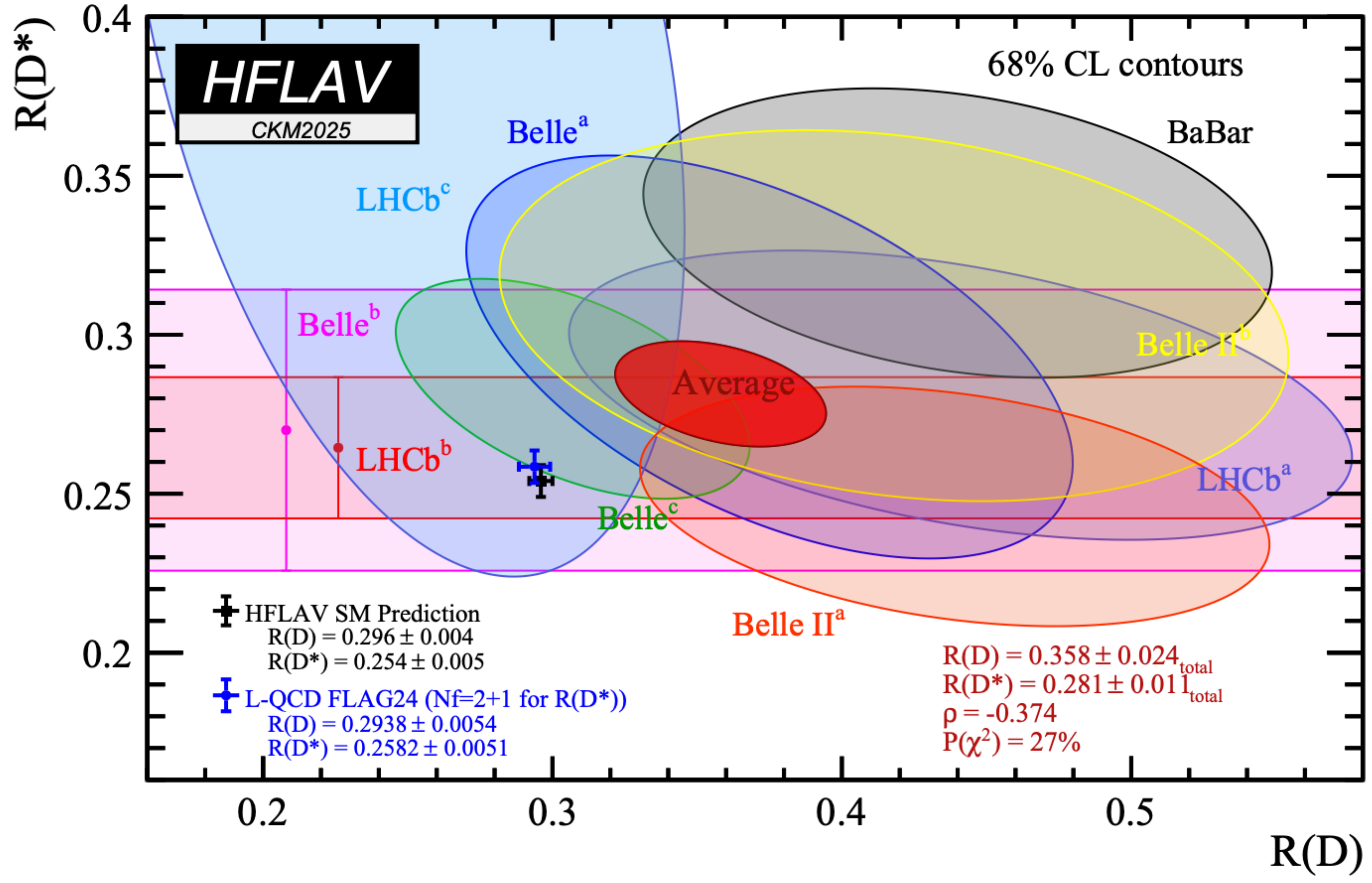
Background



Long standing 2-3 sigma tension



Experimental situation is a bit different now…

$B_{(s)} \to D_{(s)}\ell\nu$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2}{24\pi^3}\eta_{\mathrm{EW}}|V_{cb}|^2(1-\epsilon)(1+\delta_{\mathrm{EM}})\times$$

$$\left[|\mathbf{p}|^3\left(1+\frac{\epsilon}{2}\right)\boxed{|f_+(q^2)|^2}+|\mathbf{p}|M_{B(s)}^2\left(1-\frac{M_{D(s)}^2}{M_{B(s)}^2}\right)^2\frac{3\epsilon}{8}\boxed{|f_0(q^2)|^2}\right]$$

Form factors contain nonperturbative information

$$\langle D_{(s)}|\mathcal{V}^\mu|B_{(s)}\rangle \equiv \sqrt{2M_{B_{(s)}}}\left[v^\mu f_\parallel(q^2)+p^\mu f_\perp(q^2)\right],$$

$$\equiv f_+(q^2)\left(k^\mu+p^\mu-\frac{M_{B_{(s)}}^2-M_{D_{(s)}}^2}{q^2}q^\mu\right)+f_0(q^2)\frac{M_{B_{(s)}}^2-M_{D_{(s)}}^2}{q^2}q^\mu,$$

$$\langle D_{(s)}|S|B_{(s)}\rangle = \frac{M_{B_{(s)}}^2-M_{D_{(s)}}^2}{m_h-m_c}f_0(q^2).$$

Definitions

- Form factors $f_0, f_{\|}, f_{\perp}$ can be obtained directly from matrix elements

$$f_{\|}(q^2) = Z_{V^0} \frac{\langle D_s|V^0|B_s\rangle}{\sqrt{2M_{B_s}}}$$

$$f_{\perp}(q^2) = Z_{V^i} \frac{1}{p^i} \frac{\langle D_s|V^i|B_s\rangle}{\sqrt{2M_{B_s}}}$$

$$f_0(q^2) = Z_m Z_S \frac{m_b - m_c}{M_{B_s}^2 - M_{D_s}^2} \langle D_s|S|B_s\rangle$$

- f_+ is constructed from linear combinations

Definitions

- Consider all possible constructions

$$f_+(q^2) = \left(\frac{M_H - E_L}{\sqrt{2M_H}}\right)\left(1 - \frac{E_L^2 - M_L^2}{(M_H - E_L)^2}\right) f_\perp(q^2) + \left(\frac{M_H^2 - M_L^2}{M_H - E_L}\right)\frac{f_0(q^2)}{2M_H}.$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_H}}\left[f_\parallel(q^2) + (M_H - E_L)\, f_\perp(q^2)\right],$$

$$f_0(q^2) = \frac{\sqrt{2M_H}}{M_H^2 - M_L^2}\left[(M_H - E_L)\, f_\parallel(q^2) + (E_L^2 - M_L^2)\, f_\perp(q^2)\right].$$

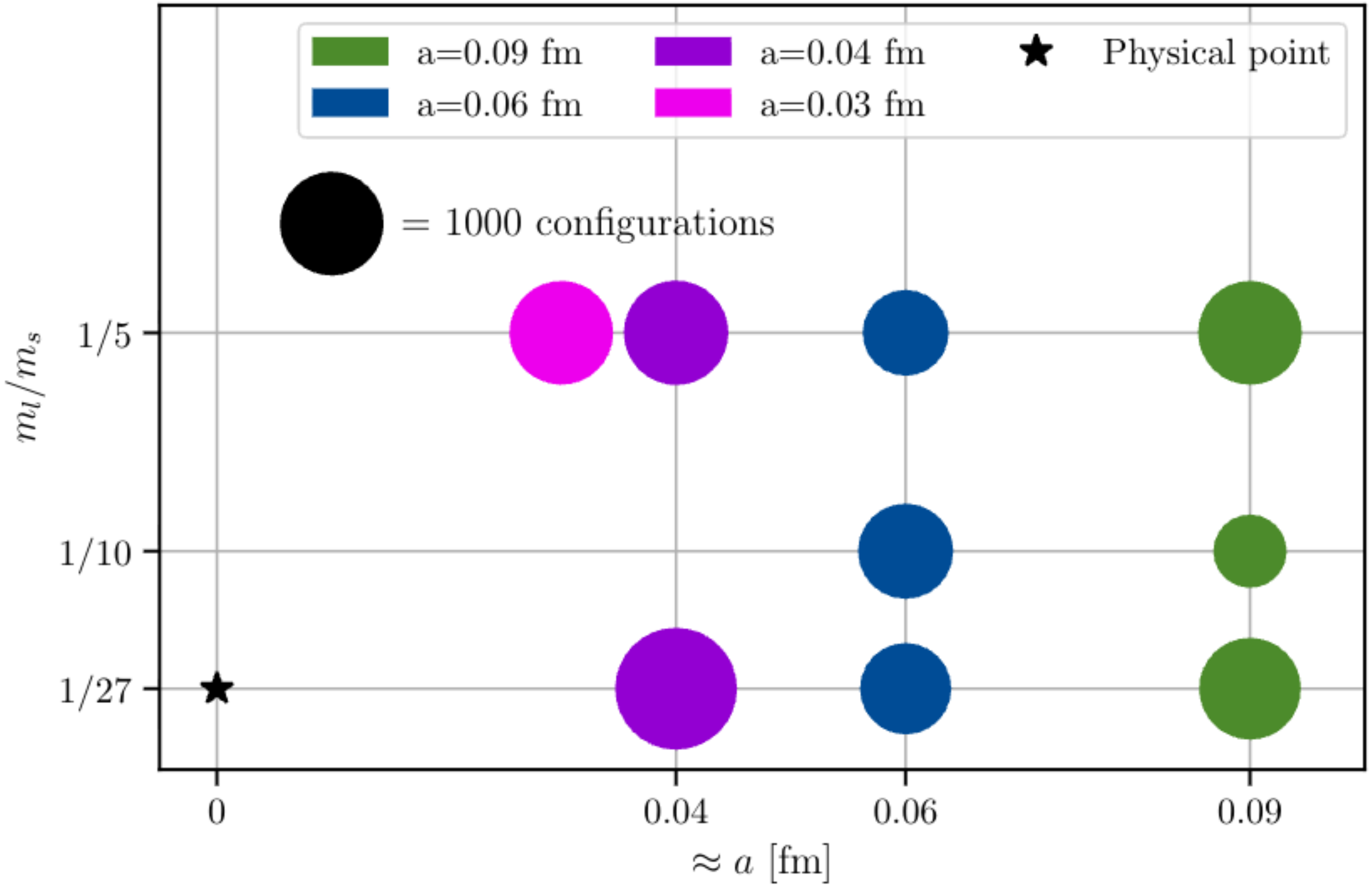
- There is a preferred order taking continuum limit based on angular momentum

Simulation Details

- 2+1+1 Flavor Highly Improved Staggered Quarks (HISQ) in both valence and sea
  - Discretization effects begin at $\mathcal{O}(\alpha_s(am)^2)$
- Valence heavy quark is varied from $1.5m_c$ to m_b
- Final state varied from $0.9m_c \rightarrow 1.1m_c$
- Analysis is blinded ($\pm 5\,\%$ shift)

Follana *et al.*, *Phys. Rev. D* **75**, 054502 (2007), arXiv:hep-lat/0610092

Ensembles

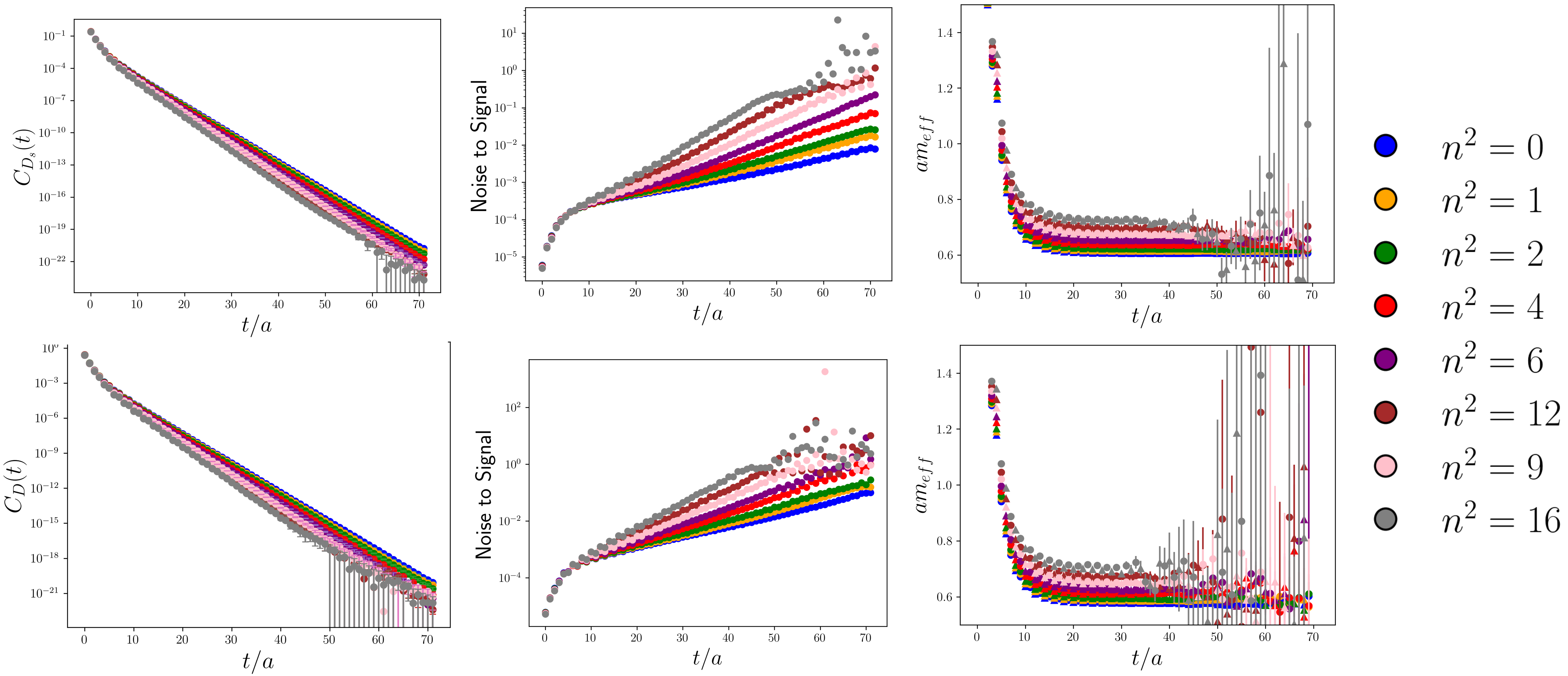


a=0.09 fm
a=0.06 fm
a=0.04 fm
a=0.03 fm
Physical point
= 1000 configurations
m_l/m_s
1/5
1/10
1/27
0
0.04
0.06
0.09
$\approx a$ [fm]

Simulation Details

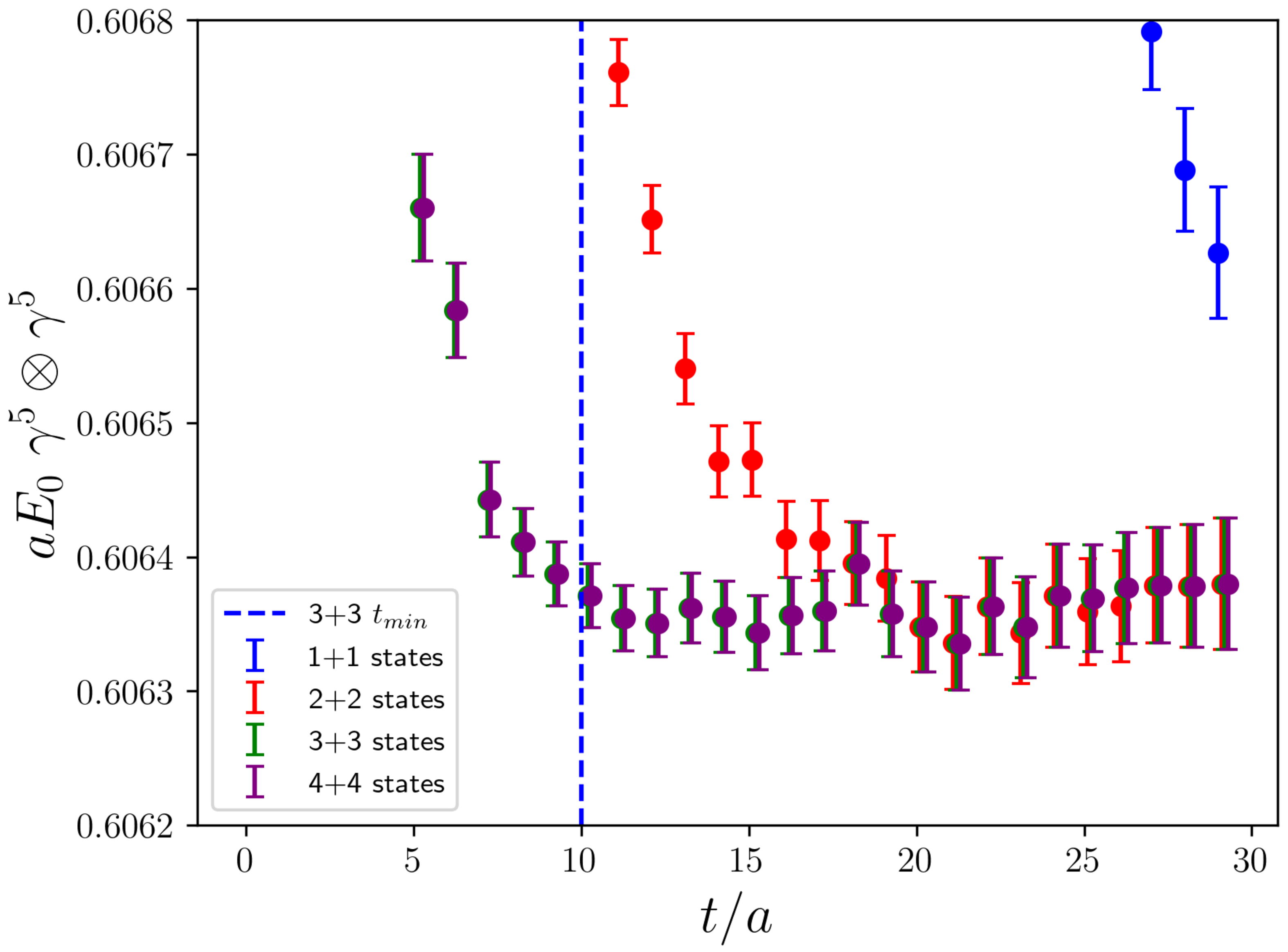
$\approx a$ [fm]	$N_s^3 \times N_t$	m_l	m_h/m_c	w_0/a	$N_{\text{src}} \times N_{\text{configs}}$	T/a
0.088	$64^3 \times 96$	physical	$0.9, 1.0, 1.5, 2.0$	1.9579(04)	24×980	{14, 15, 17, 20, 21}
0.088	$48^3 \times 96$	$m_s/10$	$0.9, 1.0, 1.5, 2.0$	1.9391(13)	24×697	{16, 19, 22, 25}
0.088	$32^3 \times 96$	$m_s/5$	$0.9, 1.0, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75$	1.9033(14)	24×1006	{14,15,17,20,21}
0.057	$96^3 \times 192$	physical	$0.9, 1.0, 1.1, 2.2, 3.3, 4.4$	3.0119(19)	32×877	{25, 28, 30, 34, 37}
0.057	$64^3 \times 144$	$m_s/10$	$0.9, 1.0, 2.0, 3.0, 4.0$	2.9478(31)	36×916	{23, 30, 34, 37}
0.057	$48^3 \times 144$	$m_s/5$	$0.9, 1.0, 2.0, 3.0, 4.0$	2.9041(58)	36×823	{23, 30, 34, 37}
0.042	$64^3 \times 192$	$m_s/5$	$0.9, 1.0, 2.0, 3.0, 4.0, 4.61$	3.9036(15)	24×1008	{34, 39, 45, 52}
0.042	$144^3 \times 288$	physical	$0.9, 1.0, 2.0, 3.0, 4.0, 4.61$	4.0366(19)	24×1170	{34, 39, 45, 52}
0.03	$96^3 \times 288$	$m_s/5$	$0.9, 1.0, 3.0, 4.0, 4.58$	5.2285(98)	24×1002	{ 38, 43, 50, 55 }

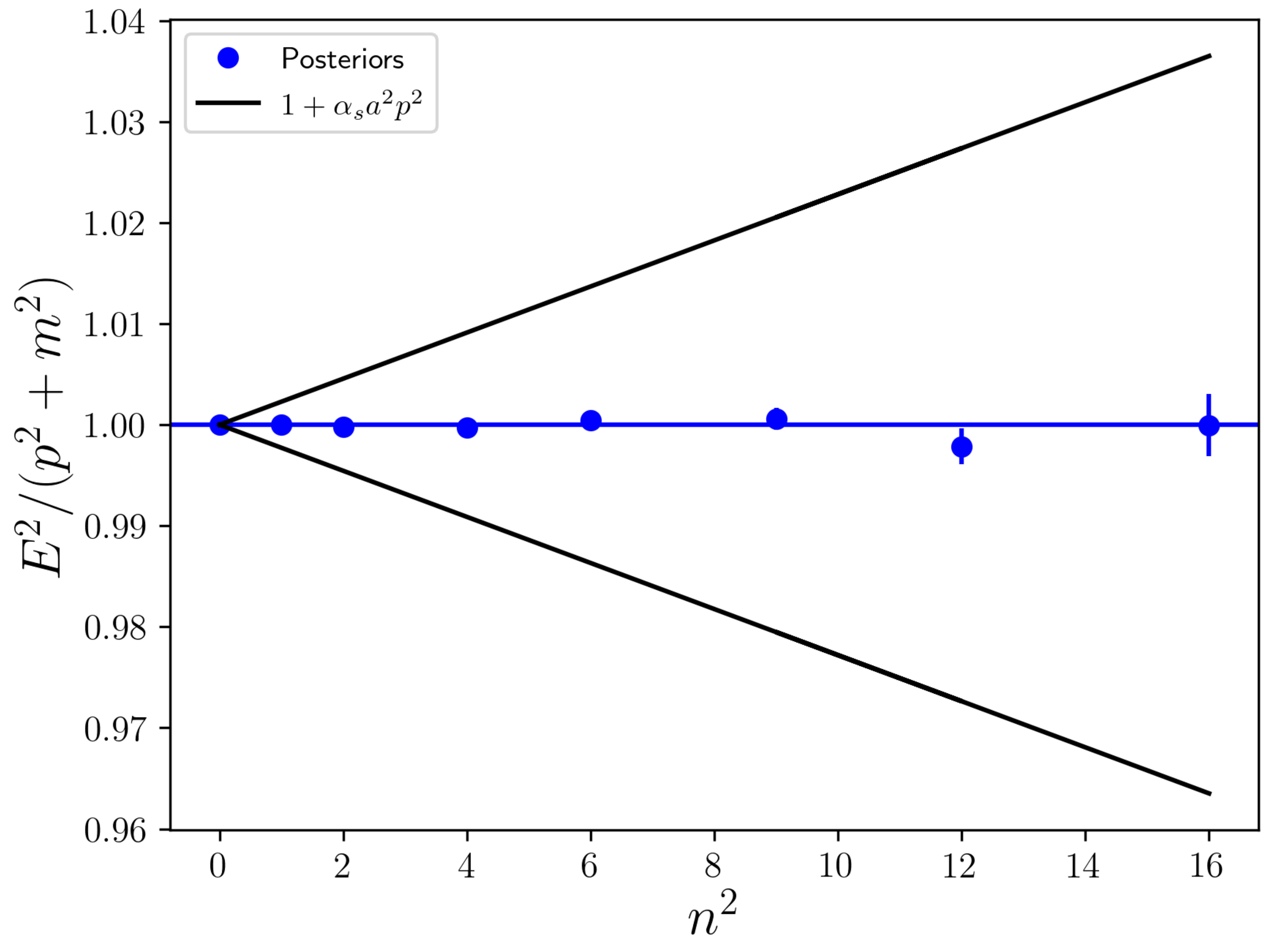
Two-point Correlators @ 0.06 fm



Correlator Analysis

- Bayesian fitting procedure
- Maximize information obtained from correlation functions
- Perform stability analysis to obtain $t_{\min}$, n_{dec}, n_{osc}

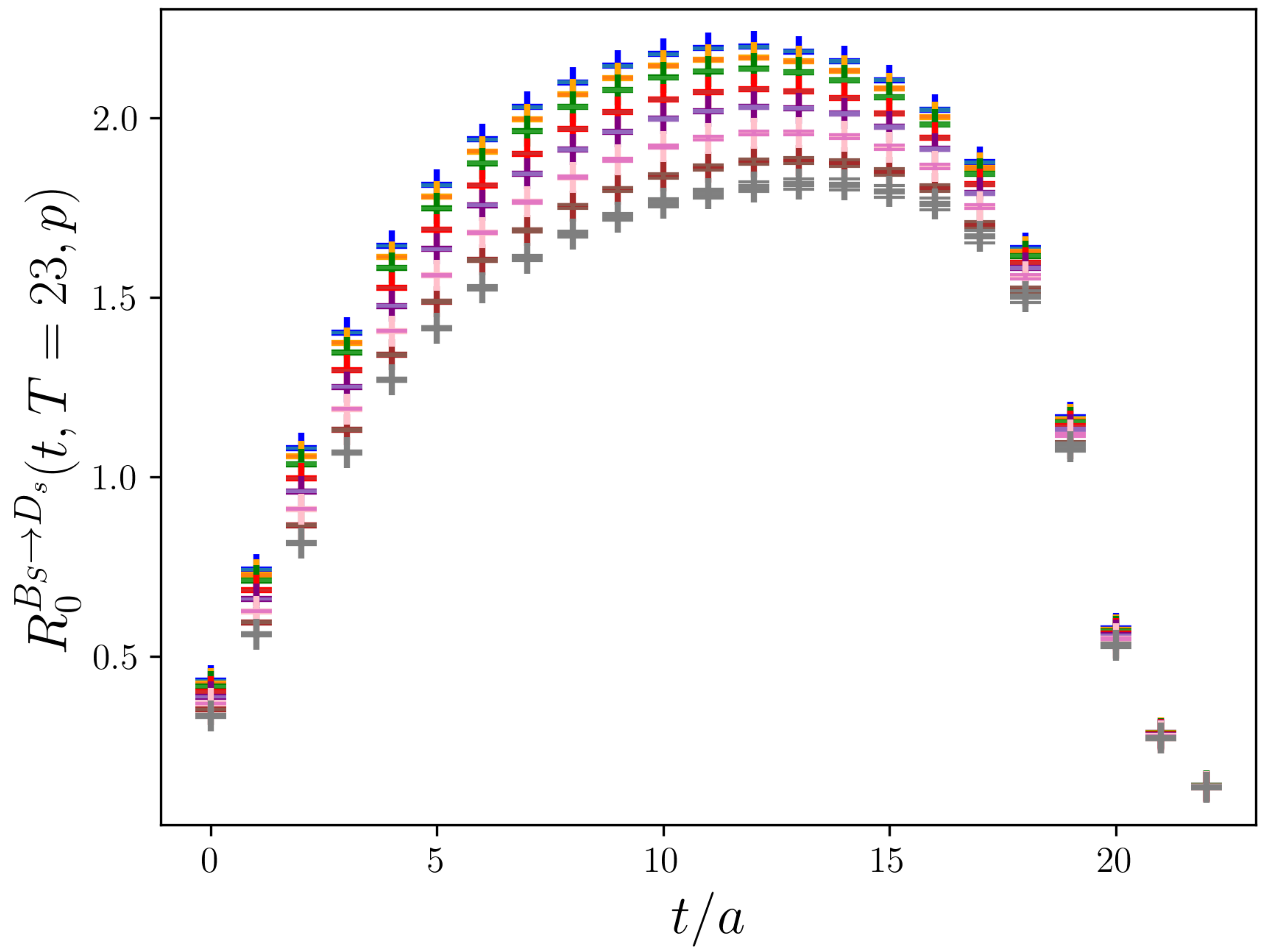




Correlator Analysis

- Construct ratio of two- and three-point functions

$$R_0(t,T,\mathbf{p}) = 2\sqrt{M_{H_s}E_{D_s}}\left(\frac{m_h - m_c}{M_{H_s}^2 - M_{D_s}^2}\right)\frac{\bar{C}^S_{H_s\to D_s}(t,T,\mathbf{p})}{\sqrt{\bar{C}^P_{D_s}(t,\mathbf{p})\,\bar{C}^P_{H_s}(T-t)\,e^{-E_{D_s}t}e^{-M_{H_s}(T-t)}}}$$

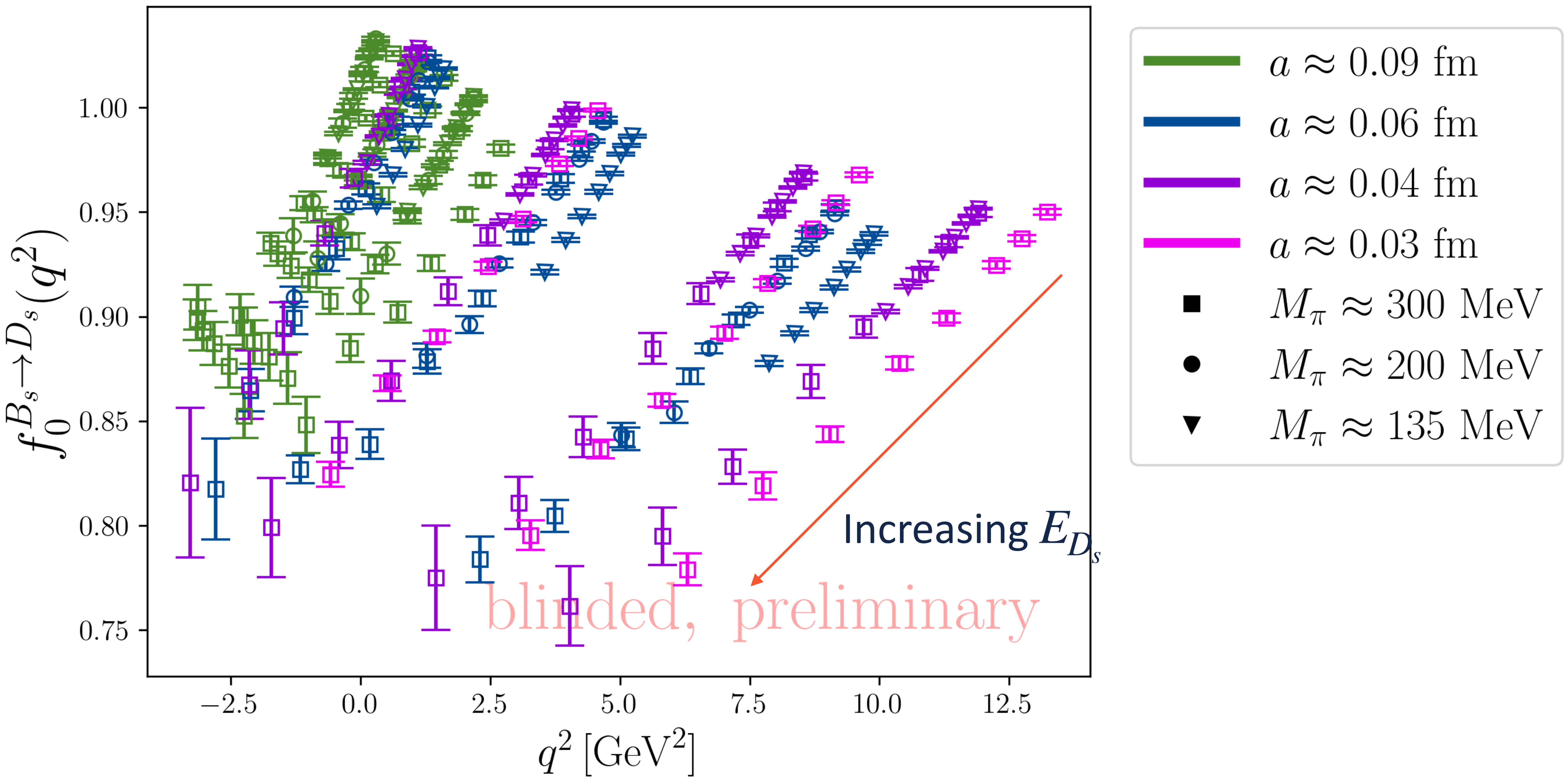


- Fit two- and three-point functions simultaneously to obtain matrix elements

See N. Cassar talk for more details on fitting and excited states*

Form Factor

Increasing m_h, decreasing a



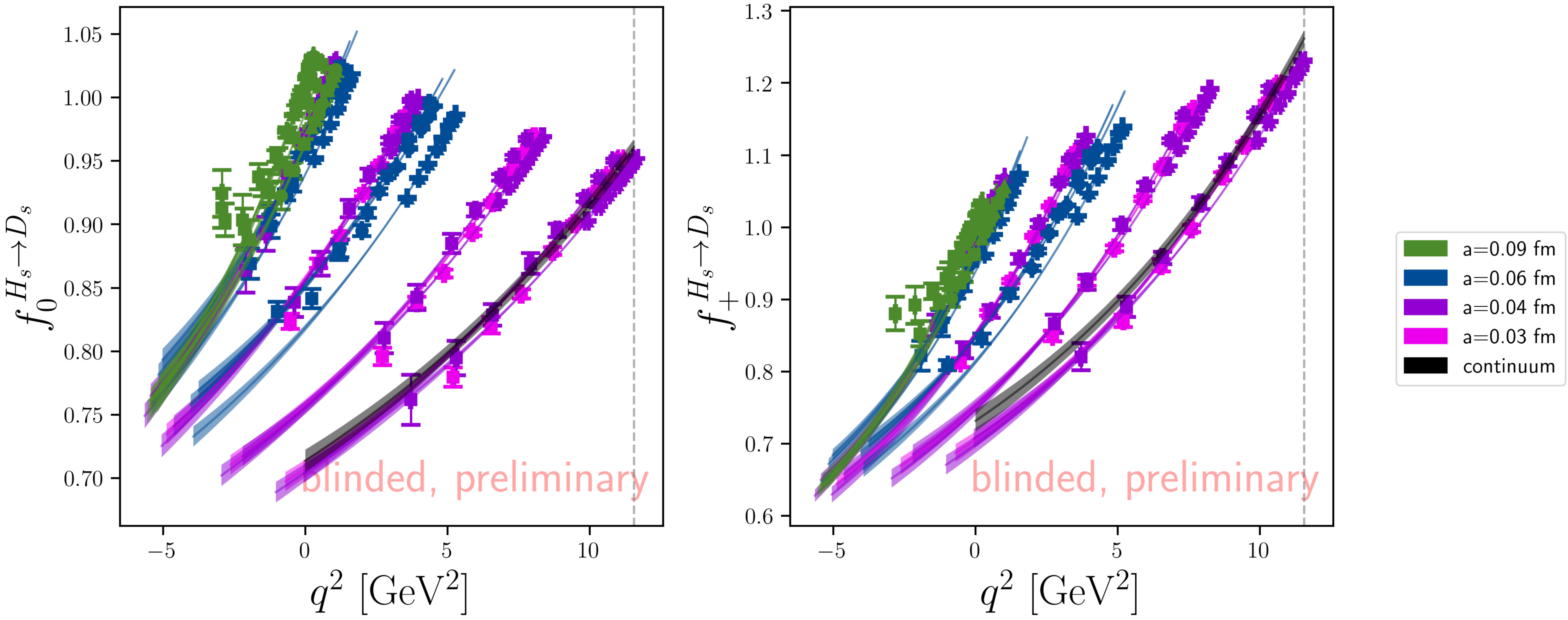
Chiral Continuum Limit

$$w_0^{d_P} f_P(E) = \frac{c_0}{w_0(E+\Delta_{xy,P})}\Bigg[1 + \delta f_{\rm logs} + c_l\chi_l + c_s\chi_s + c_C\chi_C + c_H\chi_H + c_E\chi_E$$
$$+ c_{l^2}\chi_l^2 + c_{ls}\chi_l\chi_s + c_{s^2}\chi_s^2$$
$$+ c_{lH}\chi_l\chi_H + c_{lE}\chi_l\chi_E + c_{sH}\chi_s\chi_H + c_{sE}\chi_s\chi_E$$
$$+ c_{H^2}\chi_H^2 + c_{HE}\chi_H\chi_E + c_{E^2}\chi_E^2$$
$$+ c_{C^2}\chi_C^2 + c_{CE}\chi_C\chi_E + c_{CH}\chi_C\chi_H + c_{lC}\chi_l\chi_C + c_{sC}\chi_s\chi_C$$
$$+ \delta f_{\rm artifacts}^{(a^2+h^2+c^2)}\Bigg].$$

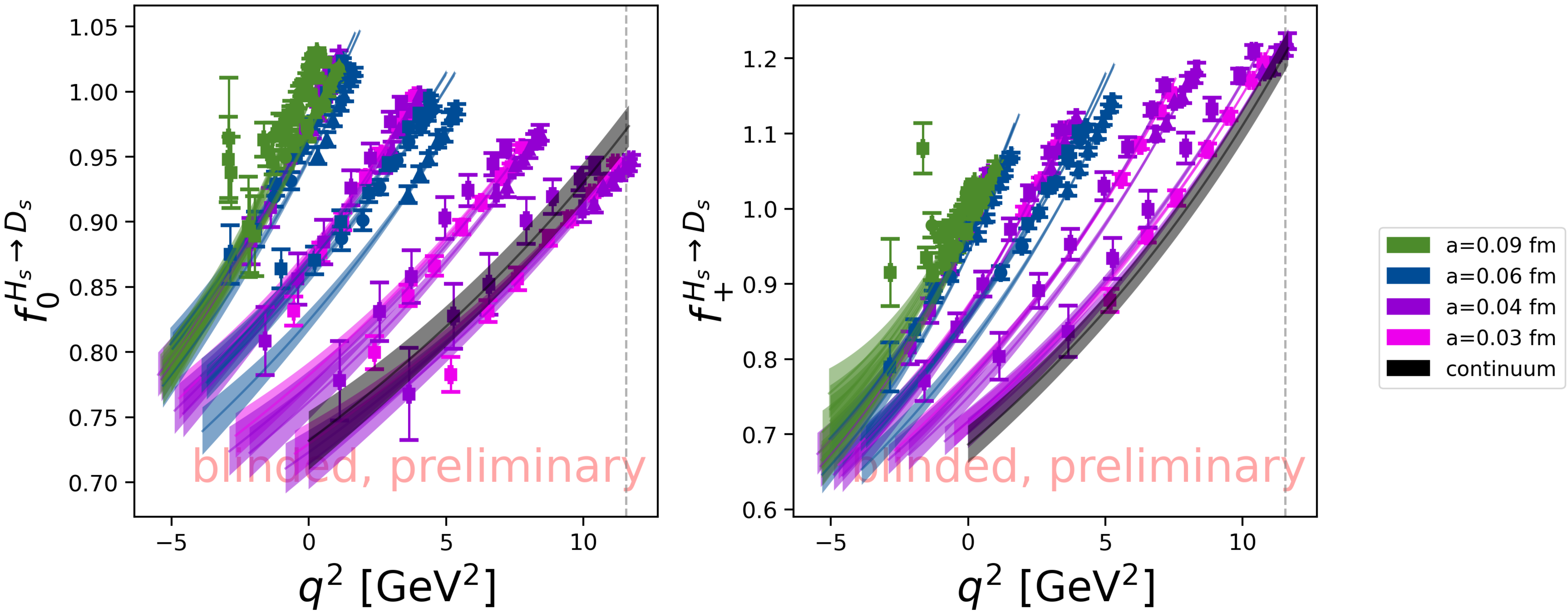
$$\text{tree} \times \left(1 + \delta_{\rm logs} + \delta^{\rm NLO}_{\rm analytic} + \delta^{\rm NNLO}_{\rm analytic} + \delta_{\rm artifacts}\right).$$

See Backup for explicit expressions*

Chiral Continuum $B_s \to D_s$



Chiral Continuum $B \to D$



z-expansion

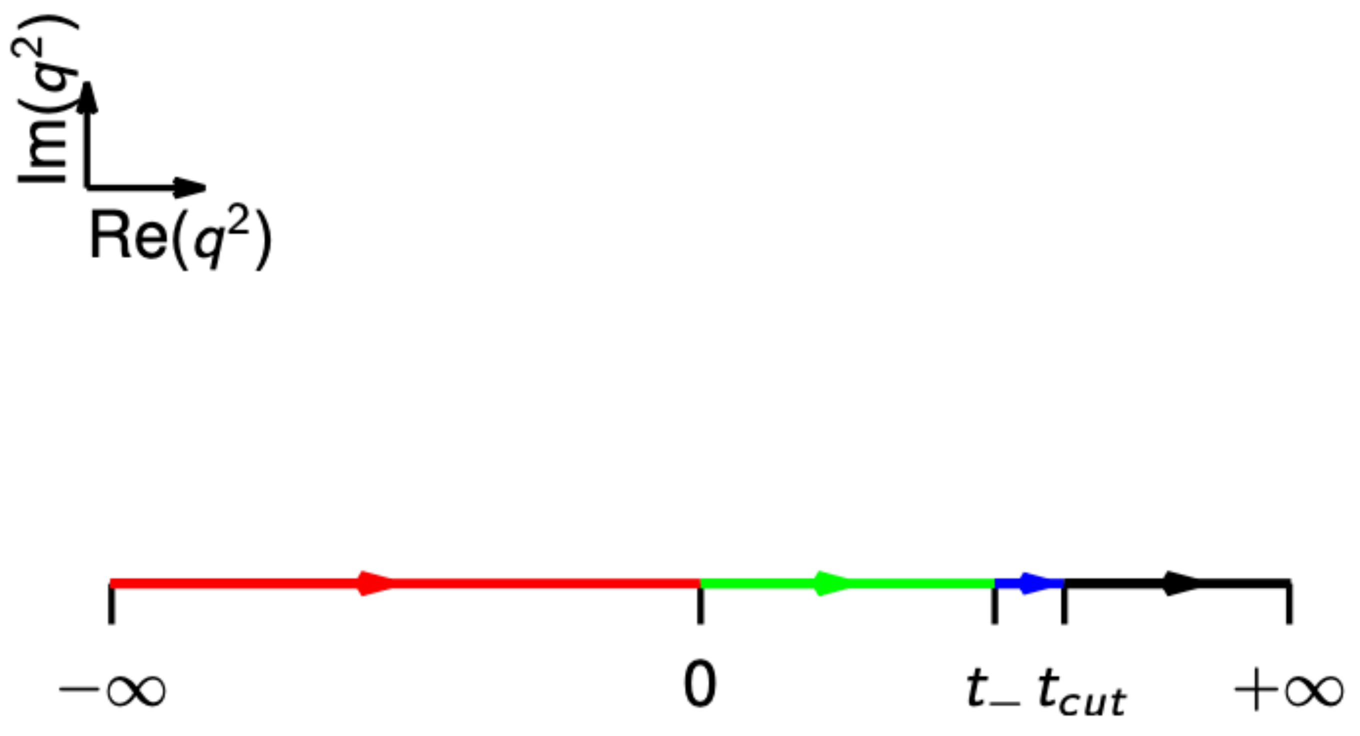
- Take N = 3 synthetic data points and apply conformal transformation

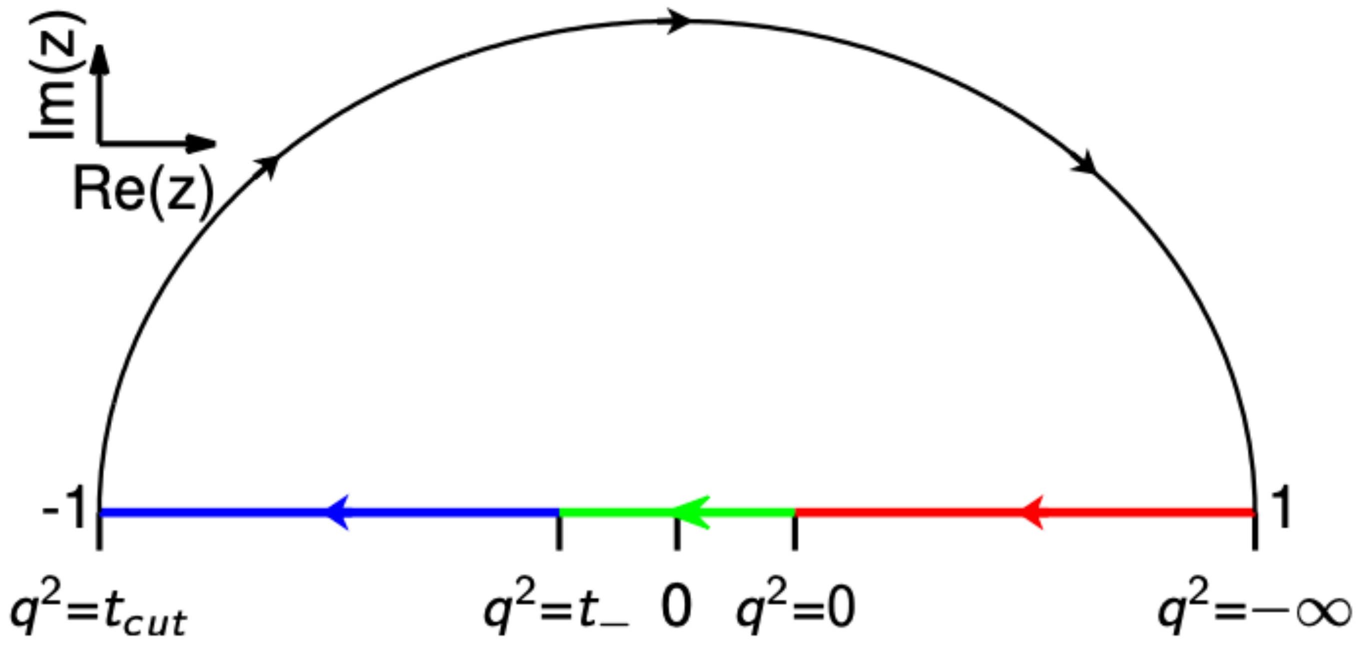
$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- Choose optimal t_0
- Explore both BCL and BGL parameterizations

Boyd, Grinstein & Lebed, *Nucl. Phys. B* **461**, 493 (1996), arXiv:hep-ph/9508211.

Bourrely, Caprini & Lellouch, *Phys. Rev. D* **79**, 013008 (2009), arXiv:0807.2722.





BCL parameterization

- Simpler choice of outer function

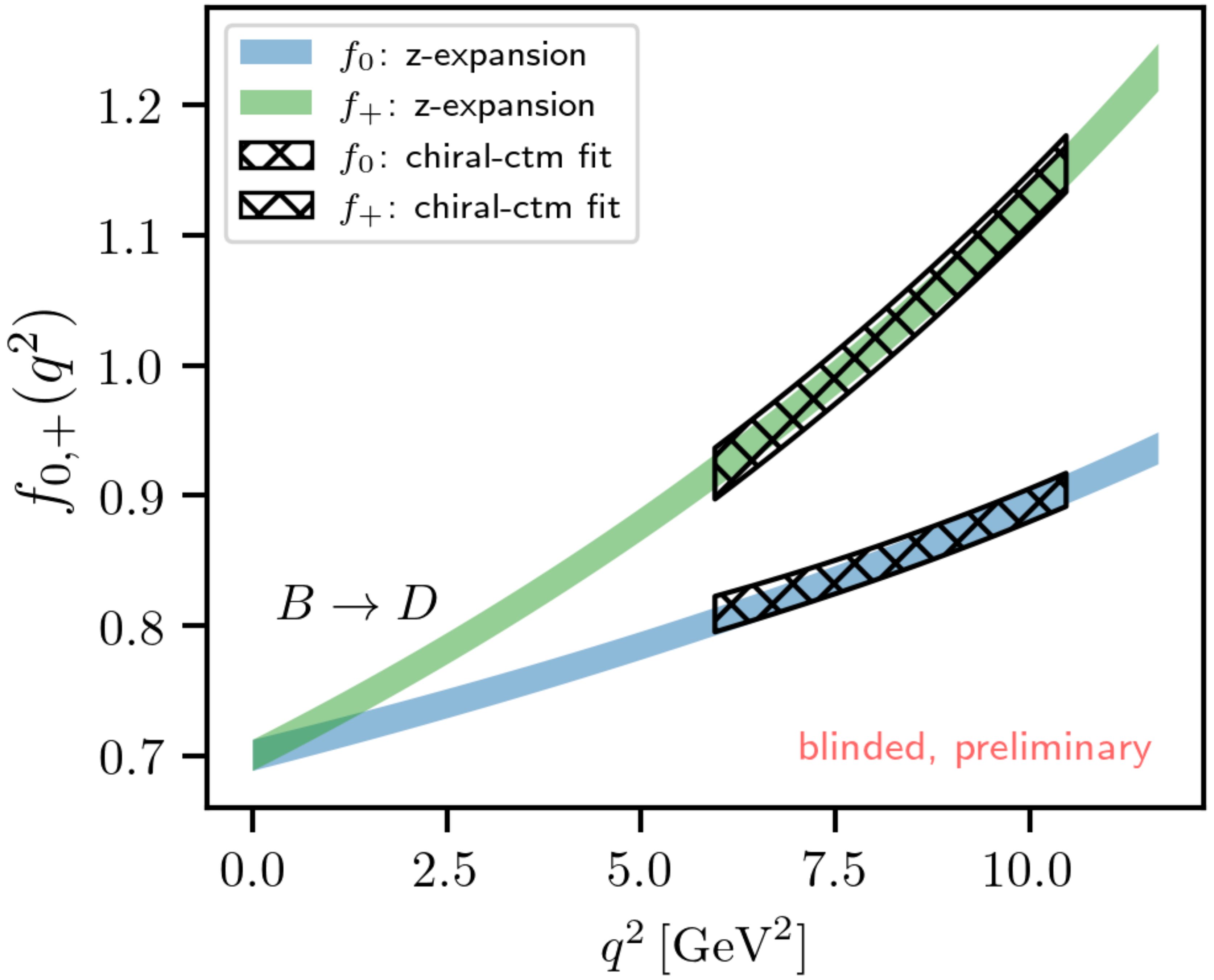
$$f_0(z) = \frac{1}{1 - q^2(z)/M_{0+}^2} \sum_{m=0}^{M-1} b_m z^m,$$

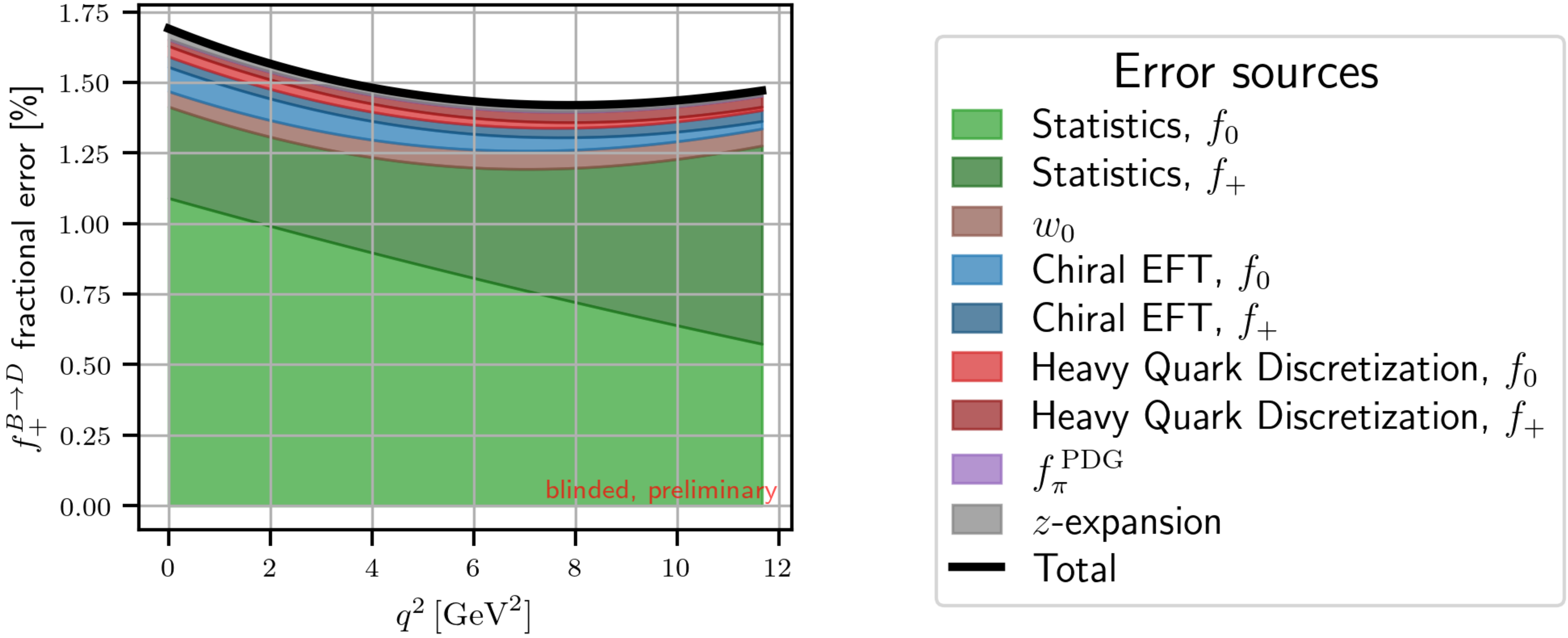
$$f_+(z) = \frac{1}{1 - q^2(z)/M_{1-}^2} \sum_{n=0}^{N-1} a_n \left(z^n - \frac{n}{N}(-1)^{n-N} z^N \right).$$

- Unitarity constraint given by

$$\sum_{j,k=0}^{N} B_{jk}(t_0) a_j^+(t_0) a_k^+(t_0) \leq 1$$

When all is said and done…






f_0: z-expansion
f_+: z-expansion
f_0: chiral-ctm fit
f_+: chiral-ctm fit
$f_{0,+}(q^2)$
$B_s \to D_s$
blinded, preliminary
q^2 [GeV^2]
$f_+^{B_s \to D_s}$ fractional error [%]
blinded, preliminary
q^2 [GeV^2]
Error sources
Statistics, f_0
Statistics, f_+
w_0
Chiral EFT, f_0
Chiral EFT, f_+
Heavy Quark Discretization, f_0
Heavy Quark Discretization, f_+
f_π^{PDG}
z-expansion
Total

Phenomenology

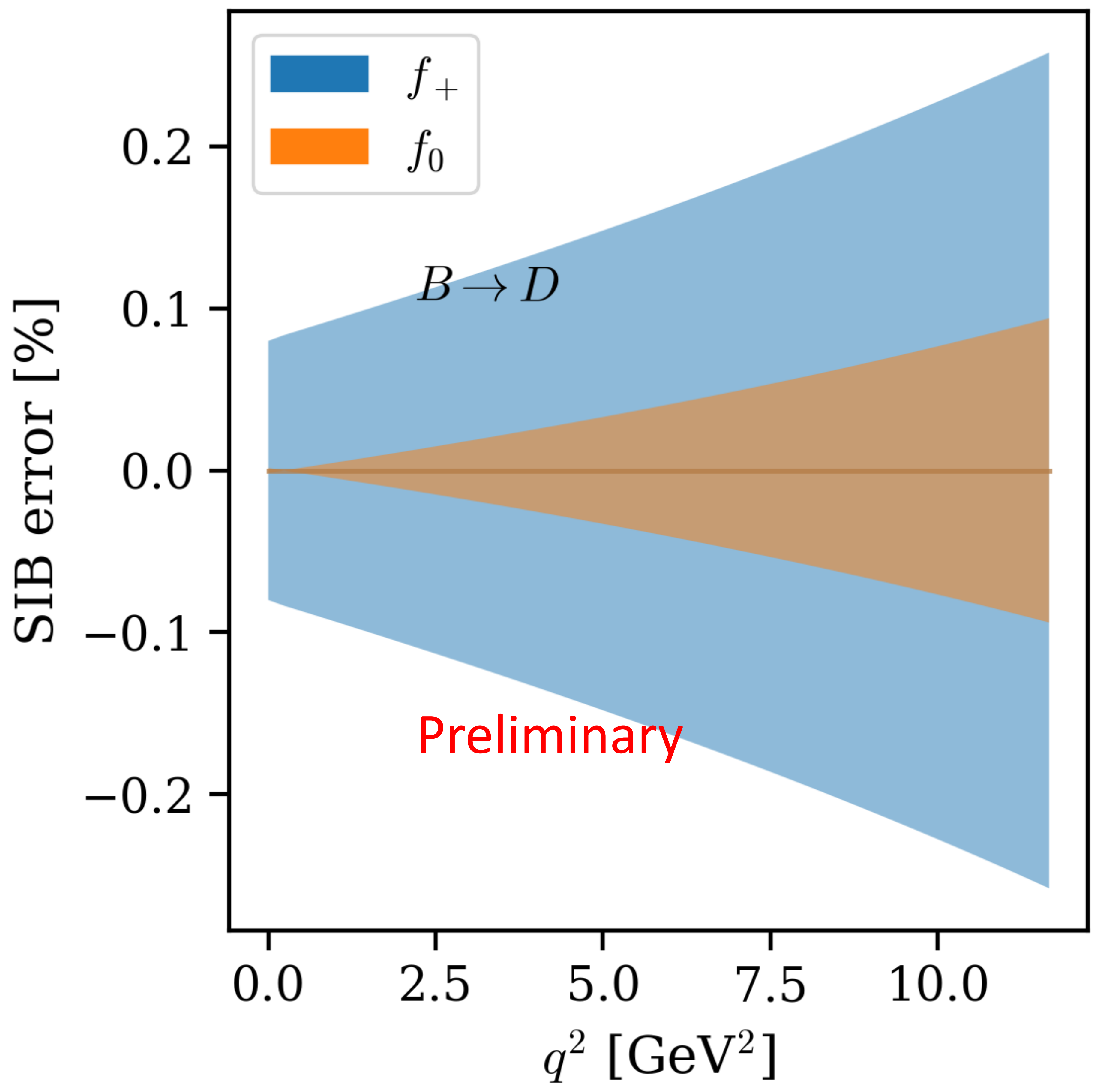
Experimental Status

- $B \to D\ell\nu$
  - Results are pure IsoQCD. Need to include isospin breaking and QED
  - Experimental data from Babar 2009, Belle 2016, BelleII 2025
    - 10 bins of w translated into q^2
    - Correlation matrices provided
- $B_s \to D_s\ell\nu$
  - No binned decay rates available
  - Can combine with $B \to D\ell\nu$

B. Aubert et al. (BaBar Collaboration), *Phys.Rev.Lett.104:011802,2010*, arXiv:0904.4063
Glattauer *et al.* (Belle Collaboration), *Phys. Rev. D* **93**, 032006 (2016), arXiv:1510.03657.
Belle II Collaboration, Phys. Rev. D 112, 112009 (2025), arXiv:2506.13842 (2025).

Isospin Breaking & QED

- Include valence correction only
  - Sea effects are negligible
- Increase errors by $\pm(1 - f_{+,0}^{\text{neutral}}/f_{+,0}^{\text{charged}})$
- QED (TBD)



Joint Fits

Joint LQCD & expt.
BELLE 2016, D^-e^+
BELLE 2016, De^+
BELLEII 2025, D^-e^+
BELLEII 2025, De^+
$|V_{cb}|^2_{\text{joint}} \times$ (LQCD only)
Blinded Preliminary
First Look
$\frac{d\Gamma}{dq^2}\left(\frac{24\pi^3}{G_F^2}\right)$
q^2 [GeV^2]
Joint LQCD & expt.
BELLE 2016, $D^-\mu^+$
BELLE 2016, $D\mu^+$
BABAR 2009, D
average e^+ and μ^+
BELLEII 2025, $D^-\mu^+$
BELLEII 2025, $D\mu^+$
$|V_{cb}|^2_{\text{joint}} \times$ (LQCD only)
Blinded Preliminary
First Look
$\frac{d\Gamma}{dq^2}\left(\frac{24\pi^3}{G_F^2}\right)$
q^2 [GeV^2]

Phenomenological Results

- $|V_{cb}|$
  - Current status: approx 1.5% uncertainty
    - Experiment > Lattice > EW > Isospin Breaking
- $R(D)_{lat}$ approx 1% uncertainty
- $R(D_s)_{lat}$ = approx 0.5% uncertainty

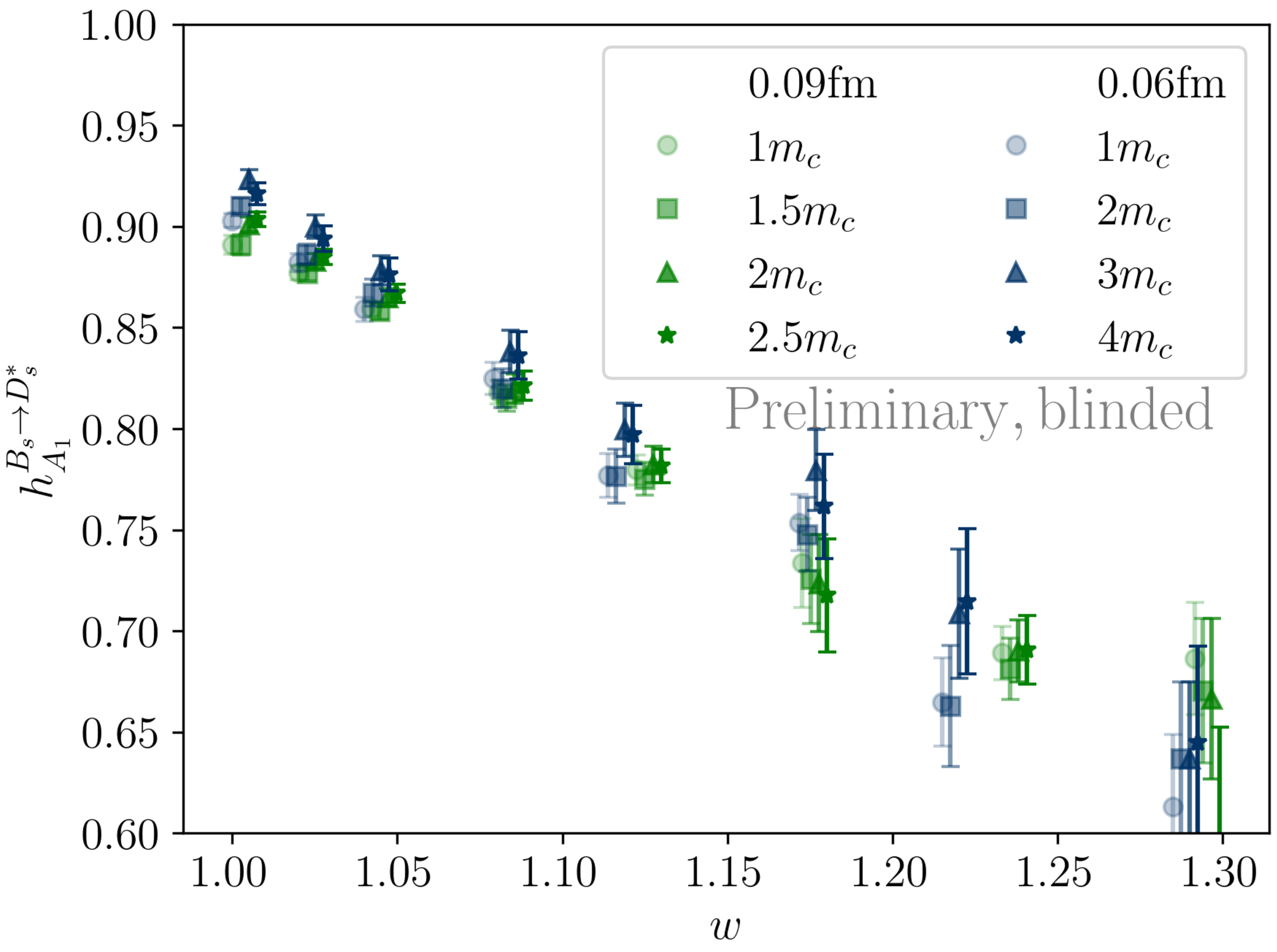
Vector Final States

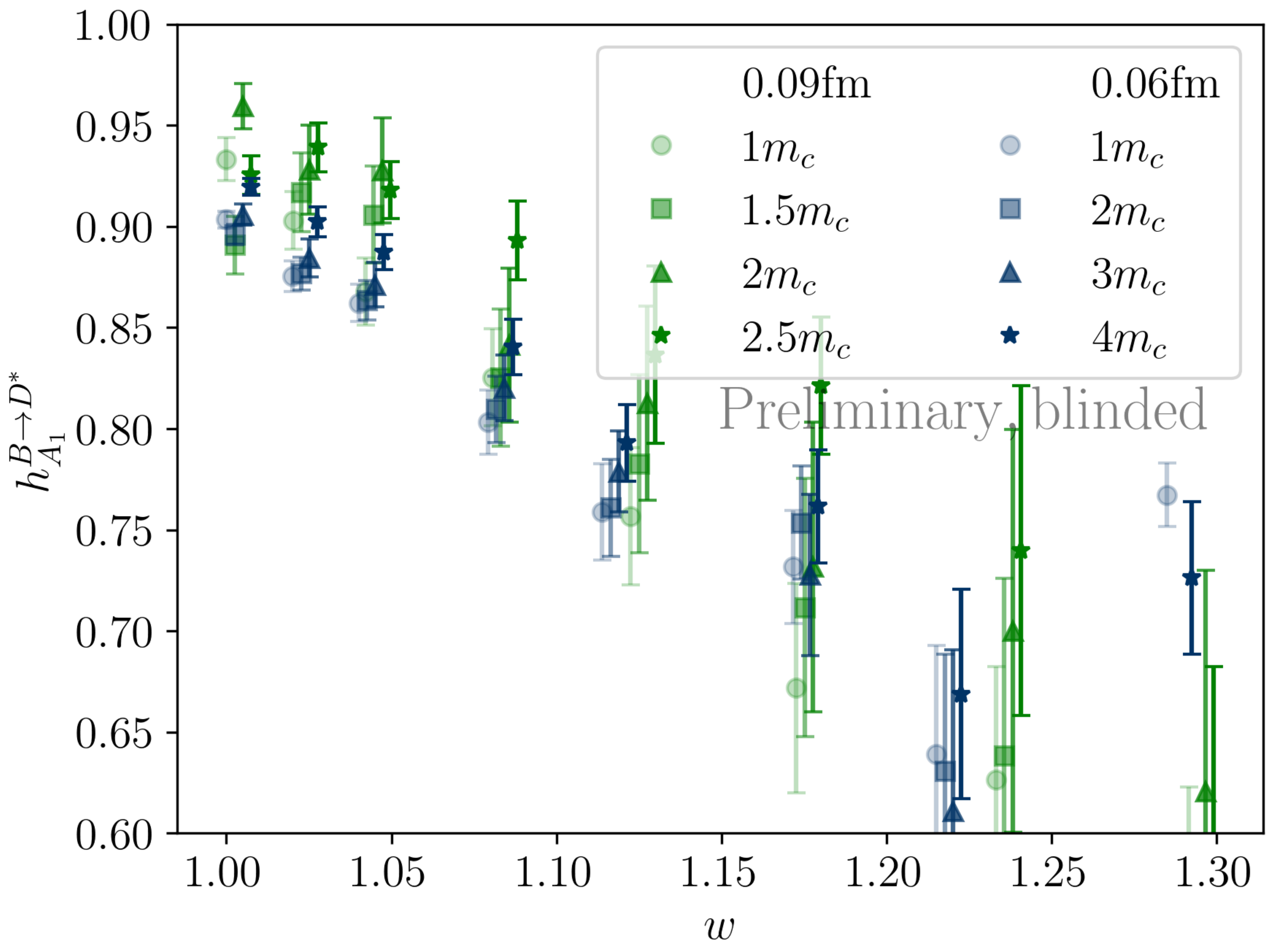
Extend Operator Structure

- Structurally the same as $P \to P$, additional spin taste operators for additional form factors

$O_{H(s)}$	$O_{H'}$	O_J	Matrix element
$\gamma_0\gamma_5 \otimes \gamma_2\gamma_3$	$\gamma_3 \otimes \gamma_3$	$\gamma_2 \otimes \gamma_2$	$\langle H'\|V_1\|H_{(s)}\rangle$
$\gamma_5 \otimes \gamma_5$	$\gamma_1 \otimes 1$	$\gamma_5 \otimes \gamma_5$	$\langle H'\|P\|H_{(s)}\rangle$
$\gamma_5 \otimes \gamma_5$	$\gamma_3 \otimes \gamma_3$	$\gamma_3\gamma_5 \otimes \gamma_3\gamma_5$	$\langle H'\|A_3\|H_{(s)}\rangle$
$\gamma_5 \otimes \gamma_5$	$\gamma_1 \otimes \gamma_1$	$\gamma_1\gamma_5 \otimes \gamma_1\gamma_5$	$\langle H'\|A_1\|H_{(s)}\rangle$
$\gamma_0\gamma_5 \otimes \gamma_2\gamma_3$	$\gamma_3 \otimes \gamma_3$	$\gamma_1\gamma_2 \otimes \gamma_2$	$\langle H'\|T_{12}\|H_{(s)}\rangle$
$\gamma_0\gamma_5 \otimes \gamma_1\gamma_3$	$\gamma_1 \otimes 1$	$\gamma_2\gamma_3 \otimes \gamma_3$	$\langle H'\|T_{23}\|H_{(s)}\rangle$
$\gamma_5 \otimes \gamma_5$	$\gamma_3 \otimes \gamma_3$	$\gamma_0\gamma_2 \otimes \gamma_3\gamma_5$	$\langle H'\|T_{02}\|H_{(s)}\rangle$

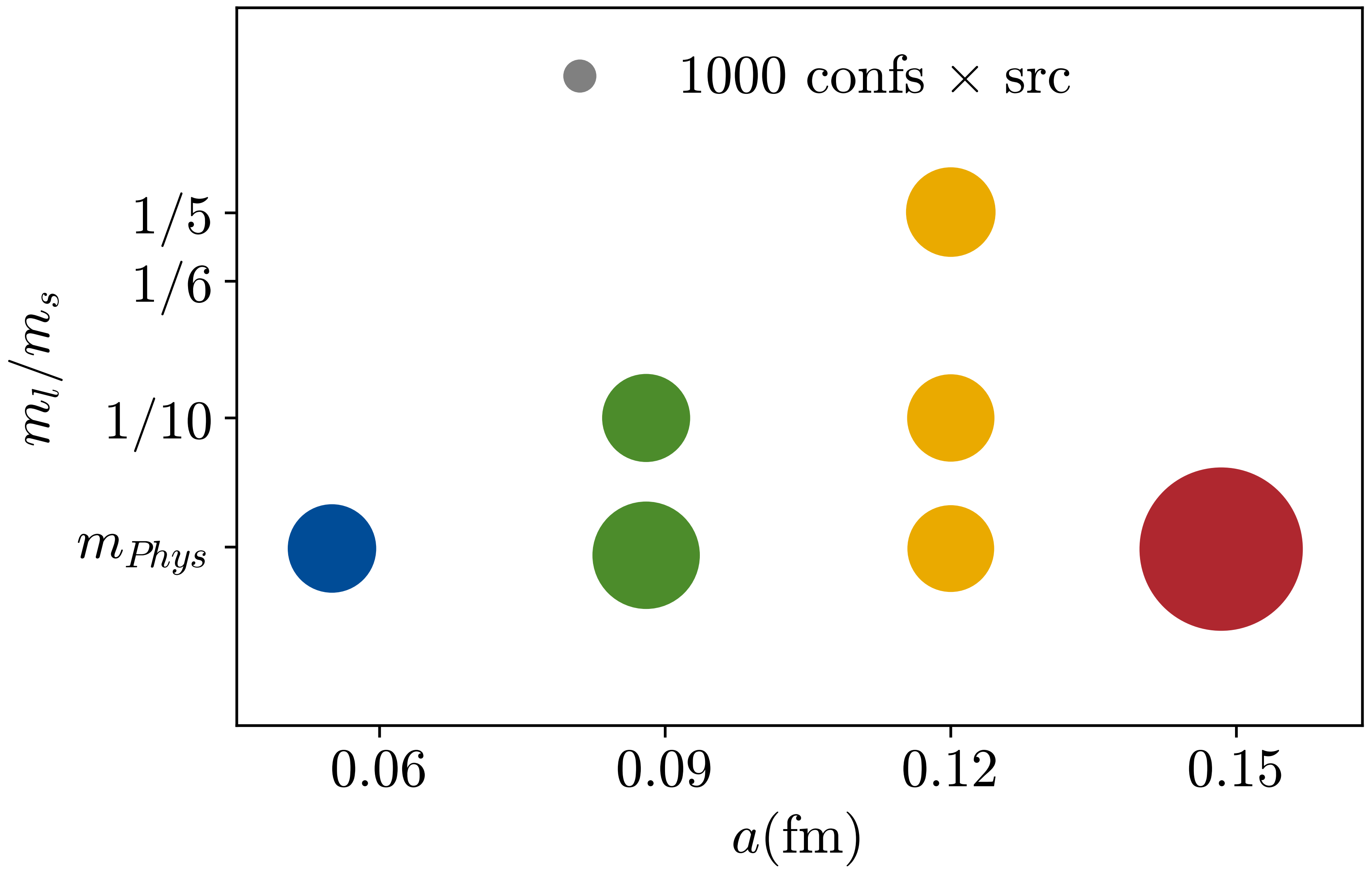
Current Status



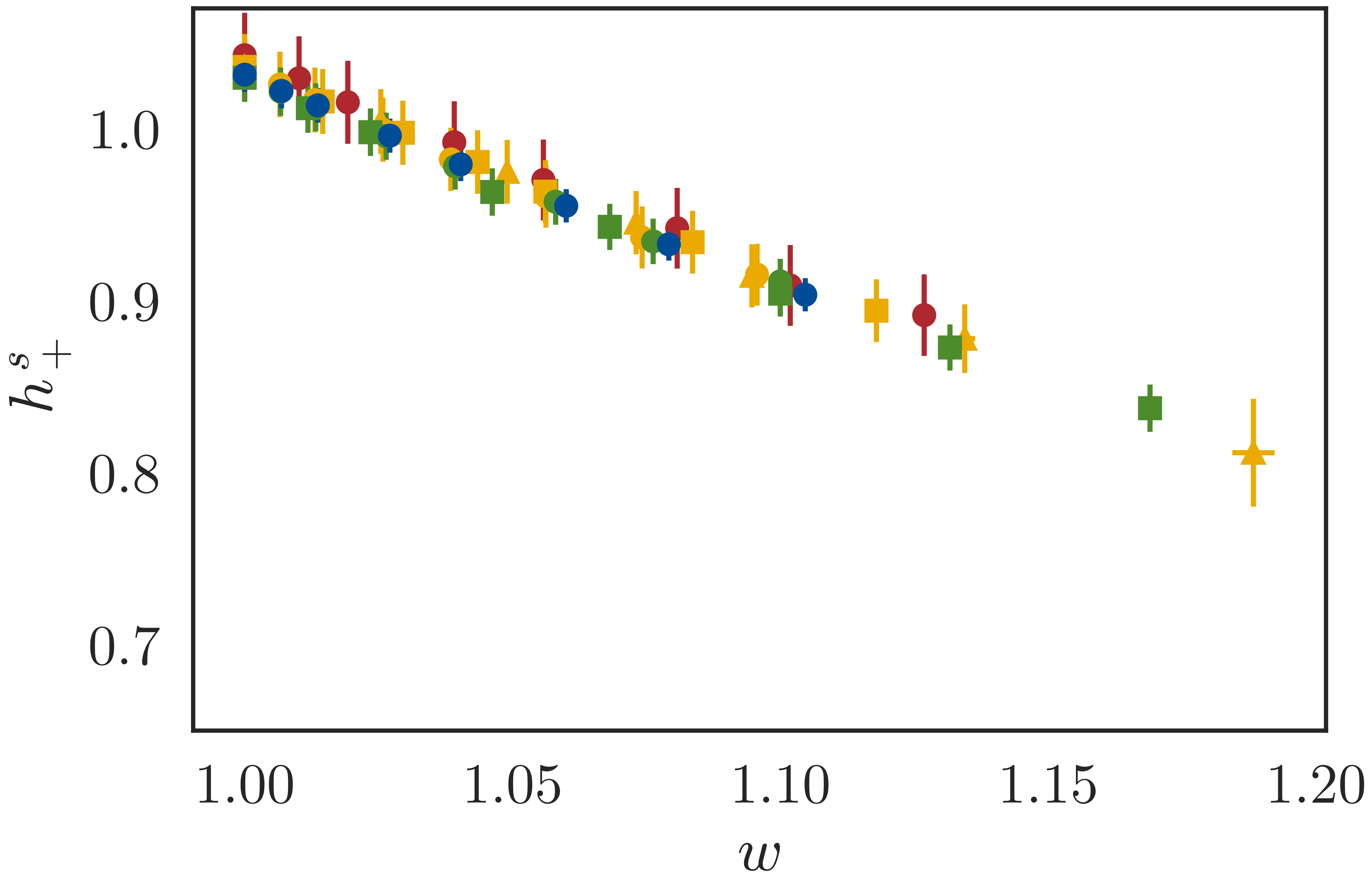


FNAL-HISQ

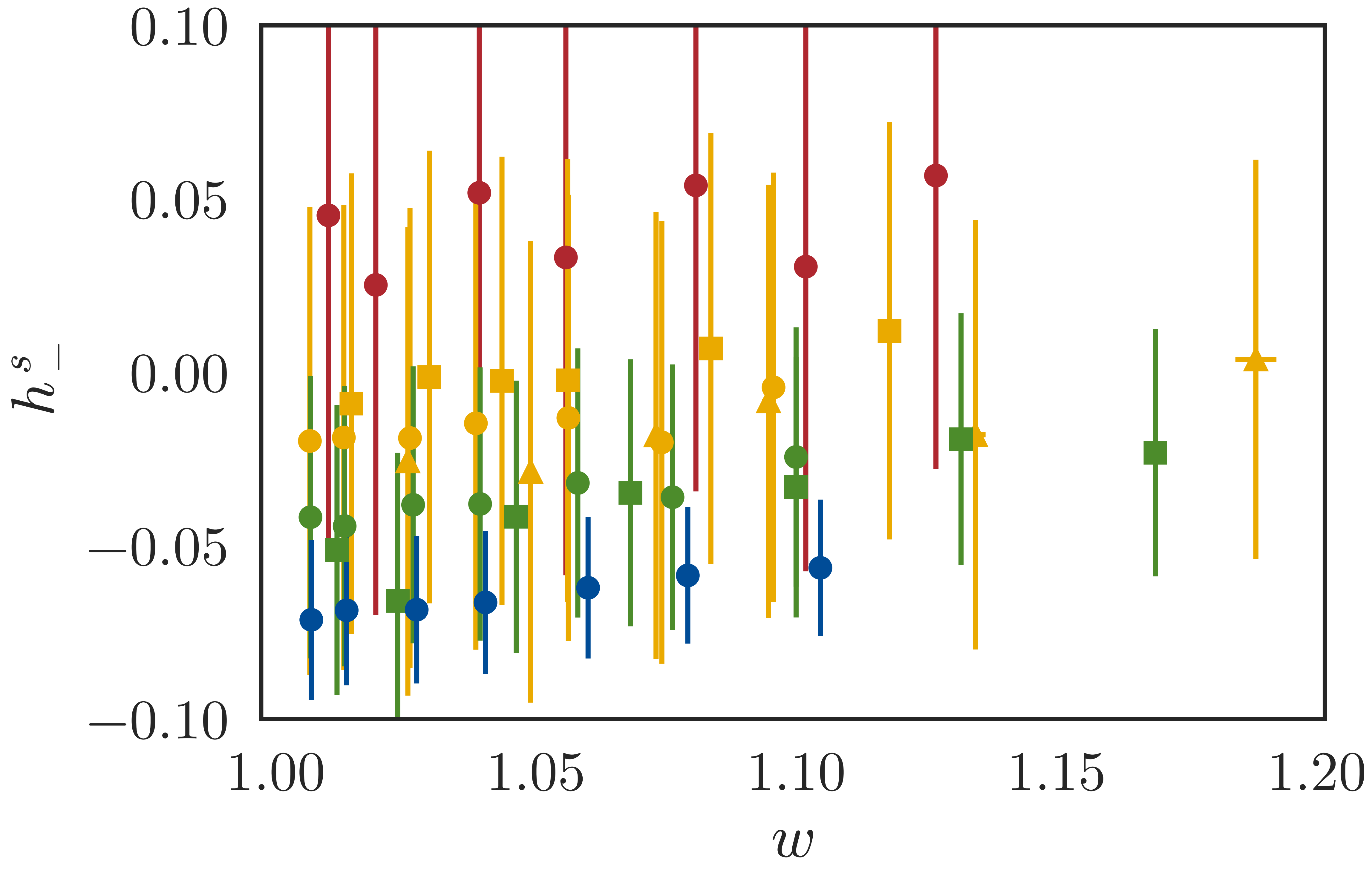
- Concurrent campaign using Fermilab heavy quark
  - Heavy to Heavy analyses done by A. Vaquero
  - Heavy to Light analysis done by H. Jeong



$B_s \to D_s$

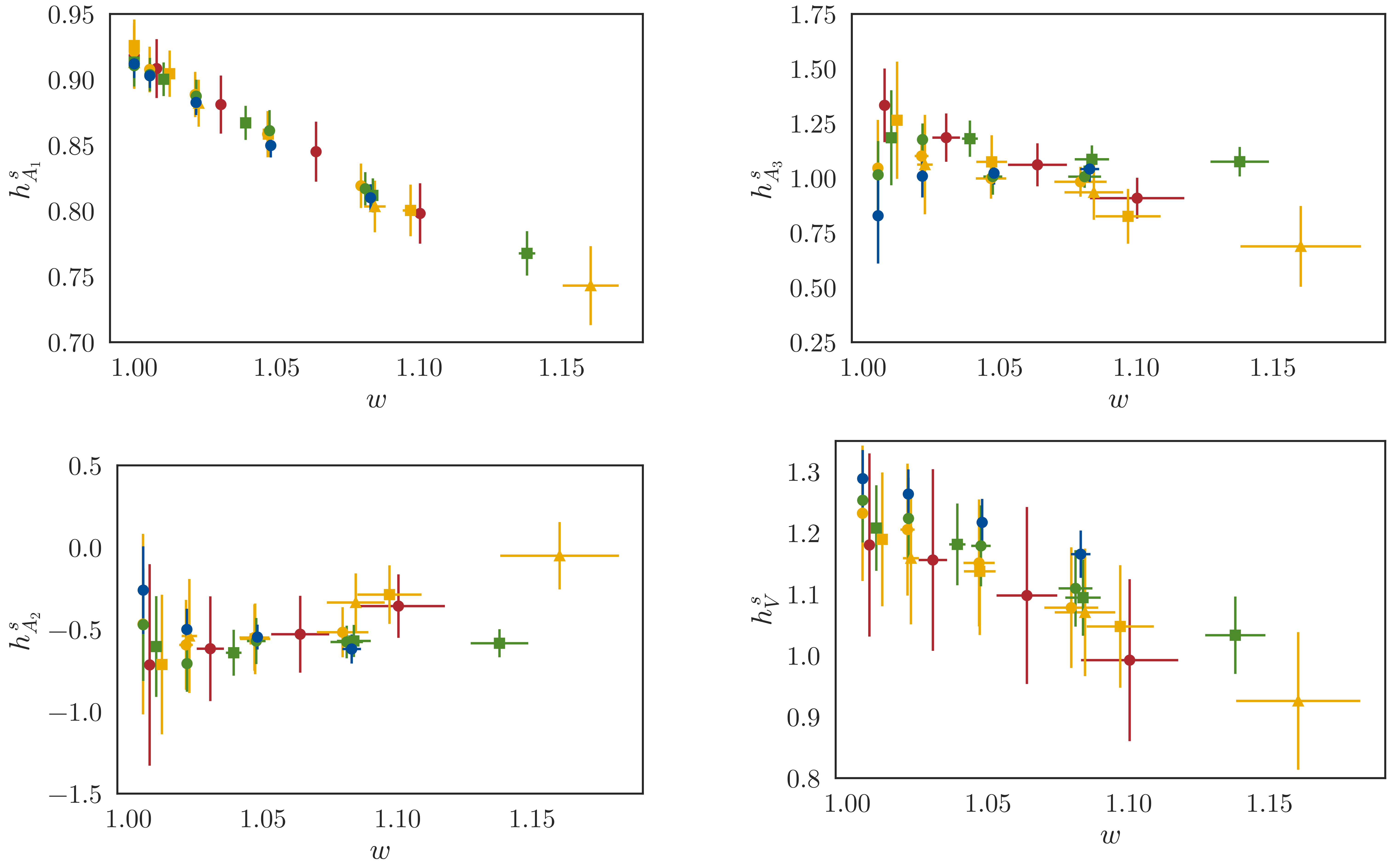


1.0
0.9
0.8
0.7
h_+^s
1.00
1.05
1.10
1.15
1.20
w



0.10
0.05
0.00
−0.05
−0.10
h_-^s
1.00
1.05
1.10
1.15
1.20
w

$B_s \to D_s^*$

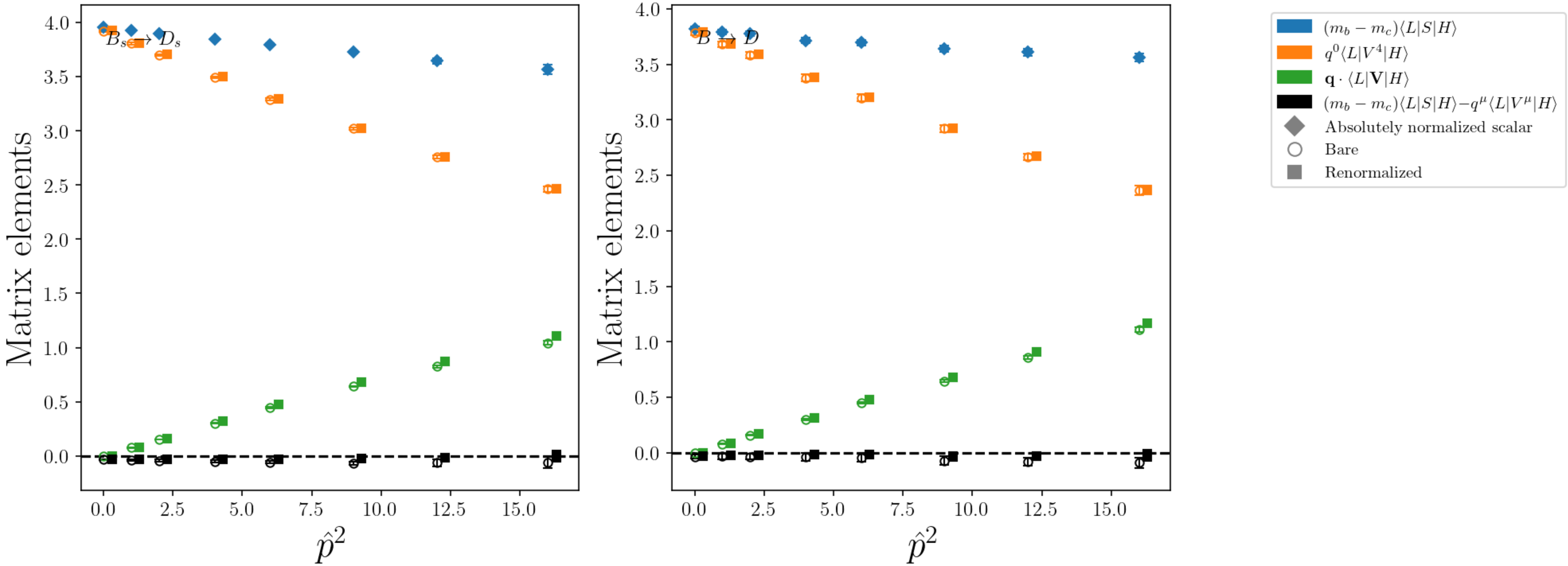


Outlook

- Able to obtain percent level form factors over entire kinematic range
- Currently combining with experimental data for phenomenology
- Finalize error budgets
- Additional
  - Spectator dependence
  - Comparison with previous results
- Publication by end of year

Backup Slides

Nonperturbative Renormalization



Expansion Parameters

$$\chi_l = \frac{(M_\pi^{\rm sim})^2}{8\pi^2 f_\pi^2},$$

$$\chi_s = \frac{(M_K^{\rm sim})^2 - (M_K^{\rm PDG})^2}{8\pi^2 f_\pi^2},$$

$$\chi_E = \frac{E}{M_{D_{(s)}}},$$

$$\chi_H = \frac{\Lambda_{\rm HQET}}{M_{H_{(s)}}^{\rm sim}} - \frac{\Lambda_{\rm HQET}}{M_{B_{(s)}}^{\rm PDG}},$$

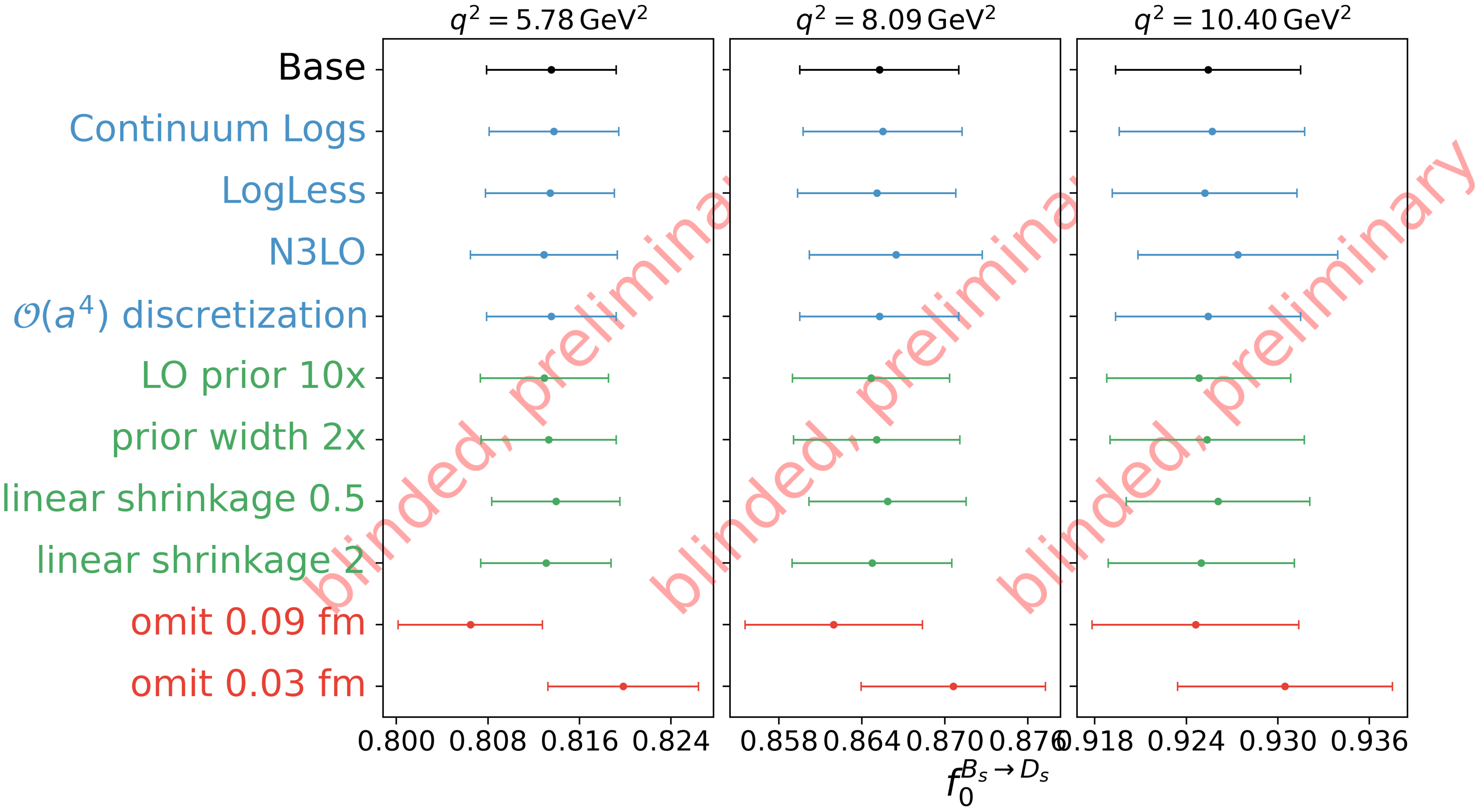
$$\chi_C = \frac{\Lambda_{\rm HQET}}{M_{D_{(s)}}^{\rm sim}} - \frac{\Lambda_{\rm HQET}}{M_{D_{(s)}}^{\rm PDG}}$$

$$x_{a^2} = \frac{a^2\bar{\Delta}}{8\pi^2 w_0^2 f_\pi^2},$$

$$x_h = \frac{2}{\pi} a m_h$$

$$x_c = \frac{2}{\pi} a m_c.$$

Stability Analysis



$B_s \to \mu^+\mu^-$

- Leading source of uncertainty from $|V_{cb}|$

$$\mathrm{BR}(B_s^0 \to \mu^+\mu^-) = \mathrm{BR}(B_q \to X)\frac{f_q}{f_s}\frac{\epsilon_X}{\epsilon_{\mu\mu}}\frac{N_{\mu\mu}}{N_X},$$

- f_q is a fragmentation fraction (probability for b-quark to hadronize into a B_q meson)

$$\frac{f_s}{f_d} = 0.0743 \times \frac{\tau_{B^0}}{\tau_{B_s^0}} \times \left[\frac{\epsilon_{DK}}{\epsilon_{D_s\pi}}\frac{N_{D_s\pi}}{N_{DK}}\right] \times \frac{1}{\mathcal{N}_a\mathcal{N}_F}$$

$$\mathcal{N}_F = \left[\frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_K^2)}\right]^2.$$

$|V_{cb}|$

- Current status: approx 1.5% uncertainty
  - Experiment > Lattice > EW > Isospin Breaking
  - QED?
- FLAG 2024:
  - $B \to D\ell\nu \; |V_{cb}| = 40.0(1.0) \times 10^{-3}$
    - 2.5% Uncertainty
  - Global fit $|V_{cb}| = 39.46(53) \times 10^{-3}$
    - 1.3% Uncertainty

$R(D_{(s)})_{lat}$

- Current status
  - $R(D)_{lat}$ = approx 1% uncertainty
  - $R(D_s)_{lat}$ = approx 0.5% uncertainty
- Flag 2024
  - $R(D)_{lat} = 0.2938(54)$
    - 1.84 % Uncertainty
  - $R(D_s)_{lat} = 0.2987(46)$
    - 1.54% Uncertainty

$B_s \to D_s \ell \nu$

- Experimentally binned differential decay rate not available
- Two q^2 bins for $(\mathscr{B}(B_s \to K)/\mathscr{B}(B_s \to D_s))$ available to obtain $|V_{ub}|/|V_{cb}|$
  - 10 q^2 bins for $B_s \to K$ will be available $\mathcal{O}(1$ year) <- N. Cassar
- Can be used for other phenomenological applications