NN scattering amplitudes at light quark masses

Miguel Salg

For the Baryon Scattering (BaSc) Collaboration

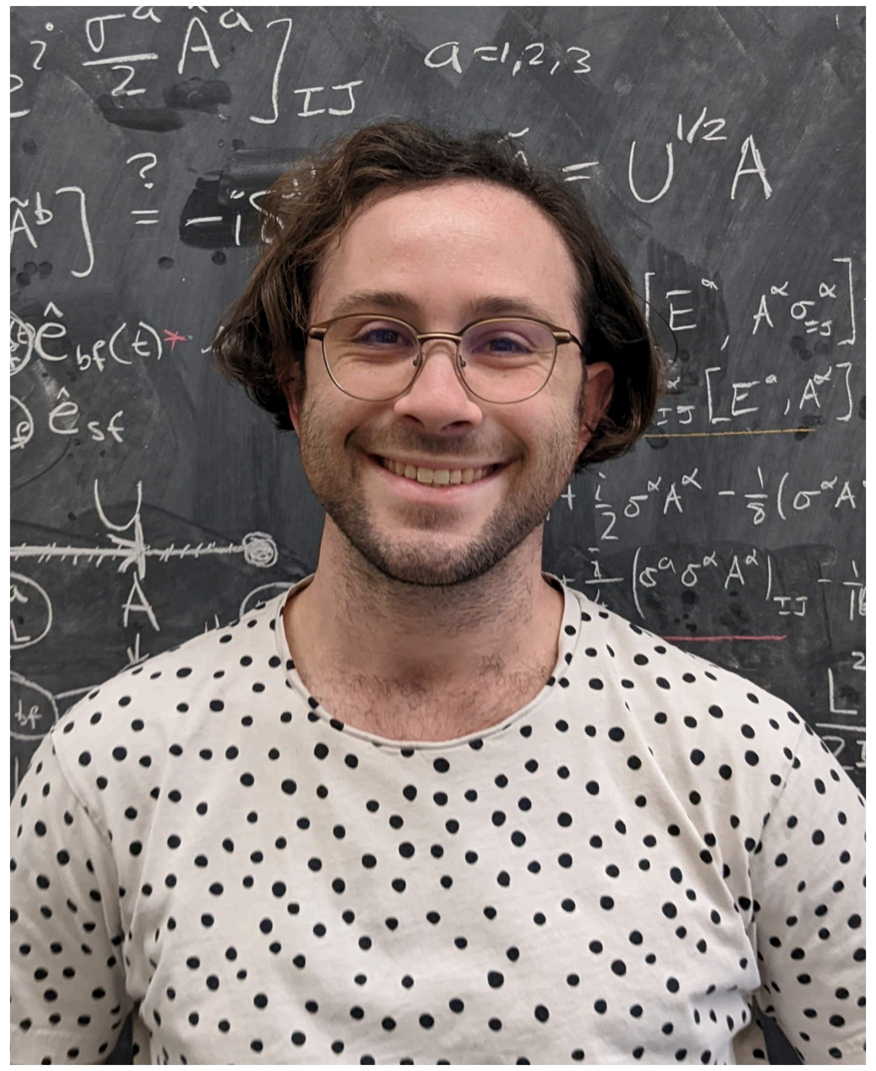
Lattice 2026, University of Maryland, College Park, July 26 — August 1, 2026

Collaborators

Jorge Baeza-Ballesteros

André Baião Raposo

Davide Laudicina



Malcolm Lazarow

John Bulava, Andrew Hanlon, Colin Morningstar, Fernando Romero-López, Hartmut Wittig

Jeremy Green, Daniel Mohler, Amy Nicholson, André Walker-Loud, + BaSc collaboration

NN scattering from lattice QCD

Deuteron towards the physical pion mass

Bulava et al. (BaSc) 2026 [PRC 113, 024002]

- Large pion masses ($M_\pi \approx 710$ MeV): both $I = 0$ and $I = 1$ *NN* interaction attractive, but not strong enough to form bound states
  —> deuteron is virtual state

Inoue et al. (HAL QCD) 2012 [Nucl. Phys. A 881, 28]
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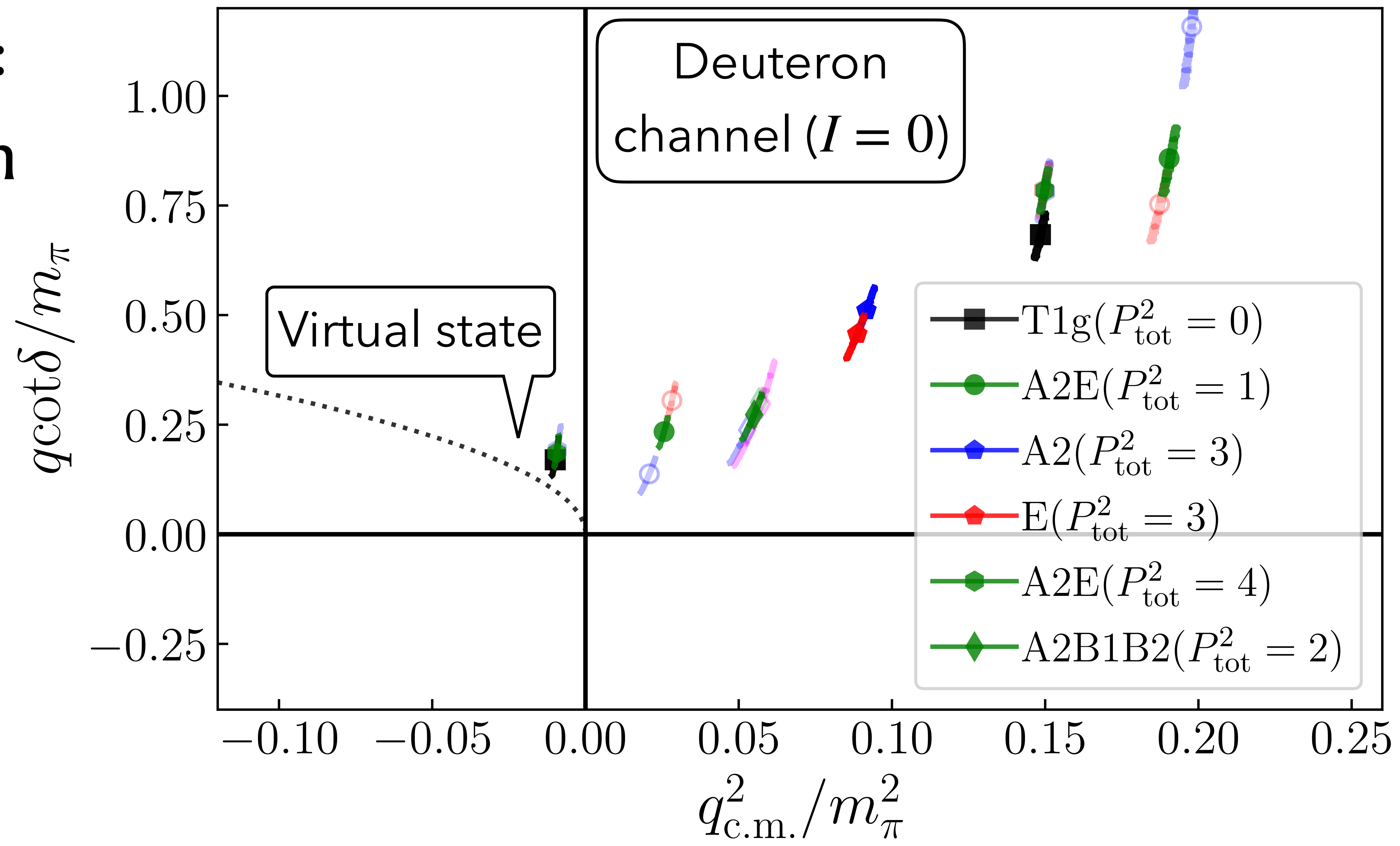

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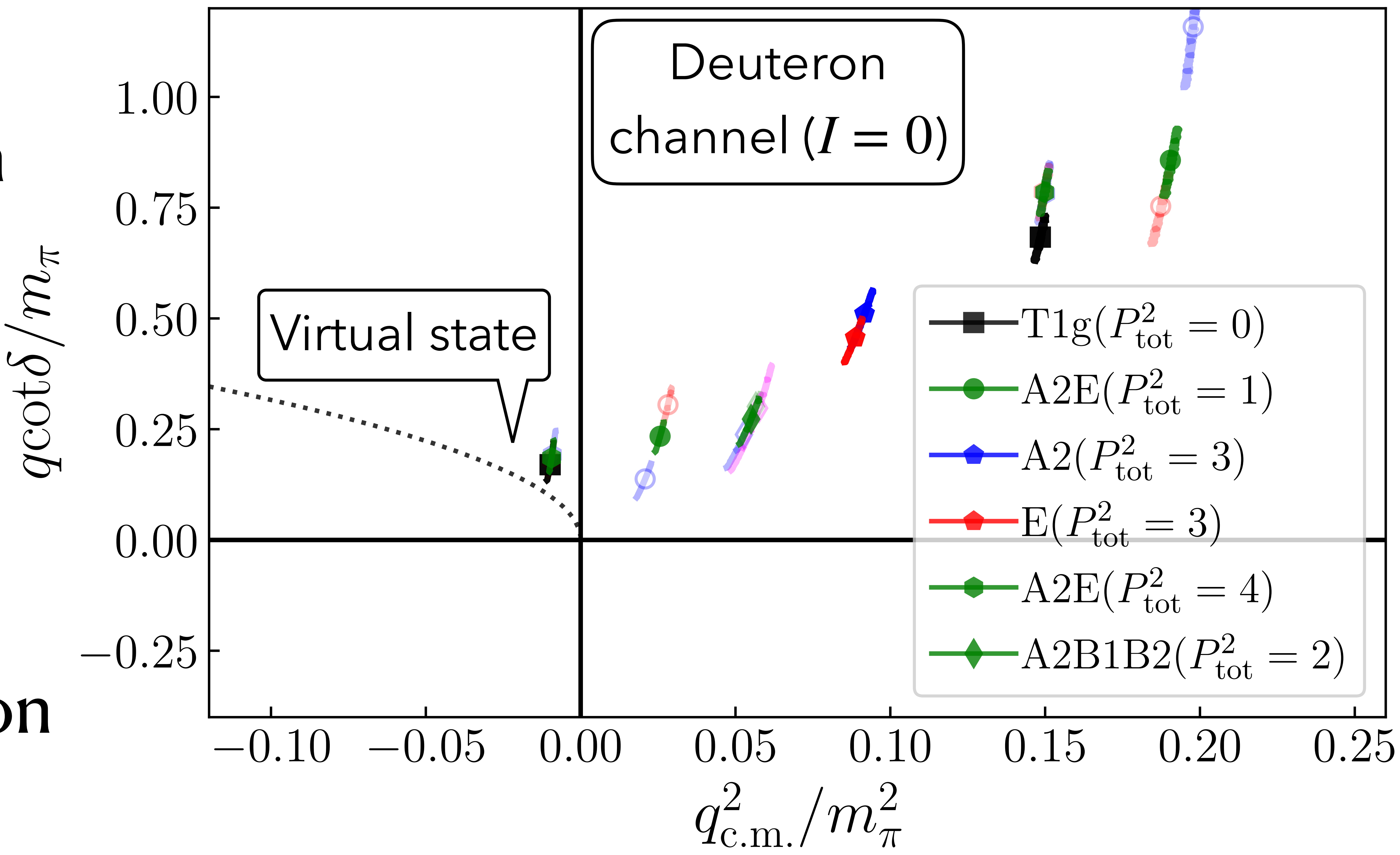

- Goal: find value of M_π where deuteron unbinds

- Now: preliminary results at $M_\pi = 280$ MeV (CLS D251) and $M_\pi = 200$ MeV (J250)

NN scattering from lattice QCD

Robert Perry, plenary Thu 09:30
Joseph Moscoso, Fri 15:00
William Detmold, Fri 16:10
Michael Wagman, Fri 16:30
Jorge Baeza-Ballesteros, poster session A

Deuteron towards the physical pion mass

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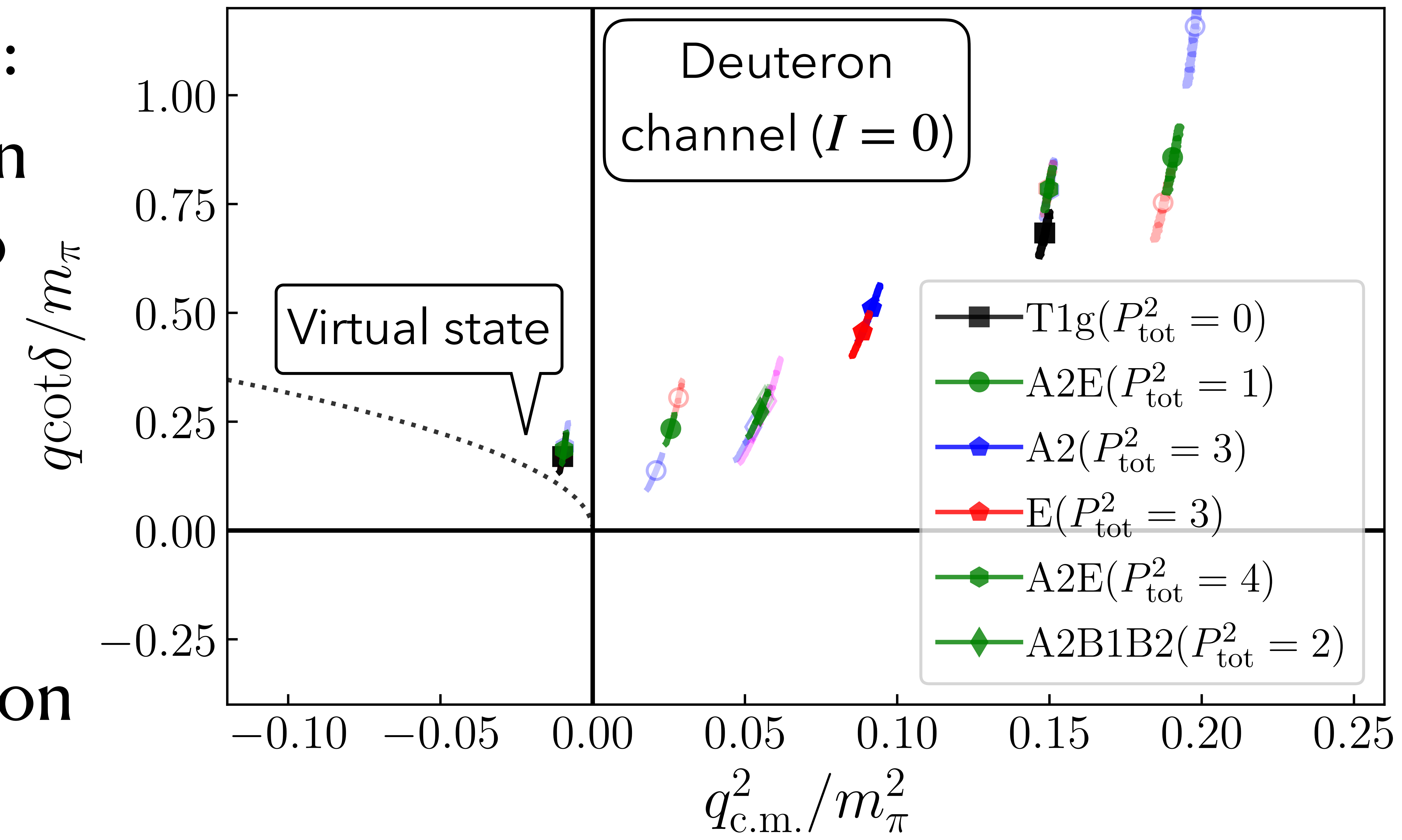

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Extraction of the finite-volume spectrum

- Build correlator matrix from basis of bilocal two-nucleon operators, $N(\mathbf{p}_1)N(\mathbf{p}_2)$ —> GEVP

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  - Exploits correlations between excited states and noise
  - Directly constrains energy shifts $\delta = E_{NN} - E_{N_1} - E_{N_2}$
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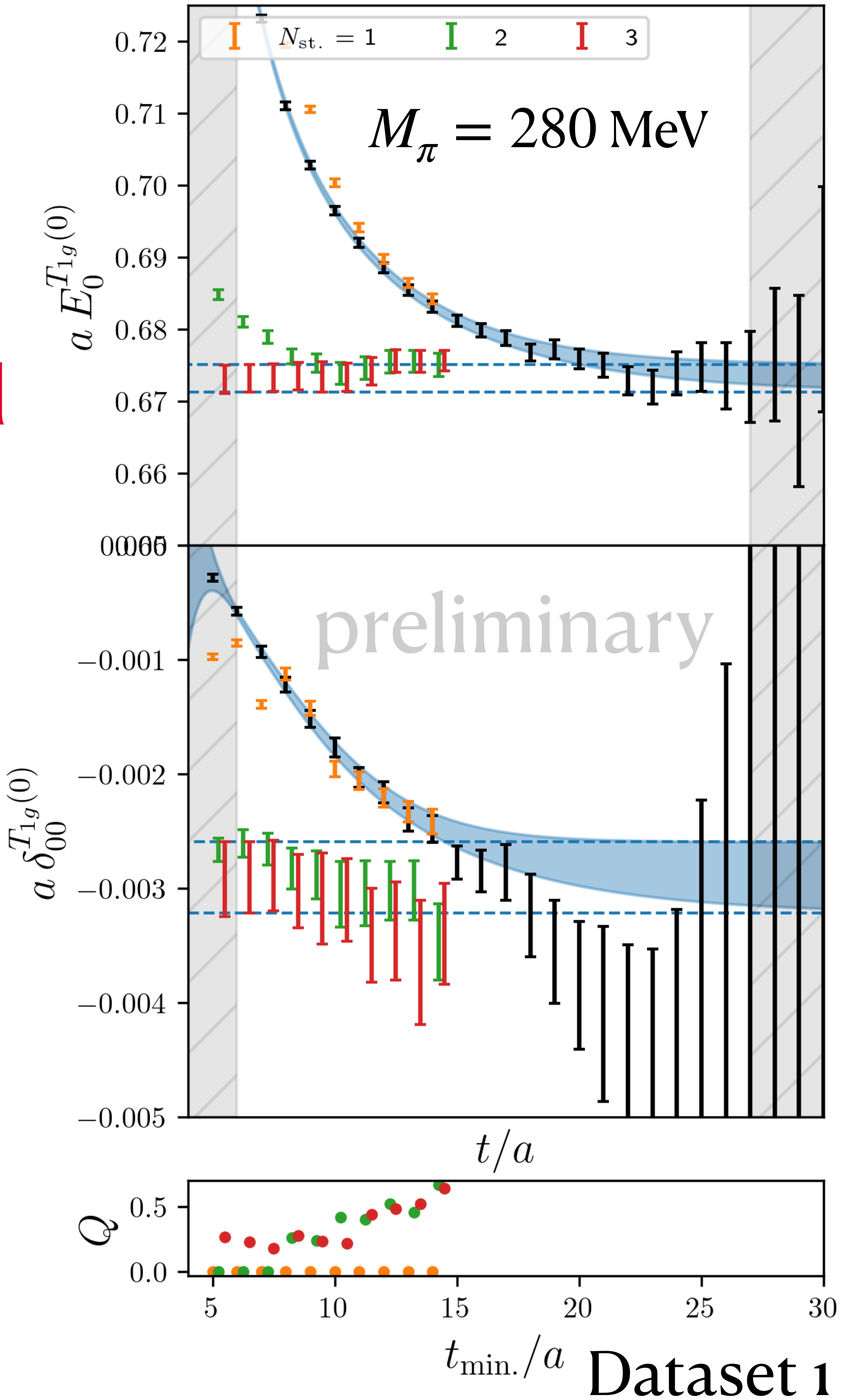

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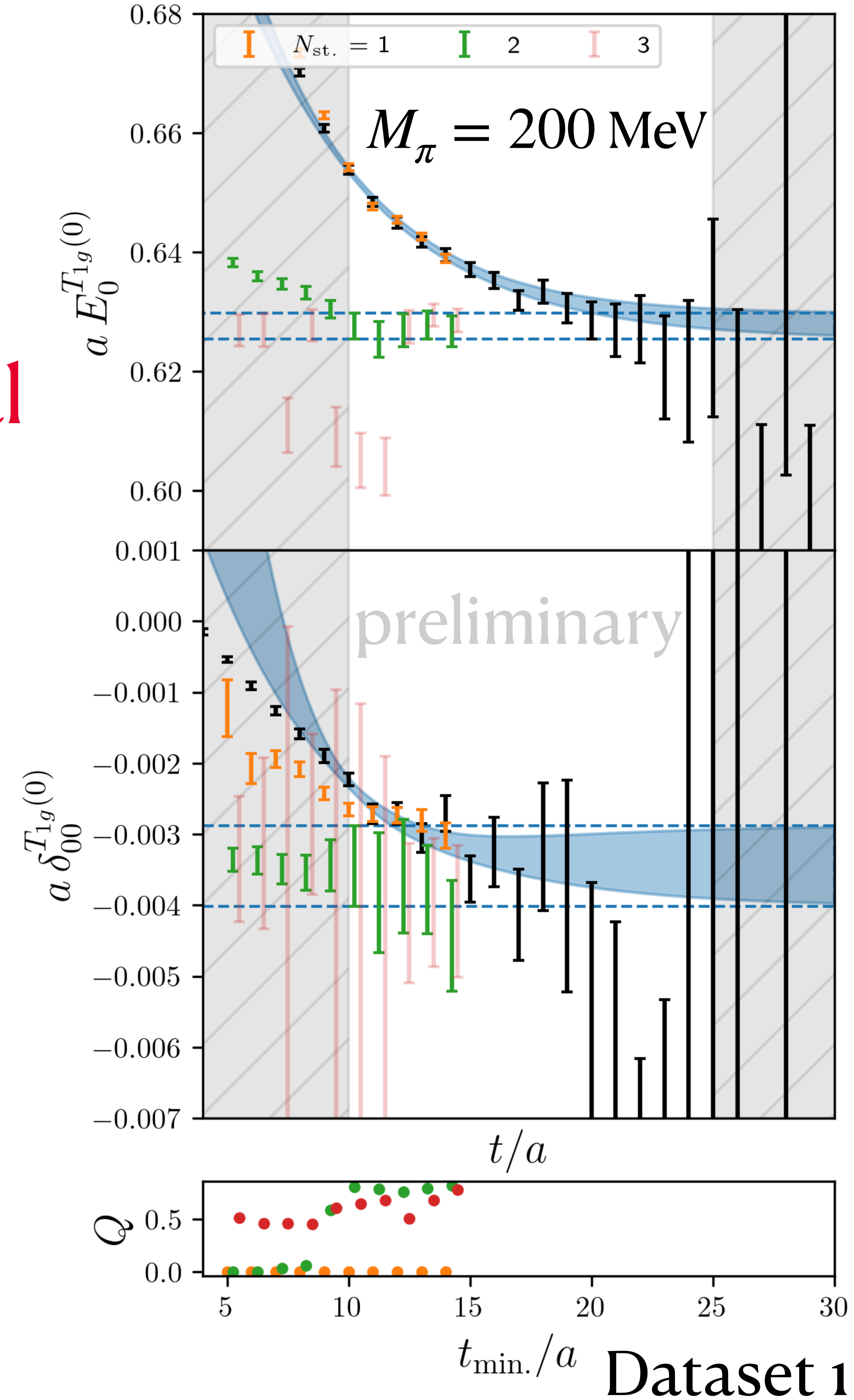
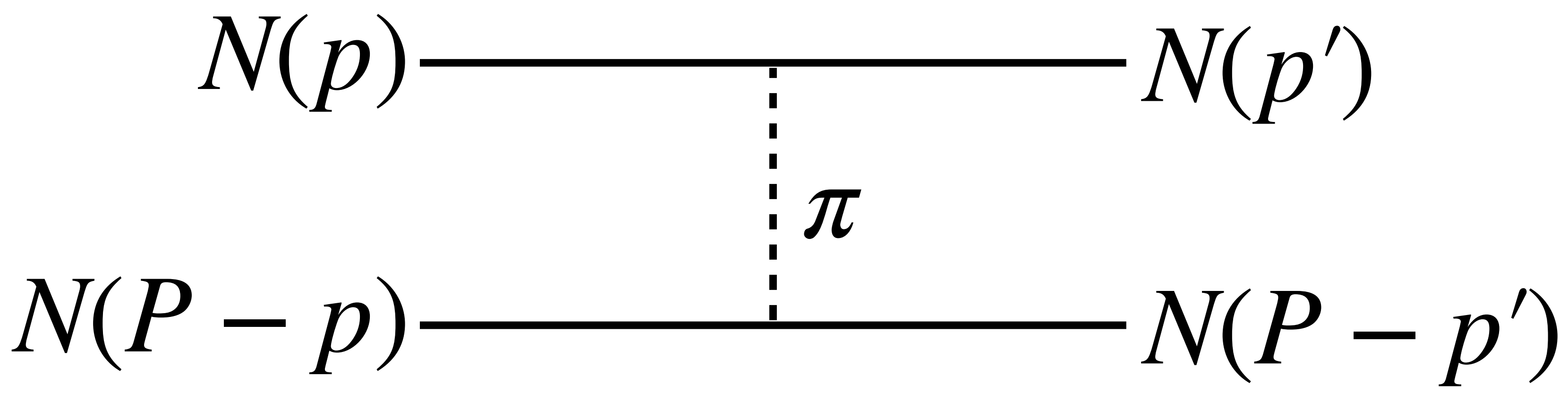


Dataset 1

NN scattering: left-hand cuts

- t-channel exchange of light particles (here: pions) leads, after partial-wave projection, to left-hand cut (lhc) at $-\infty < E^2 < 4m_N^2 - m_\pi^2$ in scattering amplitude



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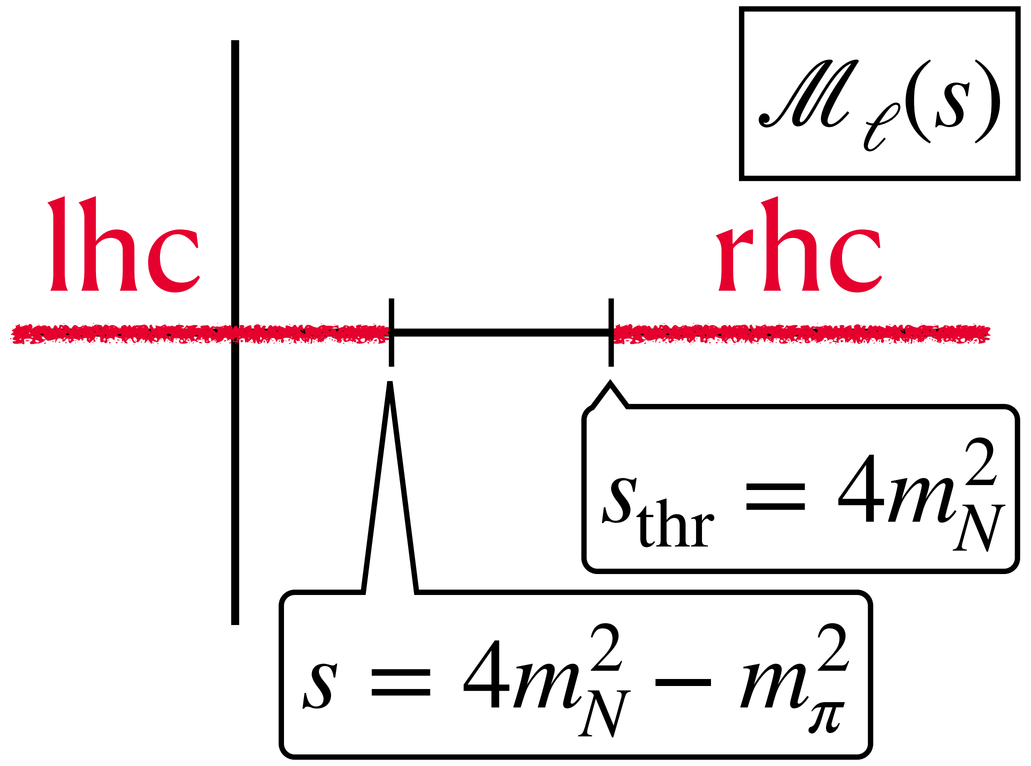

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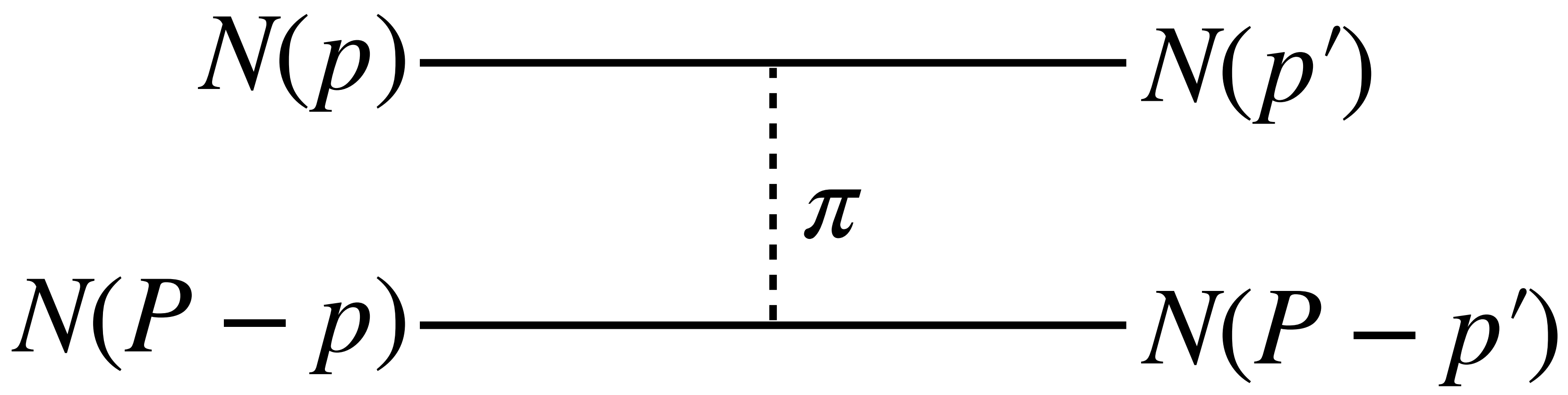

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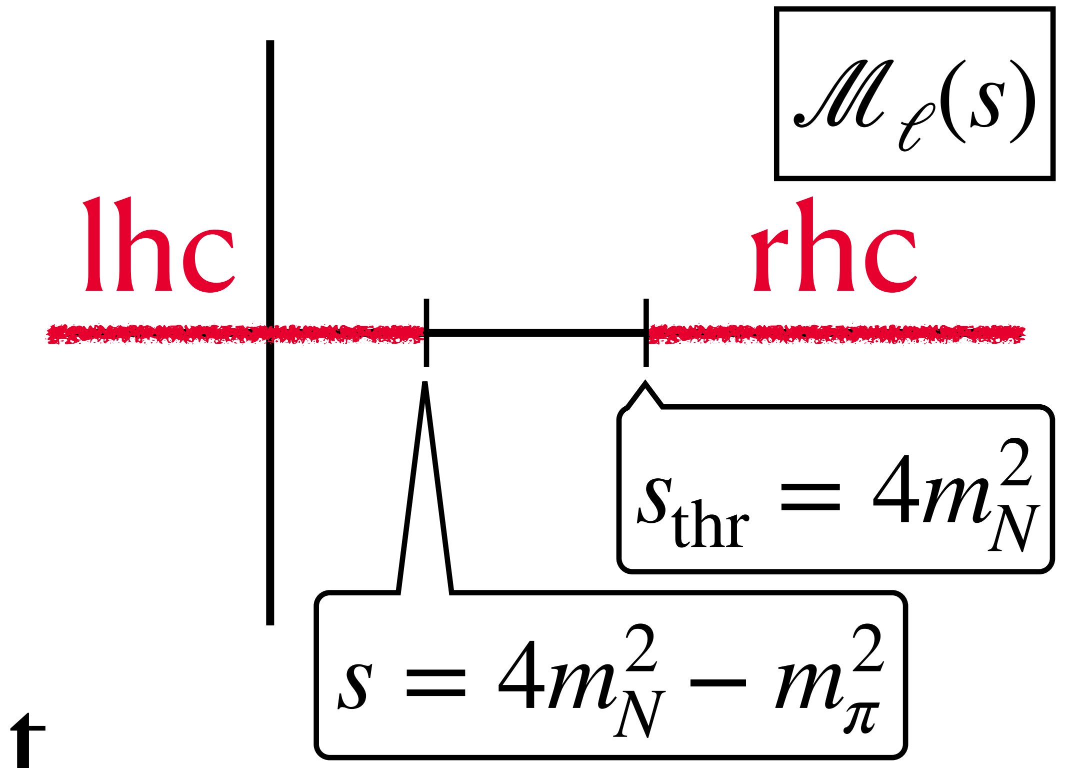

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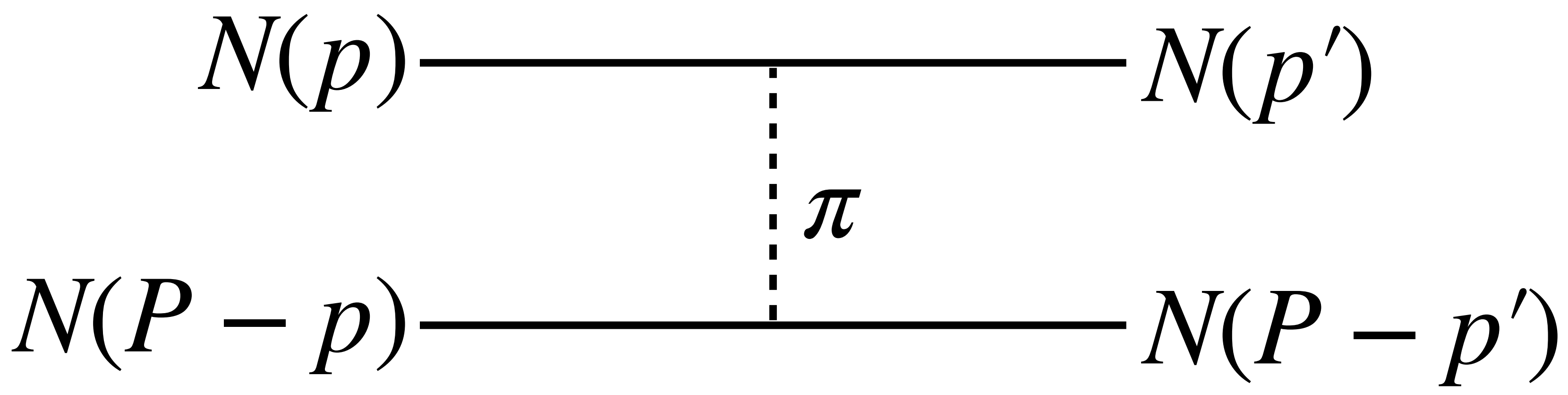

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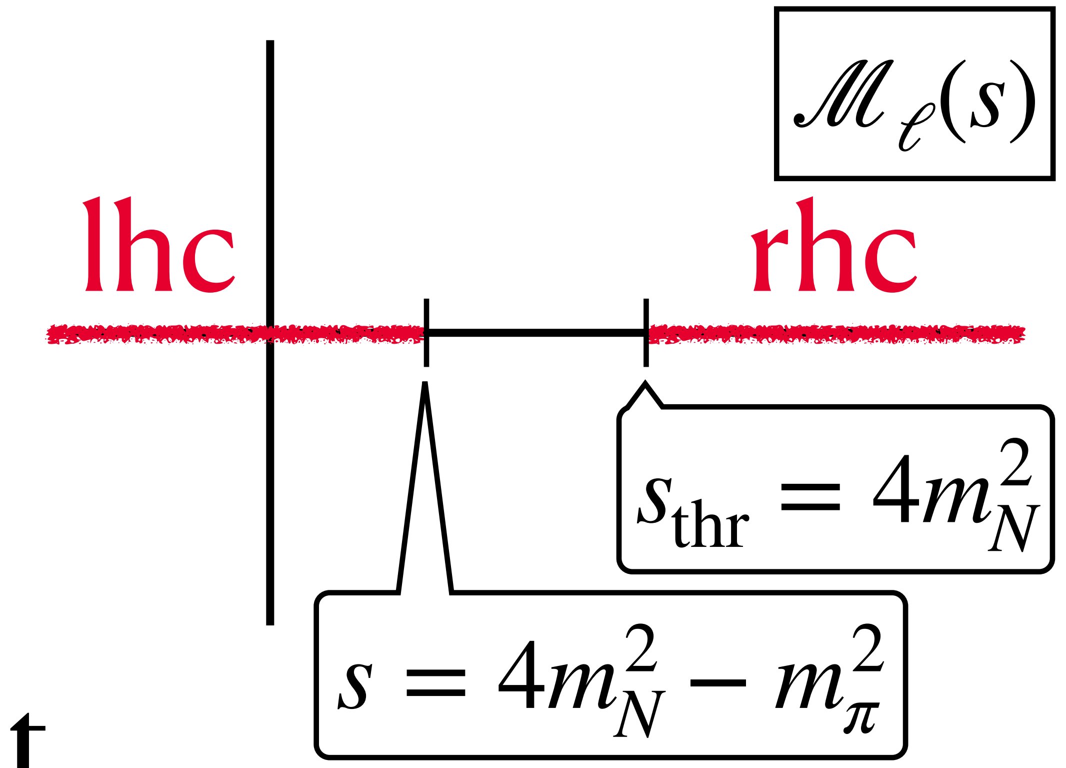

- This induces uncontrolled finite-volume effects close to the cut
- Here: two alternatives that are valid in the presence of lhcs
  - Relativistic formalism based on all-orders perturbation theory
  - QC in plane-wave basis —> based on Lippmann-Schwinger eq. with NR EFT potential

Baião Raposo and Hansen 2024 [JHEP 2024, 075]
Baião Raposo et al. 2025 [JHEP 2025, 186]

Meng and Epelbaum 2021 [JHEP 2021, 051]
Meng et al. 2024 [PRD 109, L071506]
Jorge Baeza-Ballesteros, poster session A

NN scattering: left-hand cuts

Relativistic formalism

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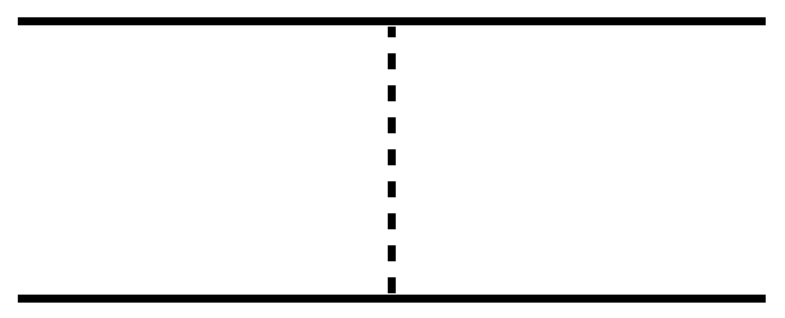
- Idea: derive modified QC taking into account one-particle exchanges (OPEs),

$$\det_{aJm_J\ell S}\left[\mathscr{K}_0^{-1}(E) + F_{2,L}(E,\mathbf{P}) + \mathscr{C}_L(E,\mathbf{P})\right]_{E=E_n} = 0$$

Modified K-matrix, $\mathscr{K}_0(E \in \mathbb{R}) \in \mathbb{R}$, non-singular

Lüscher kinematic FV function



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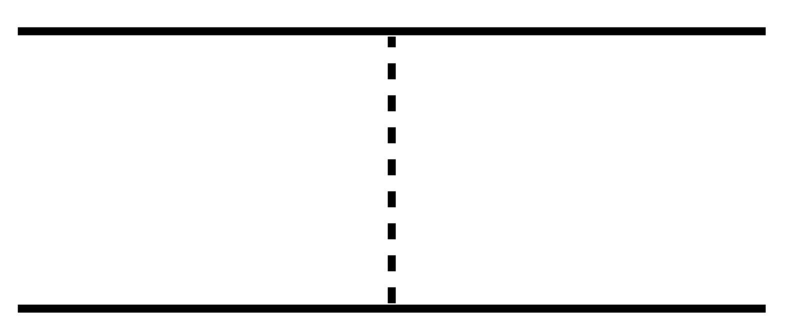

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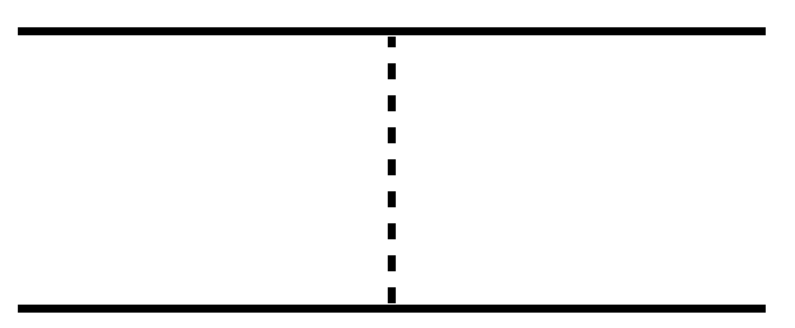

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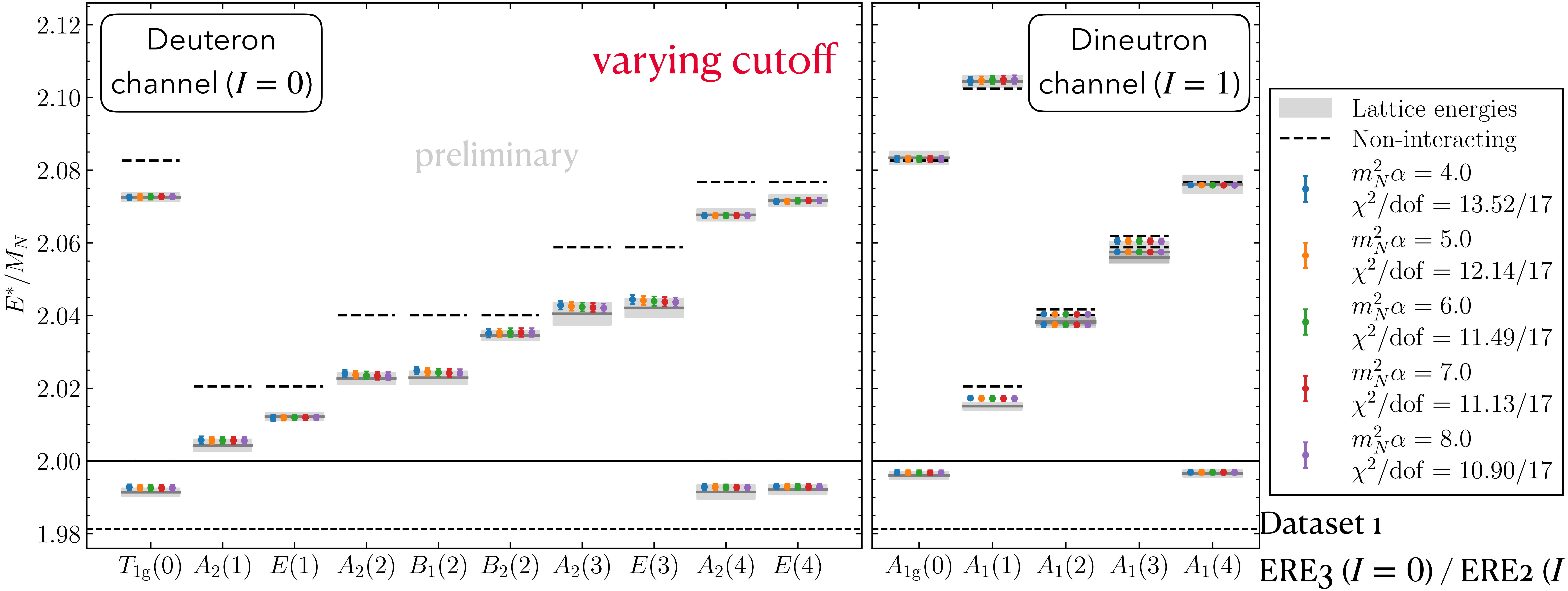
- Infinite-volume scattering amplitude can be obtained by solving known integral eqs.
- $\mathscr{C}_L$ contains inverse of matrix in plane-wave basis —> numerically more expensive
- OPE contributes to all partial waves since partial-wave projection is done only at the end

NN scattering on D251 ($M_\pi = 280$ MeV)

Finite-volume spectrum and fit stability (relativistic formalism)

Combined fit to both isospins with OPE coupling $g_{NN\pi}$ as common fit parameter



Dataset 1

ERE3 ($I = 0$) / ERE2 ($I = 1$)

no SD mixing

NN scattering on D251 ($M_\pi = 280$ MeV)

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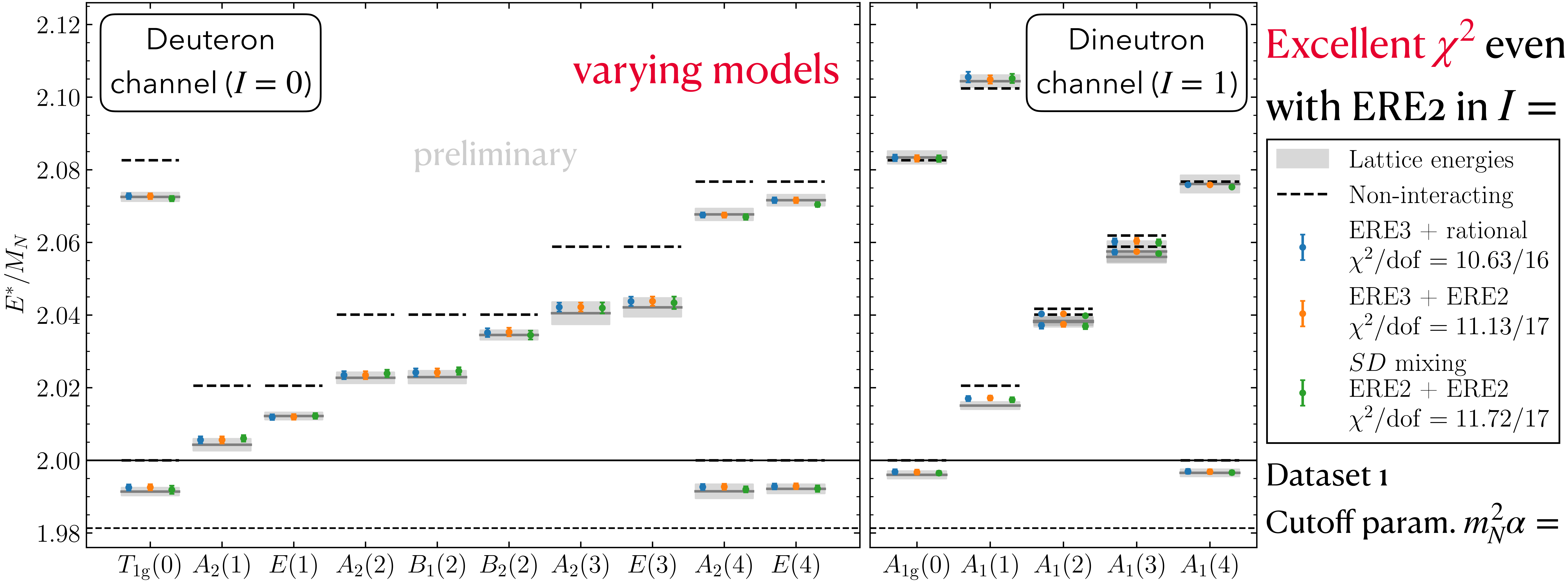


Excellent χ^2 even with ERE2 in $I = 1$

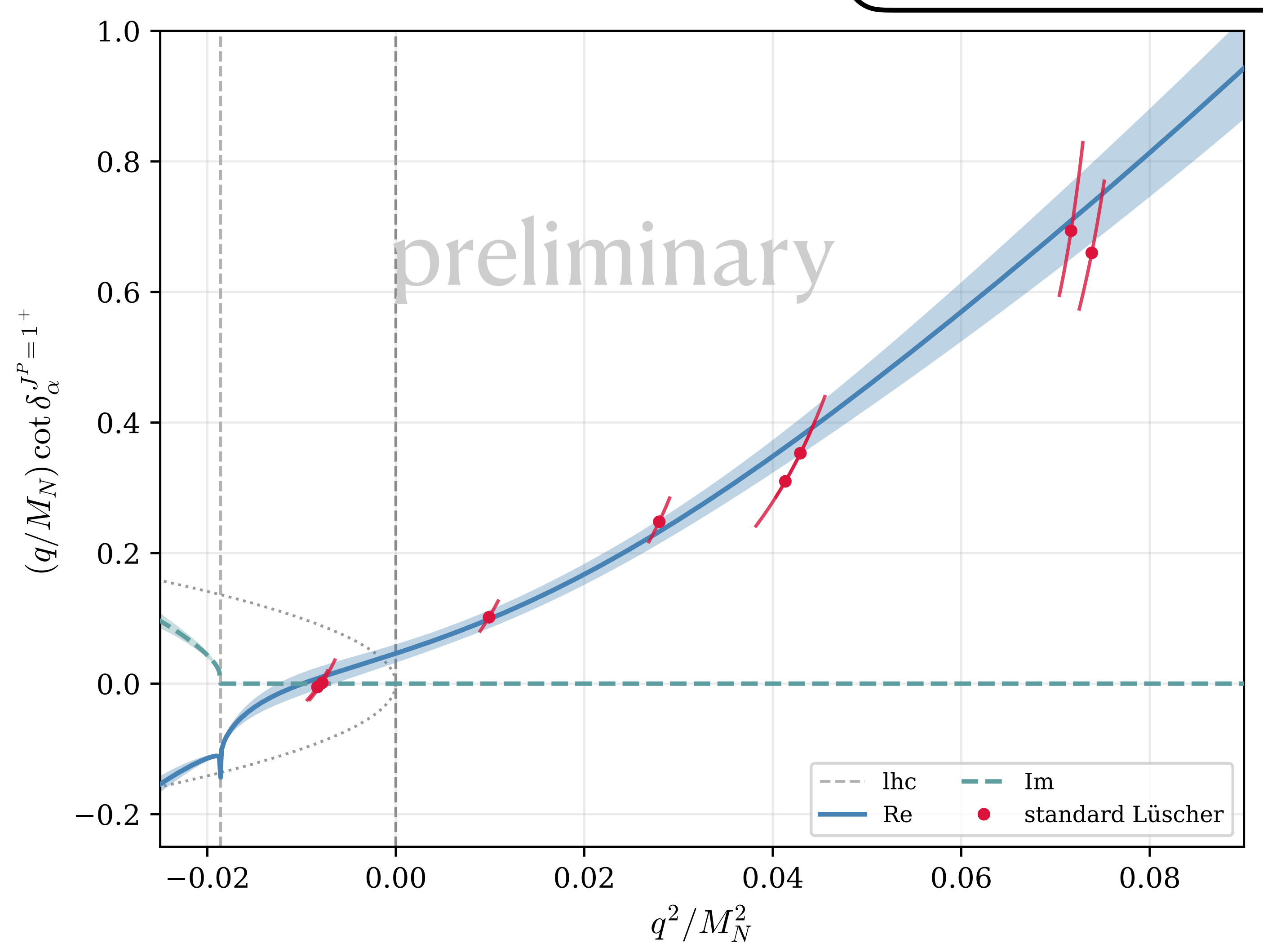
Dataset 1

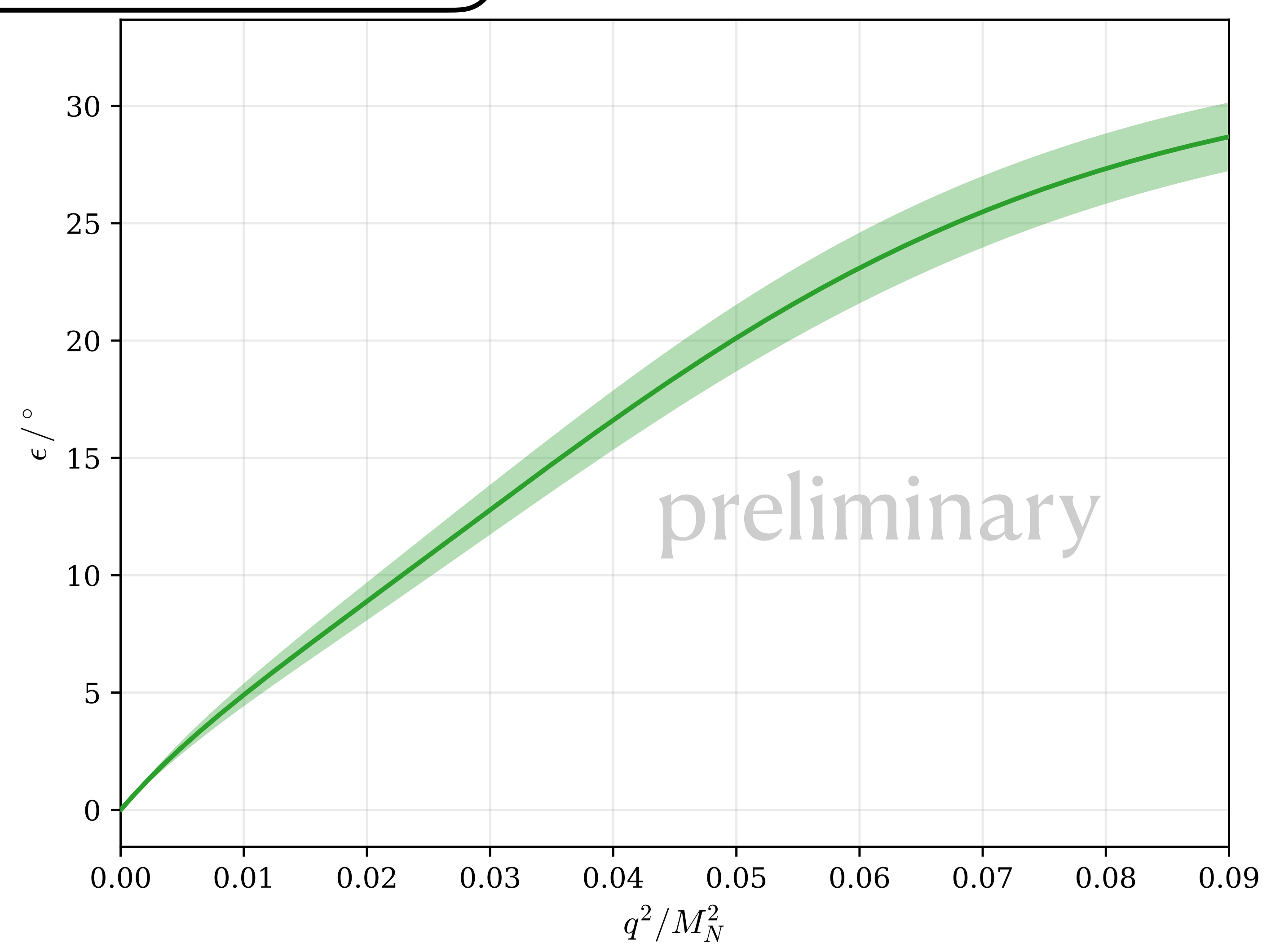
Cutoff param. $m_N^2\alpha = 7$

NN scattering on D251 ($M_\pi = 280$ MeV)

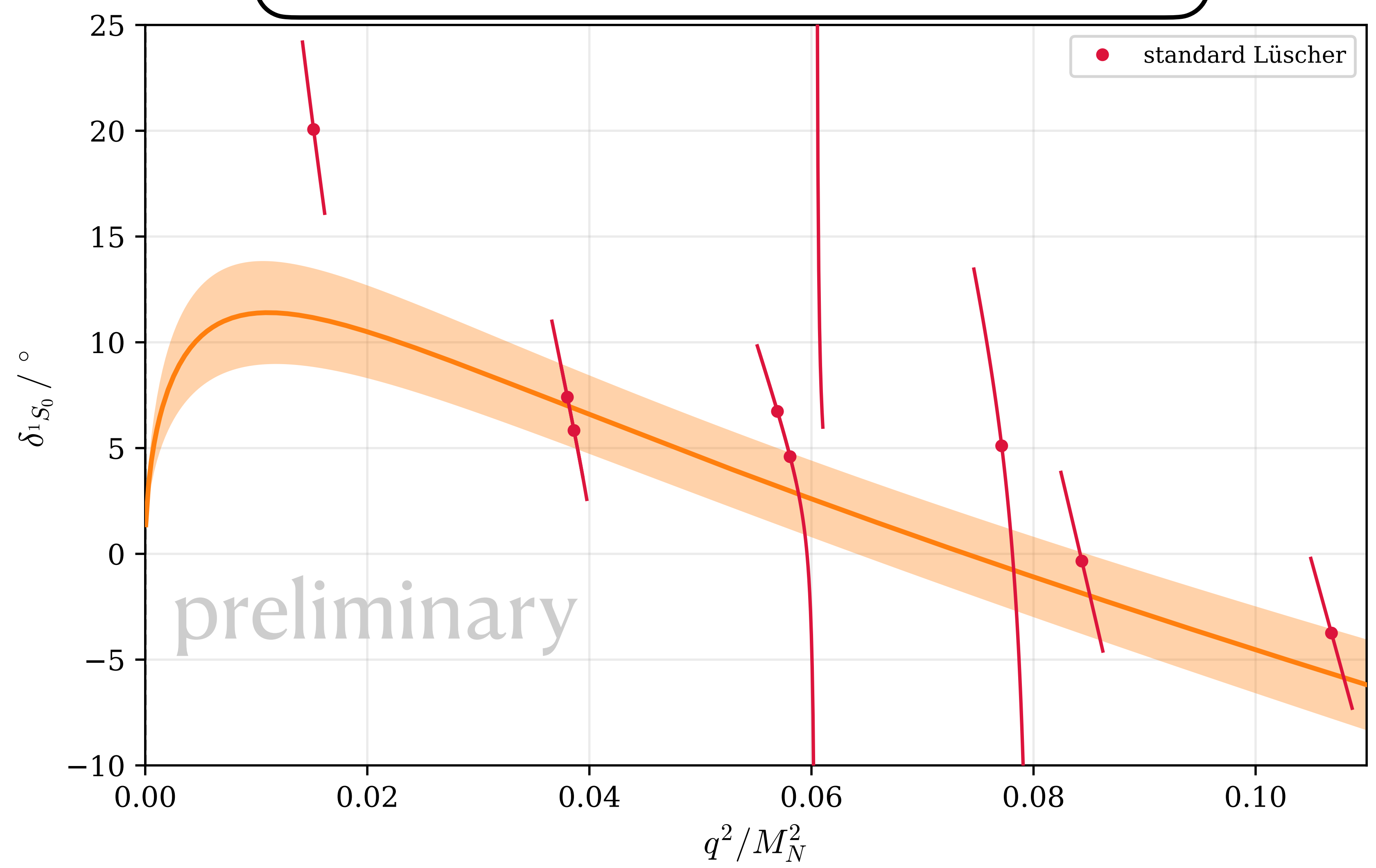
Fit results for the phase shifts (relativistic formalism)

Deuteron channel ($I = 0$)





Dineutron channel ($I = 1$)



Dataset 1

no SD mixing, ERE3 + ERE2

6 fit parameters

Cutoff param. $m_N^2\alpha = 7$

- β-wave significant in $I = 0$, even without SD mixing in $\mathcal{K}_0$
- Compares well with Lüscher at this $M_\pi = 280$ MeV and above lhc, but will become necessary at lower M_π

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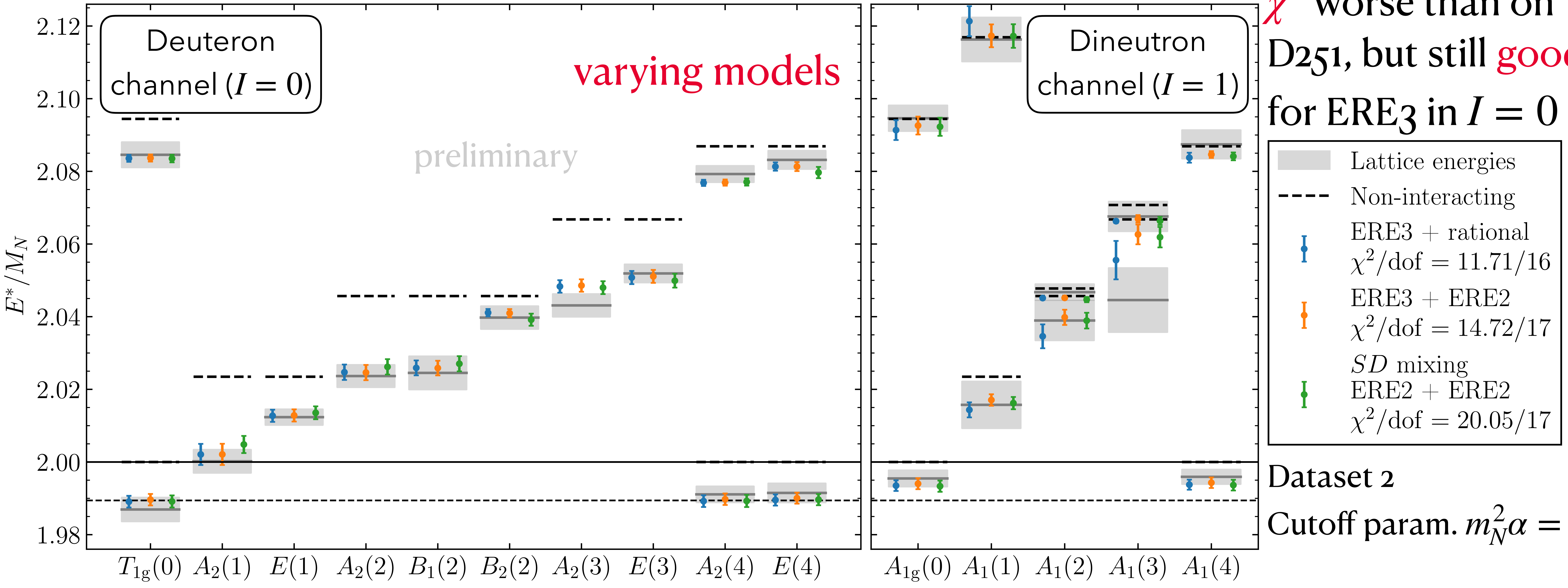

χ^2 worse than on D251, but still good for ERE3 in $I = 0$

Dataset 2

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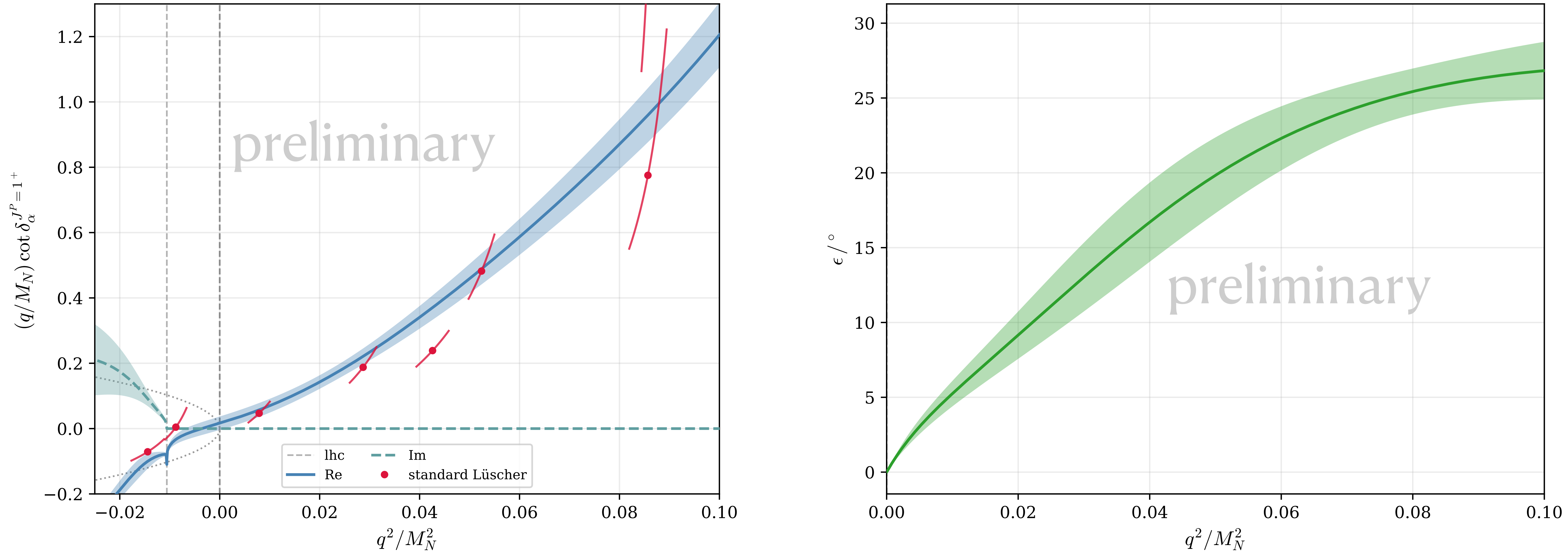

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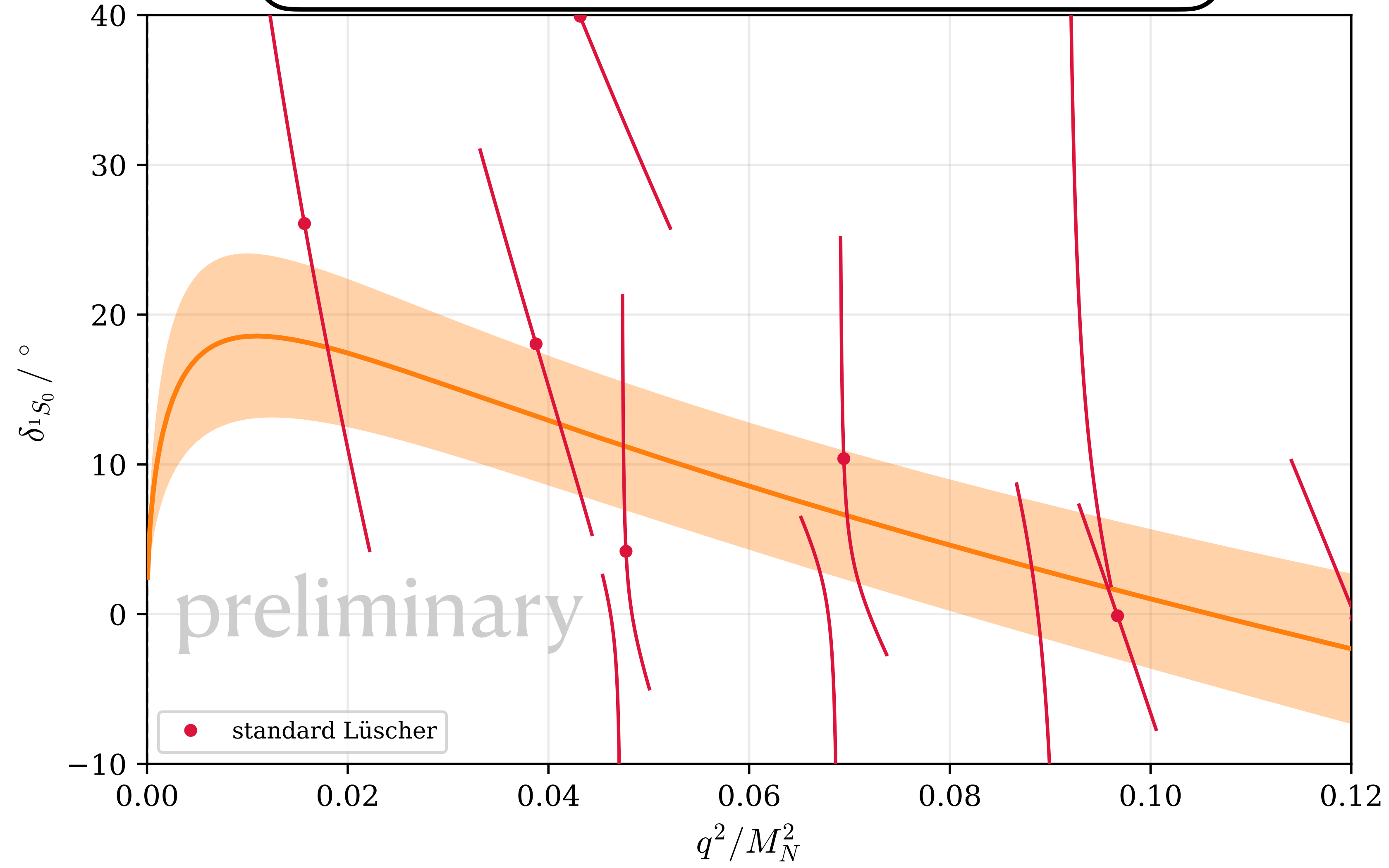

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- Significant lhc effects around threshold in $I = 0$ —> deviations from standard Lüscher for lowest few energy levels

NN scattering: left-hand cuts

Quantisation condition in the plane-wave basis

Meng and Epelbaum 2021 [JHEP 2021, 051]
Meng et al. 2024 [PRD 109, L071506]
Jorge Baeza-Ballesteros, poster session A

- Starting point: non-relativistic Lippmann-Schwinger equation (LSE),

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- Expand LSE in finite volume plane-wave basis —> matrix eq.: $\mathbb{M} = \mathbb{V} + \mathbb{V}\mathbb{J}\mathbb{G}\mathbb{M}$

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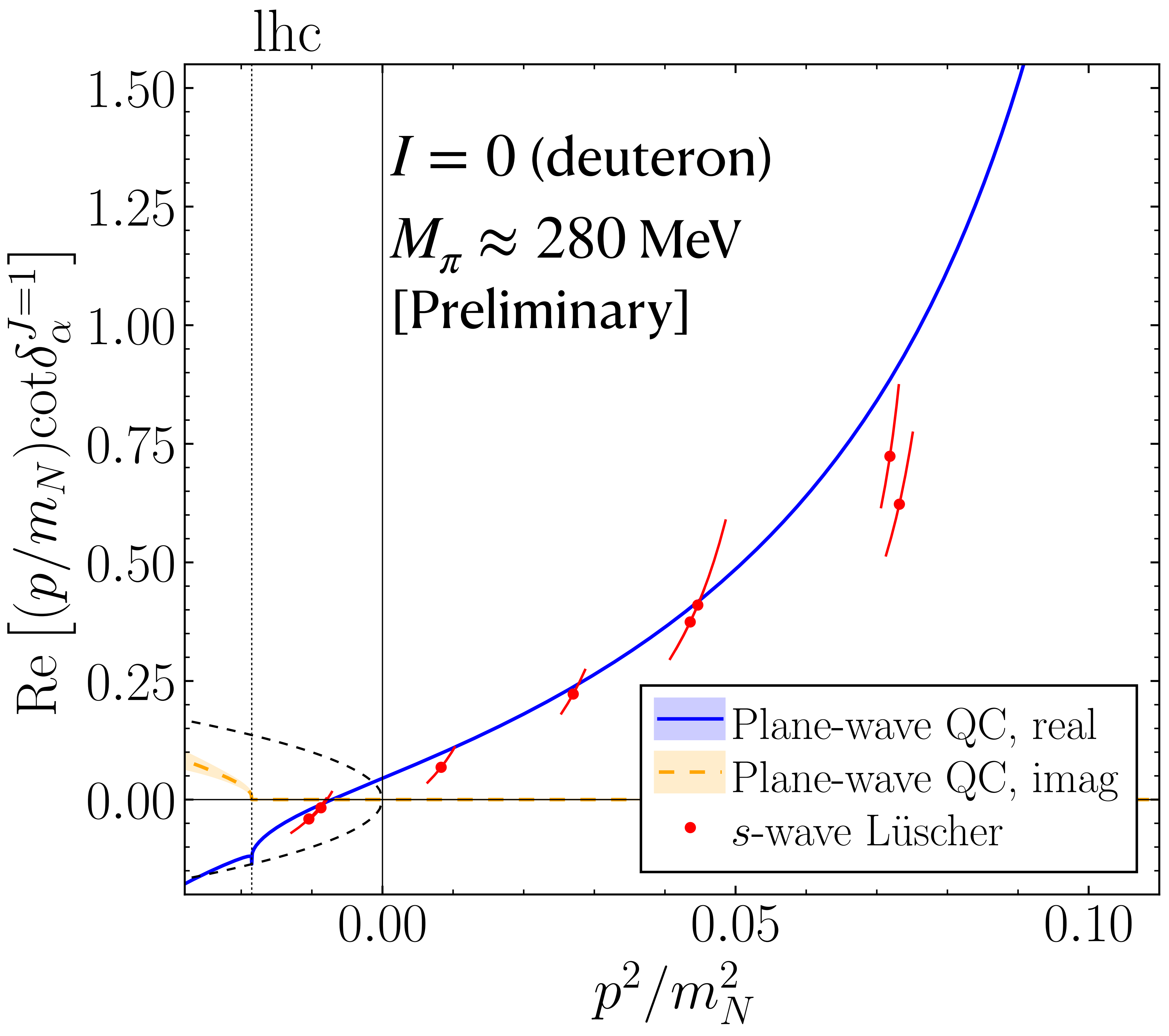
- Determine parameters of V from fit to LQCD spectrum
- Infinite-volume scattering amplitude can be obtained by solving infinite-volume LSE

NN scattering on D251 ($M_\pi = 280$ MeV)

Fit results for the phase shifts (plane-wave basis)

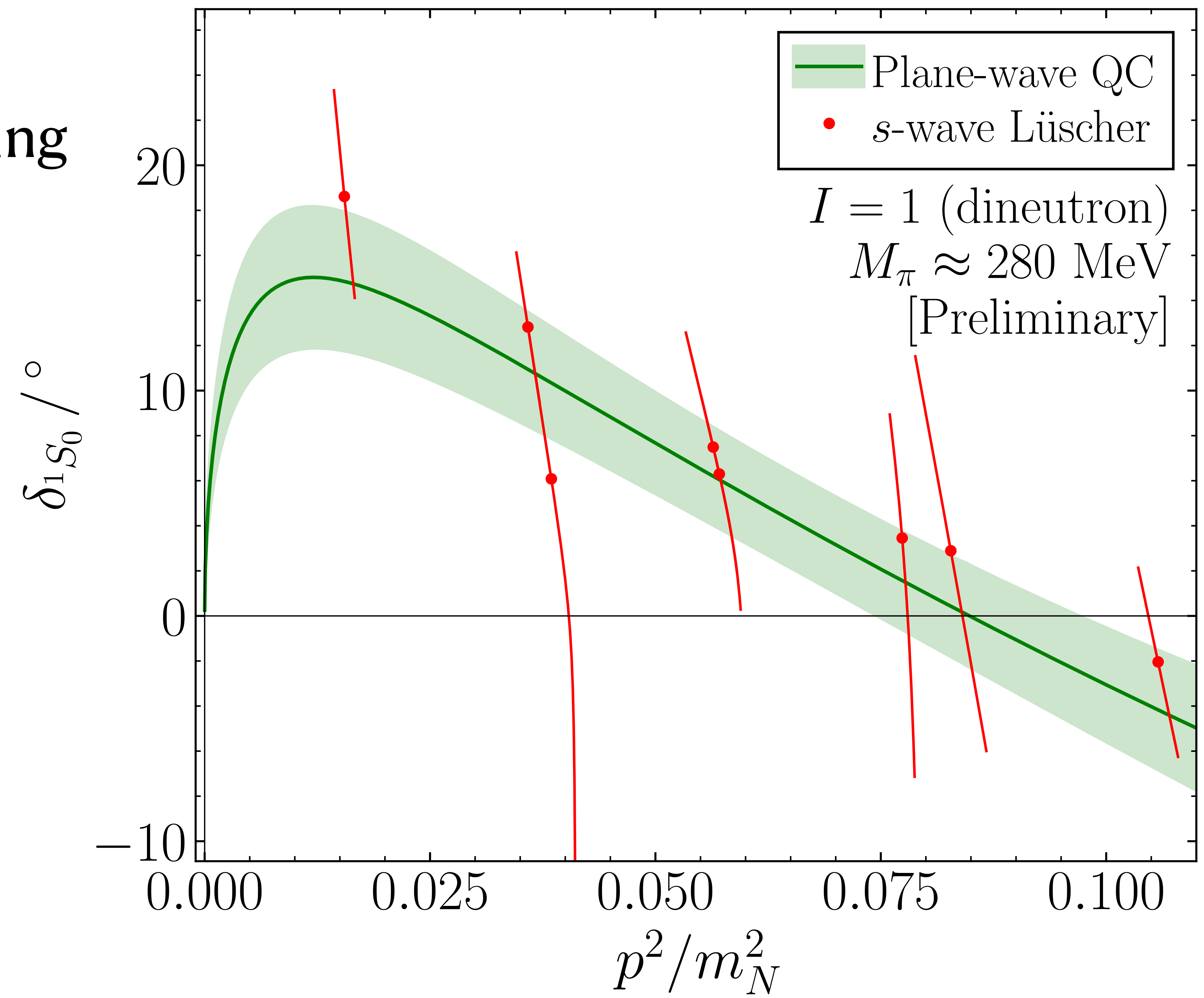


Dataset 2

NLO, no SD mixing

5 fit parameters

$\Lambda = 700$ MeV



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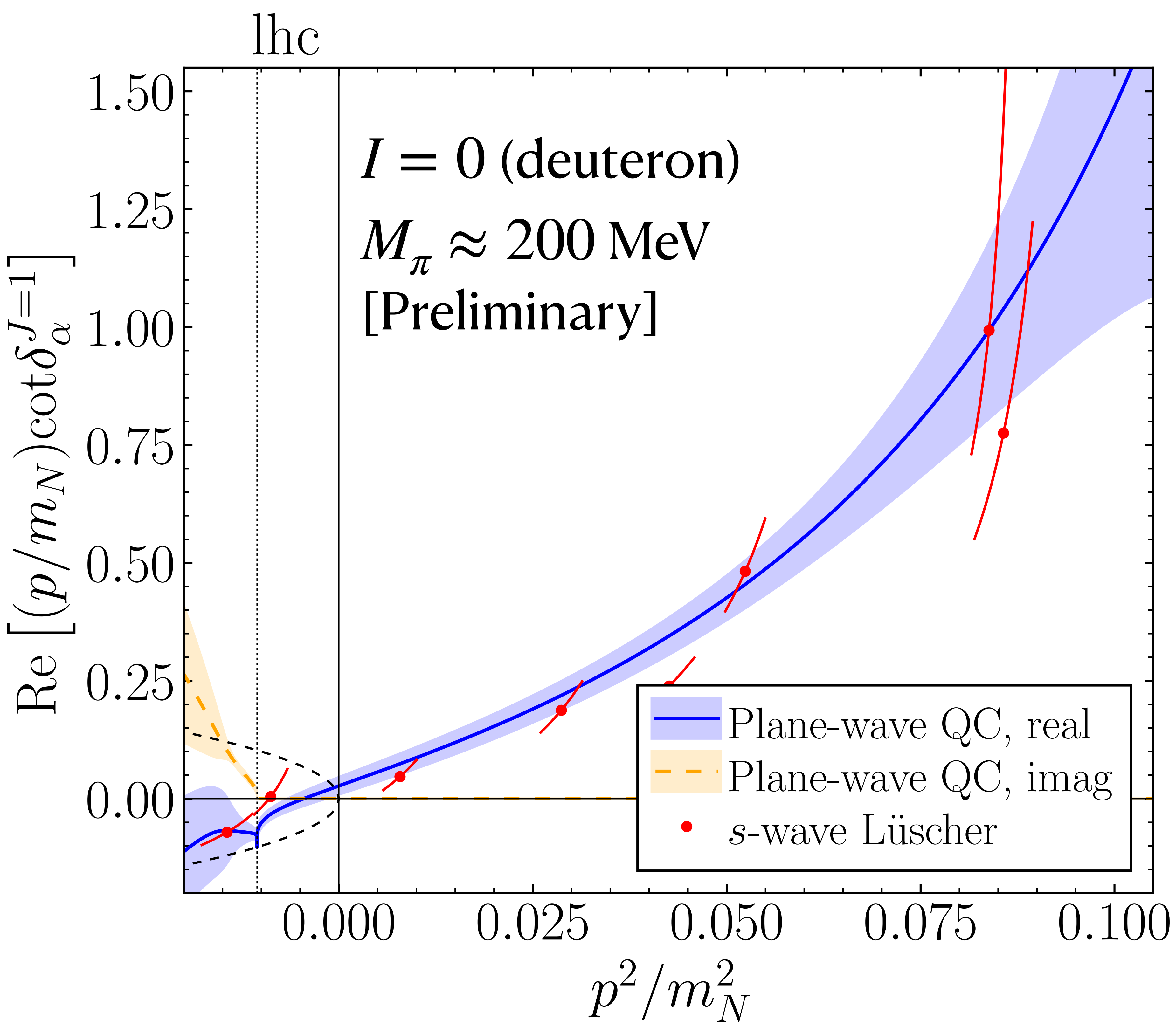

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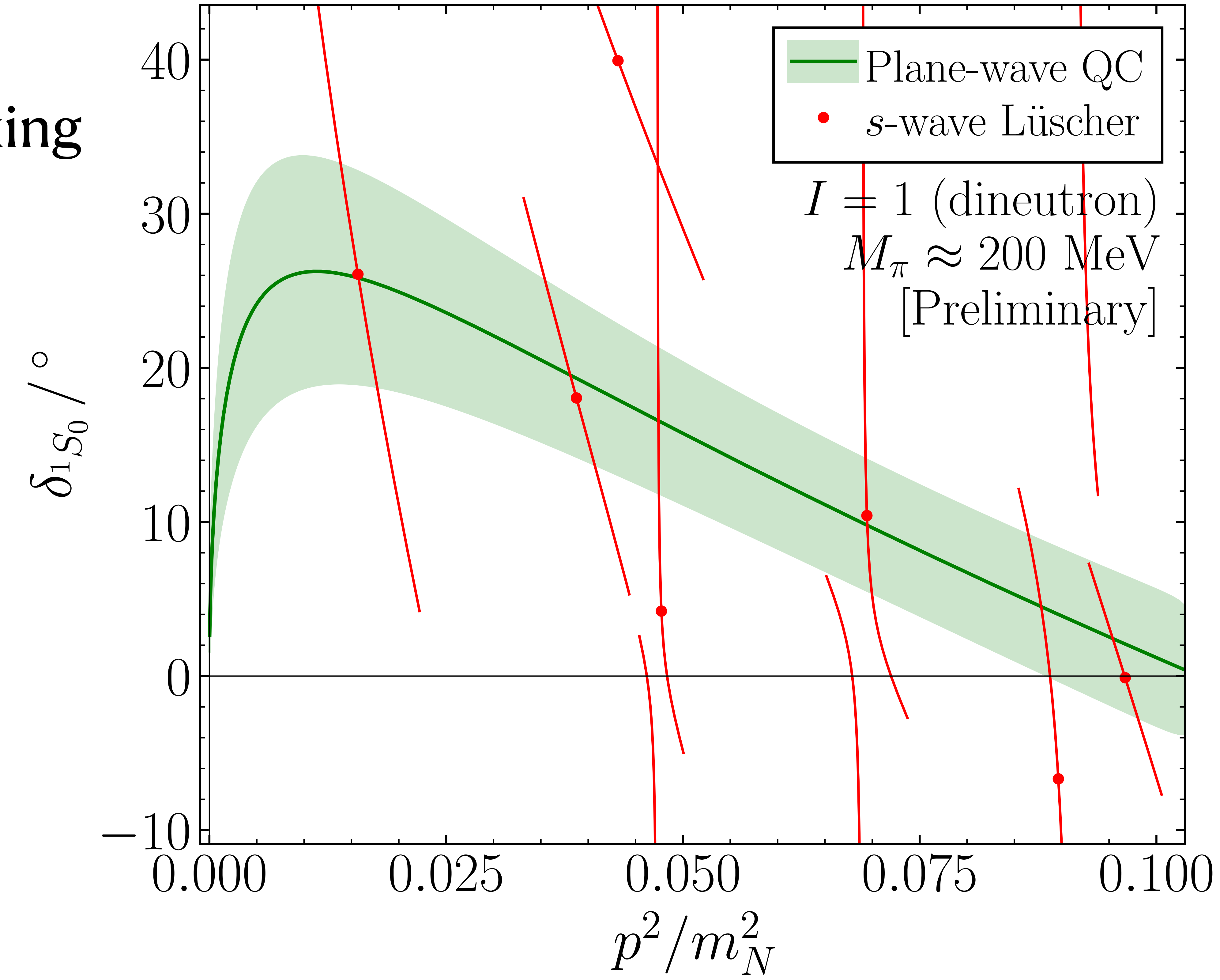
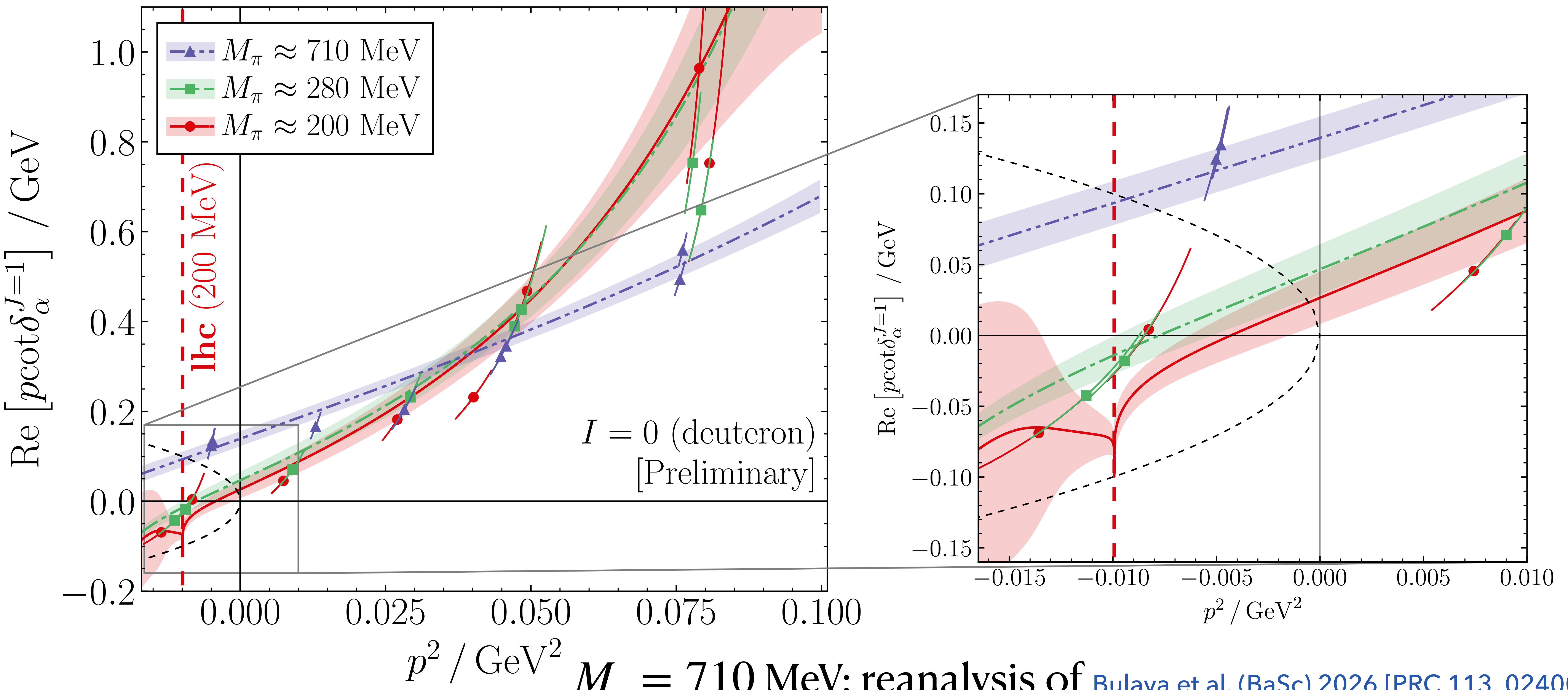


Lowest few energy levels in $I = 0$ (deuteron) severely affected by lhc

NN scattering: plane-wave basis

Comparison between the three ensembles for the deuteron

- Significant lhc effects around threshold for $M_\pi = 200$ MeV



$M_\pi = 710$ MeV: reanalysis of Bulava et al. (BaSc) 2026 [PRC 113, 024002]

$M_\pi = 280$ MeV, 200 MeV: this work (dataset 2)

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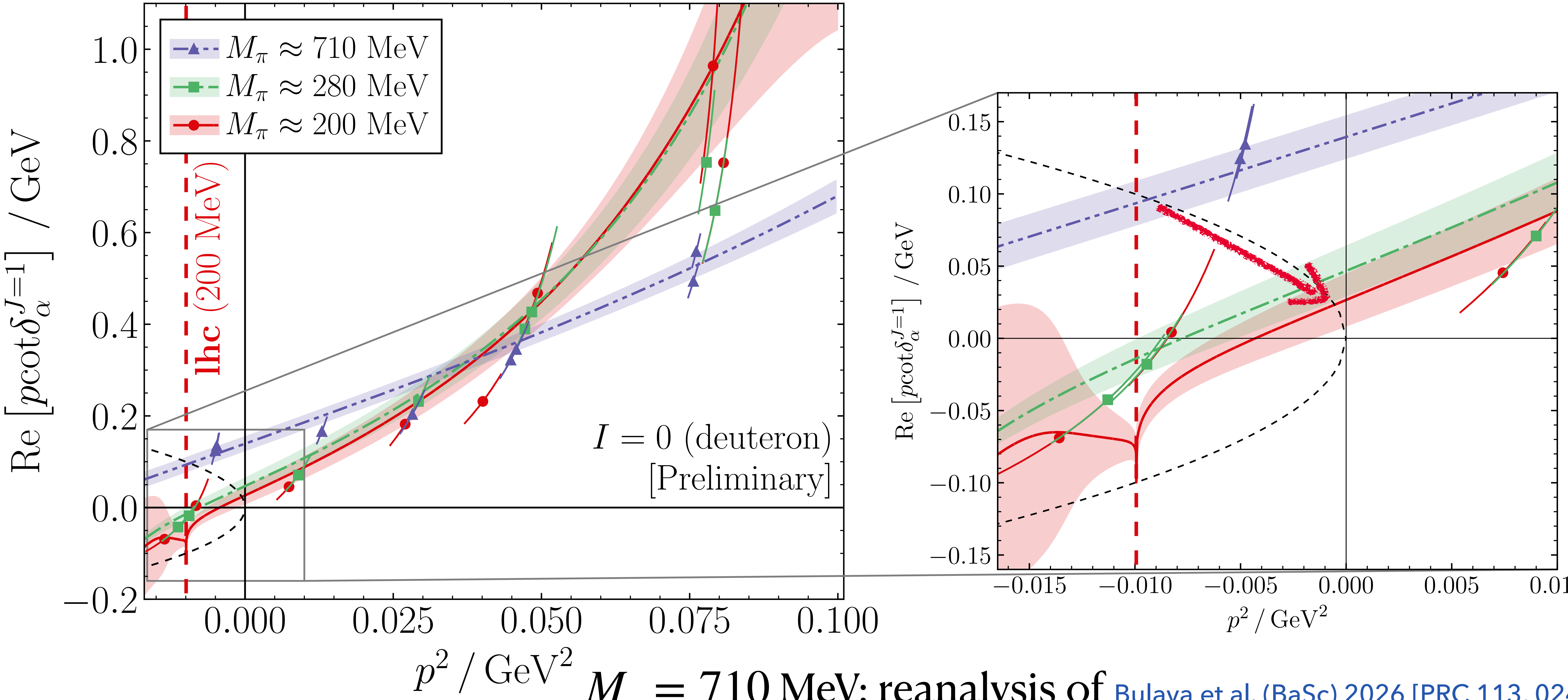

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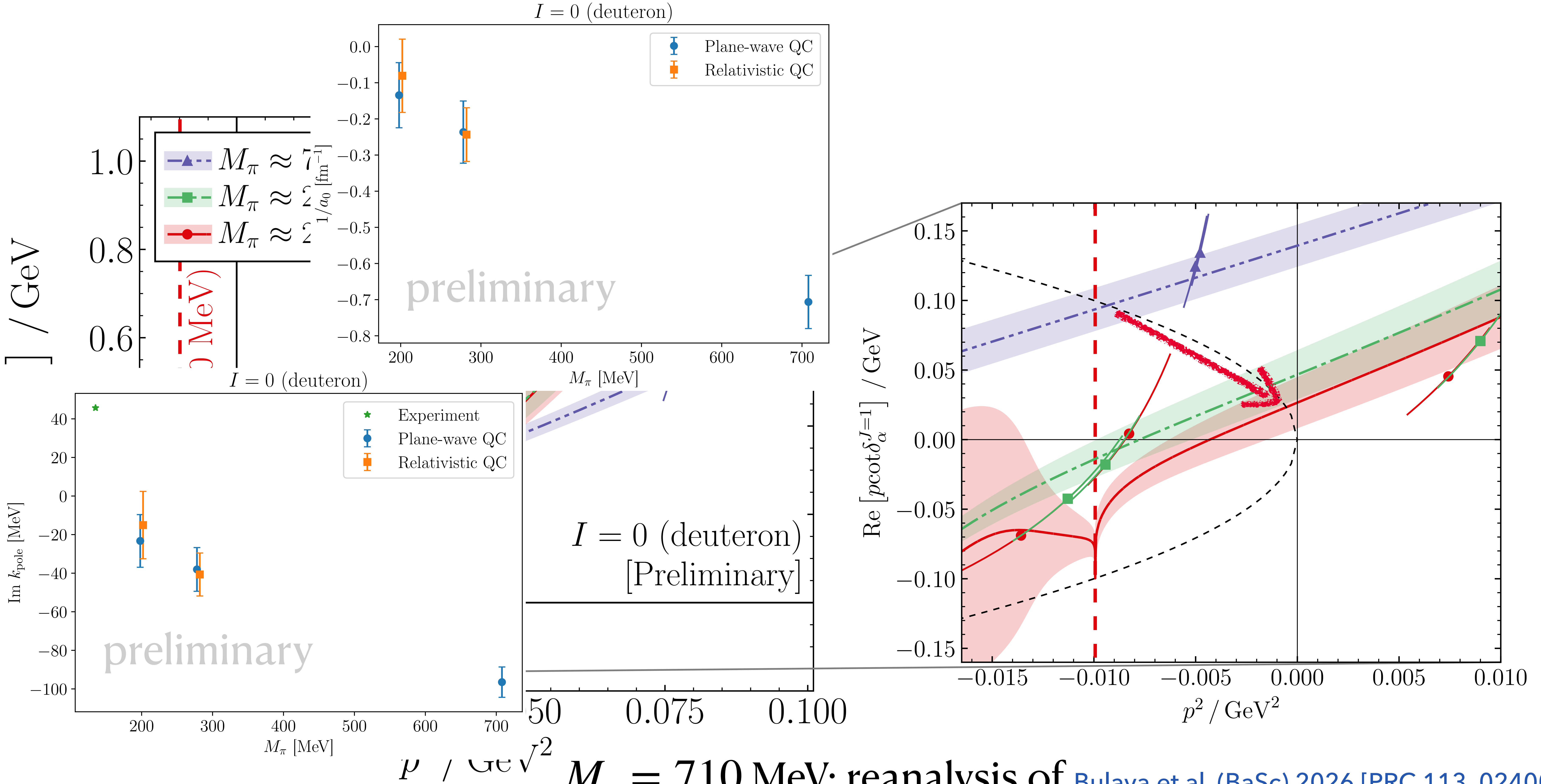
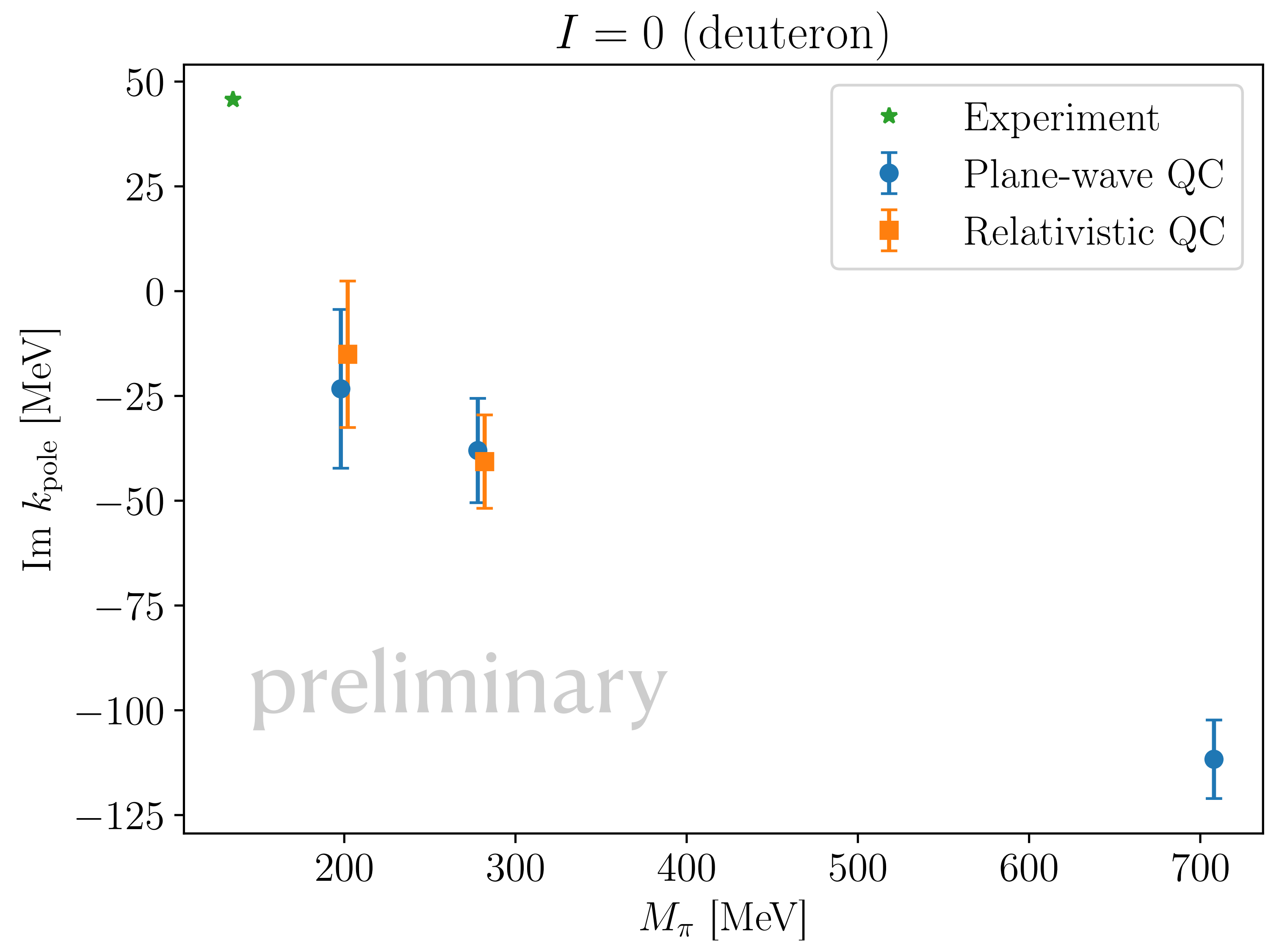


$M_\pi = 710$ MeV: reanalysis of Bulava et al. (BaSc) 2026 [PRC 113, 024002]

$M_\pi = 280$ MeV, 200 MeV: this work (dataset 2)

Summary and outlook

- Finite-volume spectrum of *NN* system can be computed reliably at light quark masses —> towards physical M_π
- Extraction of scattering amplitudes complicated by presence of left-hand cut
  - Relativistic formalism
  - Plane-wave basis QC



Summary and outlook

- Finite-volume spectrum of *NN* system can be computed reliably at light quark masses —> towards physical M_π
- Extraction of scattering amplitudes complicated by presence of left-hand cut
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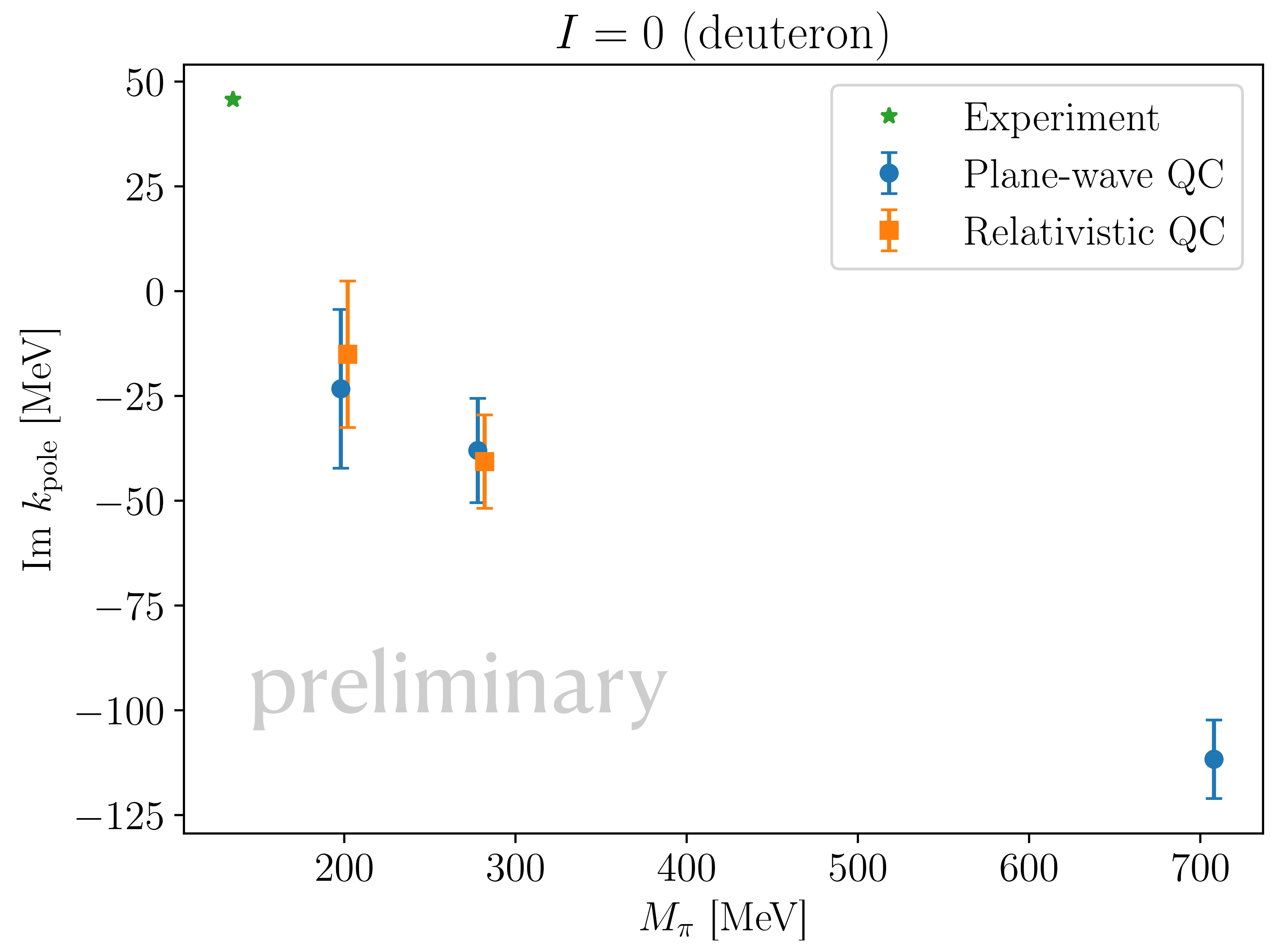
  - Plane-wave basis QC
- Future directions
  - Even smaller pion masses —> find bound deuteron
  - Continuum limit



Backup

NN scattering: relativistic formalism

Obtaining the physical scattering amplitude

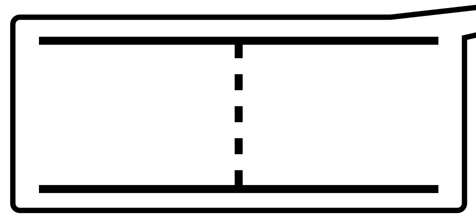
Baião Raposo et al. 2025 [JHEP 2025, 186]

- Partial-wave projected infinite-volume scattering amplitude can be written as $\mathcal{M} = \mathcal{M}_{\mathcal{E}} + (1 + i\mathcal{L})(\mathcal{M}_0^{-1} + \mathcal{C})^{-1}(1 + i\mathcal{R})$, where $\mathcal{M}_0$ is amplitude corresponding to $\mathcal{K}_0$

- $\mathcal{M}_{\mathcal{E}}$ satisfies the integral equation

Two-particle pole in IV

$$\mathcal{M}_{\mathcal{E}}(p', p) = i\mathcal{E}(p', p) + \int_0^\infty \frac{k^2 dk}{2\pi^2} i\mathcal{E}(p', k) i\Delta_{2,\infty}(k) i\mathcal{M}_{\mathcal{E}}(k, p)$$ —> solve as matrix equation



- $i\mathcal{L}(p) = \int_0^\infty \frac{k^2 dk}{2\pi^2} i\mathcal{M}_{\mathcal{E}}(p, k) i\Delta_{2,\infty}(k) \left(\frac{k}{q}\right)^{\ell}$, $i\mathcal{R}(p) = \int_0^\infty \frac{k^2 dk}{2\pi^2} \left(\frac{k}{q}\right)^{\ell'} i\Delta_{2,\infty}(k) i\mathcal{M}_{\mathcal{E}}(k, p)$,

and $i\mathcal{C} = \int_0^\infty \frac{k'^2 dk'}{2\pi^2} \int_0^\infty \frac{k^2 dk}{2\pi^2} \left(\frac{k'}{q}\right)^{\ell'} i\Delta_{2,\infty}(k') i\mathcal{M}_{\mathcal{E}}(k', k) i\Delta_{2,\infty}(k) \left(\frac{k}{q}\right)^{\ell}$

NN scattering: relativistic formalism

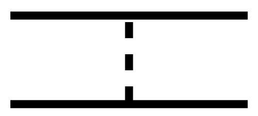
Spectrum fit: Models for $I = 0$

- Without S, D-wave mixing: ERE3 for $\mathscr{K}_0$, $q\cot(\tilde{\delta}(E)) = -\frac{1}{A} + \frac{1}{2}Rq^2 + C^{(2)}q^4$

Blatt and Biedenharn 1952 [PR 86, 399], 1954 [PR 93, 1387]

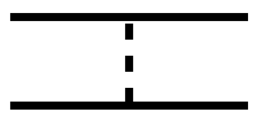
- With S, D-wave mixing: Blatt-Biedenharn parameterisation for modified K-matrix,

$\mathscr{K}_0^{-1}(E) = Q\mathscr{R}^\dagger(\theta q^2)\begin{pmatrix} q\cot(\tilde{\delta}_\alpha(E)) & 0 \\ 0 & q\cot(\tilde{\delta}_\beta(E)) \end{pmatrix}\mathscr{R}(\theta q^2)Q$, where $Q_{\ell'\ell} = \delta_{\ell'\ell}q^\ell$, $\mathscr{R}$ is a rotation matrix, and we use $q\cot(\tilde{\delta}_\alpha(E)) = -\frac{1}{A_\alpha} + \frac{1}{2}R_\alpha q^2$ and $q^5\cot(\tilde{\delta}_\beta(E)) \to \infty$

- A_α, R_α, $C^{(2)}$, and θ are parameters of $\mathscr{K}_0$ and thus inherently scheme dependent
- Additional fit parameter $g^2_{NN\pi}$ (shared with $I = 1$) is coefficient of  —> physical

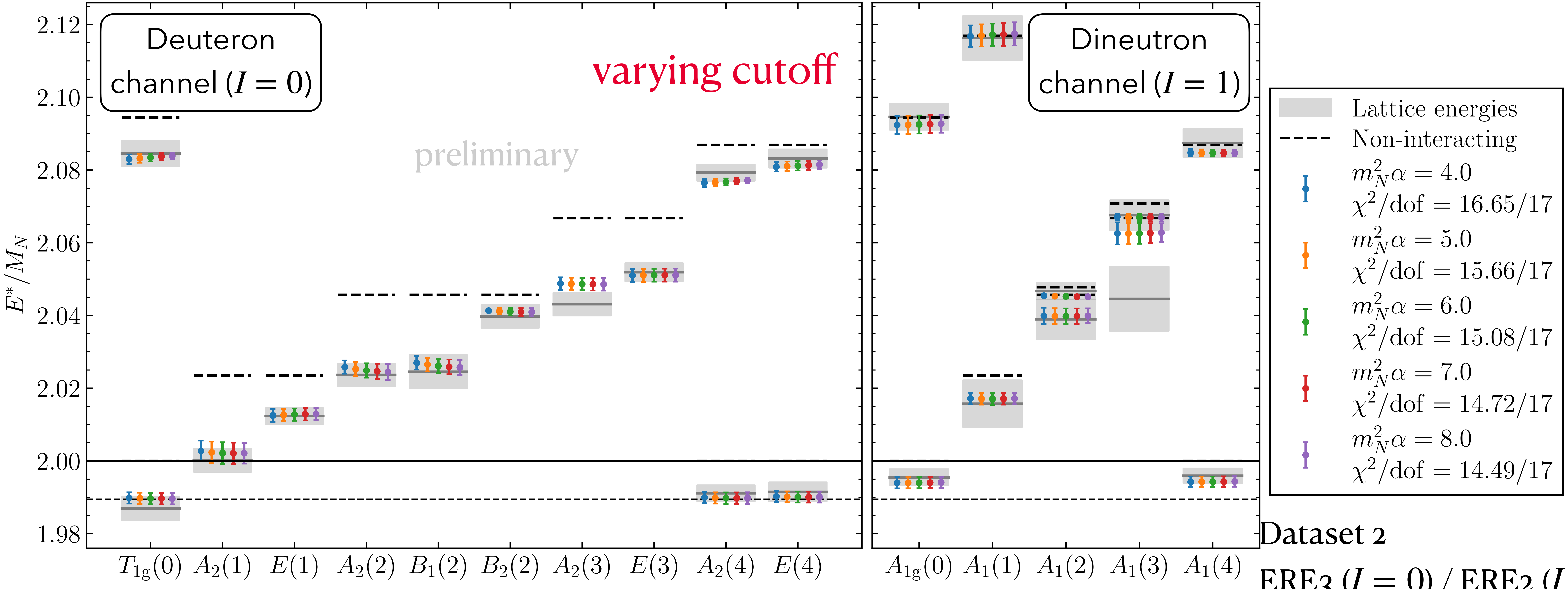
NN scattering: relativistic formalism

Spectrum fit: Models for $I = 1$

- No S, D-wave mixing, only A_1-like irreps
- Two models for modified K-matrix
  - ERE2, $q\cot(\tilde{\delta}(E)) = -\dfrac{1}{A} + \dfrac{1}{2}Rq^2$
  - Rational, $\dfrac{1}{q}\tan(\tilde{\delta}(E)) = \dfrac{C_0 + C_1 q^2}{1 + C_2 q^2}$ —> allows for both zero and pole in $q\cot\tilde{\delta}$
- A, R, and C_i are parameters of $\mathscr{K}_0$ and thus inherently scheme dependent
- Additional fit parameter $g^2_{NN\pi}$ (shared with $I = 1$) is coefficient of  —> physical

NN scattering on J250 ($M_\pi = 200$ MeV)

Finite-volume spectrum and fit stability (relativistic formalism)



Dataset 2

ERE3 ($I = 0$) / ERE2 ($I = 1$)

no SD mixing

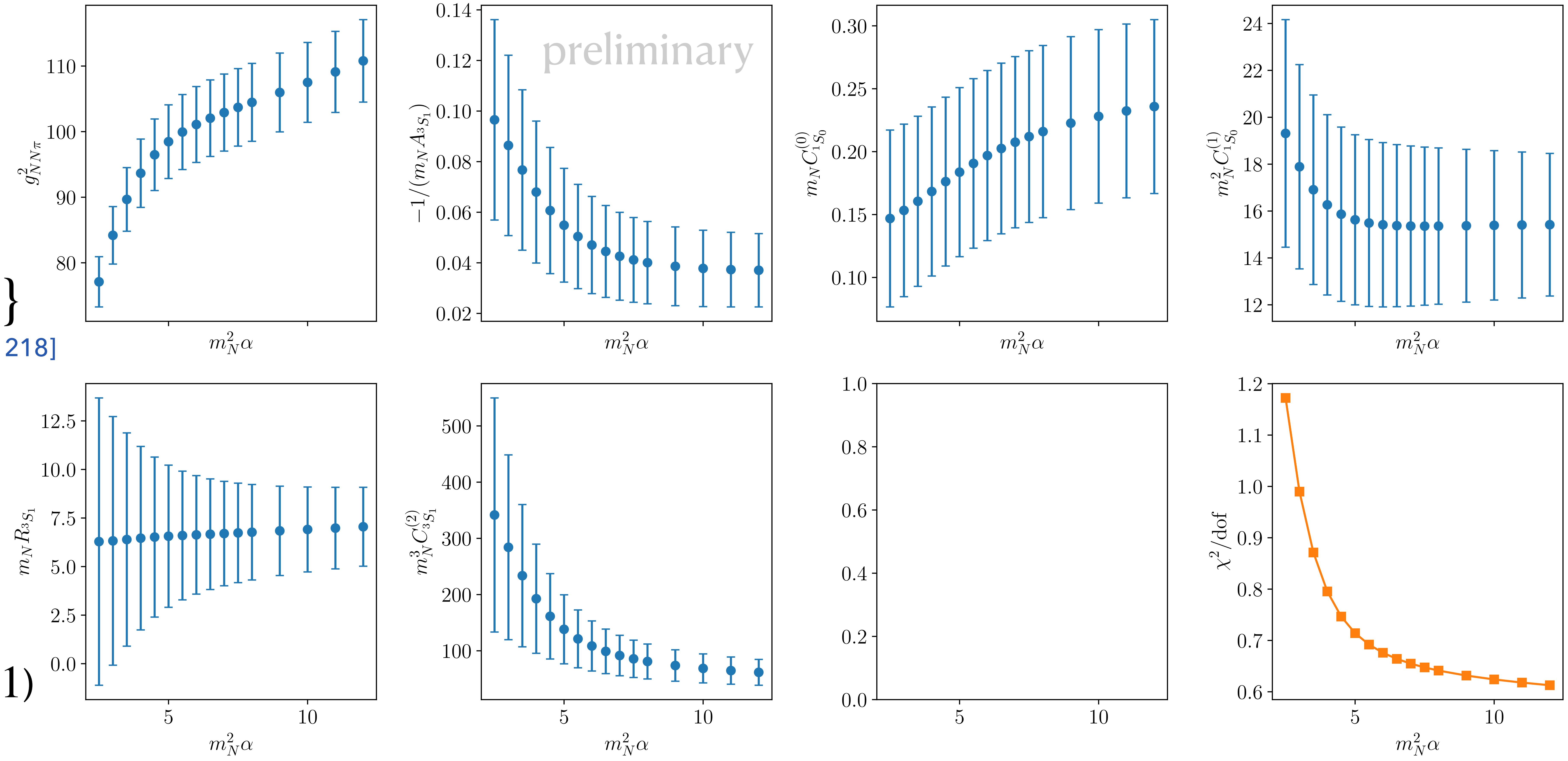
NN scattering on D251 ($M_\pi = 280$ MeV)

Cutoff dependence of fit parameters (relativistic formalism)

$\mathscr{C}_L$ and $F_{2,L}$ internally use exponential cutoff, $\exp\{\alpha[(q^*)^2 - (\mathbf{k}^*)^2]\}$

Kim, Sachrajda, and Sharpe 2005 [NPB 727, 218]

—> Larger α means stricter cutoff



Dataset 1

ERE3 ($I = 0$) / ERE2 ($I = 1$)

no SD mixing

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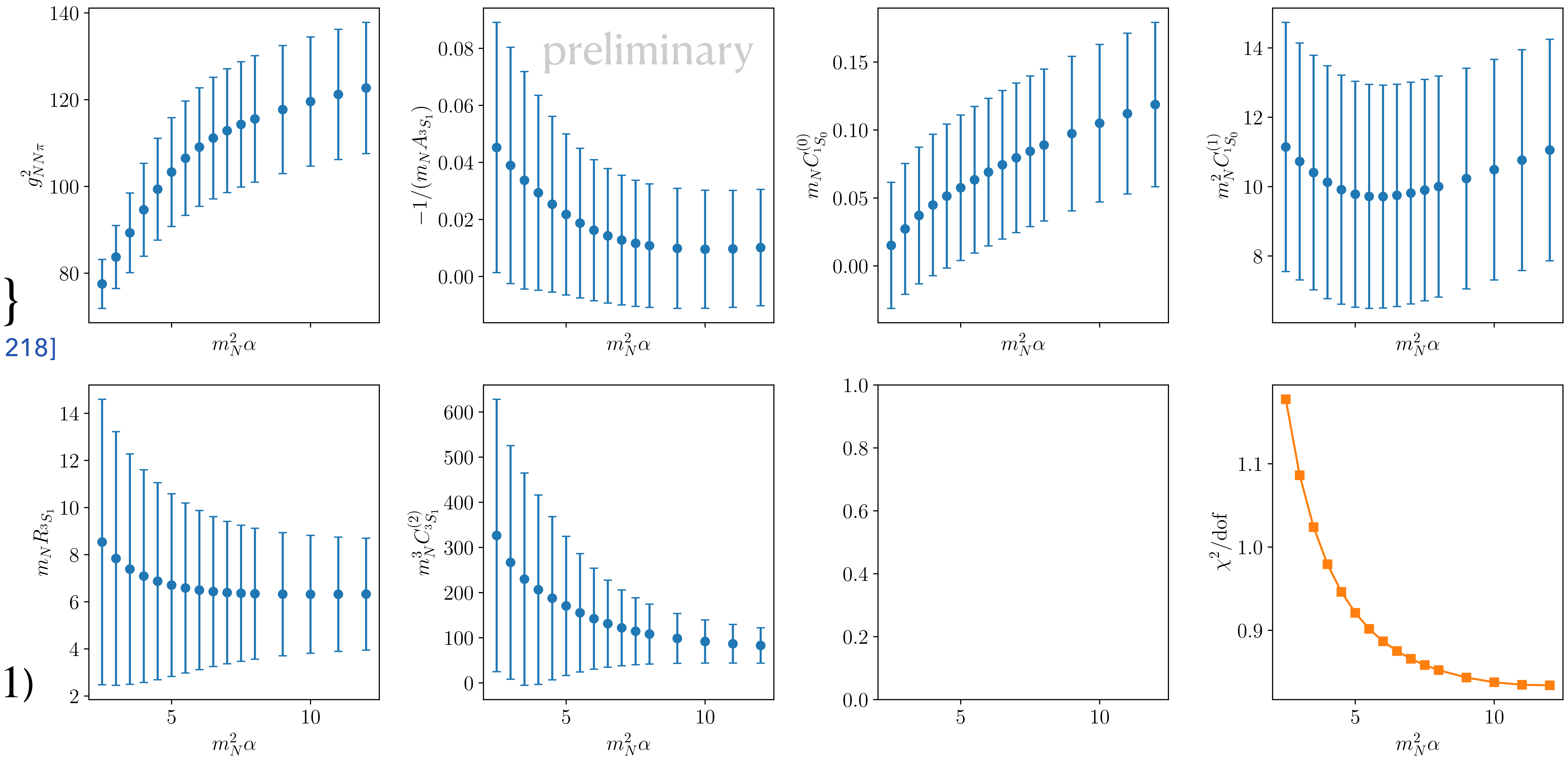
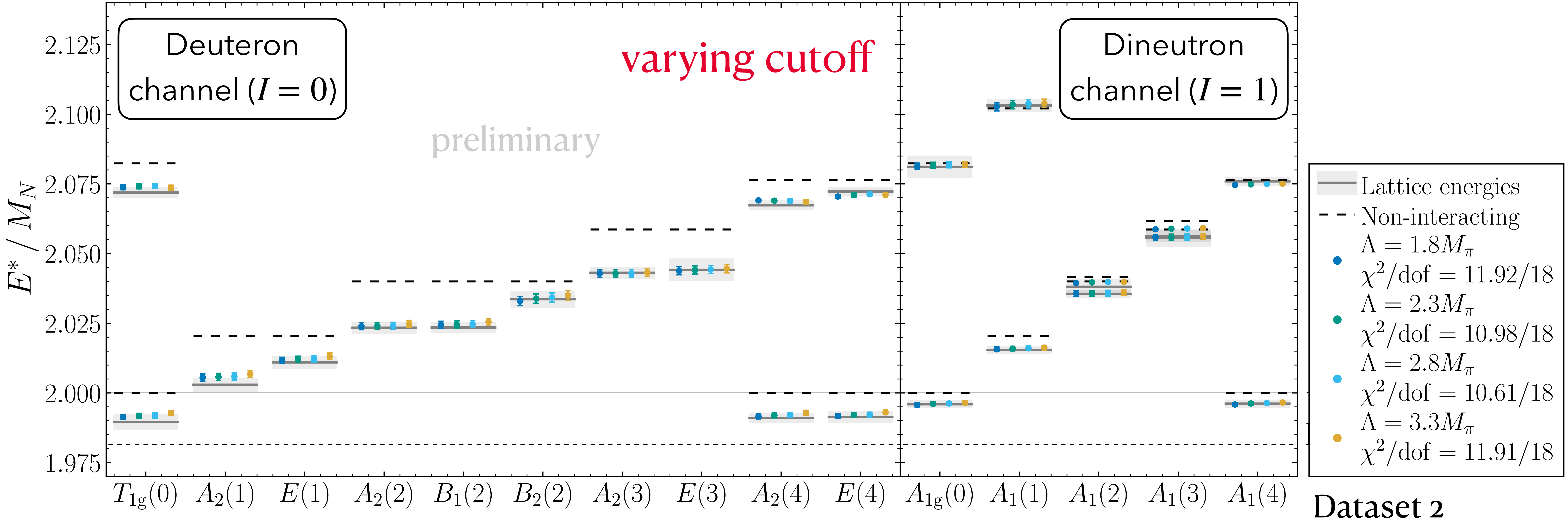


NN scattering on D251 ($M_\pi = 280$ MeV)

Finite-volume spectrum and fit stability (plane-wave basis)

Combined fit to both isospins with OPE coupling $g_{NN\pi}$ as common fit parameter



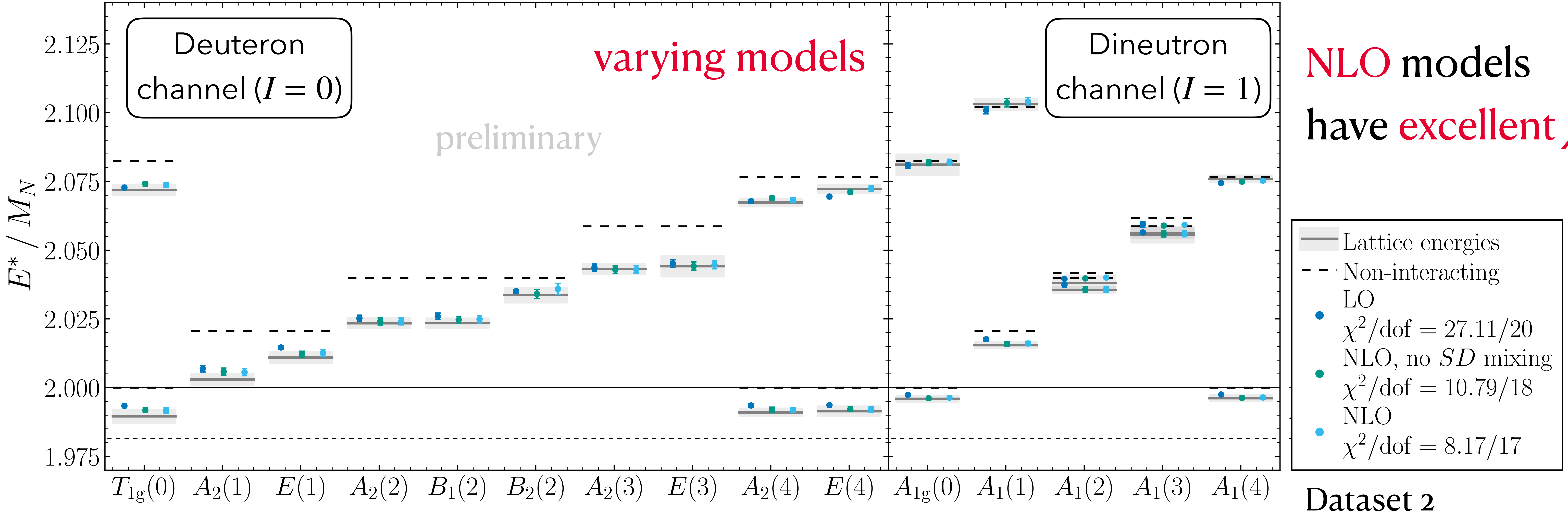
Dataset 2

NLO, no *SD* mixing

NN scattering on D251 ($M_\pi = 280$ MeV)

Finite-volume spectrum and fit stability (plane-wave basis)

Combined fit to both isospins with OPE coupling $g_{NN\pi}$ as common fit parameter



NLO models have excellent χ^2

Dataset 2

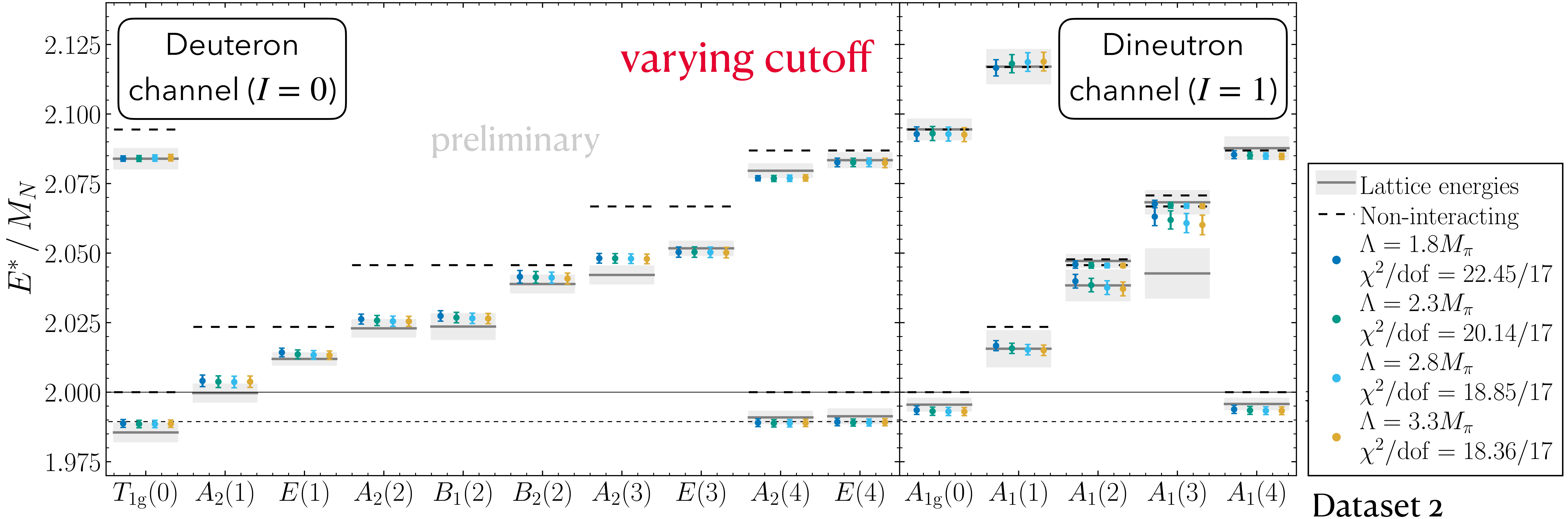
Cutoff $\Lambda = 2.5 M_\pi$

Plane-wave basis produces all partial waves

NN scattering on J250 ($M_\pi = 200$ MeV)

Finite-volume spectrum and fit stability (plane-wave basis)

Combined fit to both isospins with OPE coupling $g_{NN\pi}$ as common fit parameter



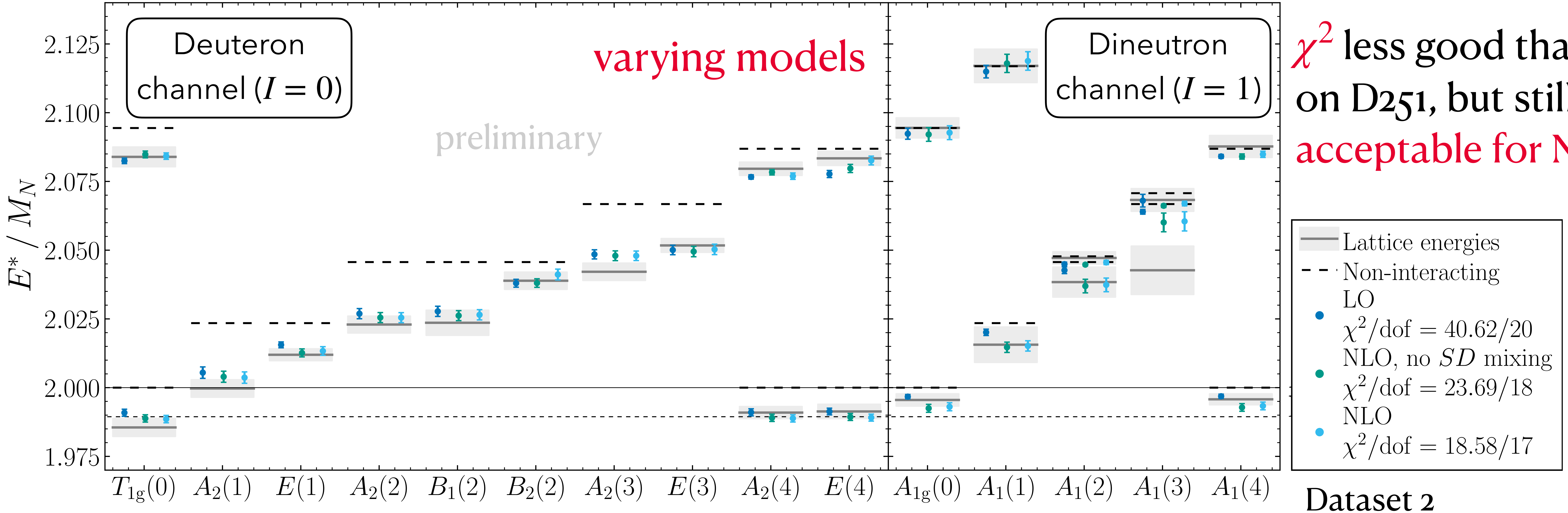
Dataset 2

NLO, with SD mixing

NN scattering on J250 ($M_\pi = 200$ MeV)

Finite-volume spectrum and fit stability (plane-wave basis)

Combined fit to both isospins with OPE coupling $g_{NN\pi}$ as common fit parameter



χ^2 less good than on D251, but still acceptable for NLO

Dataset 2

Cutoff $\Lambda = 3M_\pi$

NN scattering: $I = 0$ (deuteron)

Comparison of plane-wave and relativistic formalisms