Pion and Kaon Unpolarized Mellin Moments from Nonlocal Operators

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Work with:
J. Torsiello (Temple), **K. Cichy** (Adam Mickiewicz University),
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Outline

❖ Theoretical Formulation

❖ Lattice Formulation (focus of this talk)

- ❖ Background
- ❖ Lattice Methodology
- ❖ Results
  - Matrix Elements
  - Double Ratio
  - Fixed z Analysis and combined Analysis
  - DGLAP Evolution
  - Results Comparison
  - SU(3) Symmetry Breaking
- ❖ Summary and Future Work

PHYSICAL REVIEW D **113,** 114506 (2026)

Pion and kaon PDFs from lattice QCD via large momentum effective theory and short-distance factorization

Joshua Miller,[1,*] Joseph Torsiello,[1,†] Isaac Anderson,[2,1] Krzysztof Cichy,[3] Martha Constantinou,[1,‡] Joseph Delmar,[1] and Sarah Lampreich[1]

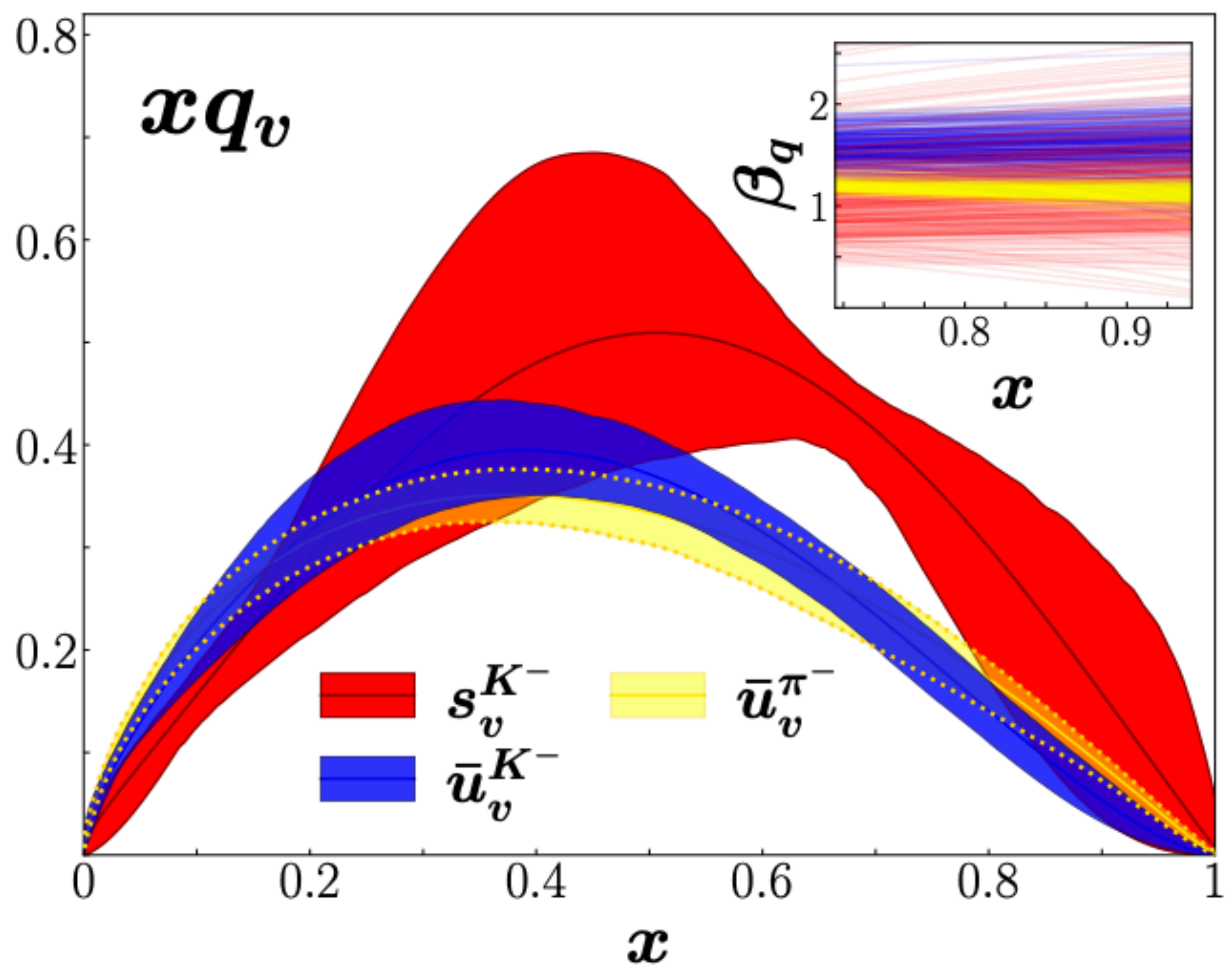
Mellin Moments of Pion and Kaon Unpolarized PDFs from Nonlocal Operators in Lattice QCD

Joshua Miller,[1,*] Joseph Torsiello,[1] Krzysztof Cichy,[2] Martha Constantinou,[1,†] and Joseph Delmar[1,3]

arXiv:2606.28102 [hep-lat]

Motivation

- The pion and kaon are pseudo-Nambu-Goldstone bosons
  - The pion is the lightest with the kaon being its SU(3) symmetry partner
  - Associated with chiral symmetry breaking, hadron structure, SU(3) symmetry breaking



[Barry et al. (JAM25), arXiv:2510.11979]

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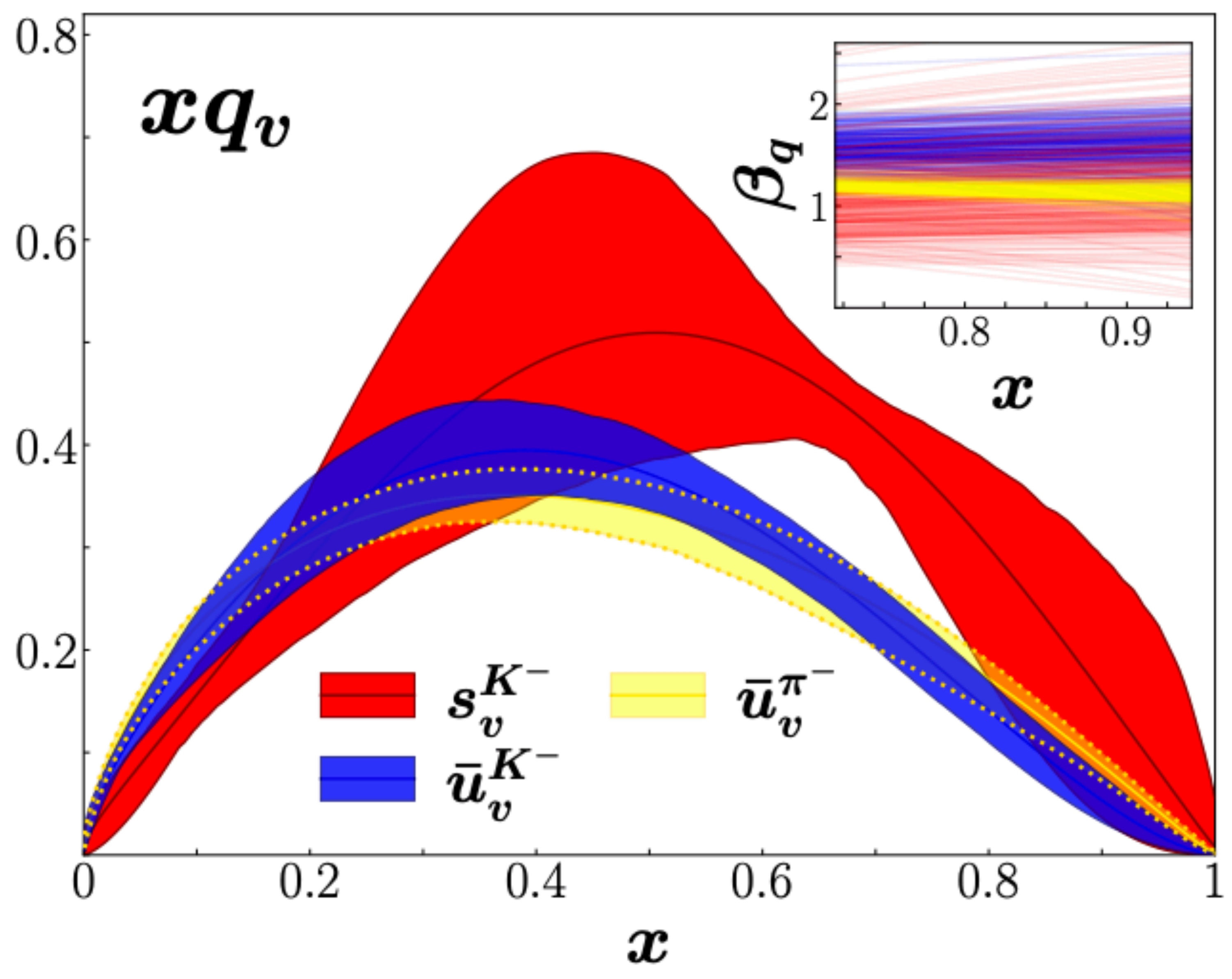
- ❖ More pion data than kaon through processes such as pion-induced Drell-Yan



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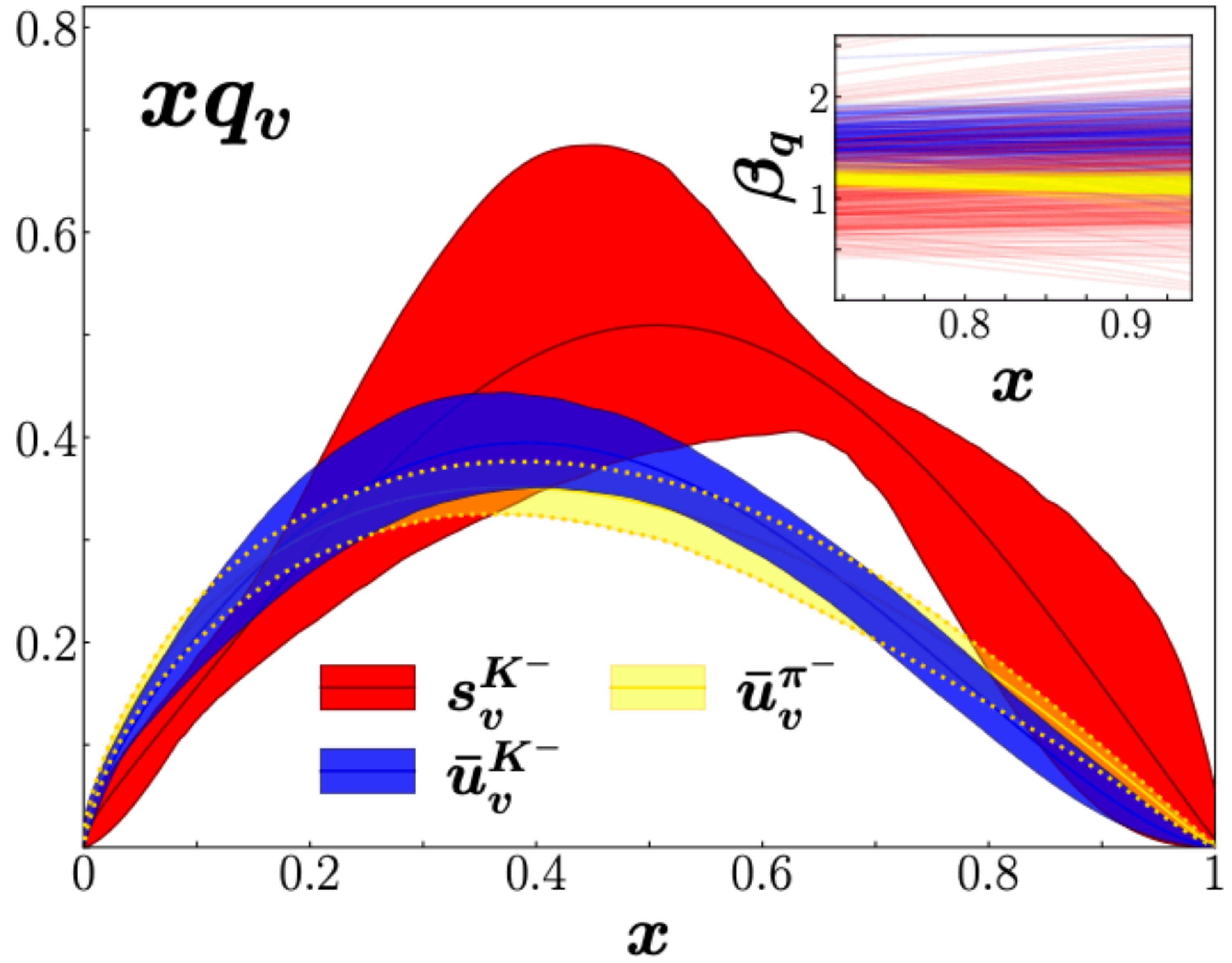
- ❖ Mellin moments contain information on the fraction of hadron momentum carried by quarks, weights results on different regions of the PDF, etc.

$$\langle x^{n-1} \rangle = \int_{-1}^{1} dx \; x^{n-1} q(x)$$



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Mellin Moments from Local Operators

- Traditionally, moments are calculated from local operators

$$\partial_\mu \psi = \frac{\psi(x+\hat{\mu}) - \psi(x-\hat{\mu})}{2a}$$

$$D_\mu = \frac{1}{2}\left(\overrightarrow{D}_\mu - \overleftarrow{D}_\mu\right)$$

$$\mathcal{O}^{\{\mu_1\mu_2\ldots\mu_i\}} = \bar{\psi}\gamma^{\{\mu_1}D^{\mu_2}\ldots D^{\mu_i\}}\psi$$

Locality

Gauge Invariance+Symmetry

Constructed Operator

- Validity comes from Operator Product Expansion (OPE) of a gauge invariant non-local matrix element

$$\langle M(p)\,|\,\bar{\psi}(0)\gamma_\mu \mathscr{W}(0,z)\psi(z)\,|\,M(p)\rangle = \bar{\psi}(0)\gamma_\mu\psi(0) + \bar{\psi}(0)\gamma_\mu D^\nu\psi(0) + \cdots$$

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- Higher order Mellin moments contain more derivatives
- Operator mixing becomes larger at higher moments
- Signal decays with increase of order of moment

Difficult to extract higher moments

- Methods proposed for accessing higher moments

[Davoudi & Savage, PRD86, 054505 (2012)]

[Shindler, PRD 110 (2024) 5, L051503]

Mellin Moments from Nonlocal Operators

Recent methods give access to $x-$dependent PDFs from non-local operators in LQCD and consequently their Mellin moments

- Large Momentum Effective Theory (**LaMET**) **[X. Ji, PRL 110 (2013) 262002; Sci. China Phys. M.A. 57 (2014) 1407]**
  - Requires momentum to be large (infinite limit ideal)
  - Matching in momentum space
  - $x \in [0.2, 0.8]$ reliable

- Short Distance Factorization (**SDF**) **[A. Radyushkin, PRD 96, 034025 (2017)]**
  - Reliable for $z \lesssim 0.3$ fm (theoretical expectations)
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Obtaining Mellin moments as integrals challenging

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Mellin moments from short-distance OPE

Complements traditional methods

(This talk)

OPE on Nonlocal Operators

- Calculate $\Pi_0 = \langle P_f | \bar{\Psi}(z)\, \gamma^0\, \mathscr{W}(z,0)\Psi(0)\, | P_i \rangle$
- Renormalize

$\nu = P \cdot z$

$$\mathscr{M}(\nu, z^2) = \frac{\Pi_0^R(\nu, z^2)/\Pi_0^R(\nu,0)}{\Pi_0^R(0,z^2)/\Pi_0^R(0,0)} = \frac{\Pi_0(\nu, z^2)/\Pi_0(\nu,0)}{\Pi_0(0,z^2)/\Pi_0(0,0)}$$

- Relate renormalized Matrix Elements to Mellin moments by Wilson Coefficients

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$$\mathscr{M}(\nu, z^2) = \sum_{n=0}^{\infty} \frac{(i\nu)^n}{n!} c_n(\mu^2 z^2) \langle x^n \rangle$$

NLO $\quad H_n^{(1)} \equiv H_n \quad H_n^{(j)} = \sum_{i=1}^{n} \frac{1}{i^j}$

$$c_n(\mu^2 z^2) = 1 + \frac{\alpha_s C_F}{2\pi} \left[\ln\left(z^2 \mu^2 \frac{e^{2\gamma_E+1}}{4} \right) \left(H_n + H_{n+2} - \frac{3}{2} \right) + 2\left(\frac{1}{n+1} - \frac{1}{n+2} - H_n^2 - H_n^{(2)} \right) \right]$$

Pre-factors from $e^{i\nu x}$

Wilson Coefficients

Light Cone Mellin Moments

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Pre-factors from $e^{i\nu x}$

Light Cone Mellin Moments

Wilson Coefficients

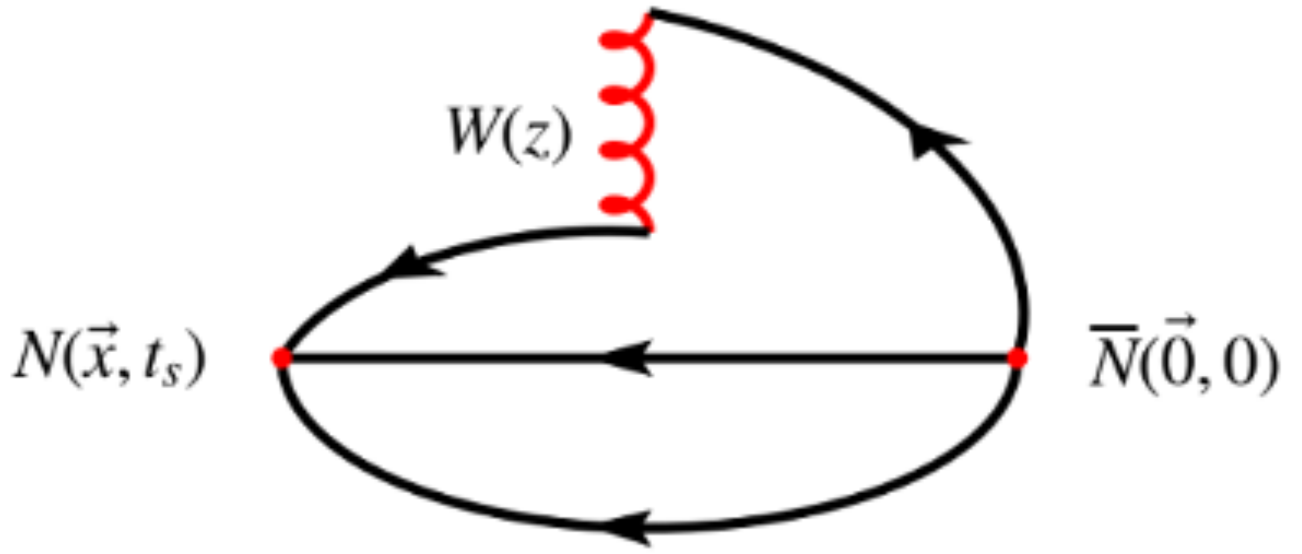
- Reconstruct the valence distribution

$$q_M^f(x) = \frac{x^\alpha (1-x)^\beta}{B(\alpha+1, \beta+1)} \longrightarrow \langle x^n \rangle = \int_0^1 dx\, x^n q_M^f(x) = \frac{B(\alpha+n+1, \beta+1)}{B(\alpha+1, \beta+1)}.$$

Ensemble - Kinematics - Statistics

❖ $N_f = 2+1+1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u, d, s, c	260	4	3.0



$\langle N(P_f) | \bar{\Psi}(z) \Gamma \mathcal{W}(z,0) \Psi(0) | N(P_i) \rangle$

❖ Large statistics required to investigate systematic effects (finite momentum effects, methodology differences, etc.)

P_3 [GeV]	0	± 0.41	± 0.83	± 1.25	± 1.66	± 2.07
t_s/a	12	12	12	$10^\star$, $12^\dagger$	10	10
N_{confs}	1,198	1,198	1,198	1,198	1,198	1,198
N_{src}^π	1	8	8	56	84	400
N_{src}^K	1	8	8	8	48	300
N_{tot}^π	1,198	9,584	9,584	67,088	100,632	479,200
N_{tot}^K	1,198	9,584	9,584	9,584	57,504	359,400

$\ast$ Pion

$\dagger$ Kaon

Analysis Methods

A. Fixed scale ($\mu = 2$ GeV)

Fixed (individual) z analysis

$$\chi^2 = \sum_{P_3} \frac{(\mathrm{Re/Im}[\mathcal{M}^{SDF}(\nu, z^2)] - \mathrm{Re/Im}[\mathcal{M}^{data}(\nu, z^2)])^2}{\sigma^2_{\mathrm{Re/Im}}}$$

❖ Fit even ($Re[\mathcal{M}]$), odd moments ($Im[\mathcal{M}]$) separately

❖ Test truncation on n for $Re[\mathcal{M}(\nu, z^2)]$ and $Im[\mathcal{M}(\nu, z^2)]$ separately

$$\left(\sum_{n=0}^{\infty} \frac{(i\nu)^n}{n!} c_n(\mu^2 z^2) \langle x^n \rangle \right)$$

❖ Allows study of residual $z-$dependence

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Combined P_3 and z analysis

$$\chi^2 = \sum_{P_3} \sum_{z=z_{min}}^{z_{max}} \left(\frac{(\mathrm{Re}[\mathscr{M}^{SDF}(\nu, z^2)] - \mathrm{Re}[\mathscr{M}^{data}(\nu, z^2)])^2}{\sigma^2_{\mathrm{Re}}} + \frac{(\mathrm{Im}[\mathscr{M}^{SDF}(\nu, z^2)] - \mathrm{Im}[\mathscr{M}^{data}(\nu, z^2)])^2}{\sigma^2_{\mathrm{Im}}} \right)$$

❖ Simultaneous fit for all P_3 and z

❖ Test different ranges for z_{min} and z_{max}

❖ Allows one to see systematic effects of higher twist contamination

Analysis Methods

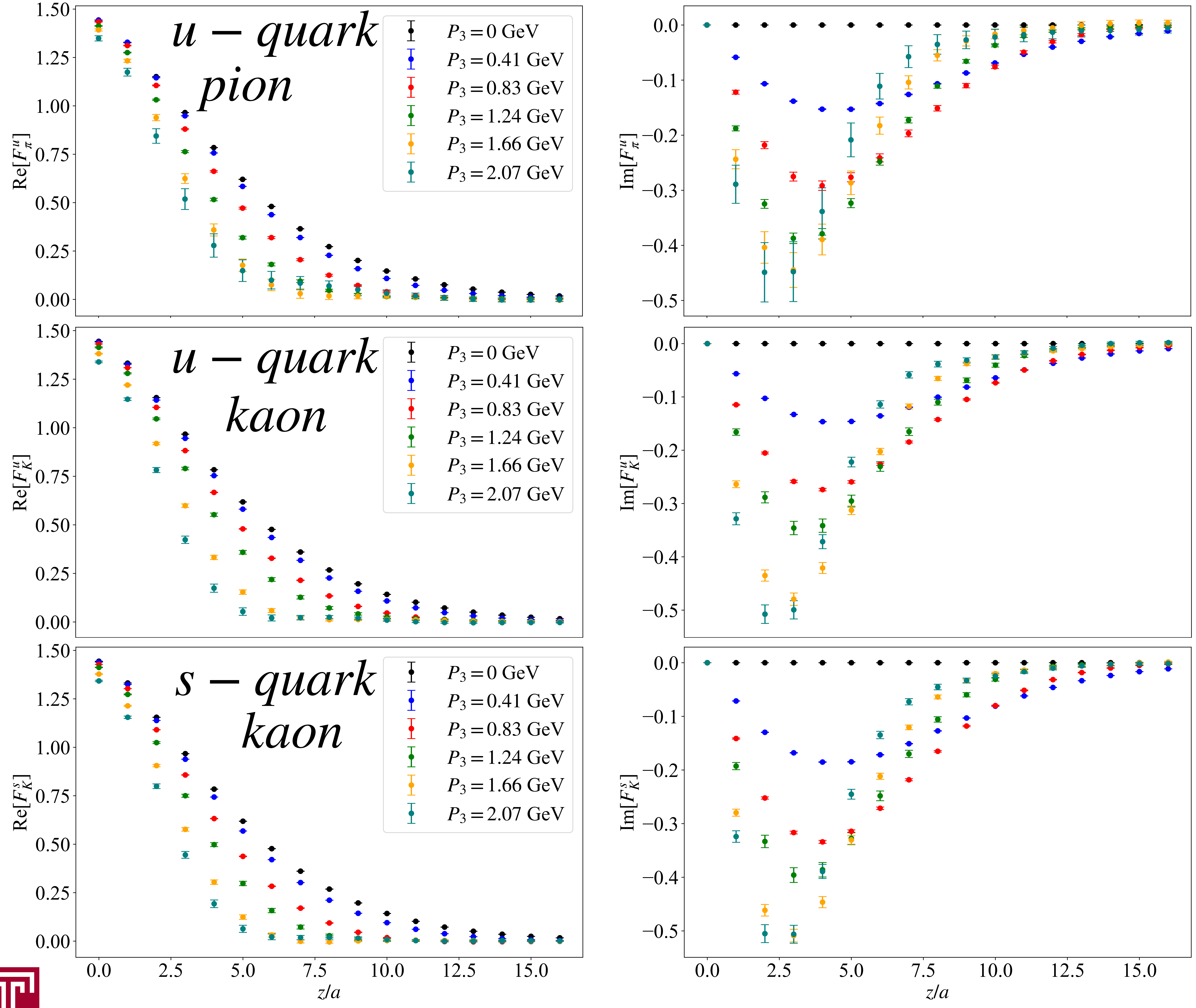
B. Scale Evolution

[Miller et al., arXiv:2606.28102]

- Unlike local operators, renormalization in SDF is not dependent on perturbation theory (PT)
  - However, Wilson coefficients (C_n) are
- Since final results are dependent on loop order, we test stability in loop order truncation
  - Utilize nonsinglet DGLAP evolution to evolve from $\mu_0 \to 2$ GeV
- Choose $\mu_0 = \frac{2\kappa}{z e^{\gamma_E}}$ such that the z-dependence in $\ln\left(z^2\mu_0^2 \frac{e^{2\gamma_E+1}}{4}\right)$ cancels → test log contributions
  - Vary the choice of $\kappa \in \left[\sqrt{2}, 2\sqrt{2}\right]$
    - Finer lattice spacings allow more choices of κ
- C_n has z-dependence in $\alpha_s(\mu_0)$: $c_n^{NLOevo}(\mu^2, z^2) = c_n^{NLO}(\mu_0^2, z^2)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\gamma_n^{(0)}/\beta_0}$

This is used to estimate systematic uncertainties.
Not covered in this talk (see backup slides)

Matrix Elements



❖ Good signal to noise ratio

❖ Larger errors for $P_3 > 1.25 GeV$, despite increased statistics

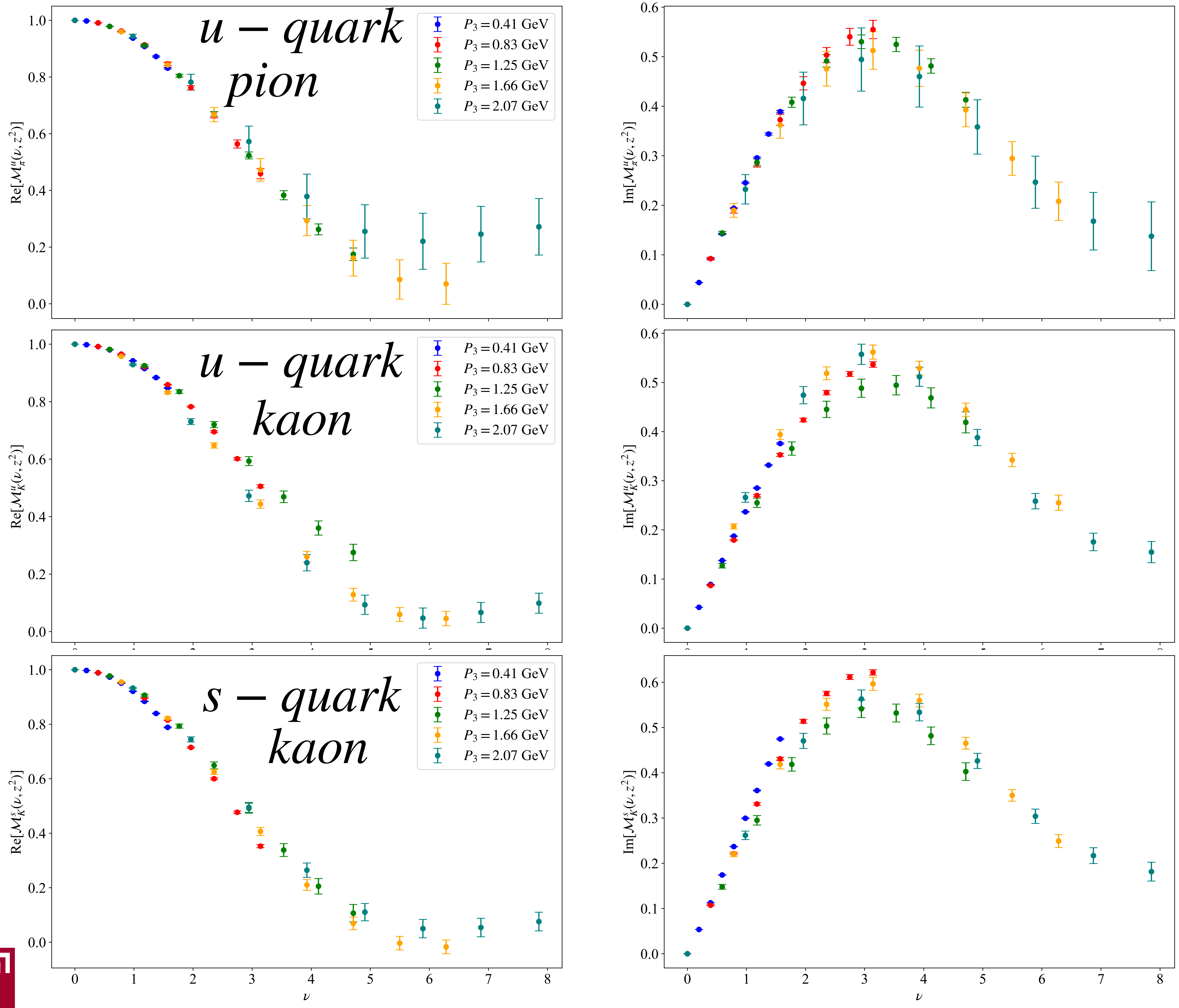
P_3 [GeV]	± 0.41	± 0.83	± 1.25	± 1.66	± 2.07
π stat. relative to $P_3 = 0$	8	8	56	84	400
K stat. relative to $P_3 = 0$	8	8	8	48	300

❖ Kaon more accurate than pion

❖ Symmetry properties implemented

$$F^f_M(-z \cdot P) = F^{f*}_M(z \cdot P)$$

Double Ratio



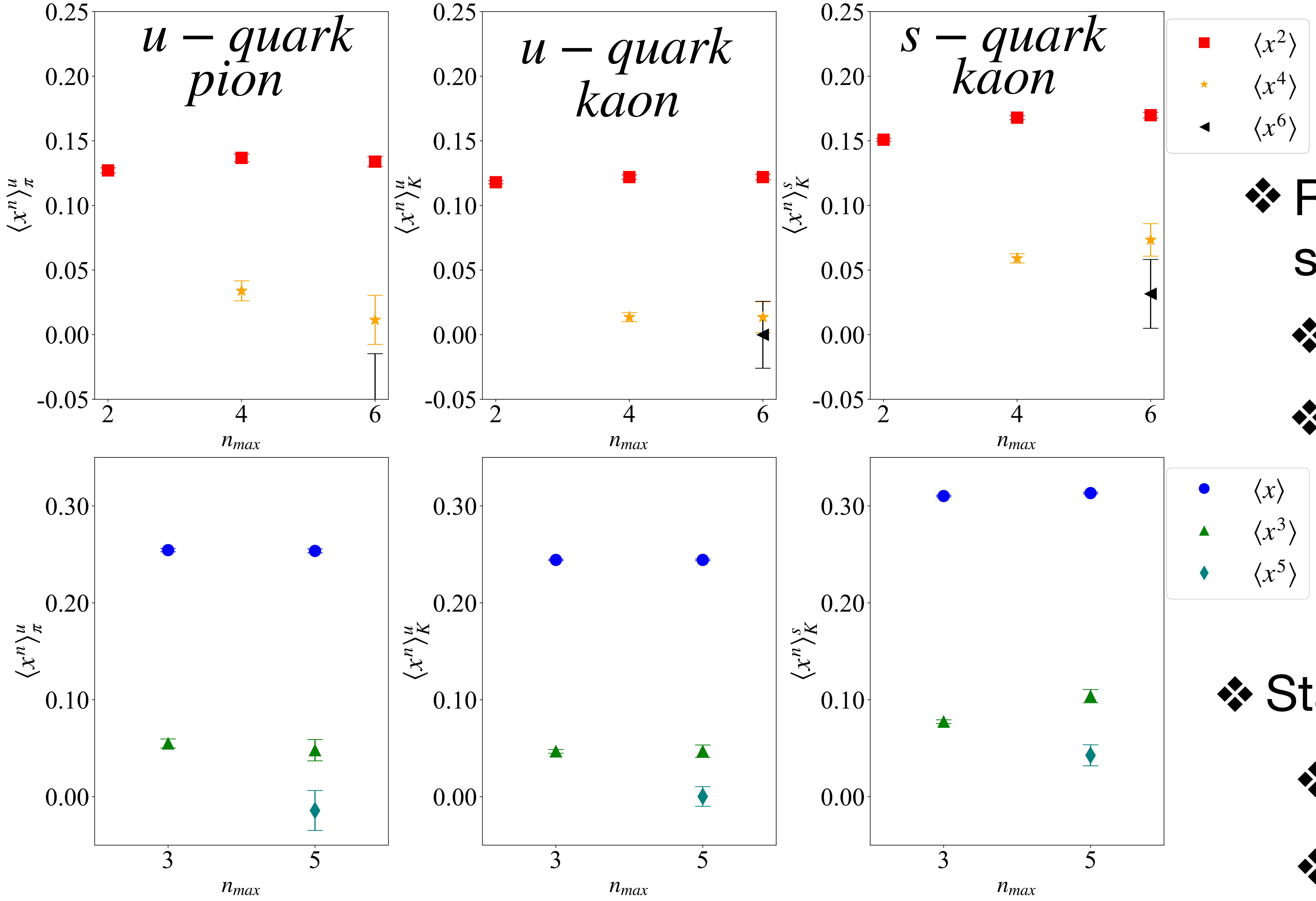
- ❖ Scheme independent renormalization

$\nu = P \cdot z$

$$\mathscr{M}(\nu,z^2) = \frac{\Pi_0^R(\nu,z^2)/\Pi_0^R(\nu,0)}{\Pi_0^R(0,z^2)/\Pi_0^R(0,0)} = \frac{\Pi_0(\nu,z^2)/\Pi_0(\nu,0)}{\Pi_0(0,z^2)/\Pi_0(0,0)}$$

- ❖ Large ν noisier for $P_3 > 1.25GeV$
- ❖ Differences between P_3 = 1.66 and 2.07 GeV at same ν
- ❖ Analysis will restrict to low z

Fixed z Analysis: n_{max} test

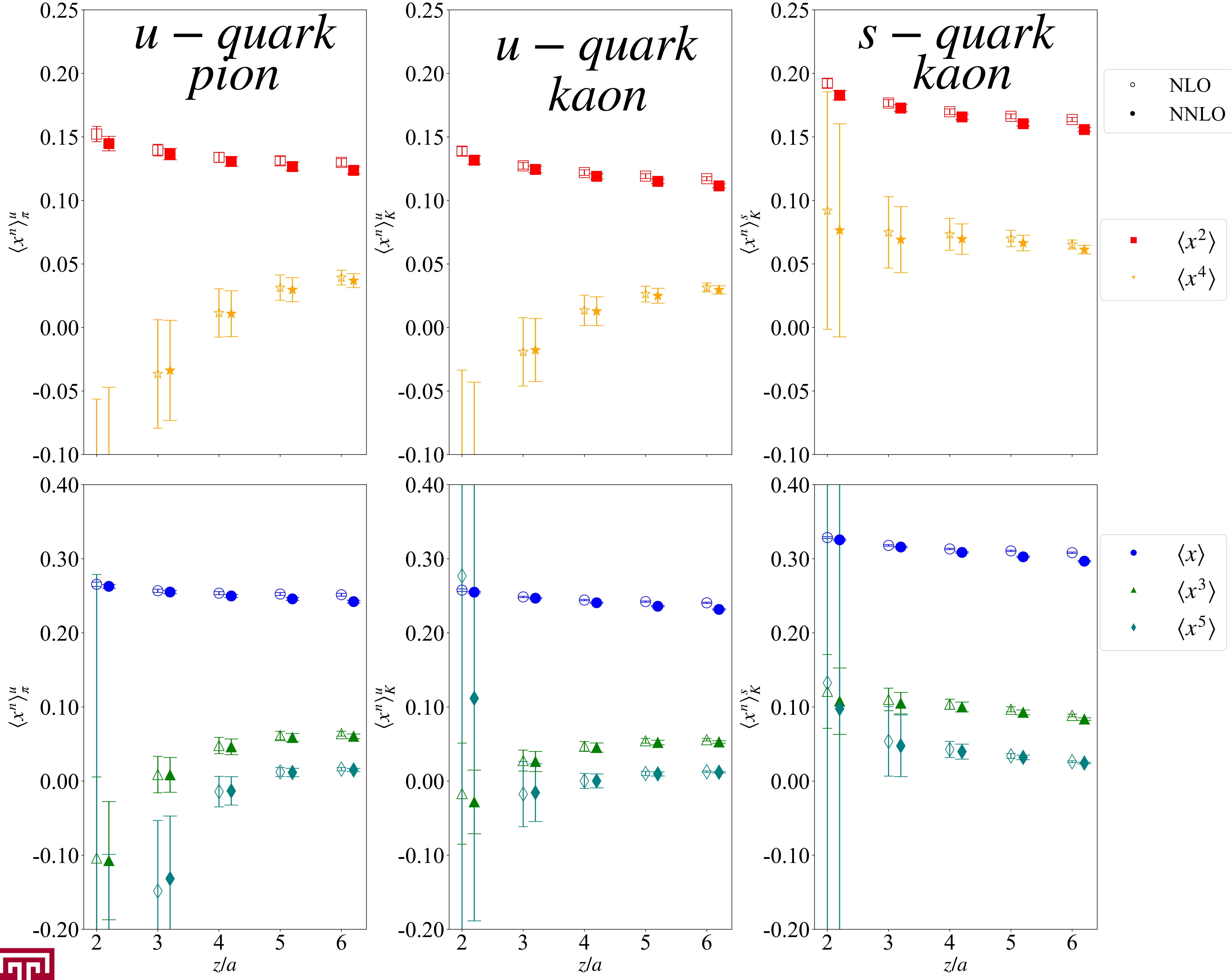


$z = 4a = 0.374$ fm

$$\mathscr{M}(\nu, z^2) = \sum_{n=0}^{\infty} \frac{(i\nu)^n}{n!} c_n(\mu^2 z^2) \langle x^n \rangle$$

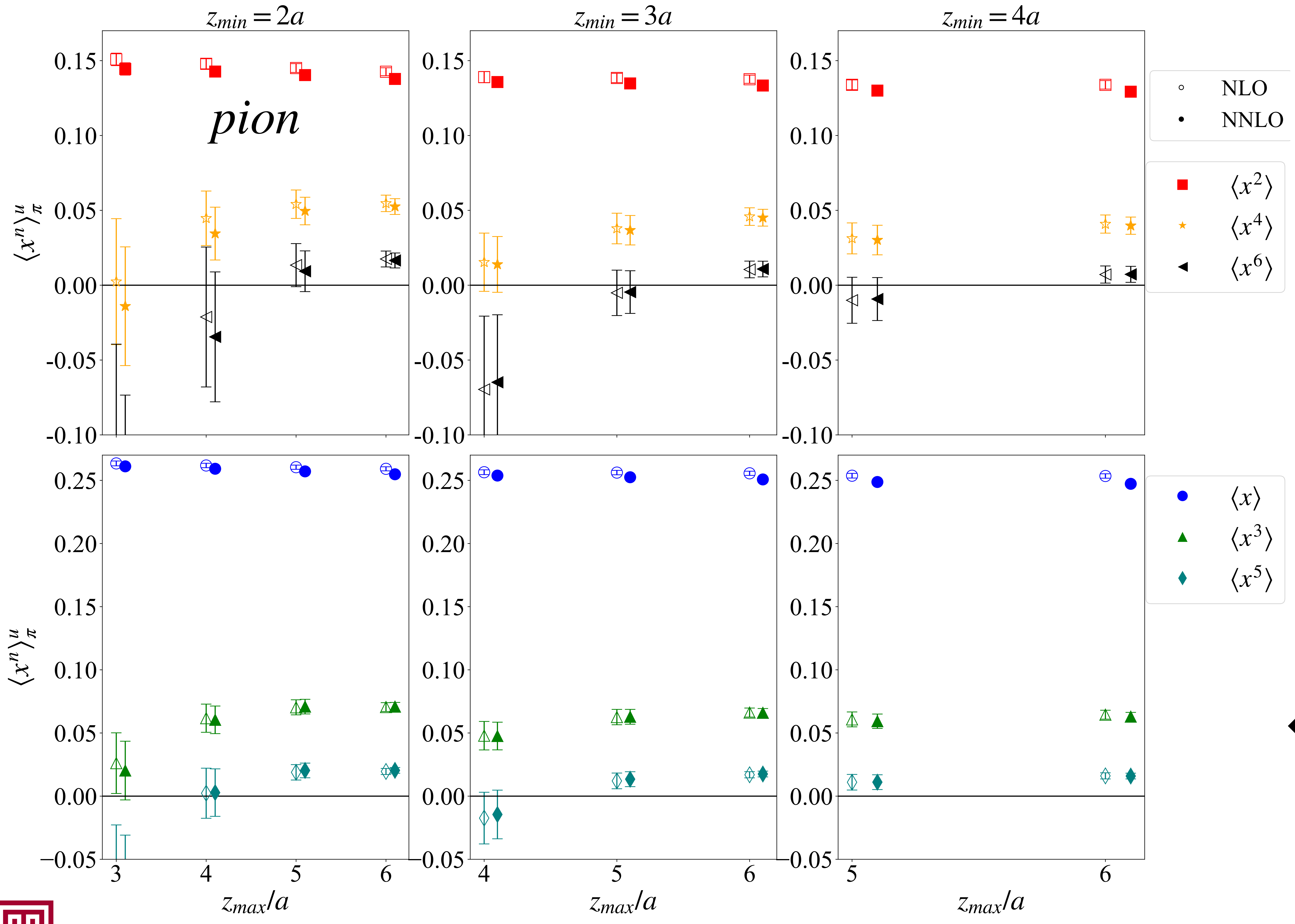
- Real and imaginary parts fit separately
  - Real part $\rightarrow \langle x^{2m} \rangle$
  - Imaginary part $\rightarrow \langle x^{2m+1} \rangle$
- Stability as n_{max} increases
  - $n_{max} = 6$ for the real part
  - $n_{max} = 5$ for the imaginary part

Fixed z Analysis: z-dependence, $c_{n,NLO}$ vs $c_{n,NNLO}$



- ❖ Moments extracted from NLO and NNLO Wilson coefficients have a minimal shift
- ❖ Nontrivial $z-$dependence of the moments
- ❖ Higher moments stabilize with larger z
- ❖ No plateau until large z values
  - ❖ Indication we need to implement combined P_3 and z fits

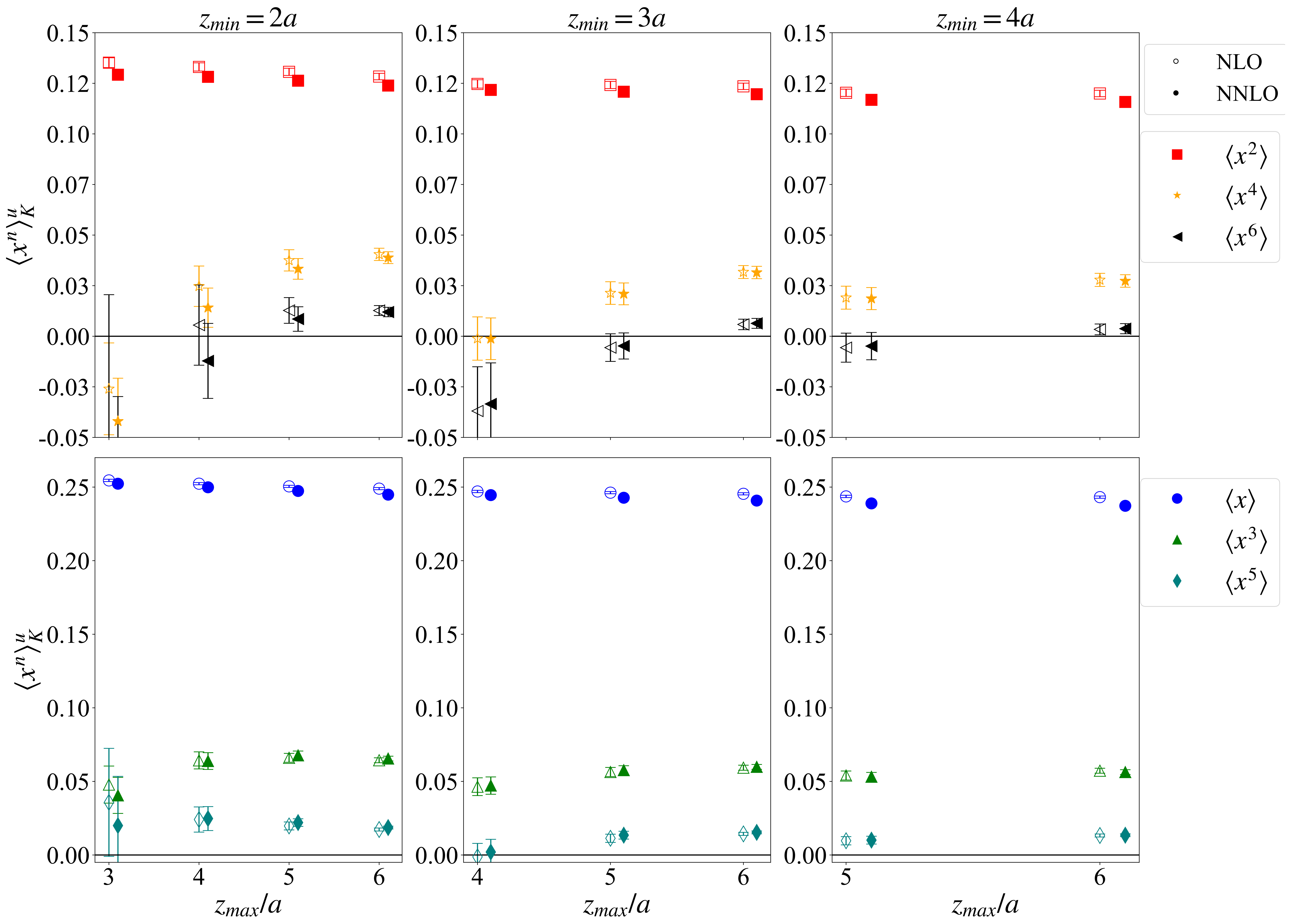
Combined P_3 and z Analysis: $z-$ ranges



- ❖ NLO Wilson coefficient results similar to NNLO
- ❖ $z_{min} = a$ not included (large discretization effects)
- ❖ Inclusion of higher z allows extraction of higher moments
- ❖ Difference due to z_{min}, z_{max}: systematic uncertainty
- ❖ Final results are a weighted average of $z_{\min} \in [3a,4a]$ and $z_{max} \in [4a,6a]$ $(z_{max} - z_{min} \geq a)$

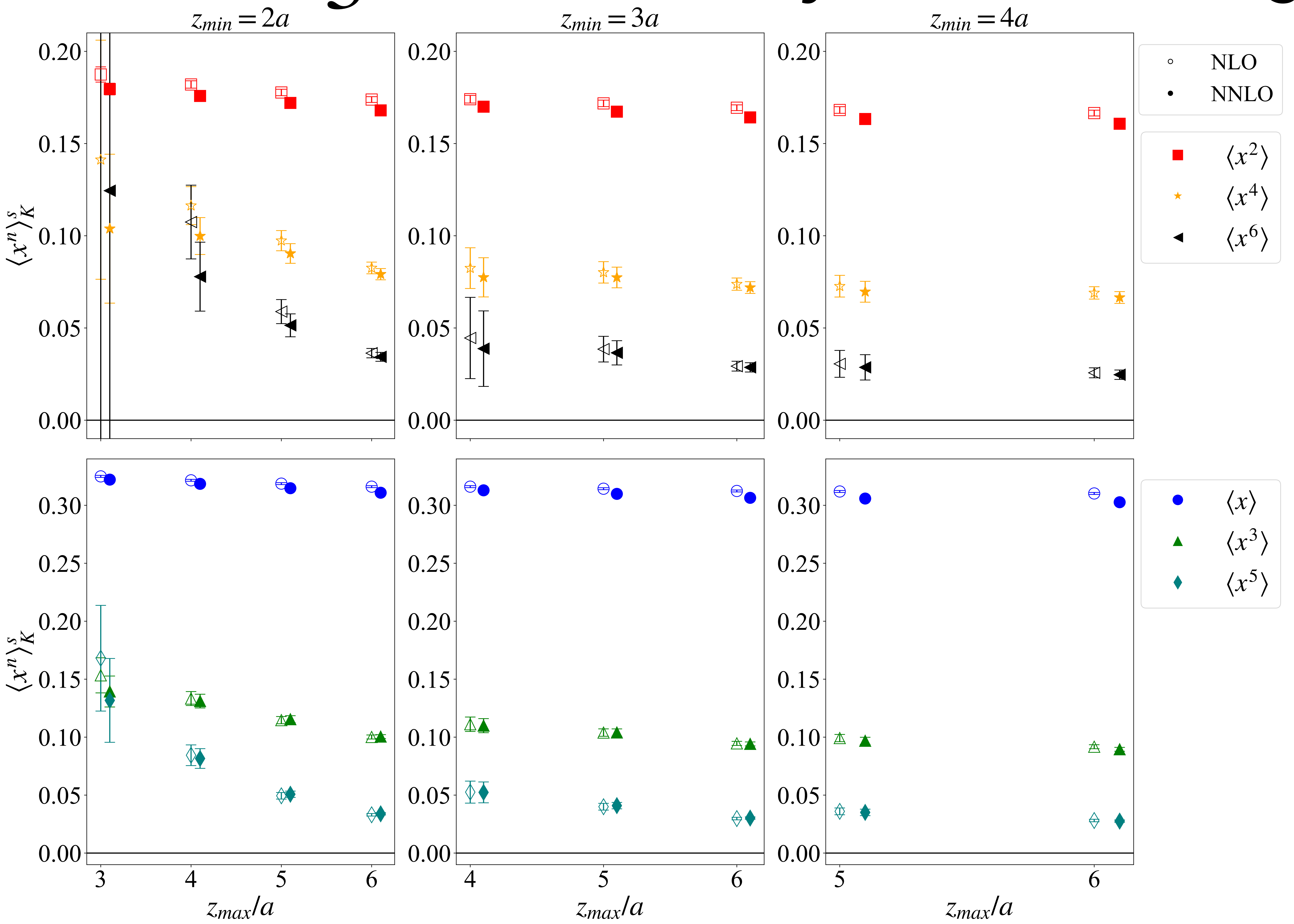
Combined P_3 and z Analysis: $z-$ranges

u − quark
kaon



Combined P_3 and z Analysis: $z-$ranges

s − quark
kaon



Final Results

Combined (P_3, z) Analysis:

NNLO	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$	$\langle x^5 \rangle$	$\langle x^6 \rangle$
π^u	0.251(2)(3)(4)	0.132(3)(3)(4)	0.063(4)(09)(1)	0.039(7)(16)(1)	0.016(3)(16)(0)	0.007(7)(38)(1)
K^u	0.241(1)(4)(4)	0.119(2)(3)(3)	0.057(2)(06)(0)	0.026(4)(16)(1)	0.014(1)(07)(1)	0.004(3)(20)(1)
K^s	0.308(1)(5)(5)	0.165(2)(5)(5)	0.094(2)(10)(2)	0.071(4)(05)(2)	0.030(1)(12)(0)	0.028(3)(07)(1)

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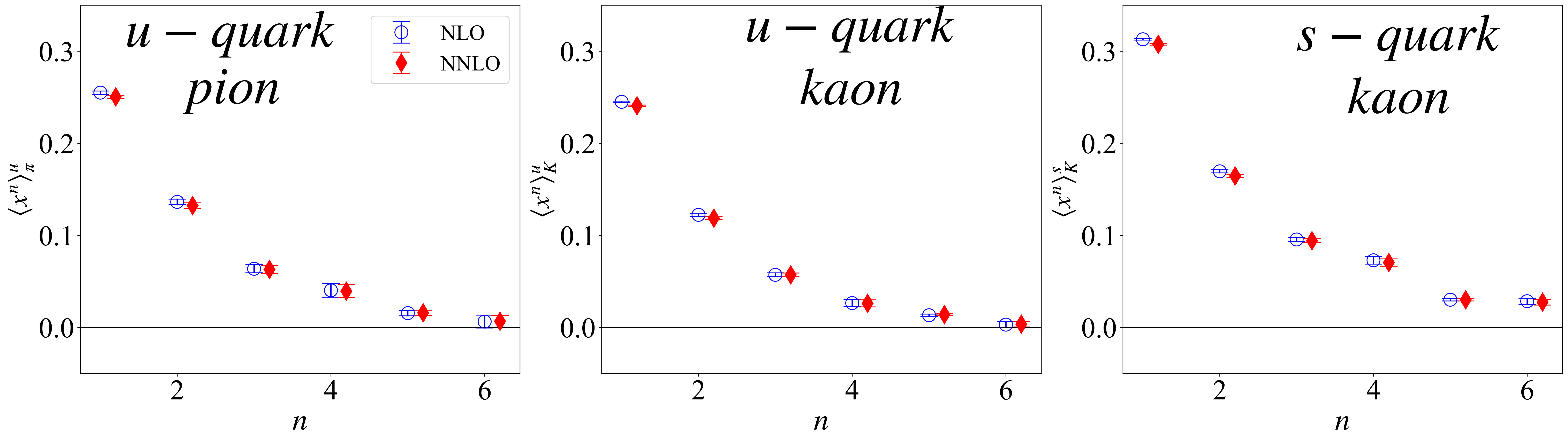


❖ Moments at both NLO and NNLO accuracy have a small differences

❖ Hierarchy of moments as anticipated (PDFs decay to zero as $x \to 1$)

(As n increases, larger weight is given to higher x: magnitude of moments decreases)

❖ The strange-quark in the kaon has larger moments at each n as anticipated

Comparison with other studies (nonlocal Oper.)

Note: different pion mass, lattice spacing, other parameters

Comparison with other studies (nonlocal Oper.)

Comparison with local operators (same ensemble)

	$\langle x \rangle$		$\langle x^2 \rangle$		$\langle x^3 \rangle$	
	Nonlocal	Local [29]	Nonlocal	Local [77]	Nonlocal	Local [77]
π^u	0.251(5)	0.273(9)	0.132(6)	0.110(6)	0.063(10)	0.026(17)
K^u	0.241(6)	0.262(3)	0.119(5)	0.101(2)	0.057(6)	0.043(7)
K^s	0.308(7)	0.332(3)	0.165(7)	0.145(2)	0.094(10)	0.079(6)

Overall good agreement, but local operators reach up to $\langle x^3 \rangle$

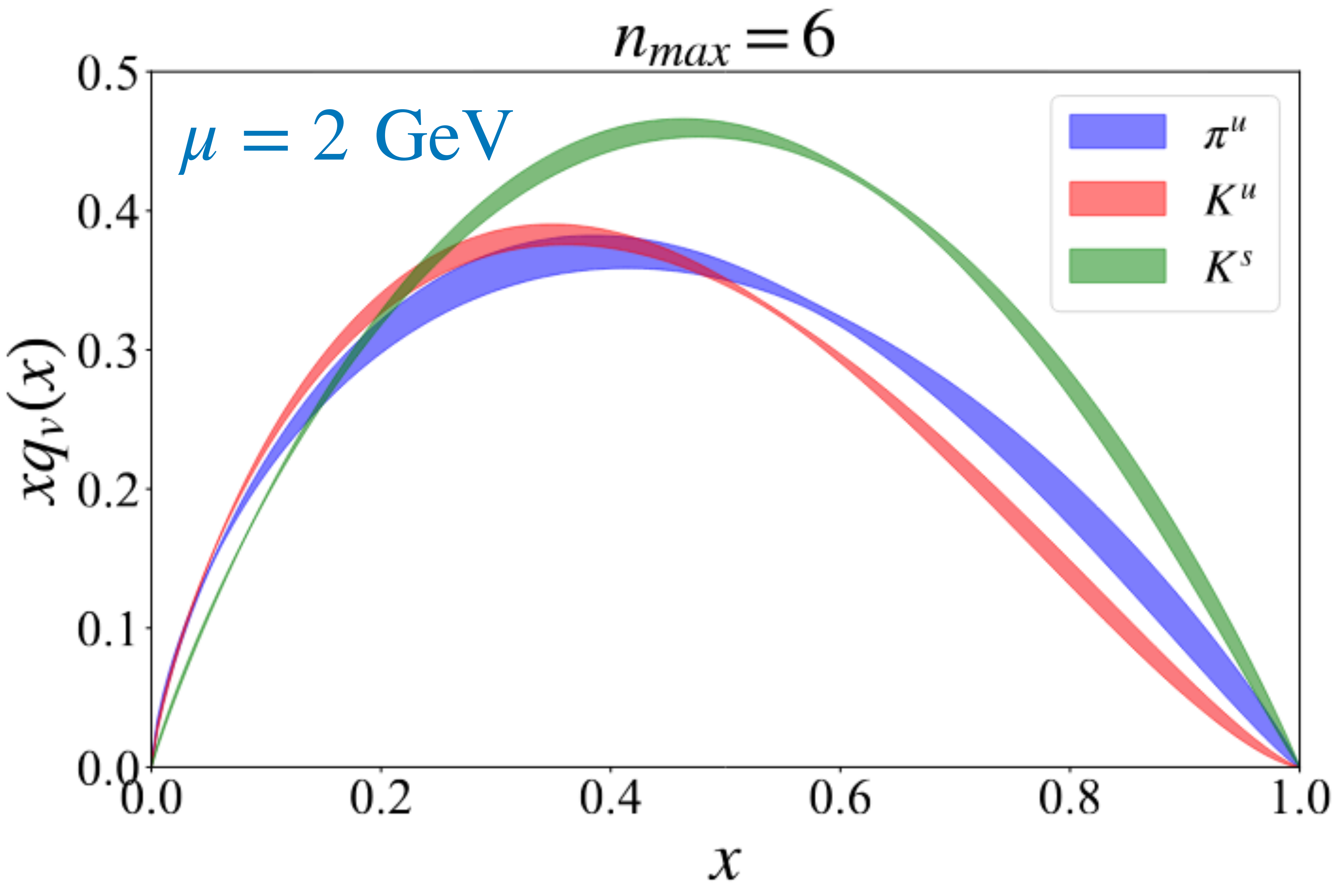
[29. PRD 103, 014508 (2021)]
[77. PRD 104, 054504 (2021)]

Reconstruction of valence PDF

❖ Parametrization:

$$q_M^f(x) = \frac{x^\alpha (1-x)^\beta}{B(\alpha+1,\beta+1)}, \qquad \langle x^n \rangle = \int_0^1 dx\, x^n q_M^f(x) = \frac{B(\alpha+n+1,\beta+1)}{B(\alpha+1,\beta+1)}.$$

❖ Odd-n moments related to q_{v2s}, but disconnected assumed small: $\langle x^{2m+1} \rangle \approx \langle x^{2m+1} \rangle_{val}$



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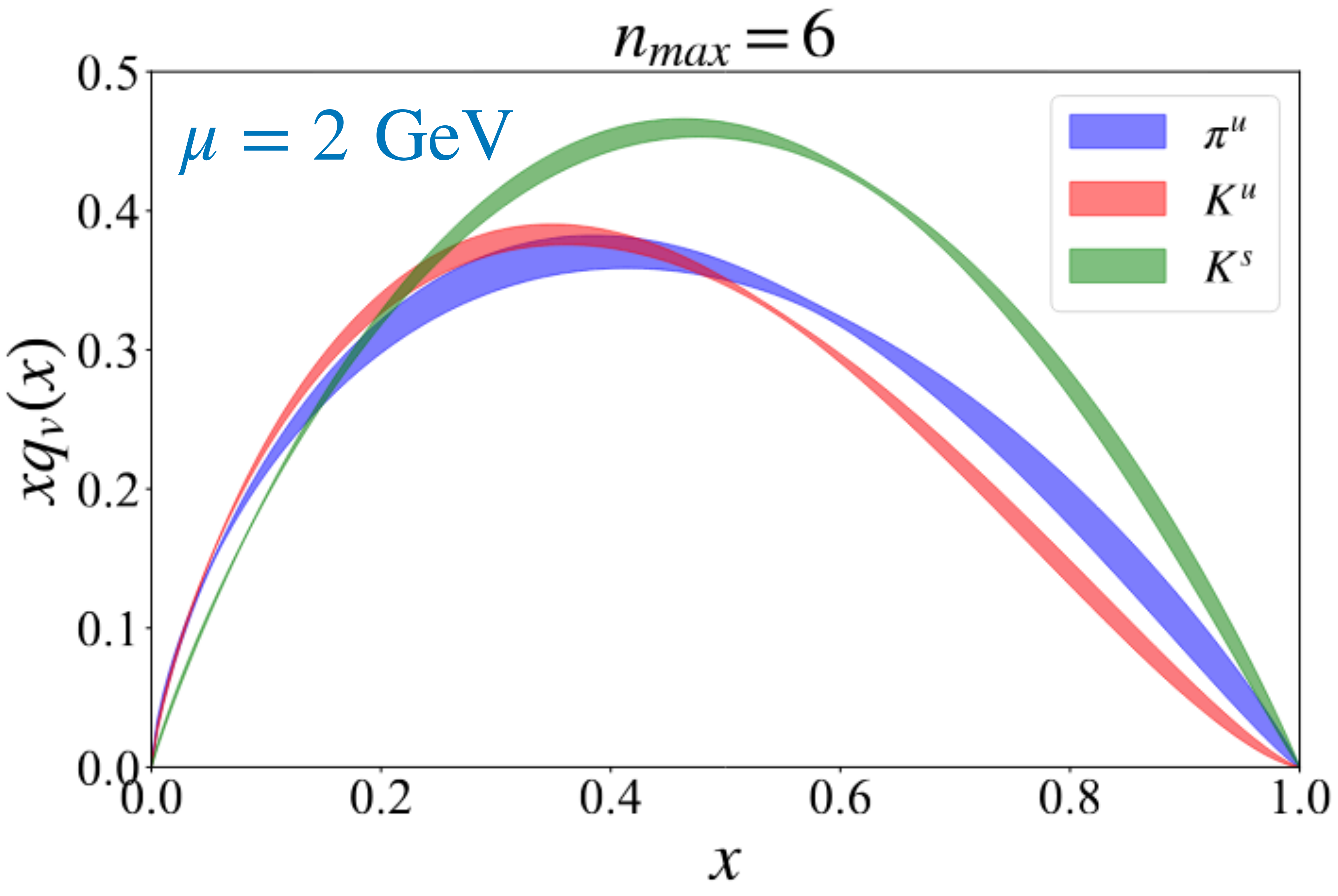
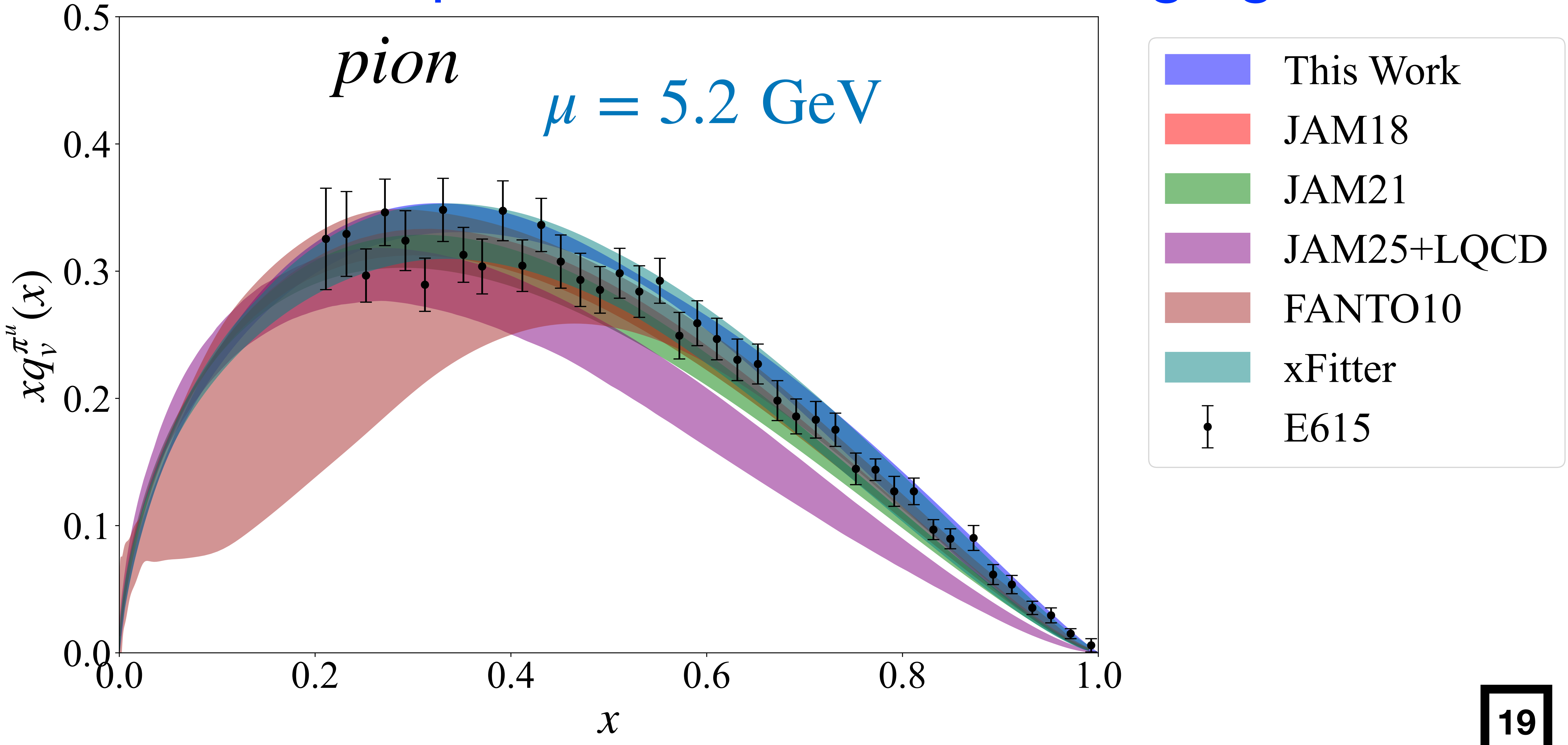
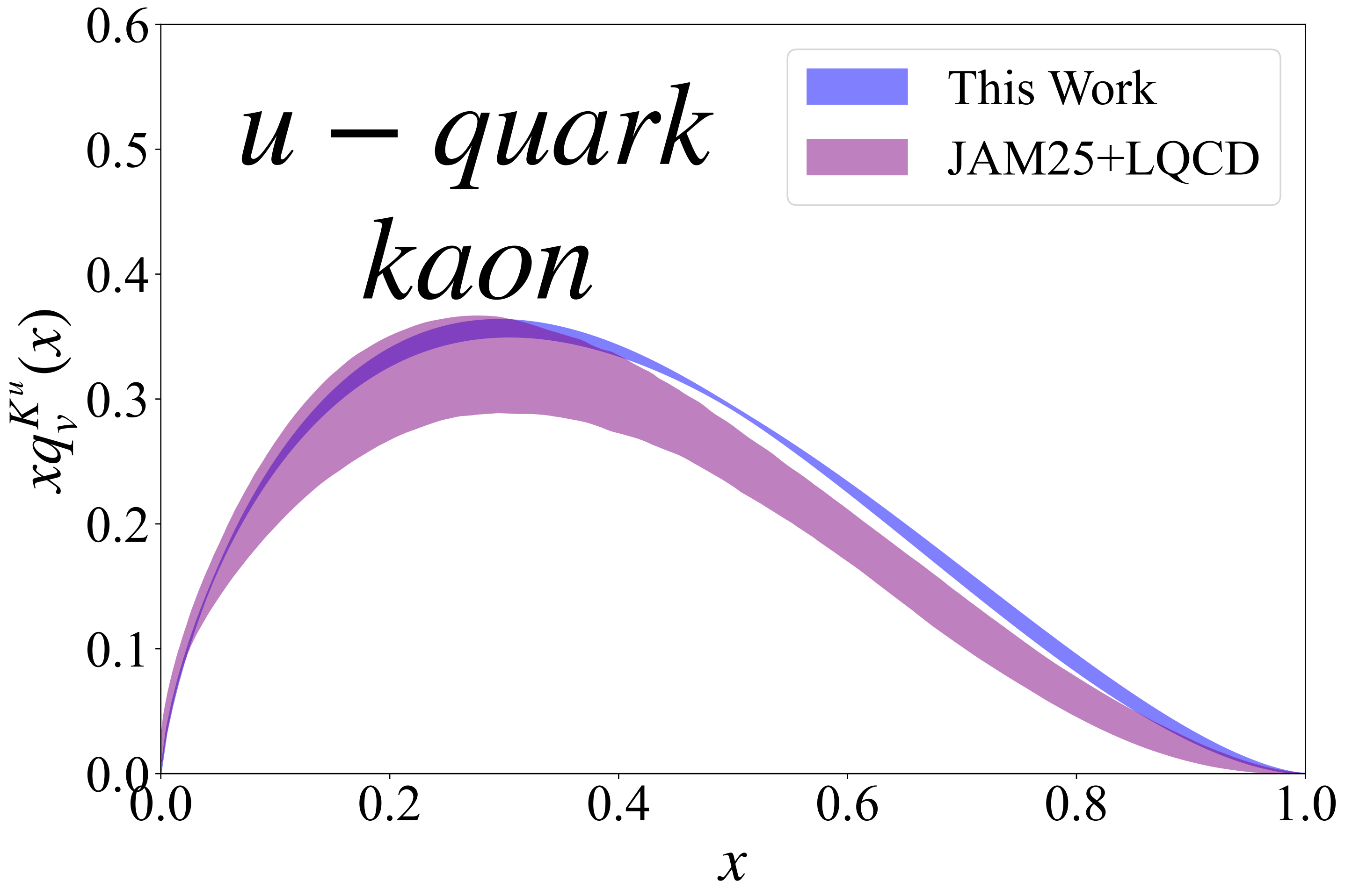


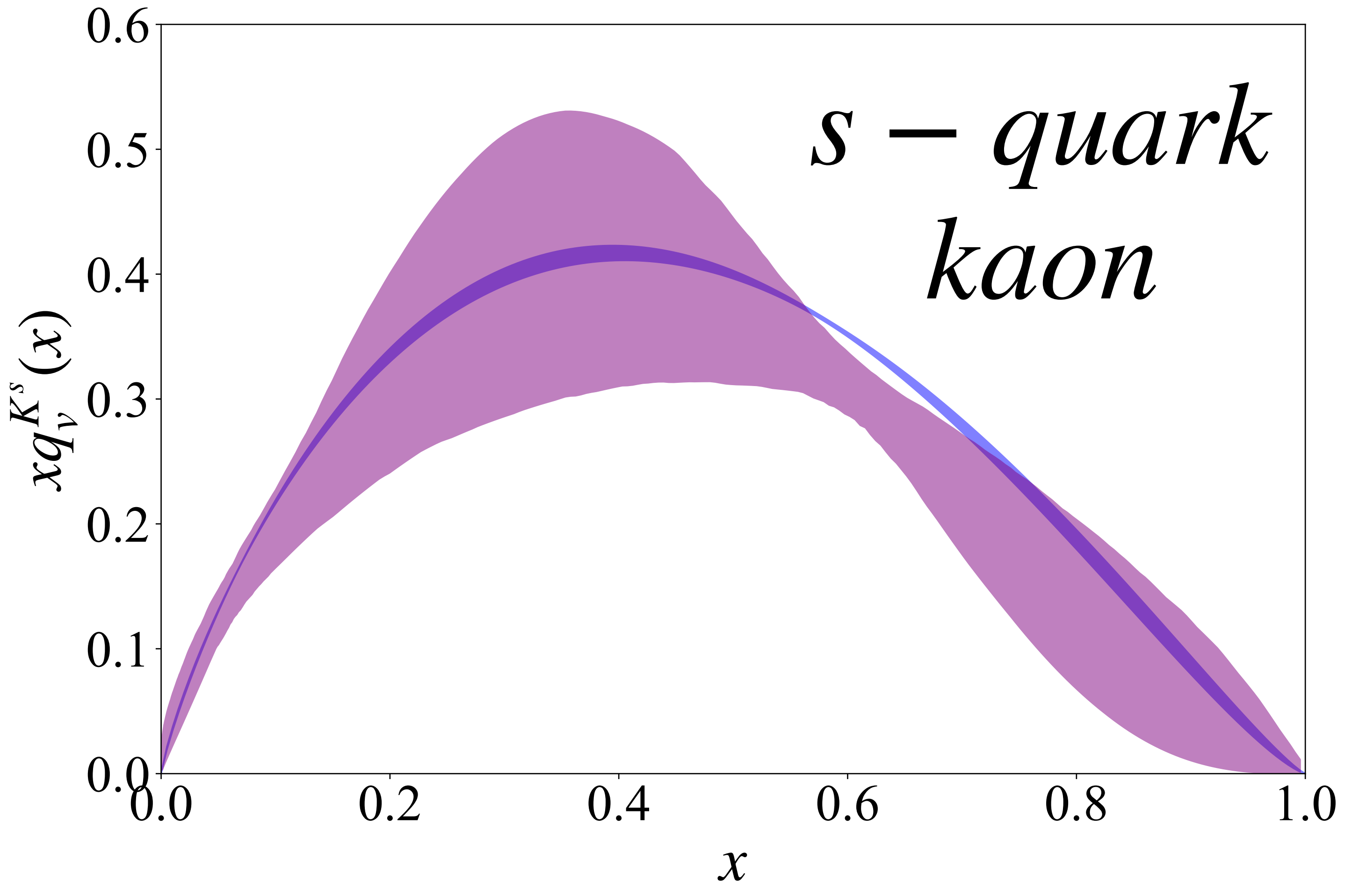
Comparison with global analyses and experimental data encouraging



Reconstruction of valence PDF

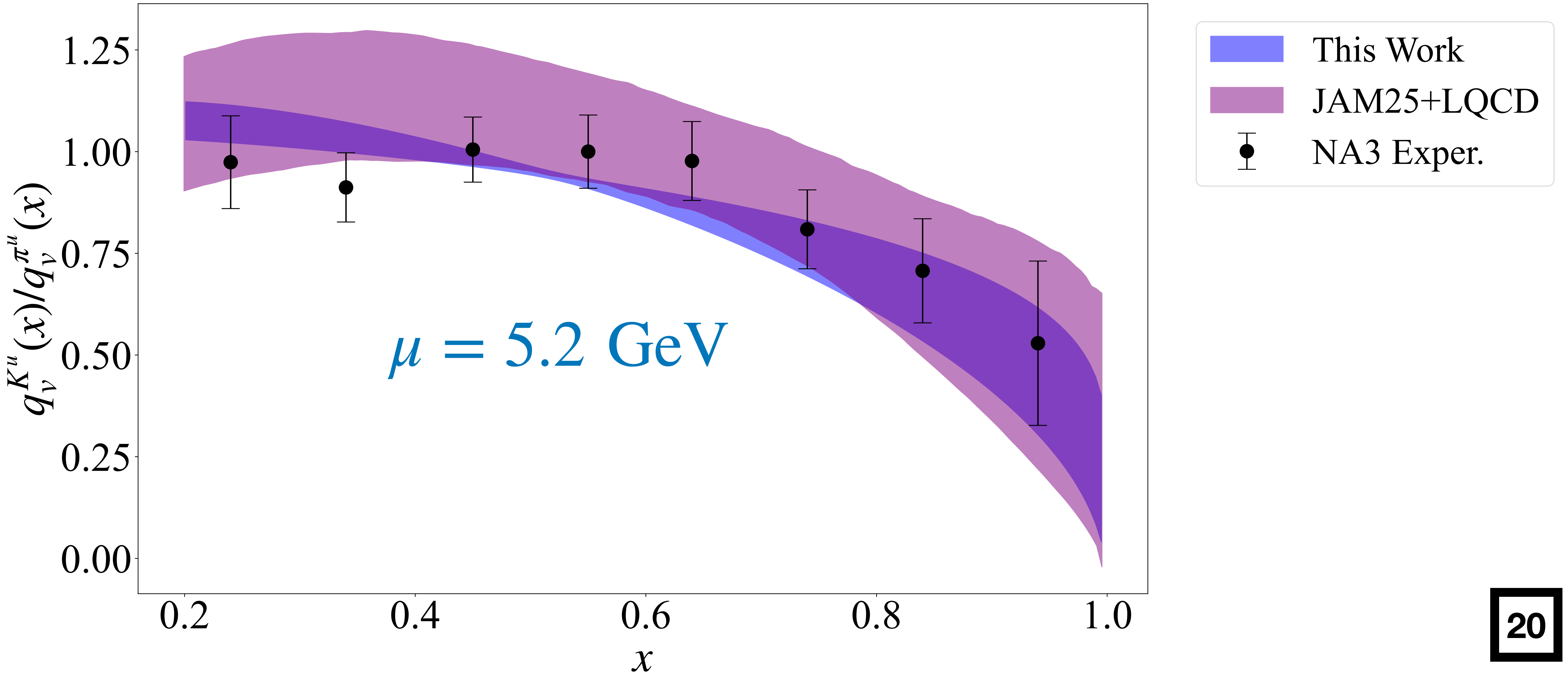
$\mu = 5.2$ GeV





- ❖ Good agreement with JAM analysis
  - ❖ Kaon distributions have limited studies
- ❖ Errors are strictly statistical

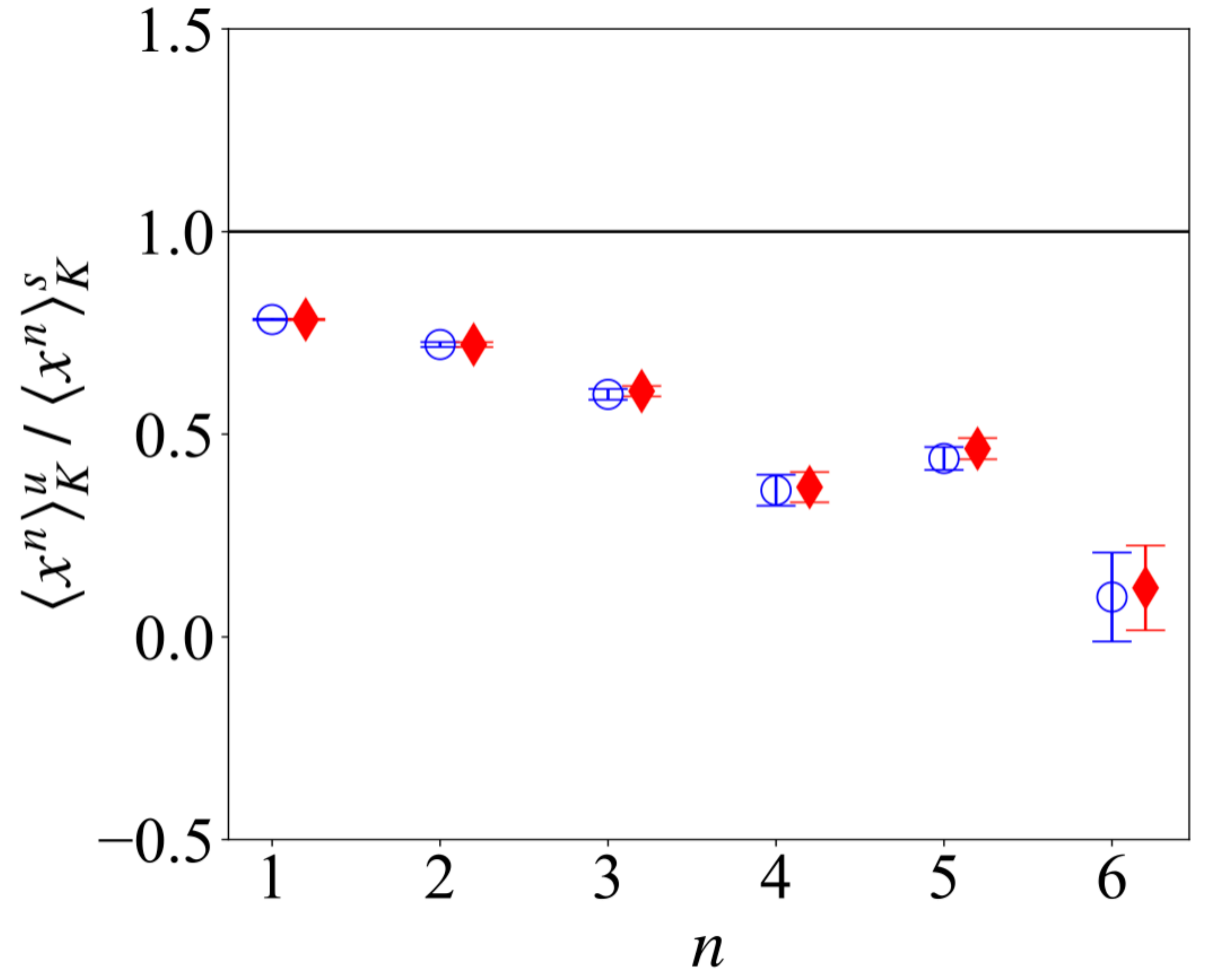
SU(3) flavor symmetry breaking



Excellent agreement with global analyses and experimental data

Summary and Future Work

- Extracting Mellin moments from nonlocal operators feasible and offers certain advantages over local operators and complementarity



- This work extracts up to $\langle x^6 \rangle$
  - Single-z and combined (P_3, z) fits
  - NLO and NNLO accuracy
  - Effect of DGLAP evolution at NLO and NNLO accuracy
  - SU(3) flavor symmetry breaking effects
    - π^u / K^u ~ 1 while π^u / K^s and K^u / K^s ~ 0.8 (see backup slides)
  - Reconstruct valence distribution

Summary and Future Work

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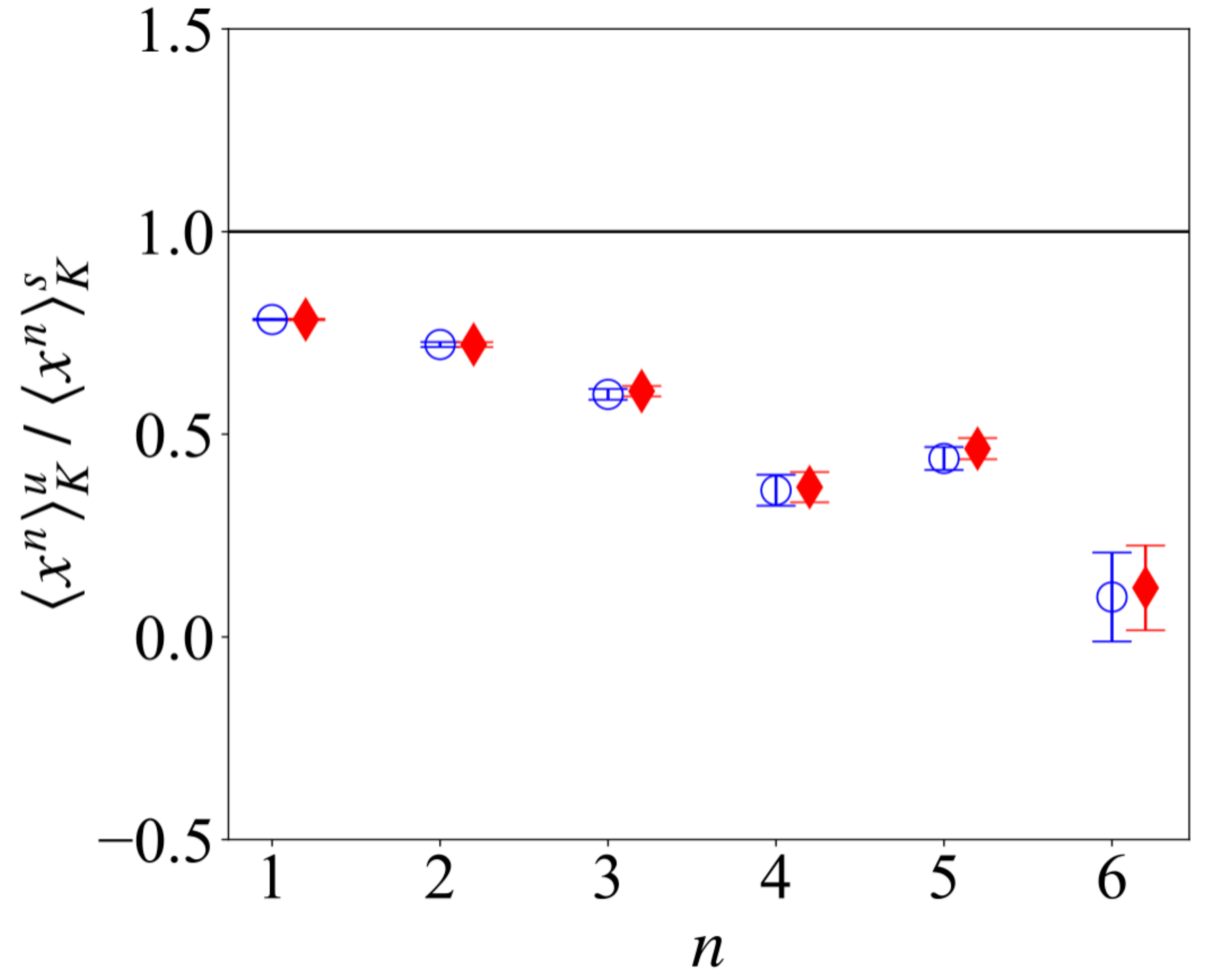

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  - ❖ Reconstruct valence distribution

Future work

- ❖ Continue work to Generalized Form Factors
- ❖ Implement Neural Networks
- ❖ Address other systematic uncertainties (require new calculations/ensembles)

See J. Torsiello's talk

Thank You!!!

Acknowledgements

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QUARK-GLUON TOMOGRAPHY COLLABORATION

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DE-SC0023646

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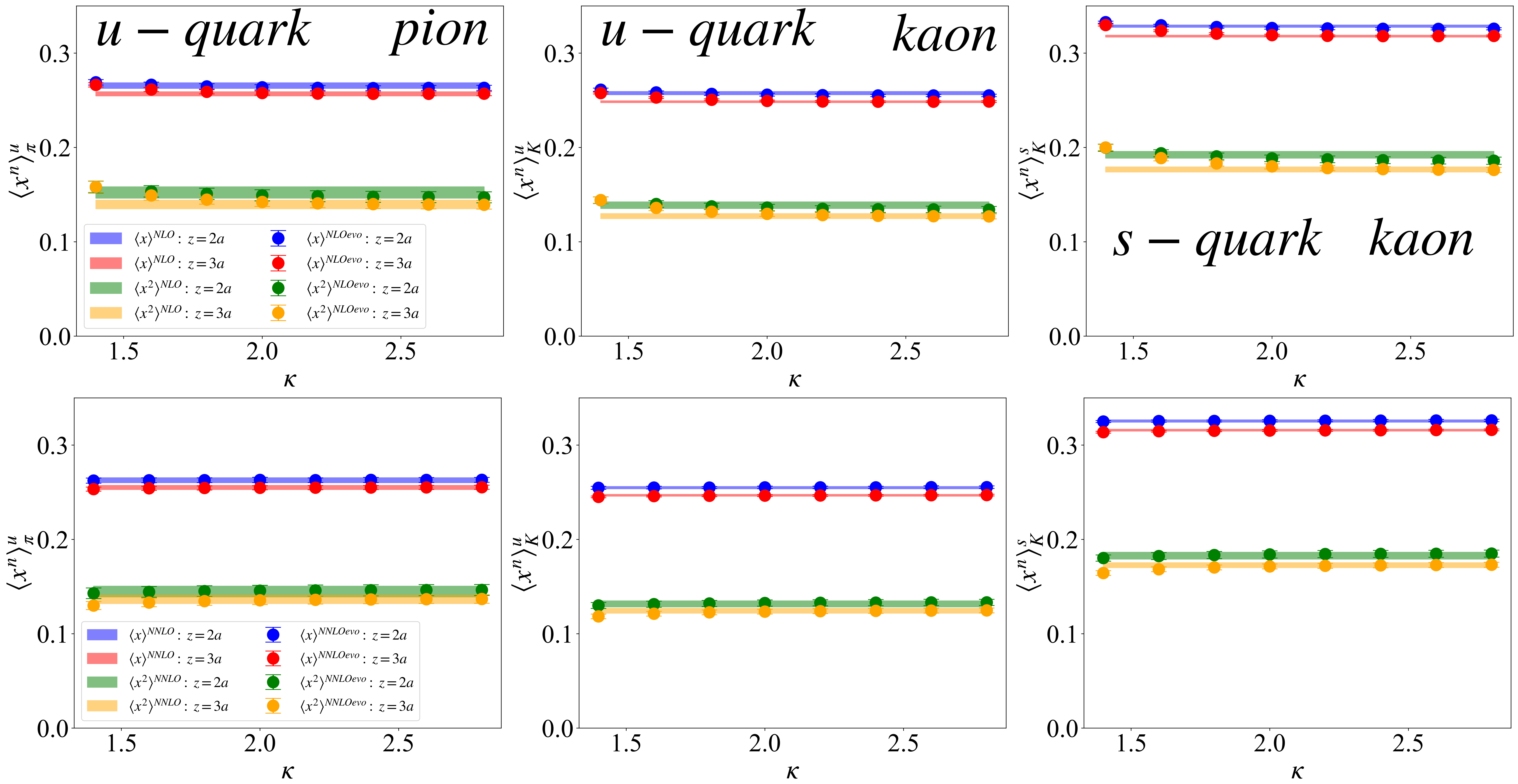
Backup Slides

DGLAP Evolution



- ❖ Less dependence on κ for NNLOevo
  - ❖ $\langle x^2\rangle$ requires larger κ to agree with the fixed μ results at NLOevo
- ❖ NNLOevo consistent with fixed μ results

Wilson Coefficient Derivation

❖ Focusing only on NLO accuracy, start with known matching relation

$$\mathscr{M}(\nu, z^2) = \mathscr{Q}(\nu, \mu^2) + \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right)\left(\frac{1+u^2}{u-1}\right)_+ + \left(4\frac{\ln(1-u)}{u-1} - 2(u-1)\right)_+ \right] \mathscr{Q}(u\nu, \mu^2)$$

$$\int dx\ [f(x)]_+ g(x) = \int dx\ f(x)(g(x) - g(1))$$

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❖ Note that $\mathscr{Q}$ is the Fourier transform on the light-cone PDF

$$\mathscr{Q}(\nu, \mu^2) = \int_{-1}^{1} dx \; e^{i\nu x} q(x, \mu^2) = \sum_{n=0}^{\infty} \frac{(i\nu)^n}{n!} \langle x^n \rangle$$

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❖ Substitute in and apply plus prescription

$$\mathcal{M}(\nu, z^2) = \sum_{n=0}^{\infty} \frac{(i\nu)^n}{n!} \langle x^n \rangle \left[1 + \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2 \mu^2 \frac{e^{2\gamma_E + 1}}{4} \right) \left(\frac{1+u^2}{u-1} \right) + \left(4 \frac{\ln(1-u)}{u-1} - 2(u-1) \right) \right] (u^n - 1) \right]$$

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❖ Solve the integrals

$$\mathscr{M}(\nu, z^2) = \sum_{n=0}^{\infty}\frac{(i\nu)^n}{n!}\langle x^n\rangle\left[1 + \frac{\alpha_s C_F}{2\pi}\left[\ln\left(z^2\mu^2\frac{e^{2\gamma_E+1}}{4}\right)\left(H_n + H_{n+2} - \frac{3}{2}\right) + 2\left(\frac{1}{n+1} - \frac{1}{n+2} - H_n^2 - H_n^{(2)}\right)\right]\right]$$

DGLAP Evolution Derivation

❖ Start with the evolution equation for Wilson coefficients

$$\ln\left(\frac{c_n(\mu)}{c_n(\mu_0)}\right) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} d(\alpha_s(\mu'))\,\frac{\gamma_n(\alpha_s(\mu'))}{\beta(\alpha_s(\mu'))}$$

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- Use the expansions of the anomalous dimension (γ_n) and β function

$$\gamma_n = \sum_{i=0} \left(\frac{\alpha_s}{4\pi}\right)^{i+1} \gamma_n^{(i)} \,, \quad \beta = -\,\alpha_s \sum_{i=0} \left(\frac{\alpha_s}{4\pi}\right)^{i+1} \beta_i$$

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❖ Substitute the order you want into the integral and solve

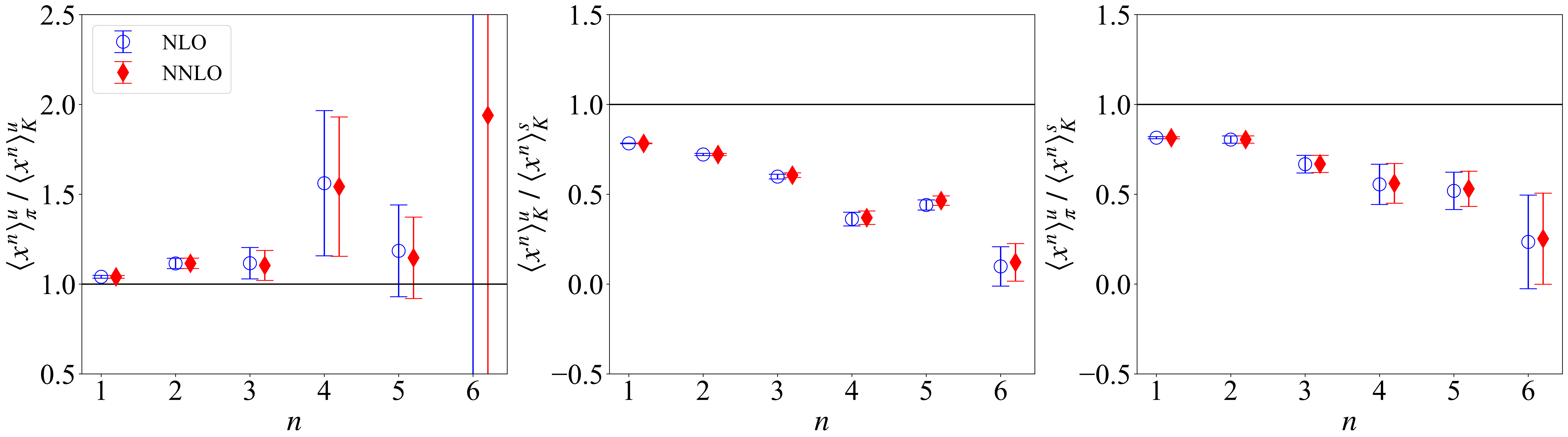
❖ NLOevo

$$\ln\left(\frac{c_n(\mu)}{c_n(\mu_0)}\right) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} d(\alpha_s(\mu'))\, \frac{-\gamma_n^{(0)}}{\alpha_s(\mu')\beta_0} = \frac{-\gamma_n^{(0)}}{\beta_0} \ln\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)$$

❖ NNLOevo

$$\ln\left(\frac{c_n(\mu)}{c_n(\mu_0)}\right) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} d(\alpha_s(\mu'))\, \frac{\frac{\alpha_s}{4\pi}\gamma_n^{(0)} + \frac{\alpha_s^2}{(4\pi)^2}\gamma_n^{(1)}}{-\frac{\alpha_s^2}{4\pi}\beta_0 - \frac{\alpha_s^3}{(4\pi)^2}\beta_1} = -\frac{\gamma_n^{(0)}}{\beta_0} \ln\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right) - \left(\frac{\gamma_n^{(1)}}{\beta_1} - \frac{\gamma_n^{(0)}}{\beta_0}\right) \ln\left(\frac{4\pi\beta_0 + \beta_1\alpha_s(\mu)}{4\pi\beta_0 + \beta_1\alpha_s(\mu_0)}\right)$$

SU(3) Symmetry Breaking



- ❖ Pion and kaon up-quark are similar with statistical fluctuations
  - ❖ Up to 4% difference from unity
- ❖ Flavor dependence much stronger in comparison of up-quark and strange-quark
  - ❖ Deviation from unity becomes stronger as n increases, but lower moments give a more robust conclusion
- ❖ Consistent with previous studies