Flux Tube Entanglement Entropy Across Temperature, Geometry, and Gauge Group

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Papers Discussed

- R. Amorosso, S. Syritsyn, and R. Venugopalan, Entanglement entropy of a color flux tube in (2+1)D Yang-Mills theory, JHEP 12 177 (2024), arXiv:2410.00112 [hep-lat].
- R. Amorosso, S. Syritsyn, and R. Venugopalan, Entanglement Enabled Tomography of Flux Tubes in 2+1-D Yang-Mills Theory, to appear in JHEP (2026), arXiv:2601.17199 [hep-lat].
- R. Amorosso, S. Syritsyn, and R. Venugopalan, Upcoming paper on flux tube entanglement at finite temperature, (2026), arXiv:2610.???? [hep-lat].

Roadmap

- Introduction and past results from Flux Tube Entanglement Entropy (FTE^2)
- Entanglement radius: an intrinsic “thickness” of the vibrating flux tube that can be measured through entanglement
- Preliminary finite temperature results

What is Flux Tube Entanglement Entropy?

- FTE^2 is the entanglement solely attributable to the color flux tube between a $q\bar{q}$ pair
- FTE^2 answers the question “How much of a region’s entanglement is due to the flux tube, and not the vacuum?”
- The flux tube’s contribution to the EE is isolated through vacuum subtraction
- Can be realized on the lattice using the replica trick and Polyakov loop correlators

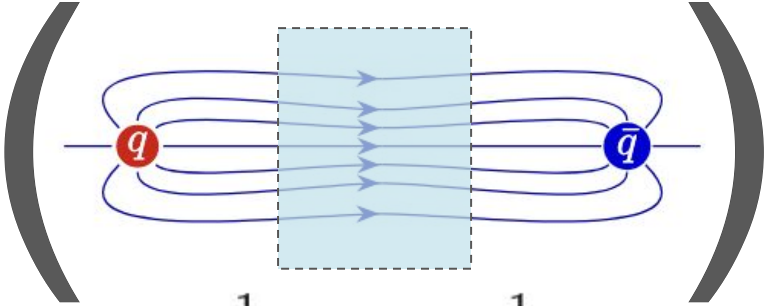
What is Flux Tube Entanglement Entropy?

$$\frac{1}{|\partial A|} S = \frac{1}{|\partial A|} S_{UV} + \frac{1}{|\partial A|} S_f$$

(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247, S. Ryu et al arXiv:hep-th/0603001, T. Nishioka et al arXiv:hep-th/0611035, I. Klebanov et al arXiv:0709.2140)

$$\tilde{S}^{(q)}_{|Q\bar{Q}} \equiv S^{(q)}_{|Q\bar{Q}} - S^{(q)}$$

Flux Tube Entanglement Entropy (FTE^2)



$$\frac{1}{|\partial A|} S_{UV} + \frac{1}{|\partial A|} S_f$$

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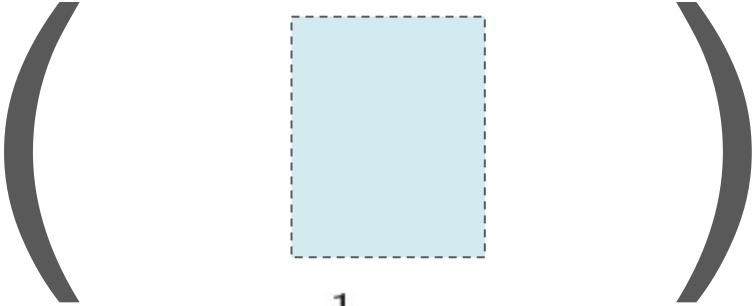
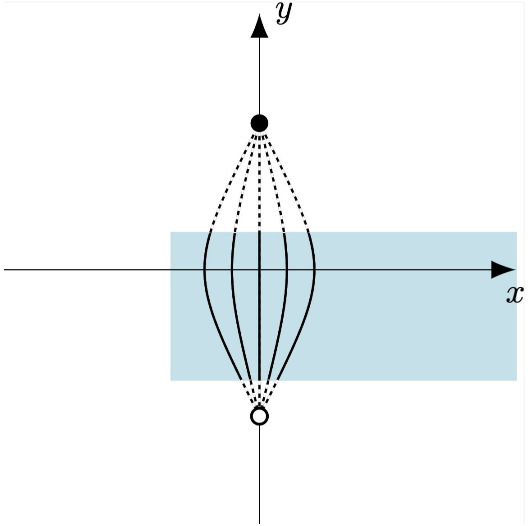
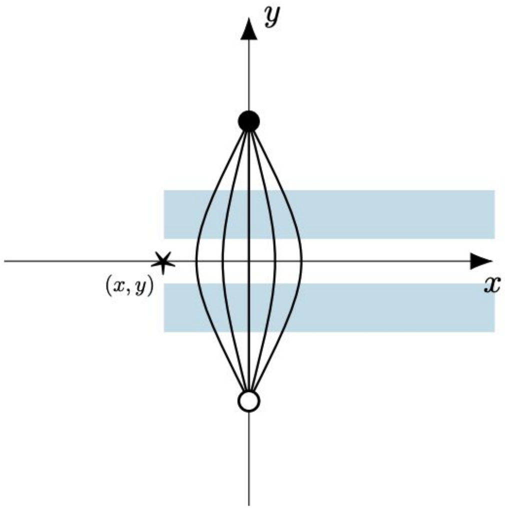


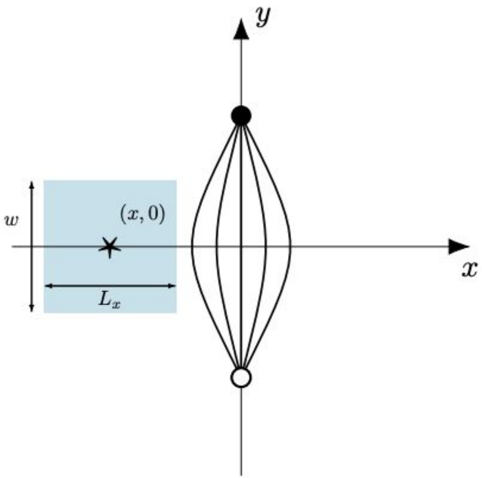
$$\frac{1}{|\partial A|} S_{UV}$$

FTE^2 possibilities

- Can study different gauge groups, entangling regions V, temperatures, etc

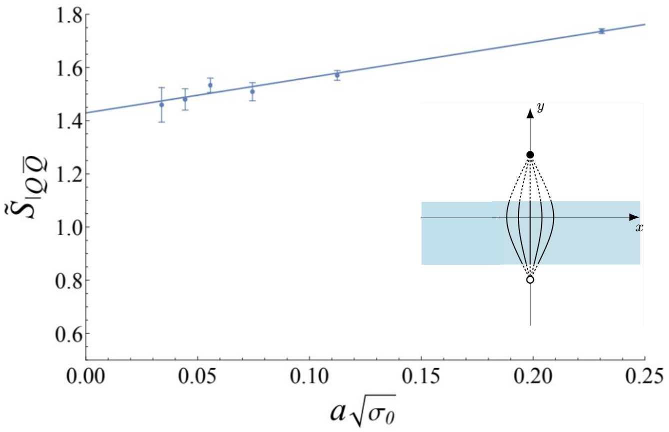


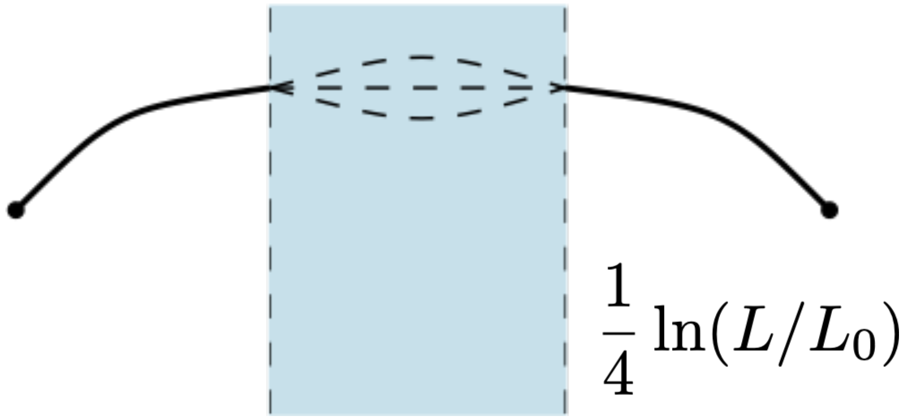


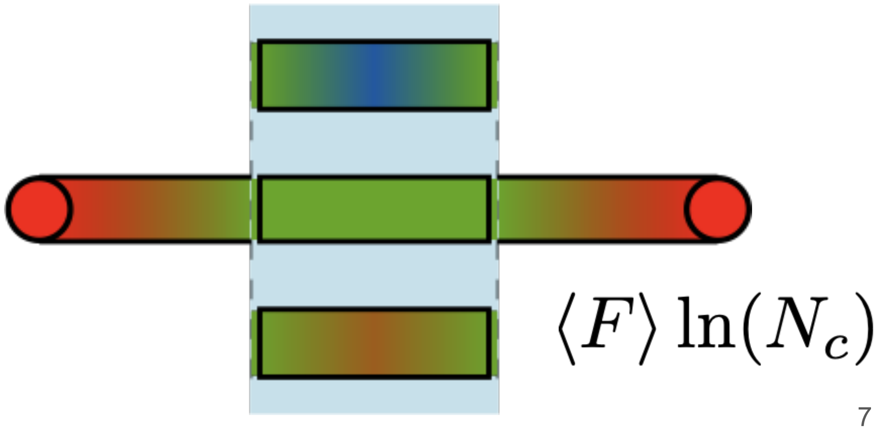


FTE^2 properties

- Finite, nonzero continuum limit
- Splits into two components following the string model
- Internal: Due to color degrees of freedom inside flux tube
- Vibrational: Due to mechanical vibrations of flux tube

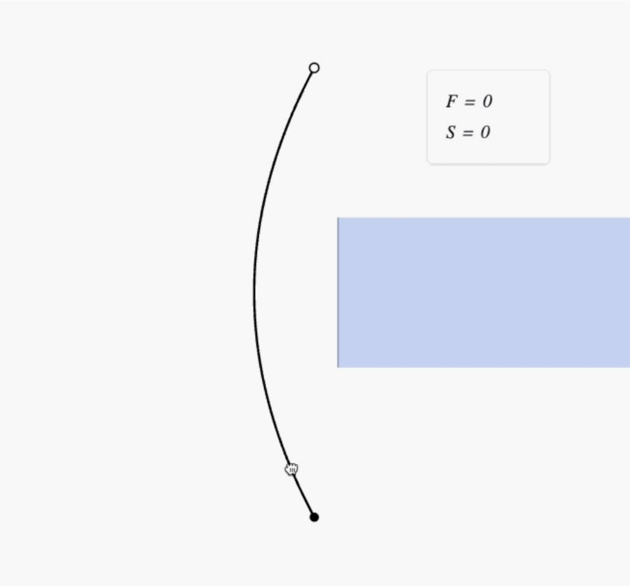


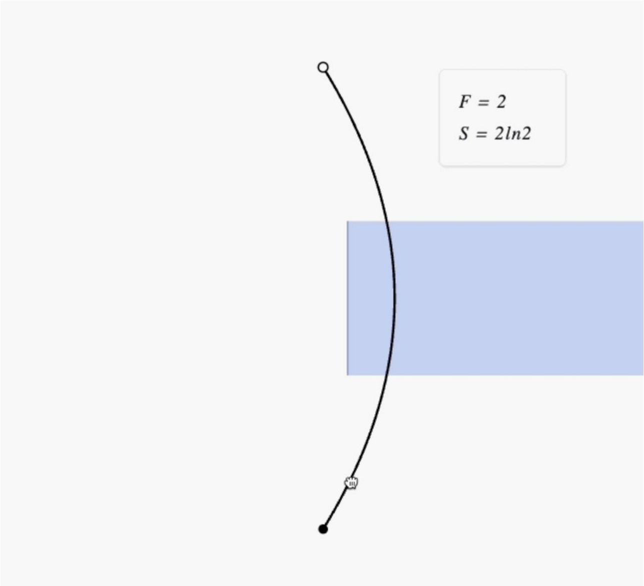




Internal Entropy

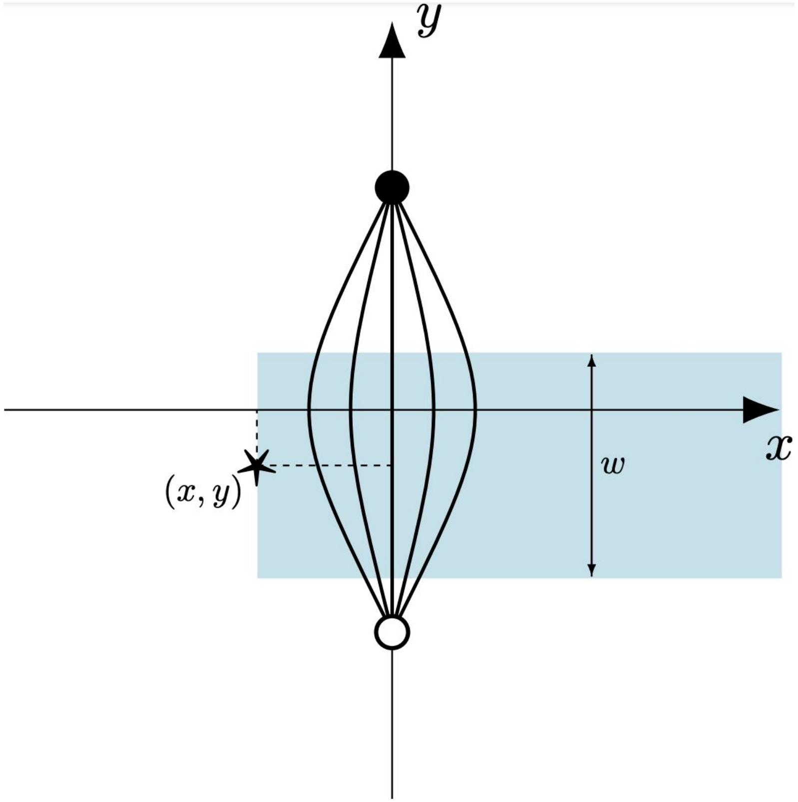
- Entanglement for every boundary crossing
- <F>*ln(Nc), where <F> is expected number of boundary crossings

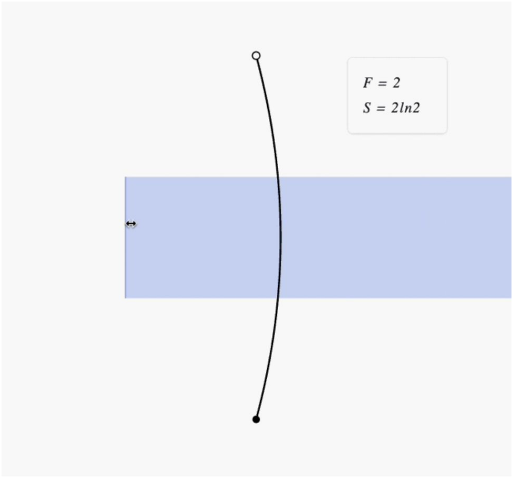


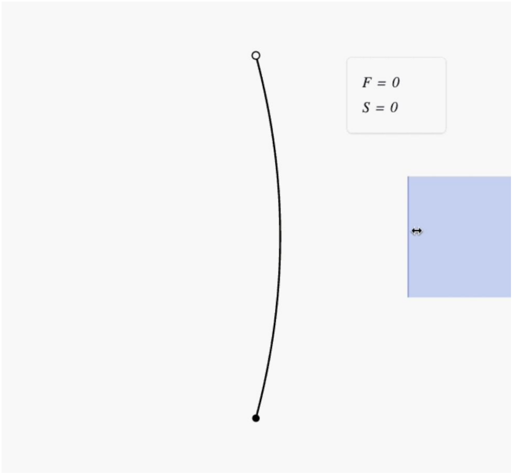


The Entanglement Radius

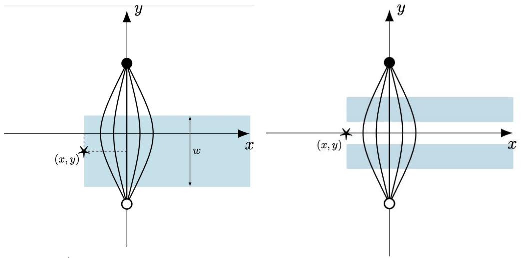
Lattice Setup: A = Half-slab

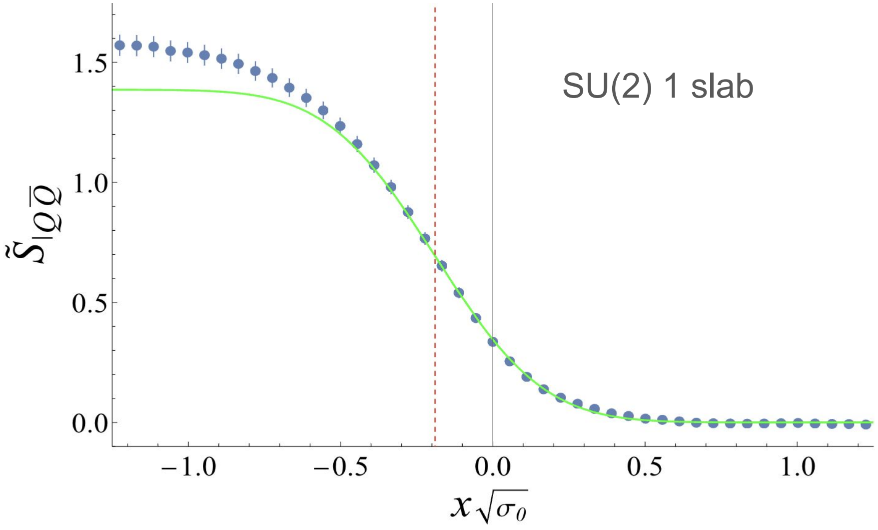


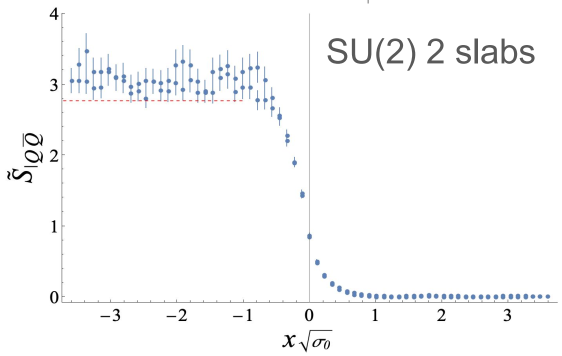


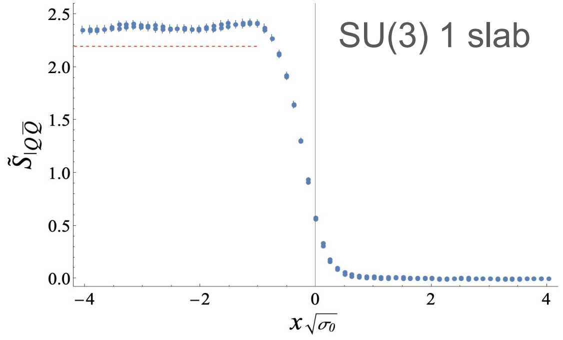


Internal Entropy

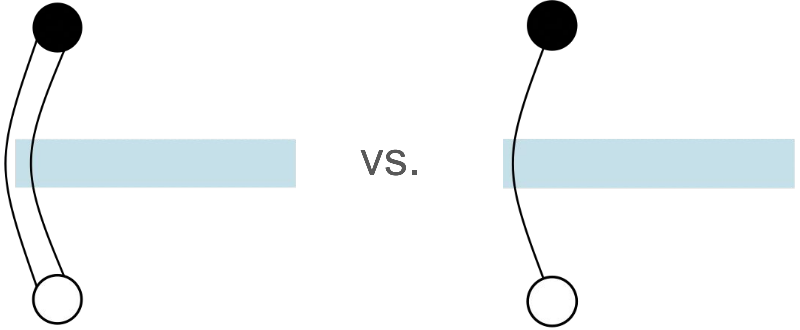


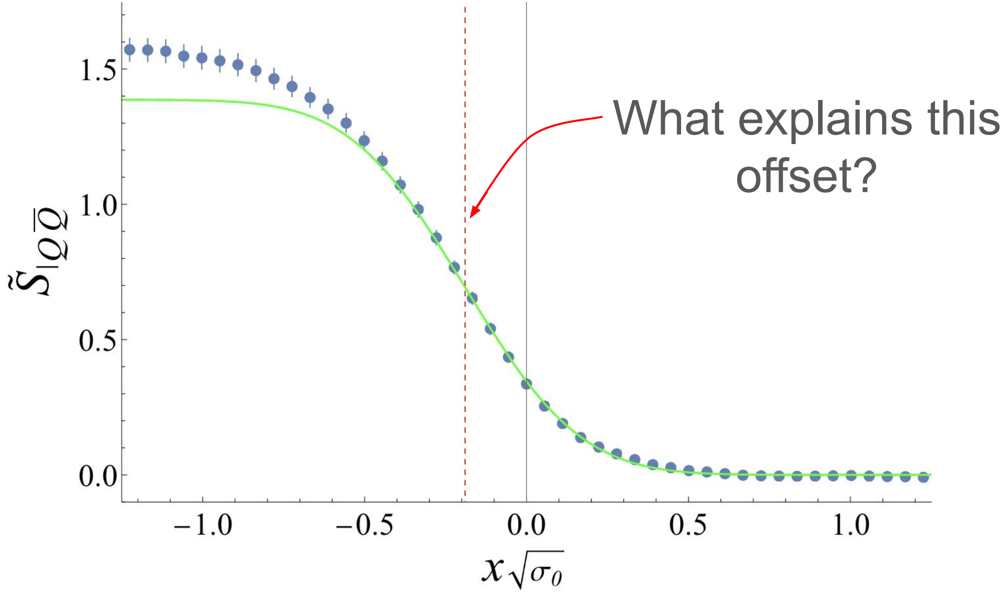






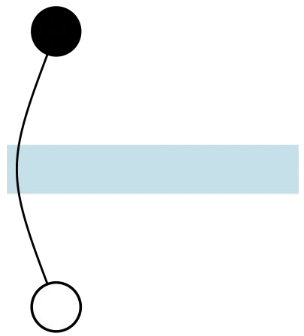
Entanglement Radius





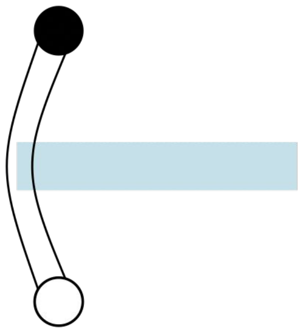
- Flux tube only seems to contribute to EE when center deeper inside region A than $\xi_0\sqrt{\sigma} \approx 0.2$
- ξ_0 has stable, non-zero continuum limit
- What is the meaning of this offset?

Entanglement Radius



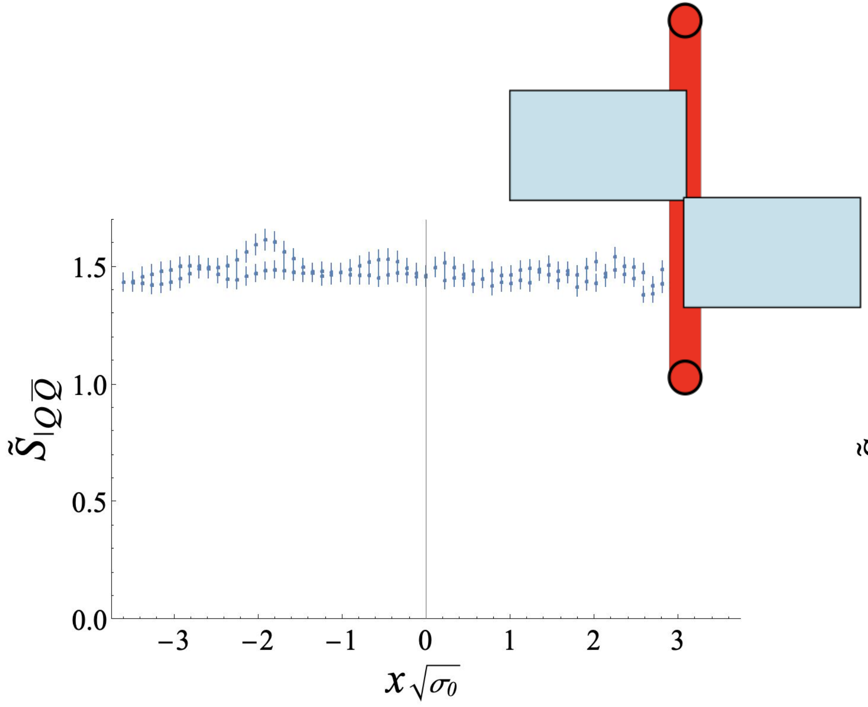
- Full boundary crossing
  - F = 2
- F*ln(Nc) entropy contribution

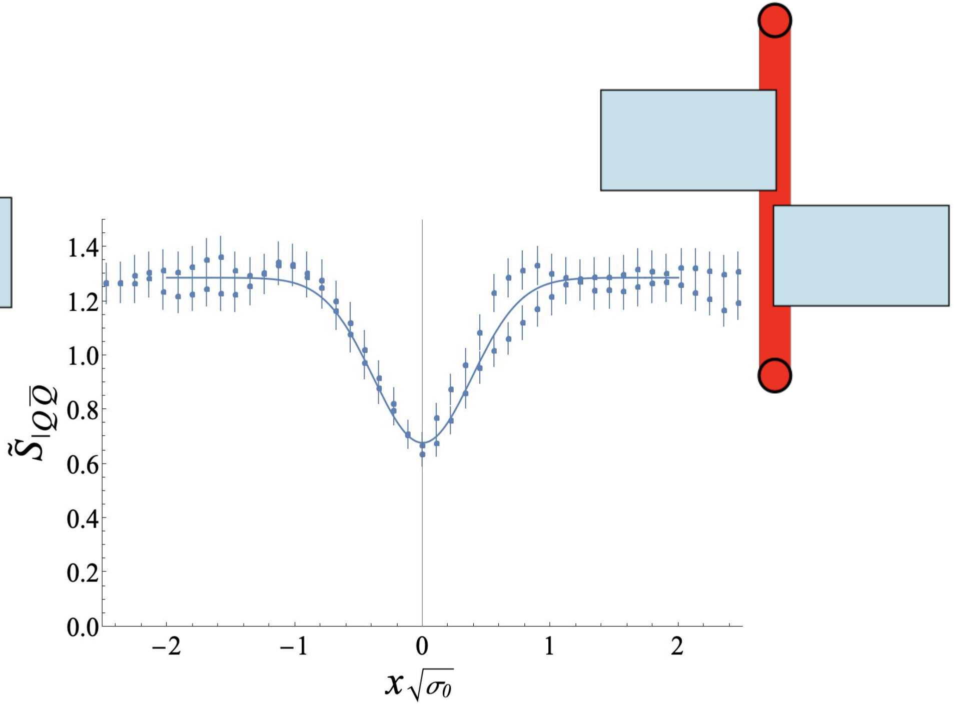
vs.



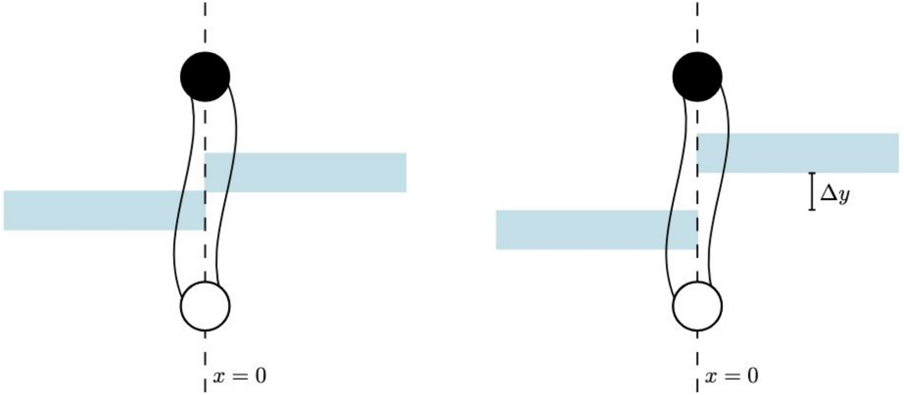
- Partial boundary crossing
  - F = ?
- If F = 0, could explain offset

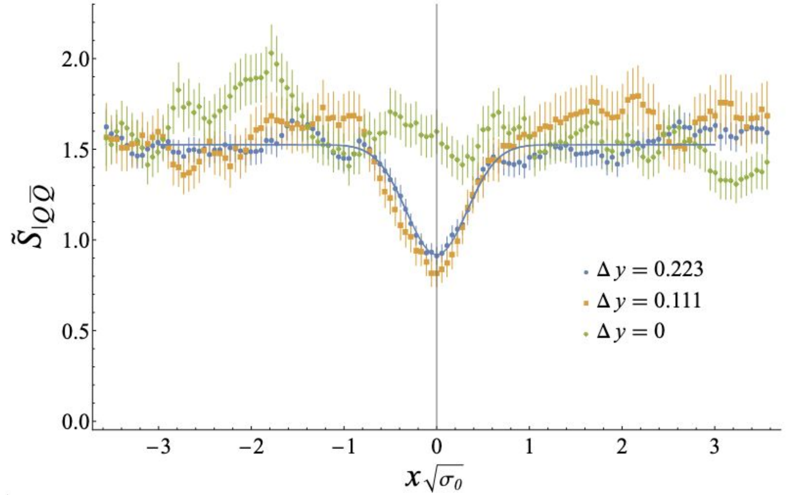
Entanglement Radius



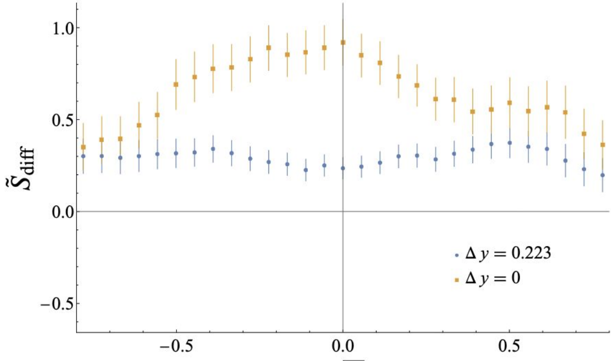


Entanglement Radius

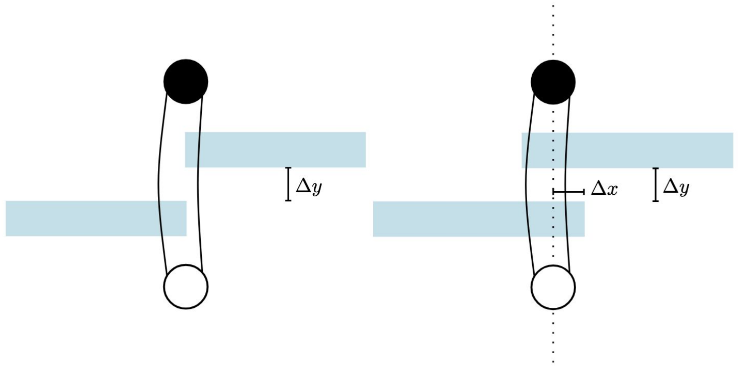


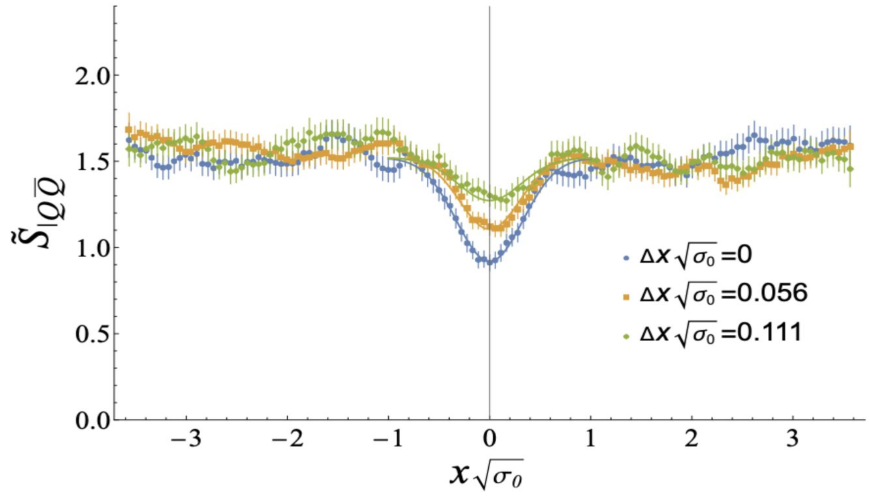


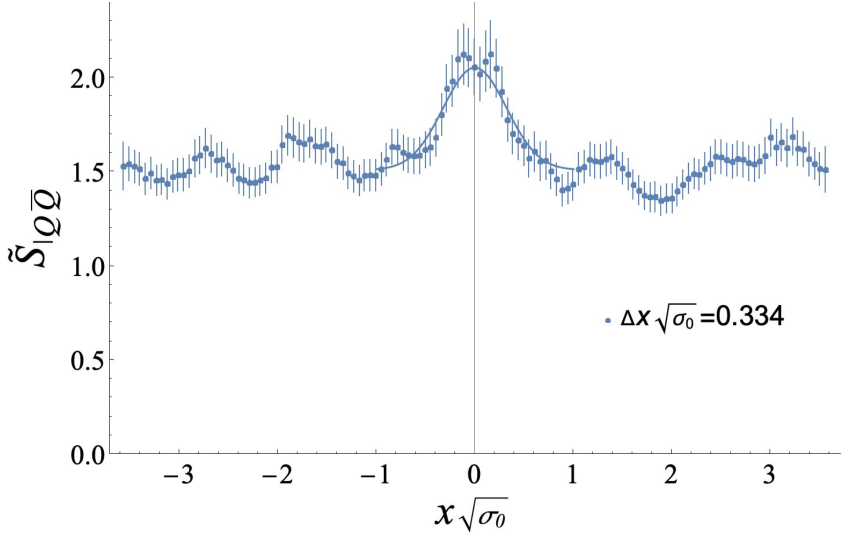
$$\tilde{S}_{\mathrm{diff}}(x) \equiv \tilde{S}_{A\cup B}(x) - \left(\tilde{S}_A(x) + \tilde{S}_B(x)\right)$$



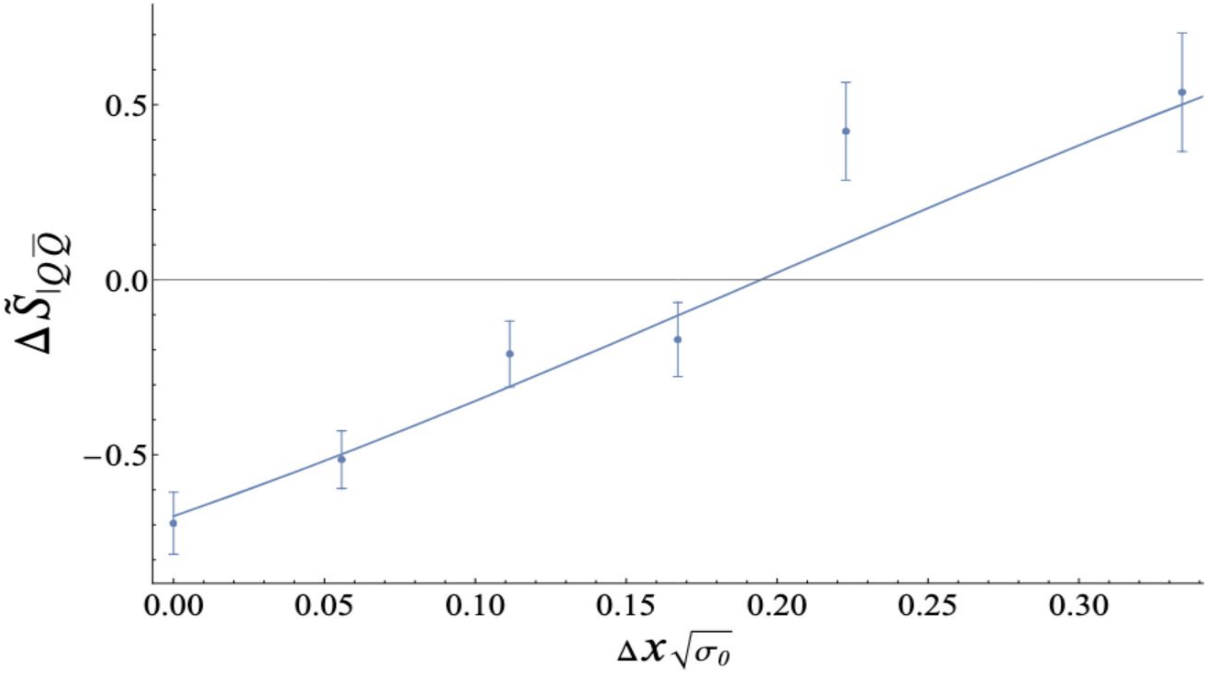
Entanglement Radius

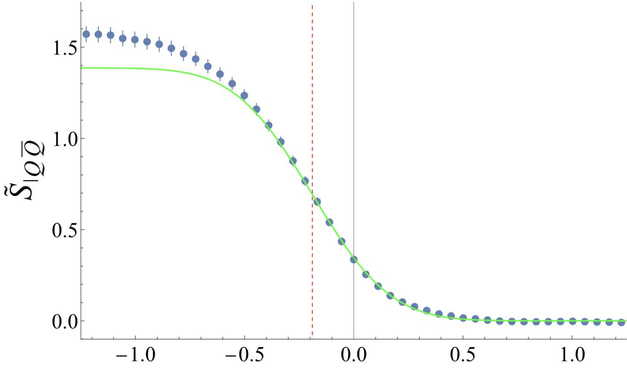


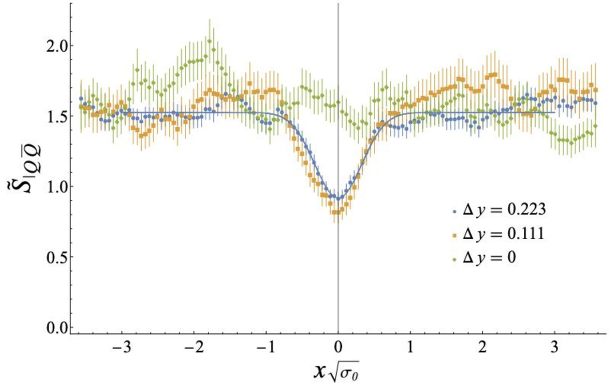




Entanglement Radius

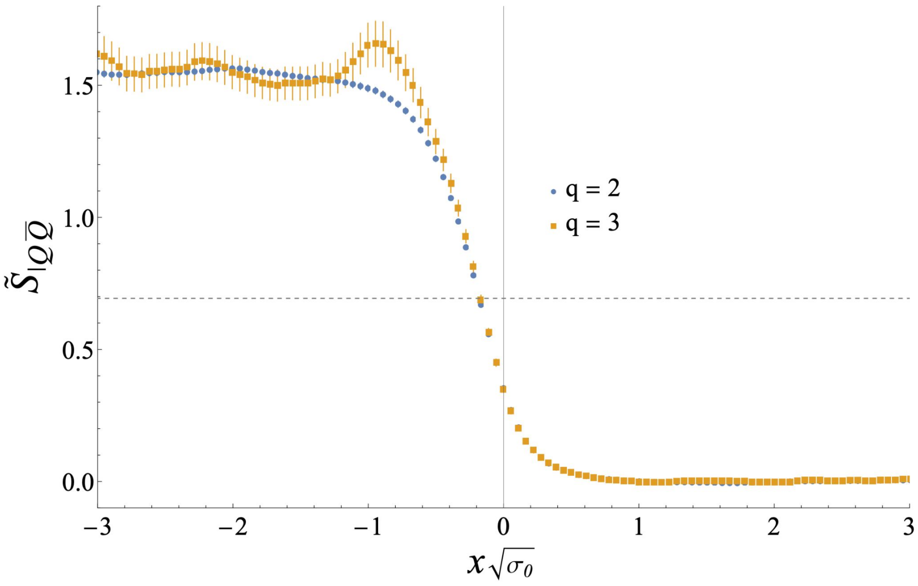


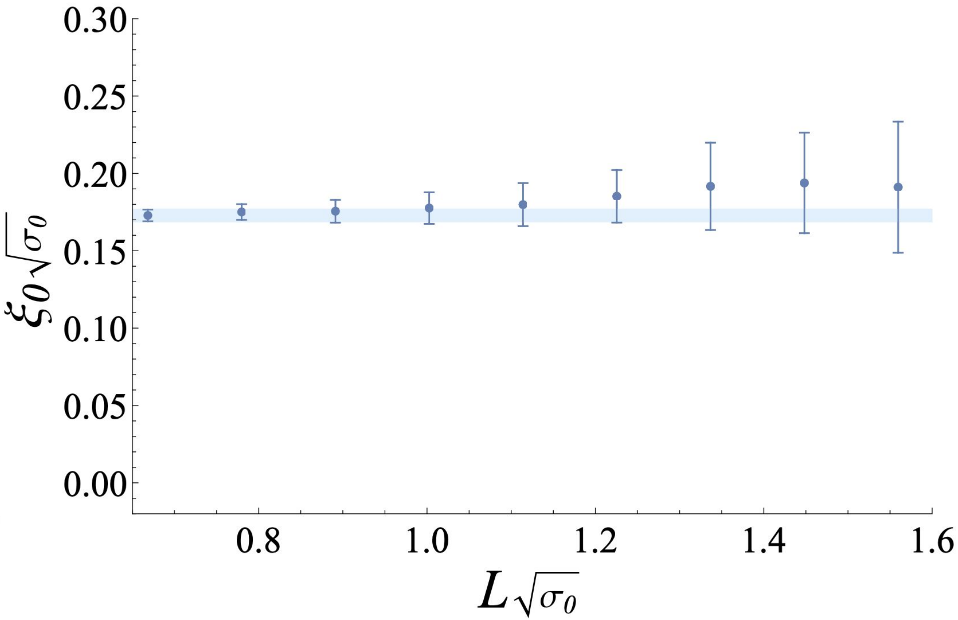




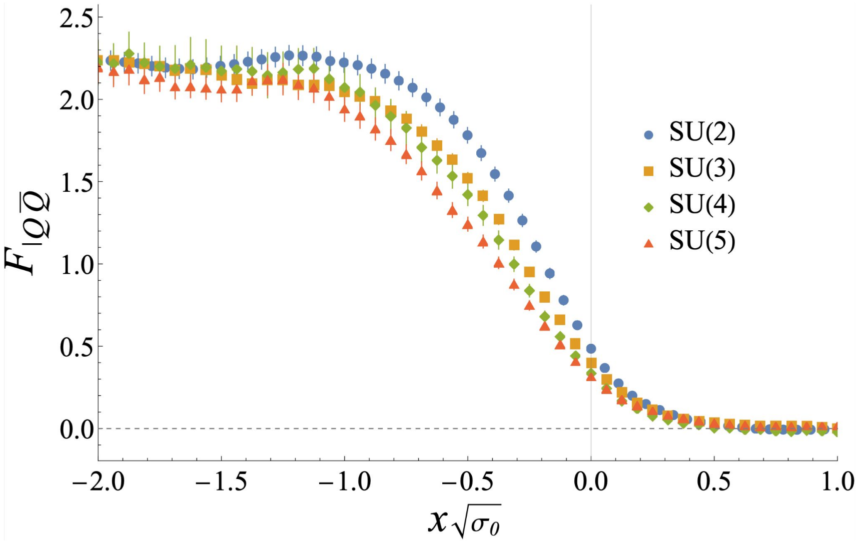
Takeaway: Entanglement Radius is consistent for multiple definitions.

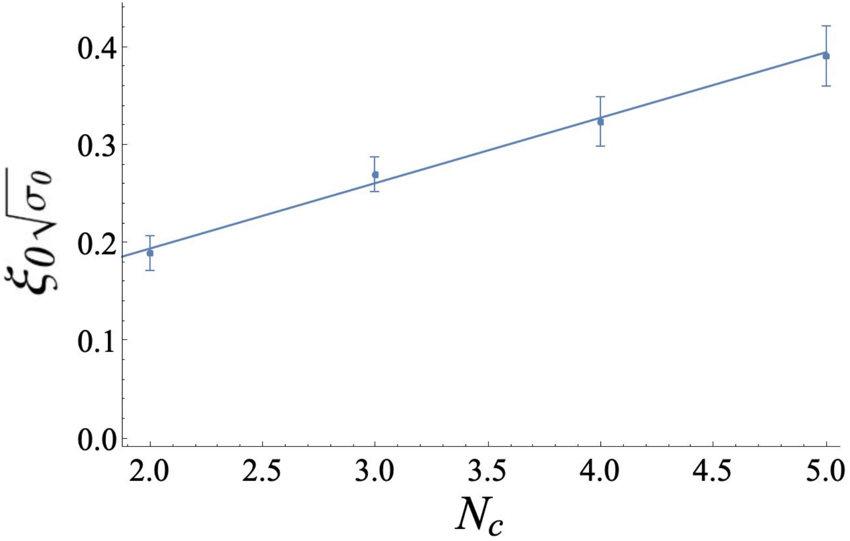
Entanglement Radius





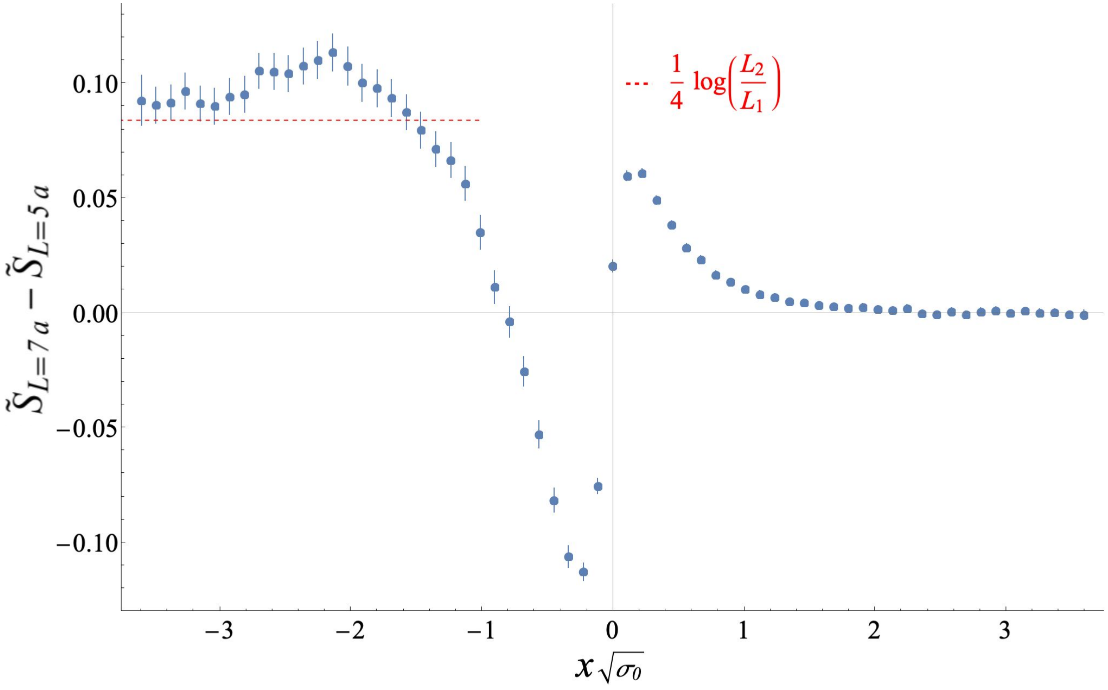
Entanglement Radius

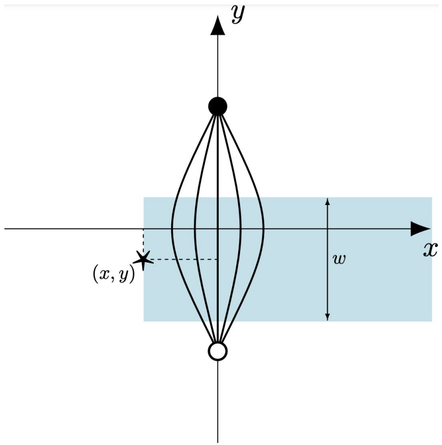


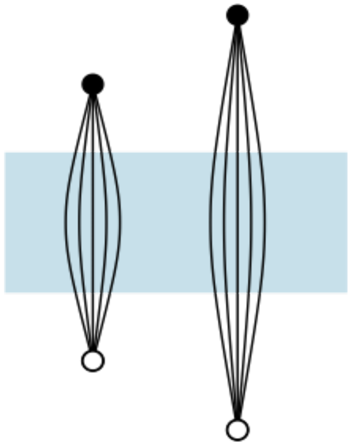


Finite Temperature

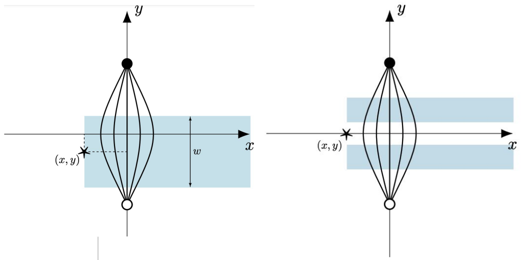
Finite Temperature (0.8 Tc)

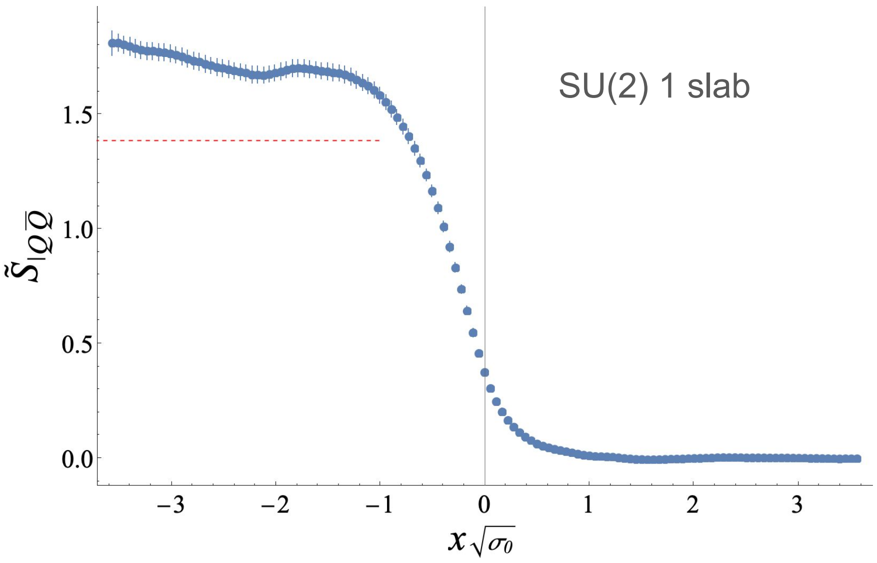


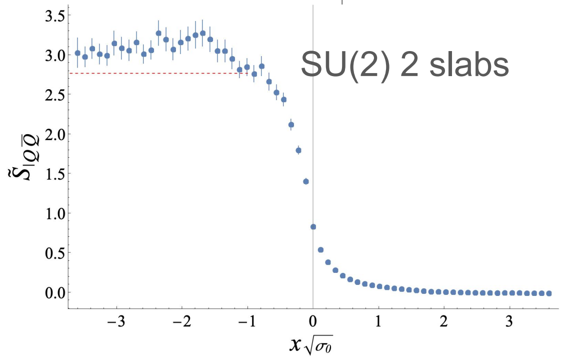


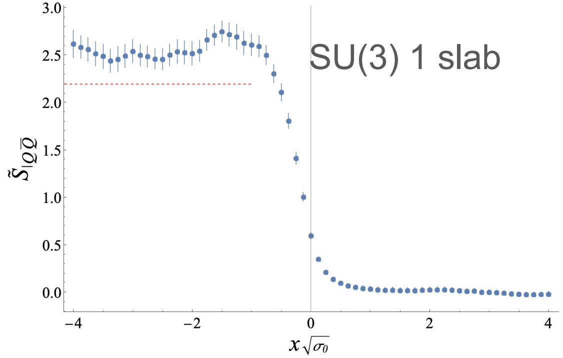


Finite Temperature (0.8 Tc)

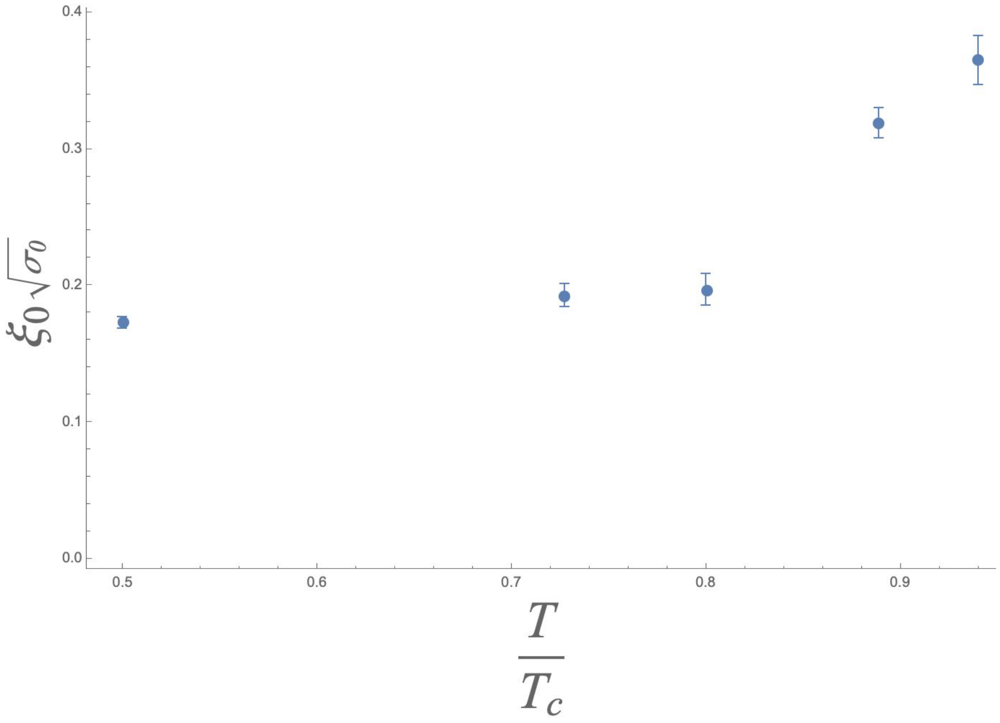






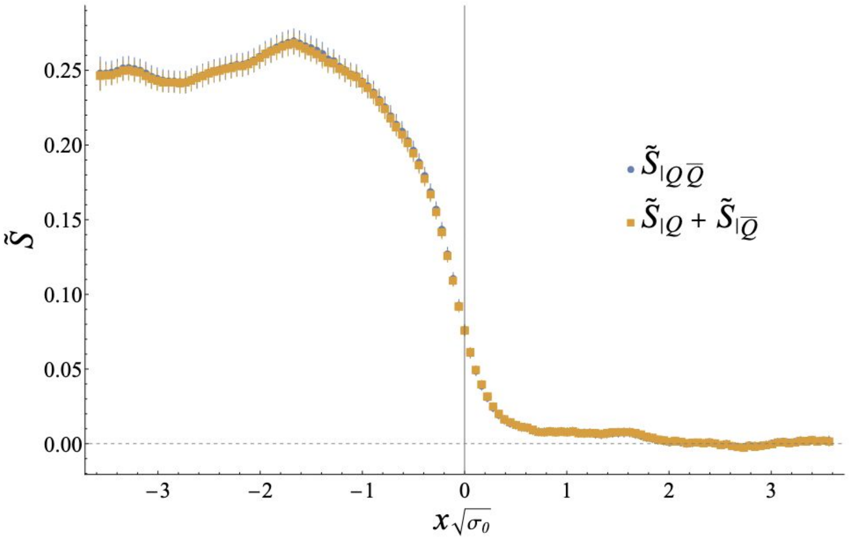


Entanglement radius at finite temperature

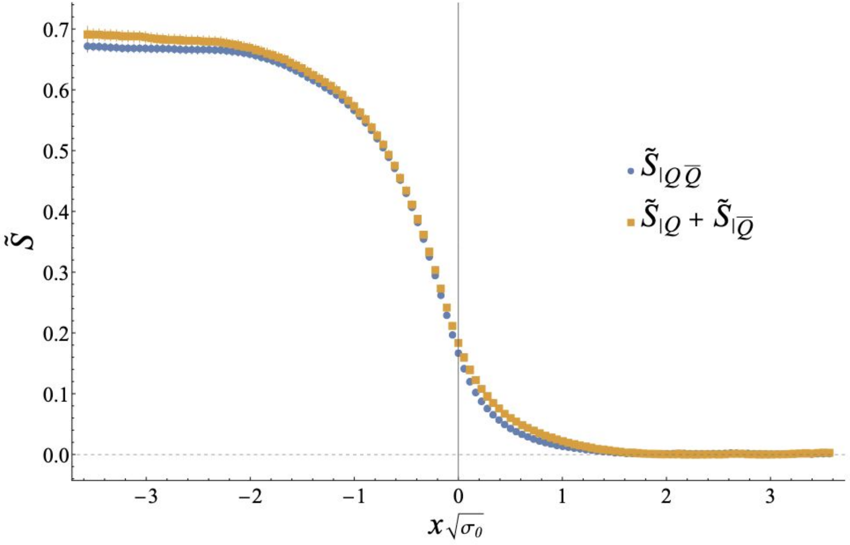


Deconfinement

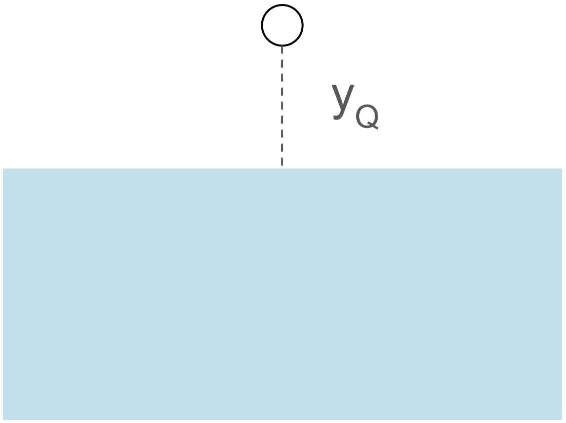
2 Tc

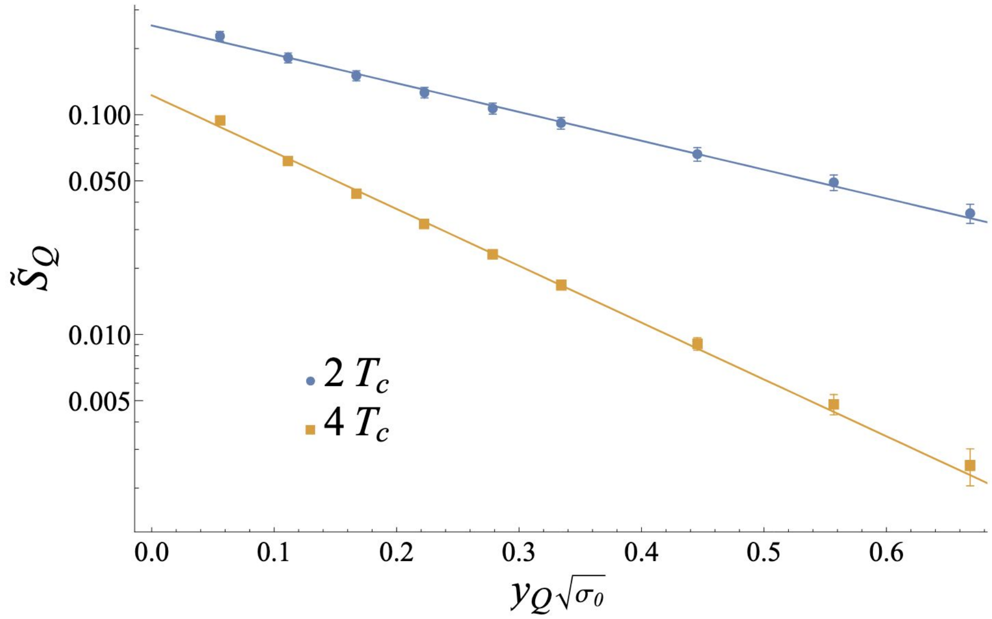


4/3 Tc



Deconfinement



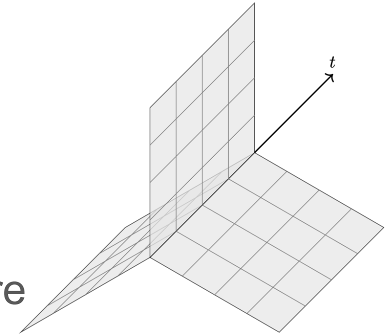


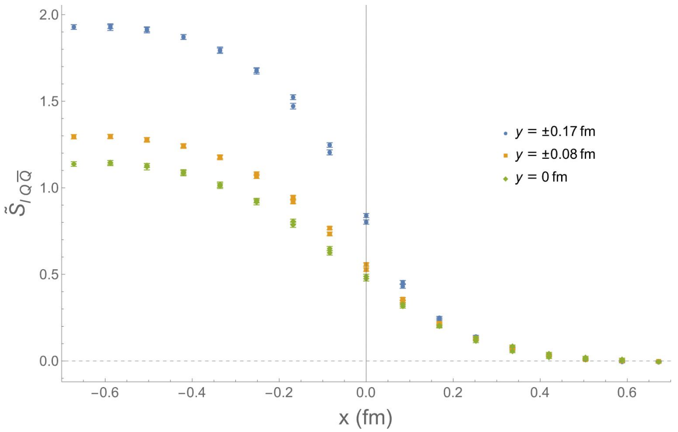
Conclusions

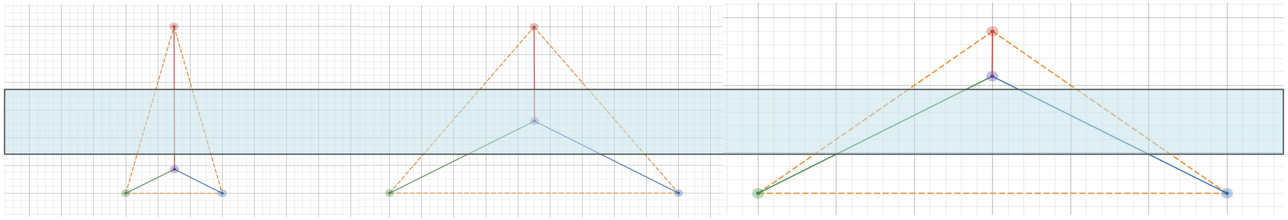
- Vacuum subtraction creates a UV-finite entanglement measure
- Flux tube largely behaves like a thin string with modifications
- Internal entropy follows <F>*ln Nc when F counts *full boundary crossings*
- Entanglement radius has continuum limit, is consistent across multiple ways of measuring it
- The flux tube still behaves as an effective string with entanglement radius at finite temperature
- Deconfinement produces much smaller flux tube entanglement

Future Directions

- FTE^2 with adjoint quarks
- FTE^2 with static baryons
- Making sense of topological nature of ξ
- Further extensions of FTE^2 (3+1, dynamical quarks)

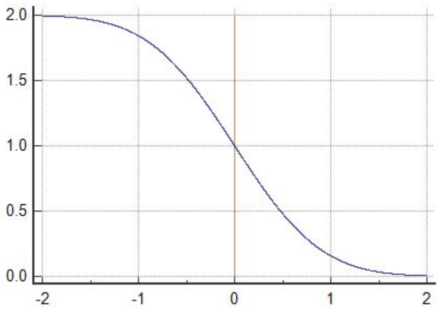


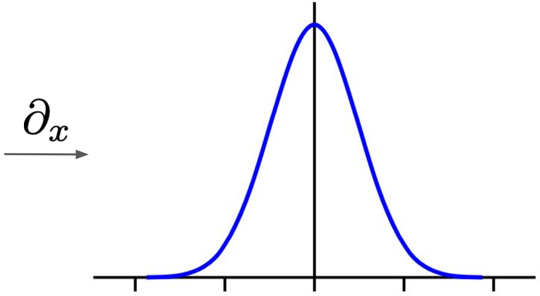


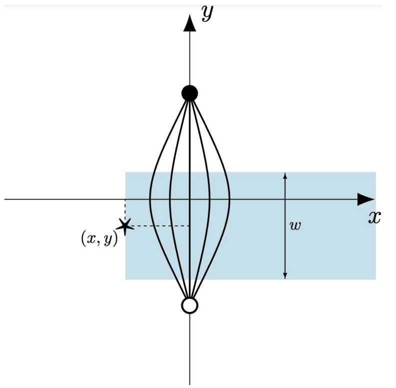


Backup

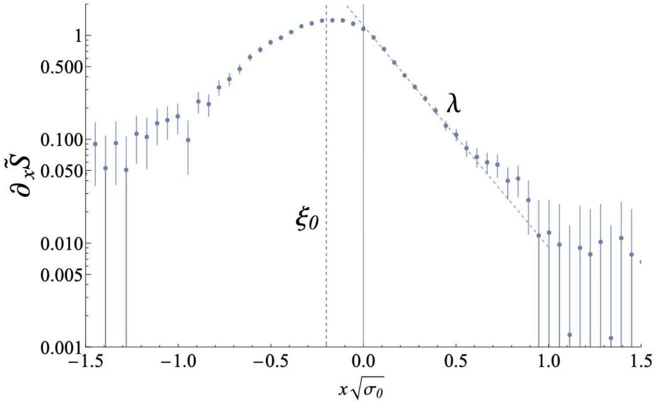
The Decay Length

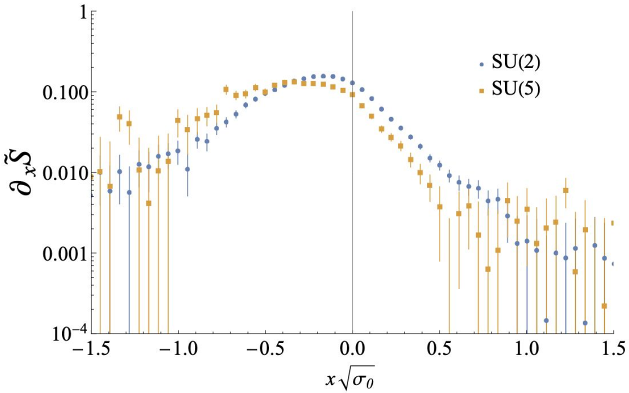




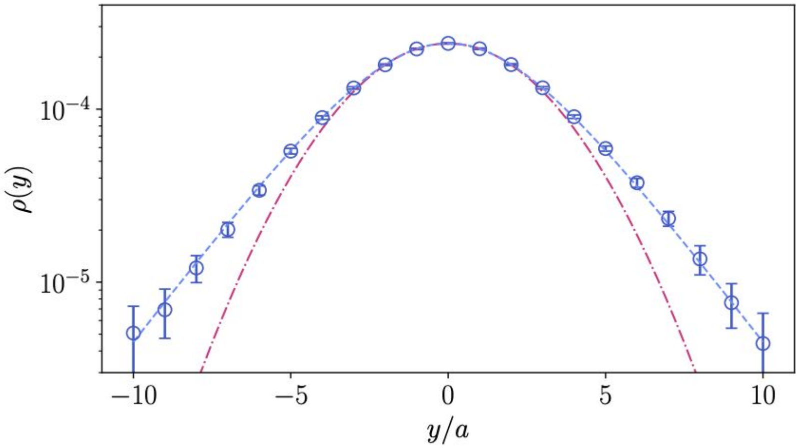


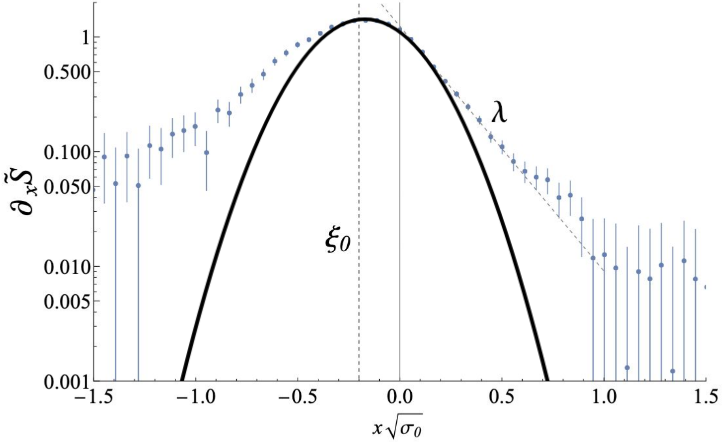
New scale: λ

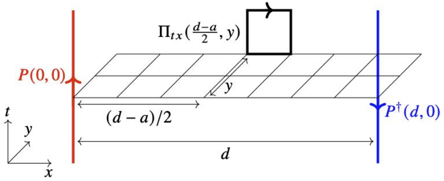




Decay Length vs Intrinsic Width







(L.Verzichelli et al arxiv:2501.01740)

Group	$\xi_0\sqrt{\sigma}$	$\lambda\sqrt{\sigma}$	$\sqrt{\sigma}/m_{0^{++}}$
SU(2)	0.189(6)	0.223(15)	0.212(2)
SU(3)	0.273(12)	0.199(45)	0.231(2)
SU(4)	0.320(8)	0.218(31)	0.236(3)
SU(5)	0.394(9)	0.191(29)	0.239(3)

UV-finite Entanglement Entropy

$$\frac{1}{|\partial A|}S = \frac{1}{|\partial A|}S_{UV} + \frac{1}{|\partial A|}S_f$$

(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247)
(T. Nishioka, T Takayanagi arXiv:hep-th/0611035)

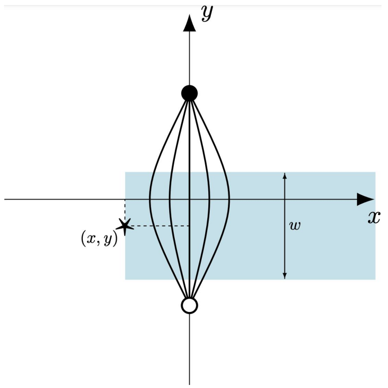
$$\tilde{S}^{E(q)}_{|Q\bar{Q}} \equiv S^{E(q)}_{|Q\bar{Q}} - S^{E(q)}$$

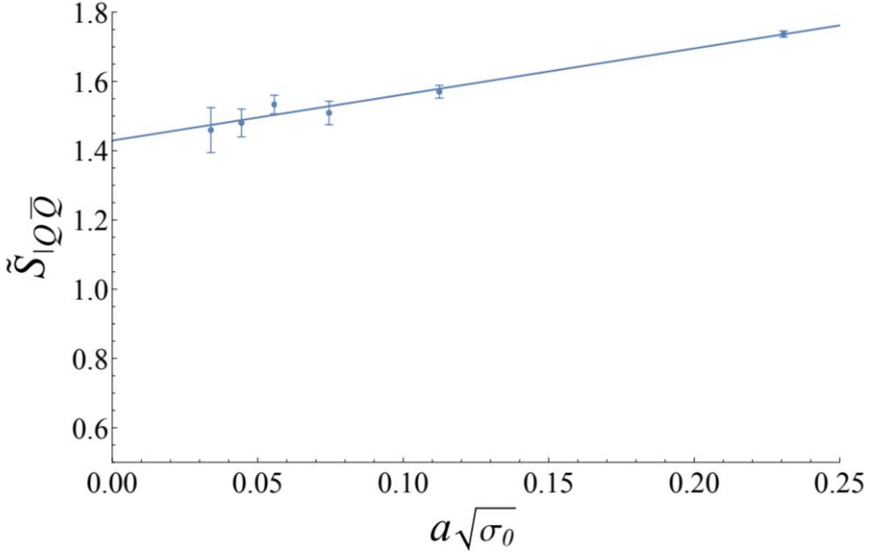
(Flux Tube Entanglement Entropy)

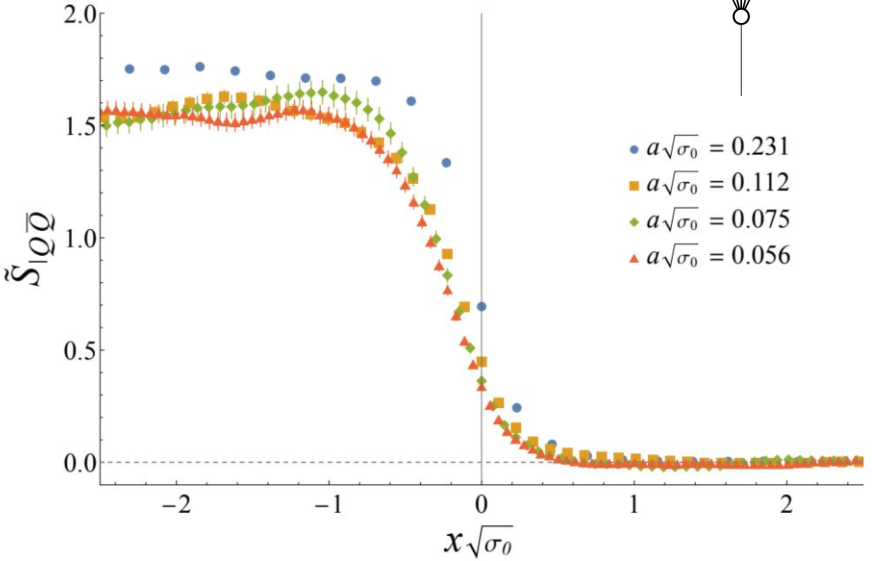
$$S^{E(q)}_X = -\frac{1}{q-1}\log \mathrm{Tr}_A\left[\left(\mathrm{Tr}_{\bar{A}}\rho_X\right)^q\right] = -\frac{1}{q-1}\log\frac{Z^{(q)}_X}{(Z_X)^q}$$

$$\boxed{-\frac{1}{q-1}\log\frac{\langle\prod_{r=1}^{q}\mathrm{Tr}P_0^{(r)}\mathrm{Tr}P_{\vec{x}}^{(r)\dagger}\rangle}{\langle\mathrm{Tr}P_0\mathrm{Tr}P_{\vec{x}}^{\dagger}\rangle^q}} = -\frac{1}{q-1}\log\frac{Z^{(q)}_{|Q\bar{Q}}/Z^{(q)}}{(Z_{|Q\bar{Q}}/Z)^q} = -\frac{1}{q-1}\log\left(\frac{Z^{(q)}_{|Q\bar{Q}}}{(Z_{|Q\bar{Q}})^q}\cdot\frac{Z^q}{Z^{(q)}}\right) = S^{E(q)}_{|Q\bar{Q}} - S^{E(q)}$$

Finite Entanglement Entropy (Scaling Study)







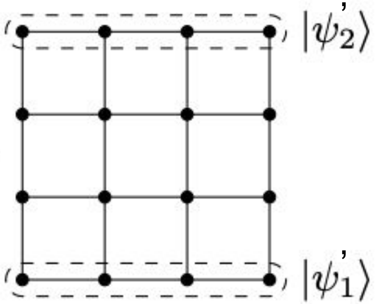
Takeaway: FTE^2 has finite non-zero continuum limit for $x \lesssim 0$

Reduced Density Matrix in Lattice Field Theory

$$\hat{\rho}_{\mathcal{A}} = \mathrm{tr}_{\bar{\mathcal{A}}}\left(\hat{\rho}\right)$$

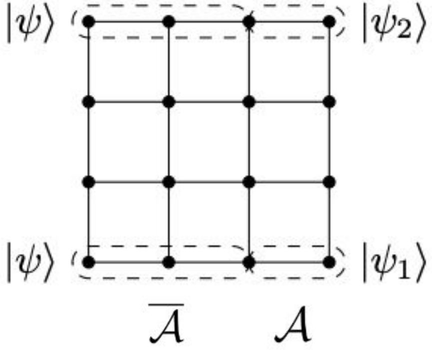
$$\langle\psi_1^{'}|\,\hat{\rho}\,|\psi_2^{'}\rangle = \frac{1}{Z}\,\langle\psi_1^{'}|\,e^{-\beta\hat{H}}\,|\psi_2^{'}\rangle$$

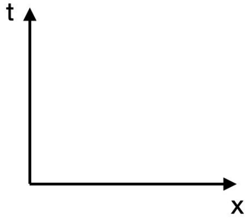
$=$



$$\langle\psi_1|\,\hat{\rho}_{\mathcal{A}}\,|\psi_2\rangle$$

$= \sum_{|\psi\rangle}$

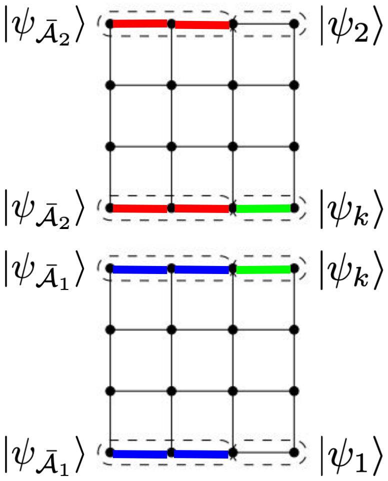




(A. Rabenstein et al arXiv:1812.04279)

Reduced Density Matrix squared

$$\langle\psi_1|\hat{\rho}^2_{\mathcal{A}}|\psi_2\rangle = \langle\psi_1|\hat{\rho}_{\mathcal{A}}|\psi_k\rangle\langle\psi_k|\hat{\rho}_{\mathcal{A}}|\psi_2\rangle = \sum_{|\psi_k\rangle,|\psi_{\bar{\mathcal{A}}_1}\rangle,|\psi_{\bar{\mathcal{A}}_2}\rangle}$$



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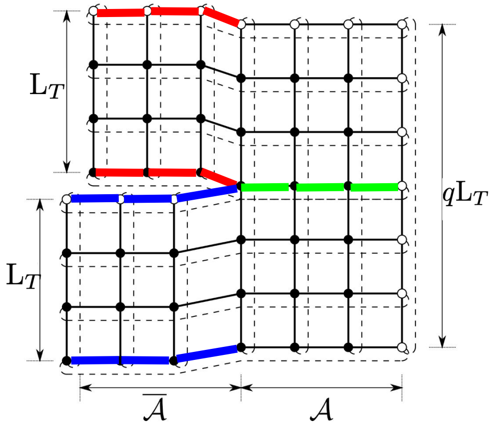
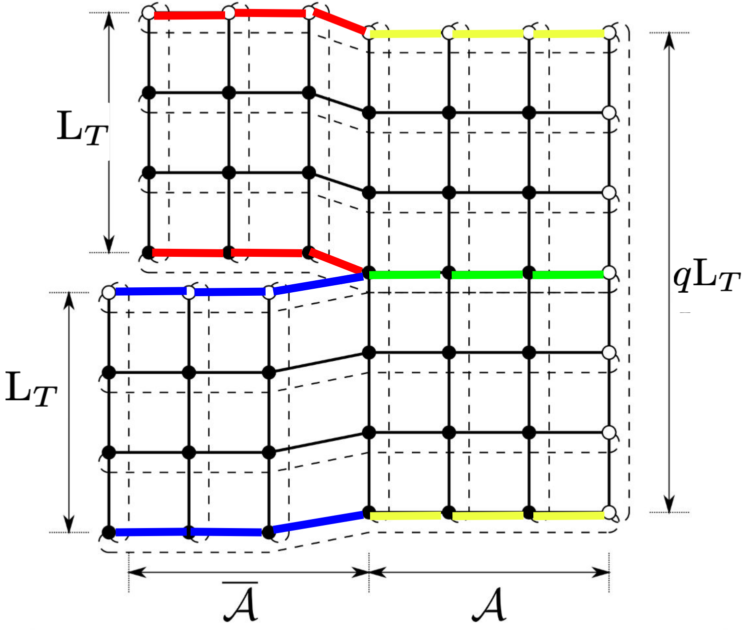
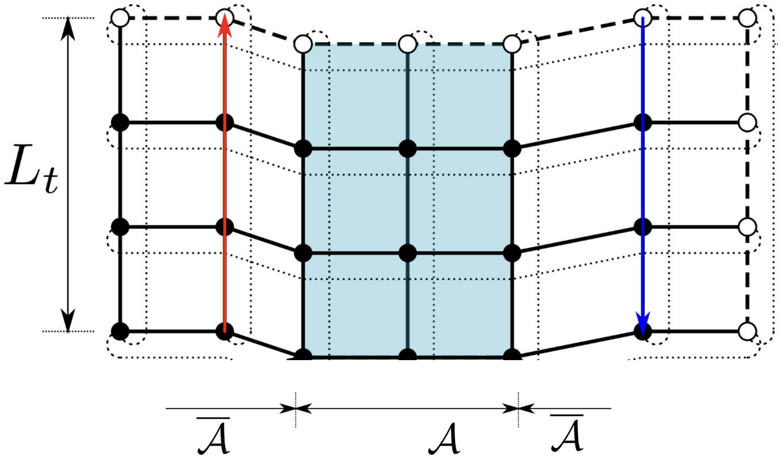


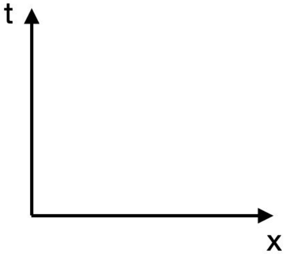
Reduced Density Matrix squared

$$\mathrm{Tr}\hat{\rho}_{\mathcal{A}}^2 = \sum_{\psi_{\mathcal{A}}} \langle \psi_{\mathcal{A}} | \hat{\rho}_{\mathcal{A}}^2 | \psi_{\mathcal{A}} \rangle = \frac{Z_2}{(Z_1)^2}$$



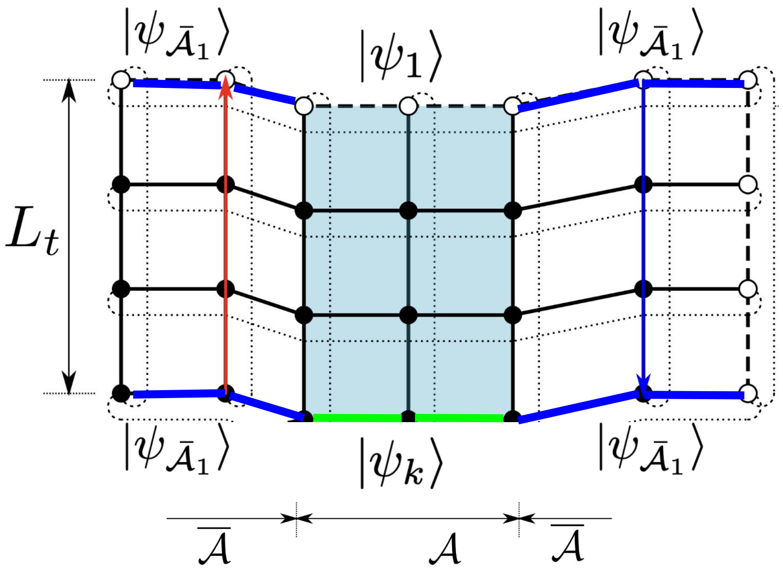
Polyakov Lines

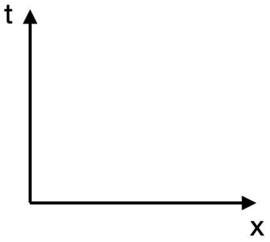




Polyakov Lines: Reduced Density Matrix

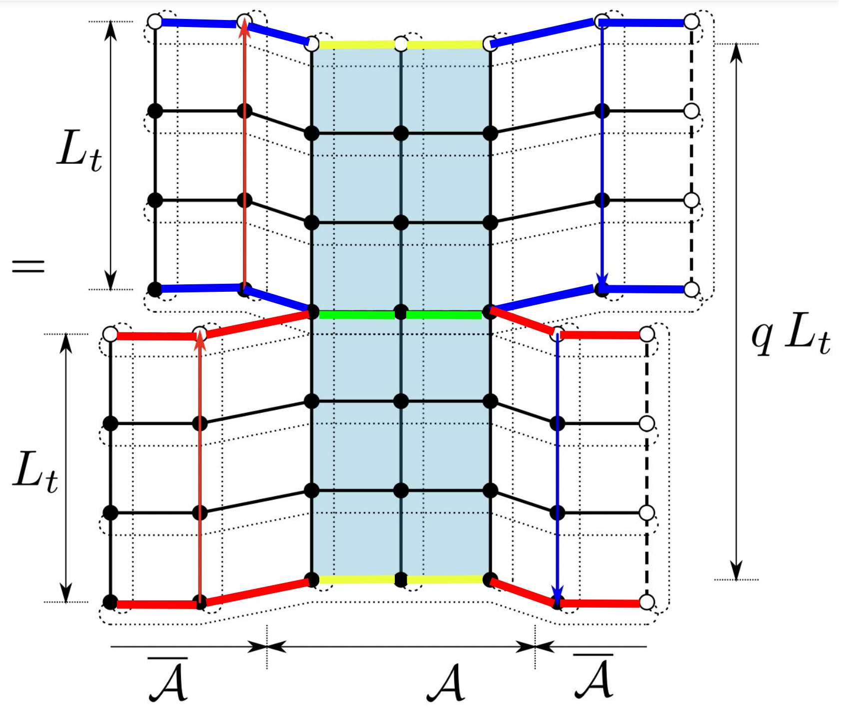
$$\langle \psi_1 | \hat{\rho}_{\mathcal{A}} | \psi_k \rangle =$$

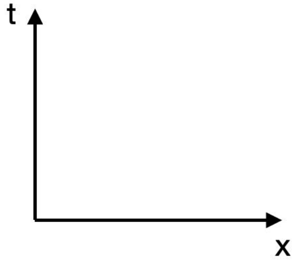




Polyakov Lines: Reduced Density Matrix squared

$$\mathrm{Tr}\hat{\rho}_{\mathcal{A}}^2 = \sum_{\psi_{\mathcal{A}}} \langle \psi_{\mathcal{A}} | \hat{\rho}_{\mathcal{A}}^2 | \psi_{\mathcal{A}} \rangle =$$





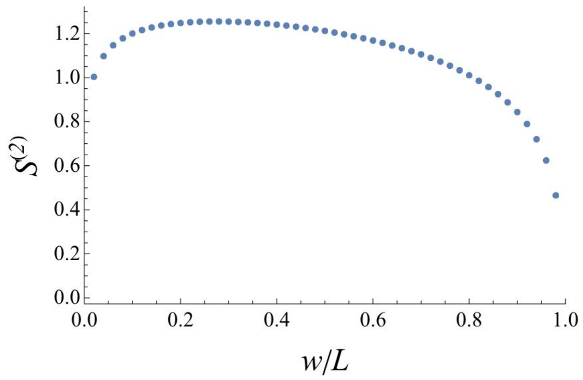
String Model Expectations (Vibrational)

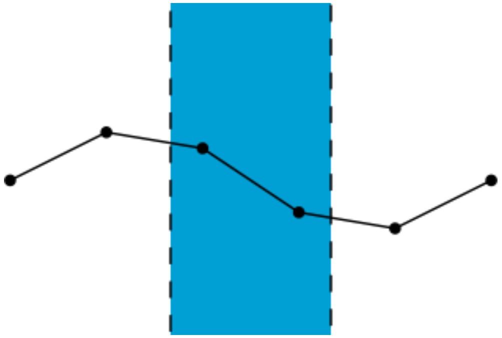
M. Luscher et al , Nucl. Phys. B 180, 1 (1981)

- Cut string into small pieces of size ε
- Numerically calculate entropy taking L/ε to infinity
- Splits into cutoff scaling and string proportion terms

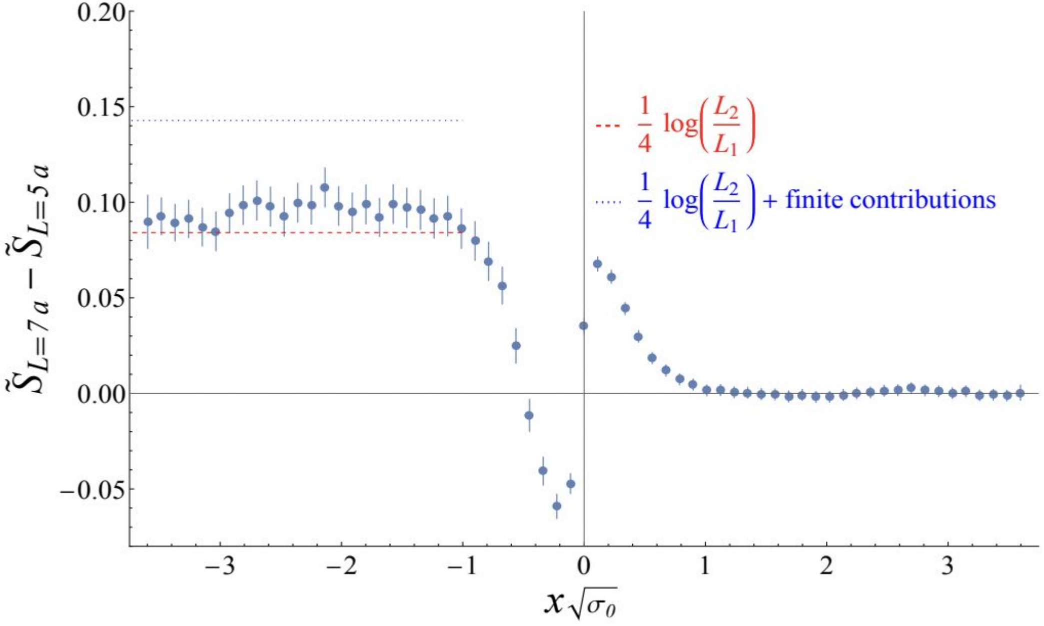
$$H = M^2 L + \frac{\pi}{2M^2 L}\int_0^\pi ds(p^2 + M^4 x'(s)^2)$$

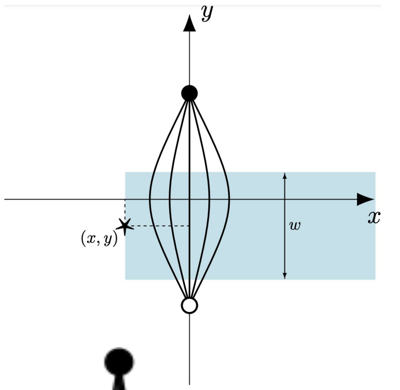
$$S^{(2)} = \frac{1}{4}\log(L/\epsilon) + \text{finite contributions}$$

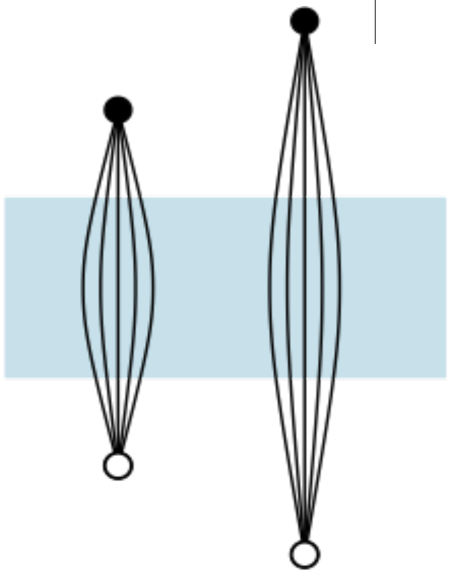




Entropy String Length Dependence

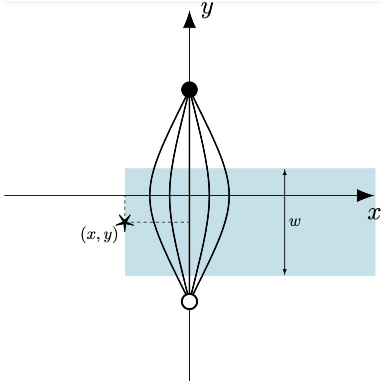


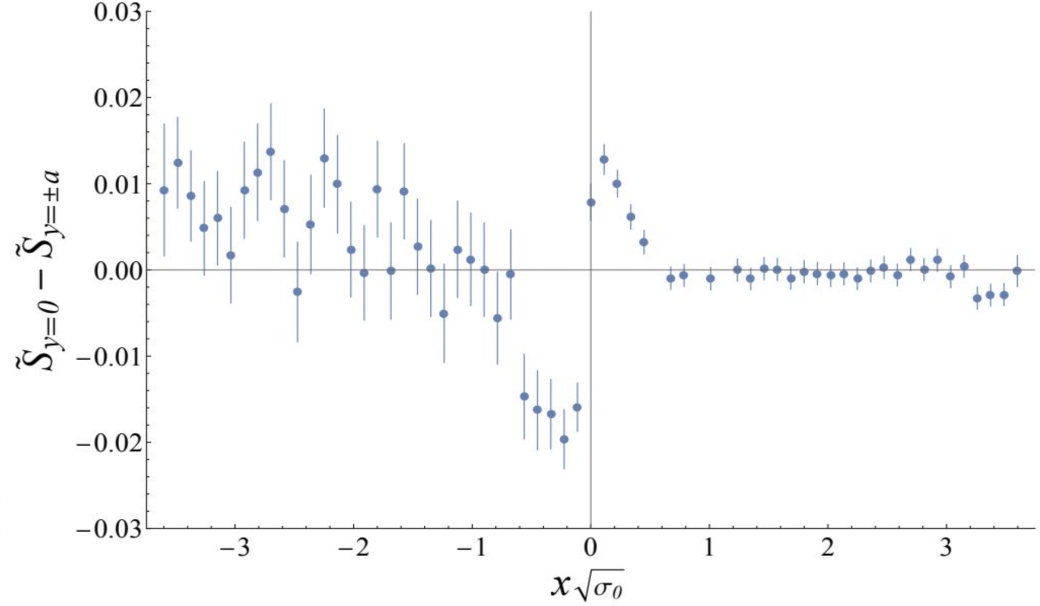


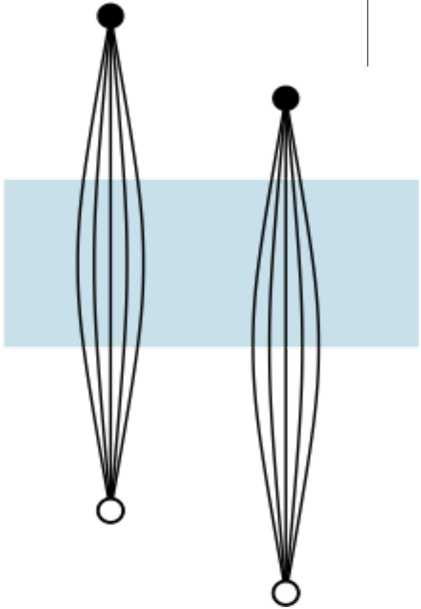


Takeaway: Present length scaling term, other finite contributions small/absent.

Entropy Location along the String Dependence







Takeaway: Finite contributions small/absent from vibrational entropy.