Pion and kaon quark GPDs via LaMET and SDF

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In collaboration with

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Outline

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2. Methodology: **LaMET**
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   - ❖ Analysis of Approach
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   - ❖ Large z-extrapolation

3. Results
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   - ❖ Quasi-GPDs
   - ❖ Light Cone GPDs

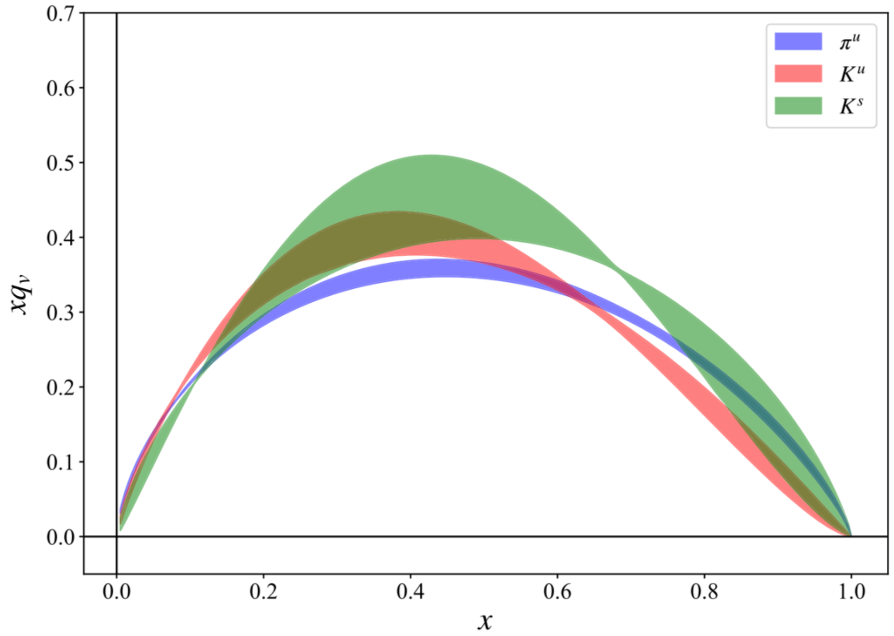
4. Summary and Future Work

Extension of PDFs calculation

PHYSICAL REVIEW D **113,** 114506 (2026)

Pion and kaon PDFs from lattice QCD via large momentum effective theory and short-distance factorization

Joshua Miller,[1,*] Joseph Torsiello,[1,†] Isaac Anderson,[2,1] Krzysztof Cichy,[3] Martha Constantinou,[1,‡] Joseph Delmar,[1] and Sarah Lamprecht[1]



Motivation and
x-dependent GPDs from LQCD

Motivation

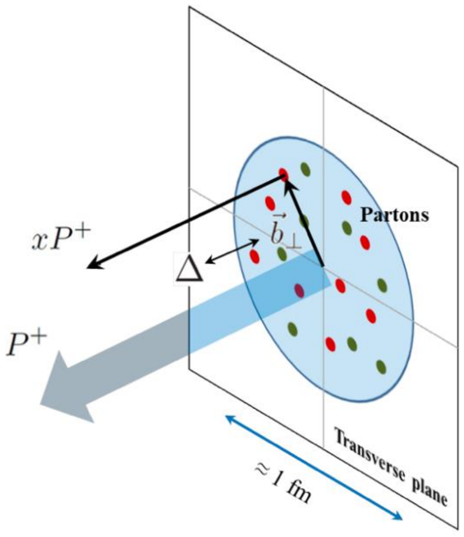
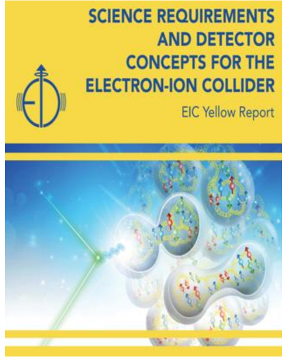


Image Credit: Cédric Lorcé

Why GPDs?

- Correlate longitudinal momentum and transverse position
- Unify information from PDFs, form factors (ffs), and generalized ffs
- 3D tomography of hadrons (together with TMDs)
- Central goal of the EIC and Jefferson Lab science programs
- Lattice results can constrain global analyses and GPD parametrizations



https://qgtcollab.github.io/

U.S. DEPARTMENT OF ENERGY Office of Science
QUARK-GLUON TOMOGRAPHY COLLABORATION
Award Number: DE-SC0023646

Why the pion and kaon?

- Laboratories for understanding emergent hadron mass
- Pion: special role as a pseudo-Goldstone boson
- Kaon: interplay of a light quark and a heavier strange quark
- Direct probe of SU(3)-flavor-symmetry breaking

Jefferson Lab

x-dependent GPDs from lattice QCD

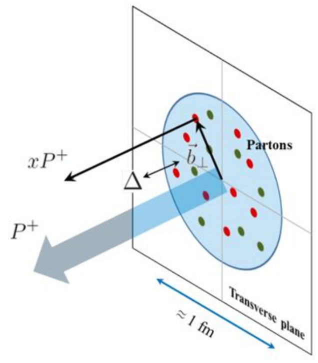


Image Credit: Cédric Lorcé

- Extracted from off-forward kinematic and multi-dimensional (longitudinal x, momentum transfer squared $-t$, and skewness ξ)

- Light-cone quantity (cannot be accessed directly from Euclidean lattice)

$$F_q(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p_f | \bar{q}(-\tfrac{z}{2}) \gamma^\mu \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

- **LaMET** and **SDF** connect spatial nonlocal matrix elements to light-cone GPDs
  [X. Ji, PRL 110 (2013) 262002; Sci. China Phys. M.A. 57 (2014) 1407] **[A. Radyushkin, PRD 96, 034025 (2017)]**

- Required hadron-momentum boost and momentum transfer increase statistical noise
  - Momentum transfer: major computational challenge (conventionally in symmetric frame)
  - Reconstruction of the x-dependence is an inverse problem (multi-dimensionality is additional complication)

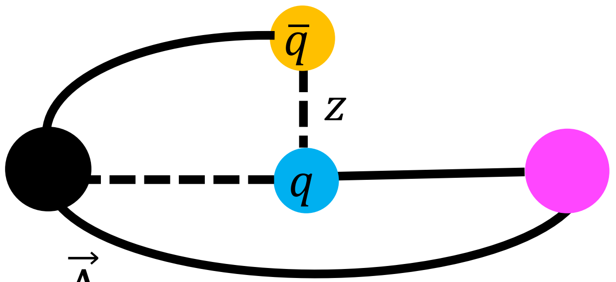
LaMET focus of this talk

Lattice Methodology for GPDs:

LaMET (quasi-GPDs approach)

Kinematic setup

Traditional Briet Frame (Symmetric)

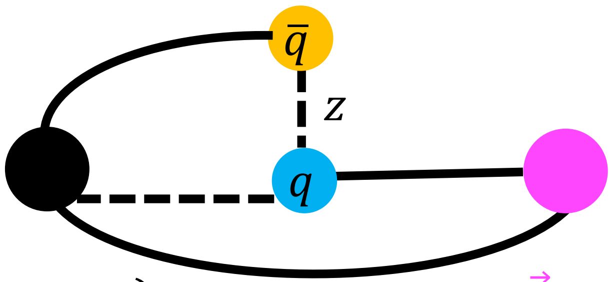


$\vec{p}_f = \bar{P} + \frac{\vec{\Delta}}{2}$ $\quad$ $\vec{p}_i = \bar{P} - \frac{\vec{\Delta}}{2}$

- ❖ $\vec{p}_f$ fixed
  - Only one $-t = \Delta^2$ is useful
- ❖ Huge computational cost
- ☹ Impractical for GPDs

Asymmetric Frame (alternative parametrization)

[S. Bhattacharya et al., PRD **106** (2022) 11, 114512]

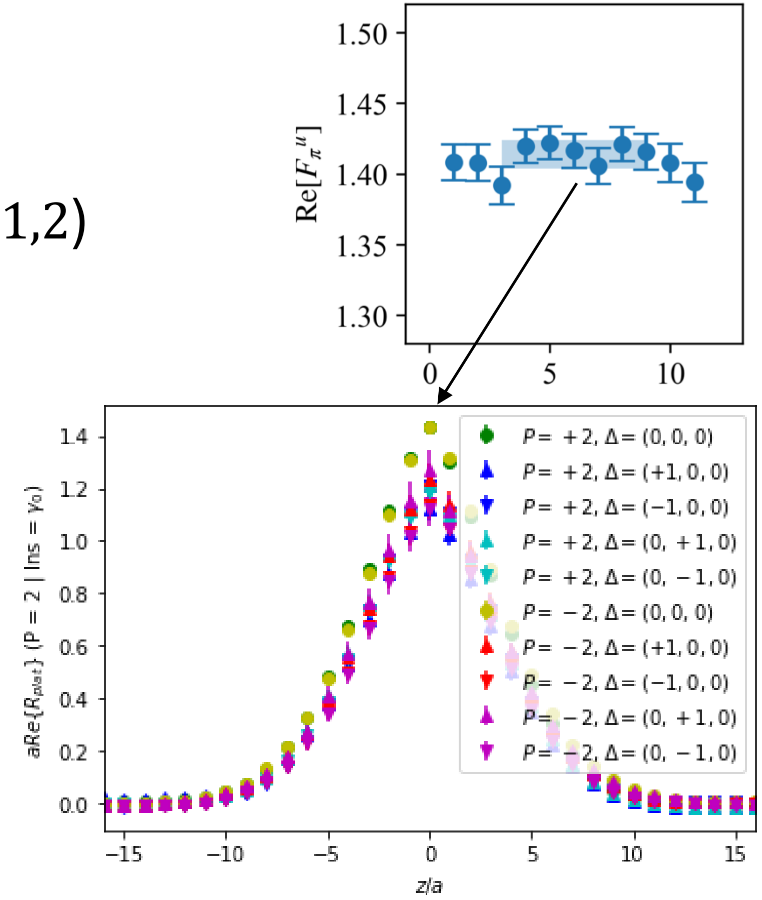


$\vec{p}_f = (0,0,P_z)$ $\quad$ $\vec{p}_i = \vec{p}_f - \vec{\Delta}$

Method of choice

- ❖ $\vec{p}_f$ fixed:
  - Several $-t = \Delta^2$ is useful
- ❖ Reduced computational cost
- ☺ - Ideal for extracting GPDs!

Analysis of Method



- Calculation in asymmetric frame for vector operators: γ^μ $(\mu = 0,1,2)$

$$M_\mu^f = \langle p_f | \bar{q}(z)\gamma^\mu \mathcal{W}(z,0) q(z) | p_f - \Delta \rangle$$

- Extraction of matrix element through a plateau fit Π_μ^f

$$R_M(z,p_i,p_f;t,t_s) = \frac{C_M^{3pt}(z,p_i,p_f,t_s,t)}{C_M^{2pt}(p_f,t_s)} \sqrt{\frac{C_M^{2pt}(p_i,t_s-t)C_M^{2pt}(p_f,t)C_M^{2pt}(p_f,t_s)}{C_M^{2pt}(p_f,t_s-t)C_M^{2pt}(p_i,t)C_M^{2pt}(p_i,t_s)}} \xrightarrow[t \gg a]{t_s - t \gg a} \Pi_\mu^f(z,p_i,p_f;t_s)$$

- Decomposition into Lorentz invariant amplitudes (A_1, A_3) and extraction of quasi-GPD

- Hybrid renormalization and exploration of large-z behavior

- x-dependence reconstruction (Backus-Gilbert and Neural Networks)

- Extraction of light-cone GPDs via a 1-loop matching kernel for hybrid

[S. Bhattacharya et al., PRD 106 (2022) 11, 114512]

Extracting Amplitudes

$$\Pi_0^a = \frac{1}{\sqrt{4E_f E_i}}\left(\frac{E_f+E_i}{2m}A_1 + \frac{E_f-E_i}{m}A_3\right)$$

$$\Pi_1^a = \frac{-i}{\sqrt{4E_f E_i}}\left(\frac{\Delta_1}{2m}A_1 + \frac{\Delta_1}{m}A_3\right)$$

$$\Pi_2^a = \frac{-i}{\sqrt{4E_f E_i}}\left(\frac{\Delta_2}{2m}A_1 + \frac{\Delta_2}{m}A_3\right)$$

- Three MEs ($\gamma_0, \gamma_1, \gamma_2$) used to disentangle Lorentz invariant amplitudes A_i
- γ_3 operator not used due to mixing and related to twist-3
- A_i have definite symmetries

$$+A_1^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_1(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$$

$$+A_3^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_3(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$$

- Quasi H-GPD related to A_i (definition not unique for quasi-GPDs)

$$\mathcal{H}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \equiv A_1(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \cancel{+ \frac{z\cdot\Delta}{z\cdot P}A_3(z\cdot P, z\cdot\Delta, \Delta^2, z^2)}\,.$$

Zero kinematic coefficient at ξ=0 ($\Delta_3 = 0$)
A_3 odd under ξ ($A_3(\xi = 0) = 0$)

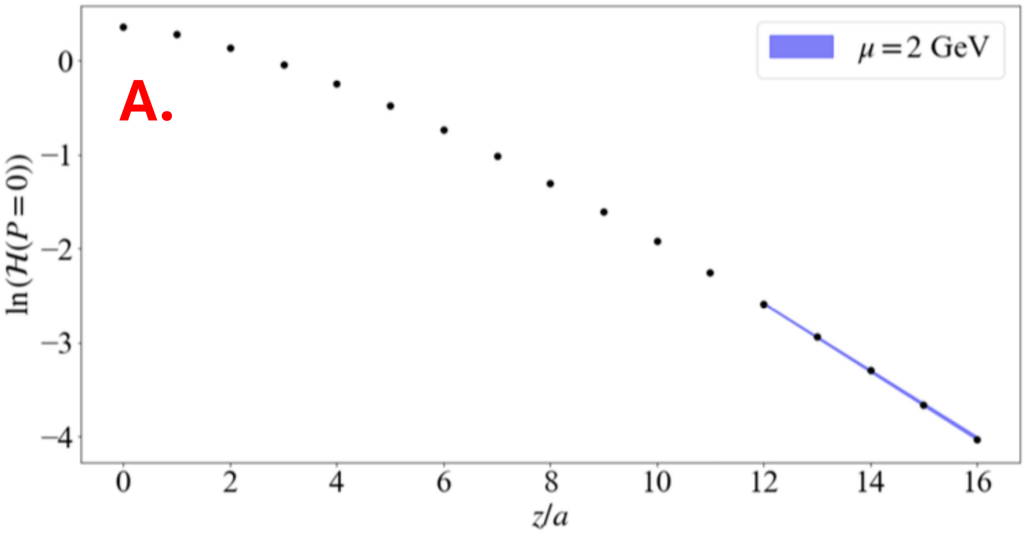
Hybrid Renormalization

[X. Ji et al., NPB 964 (2021) 115311]

Additional UV divergence in nonlocal operators with Wilson line: Linear power divergence $e^{-a\,(\delta m + m_0) z}$

Hybrid renorm. treats short- and long-z differently

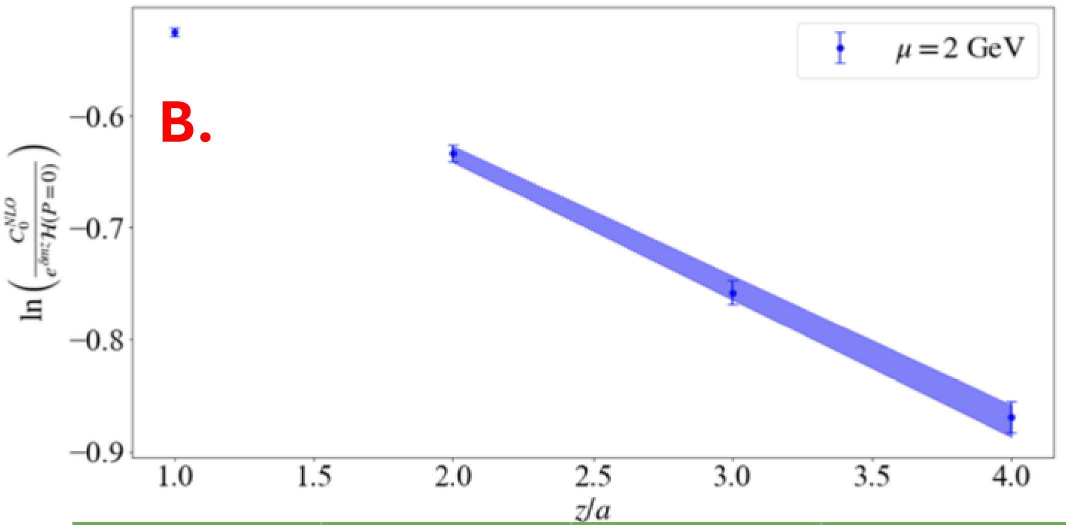
$$\mathcal{H}_R = \begin{cases} \dfrac{\mathcal{H}_B(z, P_z, \vec{\Delta})}{\mathcal{H}_B(z, 0, 0)} & z \le z_s \\[2ex] \dfrac{\mathcal{H}_B(z, P_z, \vec{\Delta})}{\mathcal{H}_B(z_s, 0, 0)} e^{(\delta m + m_0)(z - z_s)} & z \ge z_s \end{cases}$$



Approach: Fit bare quasi PDF $\mathcal{H}_{(P=0)}$ in two regions

A. Extract δm from high z $(12a - 16a)$

$$\ln \mathcal{H}_{(p=0)} = \delta m\, z + A$$



B. Extract m_0 from low z $(2a - 4a)$

$$\ln \frac{C_0}{e^{\delta m z} \mathcal{H}_{(P=0)}} = m_0 z + A$$

(δm from previous fit and Wilson coefficient C_0)

$$C_0^{NLO} = 1 + \frac{\alpha_s(\mu)}{2\pi}\left(\frac{3}{2} \ln z^2 \mu^2 \frac{e^{2\gamma_E}}{4} + \frac{5}{2}\right)$$

C.

$z \in [2a, 4a]$	π^u	K^u	K^s
$a(\delta m + m_0)$	0.2394(3)	0.2415(3)	0.2411(3)

D. Use δm and m_0 in renormalization with $z_s = 3a$

Large z-extrapolation

[X. Ji et al., arXiv:2601.12189]

- Replacement of noisy data with physics-motivated asymptotic form that may improve reconstruction of x-space distributions.

- At zero skewness:
  LA: $\mathcal{H}_R = (C_1 e^{-izP_z} + C_2) e^{-\Lambda_{QCD}\, z}$
  NLA: $\mathcal{H}_R = (C_1 e^{-izP_z} + C_2) e^{-\Lambda_{QCD}\, z} + (C_3 e^{-izP_z} + C_4) \frac{e^{-\Lambda_{QCD}\, z}}{z}$

- Investigations of z_{min} and z_{max} entering the fits, e.g.,
  LA : $[z_{min}, z_{max}] \sim 0.75 \; - 1\; fm$
  NLA: $[z_{min}, z_{max}] \sim 0.55 \; - 0.85 fm$

- Fine lattice spacing important in interplay of fit-range and number of data points

- Λ_{QCD} may be fixed, loosely constrained ($200 - 800$ MeV) or left free parameter

Reconstructing x-space and matching

x-dependence reconstruction

[M.-H. Chu et al., JHEP 05 (2026) 210]

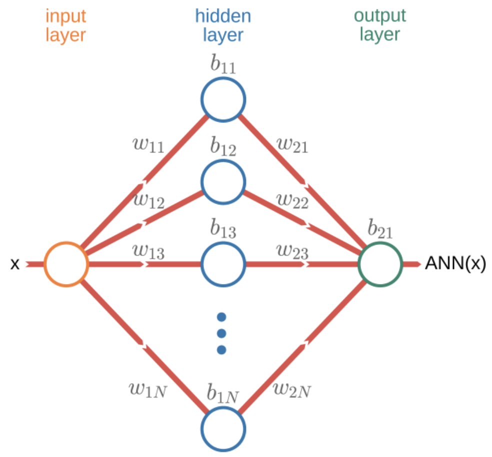
Target parametrization: $q^{(0)}(x) = x^{\delta^{(0)}}(1-x)^{\rho^{(0)}}\,\mathrm{ANN}^{(0)}(x)$

Parameters δ, ρ, b, w: fitted to lattice data

ANN replaces a low-parameter Ansatz and provides a flexible continuous parametrization of the distribution

ANN single neural network

$$\mathrm{ANN}^{(0)}(x) = \quad b_{21}^{(0)} + \sum_{i=1}^{N^{(0)}} \left[w_{2i}^{(0)} \ln\left(1+\exp(b_{1i}^{(0)} + w_{1i}^{(0)} x)\right)\right]$$



Light-cone PDF parametrized with an ANN and fitted through: inverse matching, Fourier transformation, and comparison with the lattice matrix elements.

Comparison with Backus-Gilbert [G. Backus et al., Geophysical Journal International 16 (2) 169]

Matching formalism

One-loop accuracy:

$$C_{\mathrm{hybrid}}(\xi, \mu^2/(p^z)^2, z_S^2\mu^2) = C_{\mathrm{ratio}}(\xi, \mu^2/(p^z)^2) + \frac{\alpha_s C_F}{2\pi}\frac{3}{2}\left[-\frac{1}{|1-\xi|_+} + \frac{2\mathrm{Si}((1-\xi)\lambda_S)}{\pi(1-\xi)}\right],$$

[X. Ji et al., NPB 964 (2021) 115311]

C_{ratio} from:

[T. Izibuchi et al, PRD98 056004 (2018)]

Results

Lattice Setup

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.30.32	1.726	0.09471(39)	$32^3 \times 64$	2+1+1	265	4	3.0

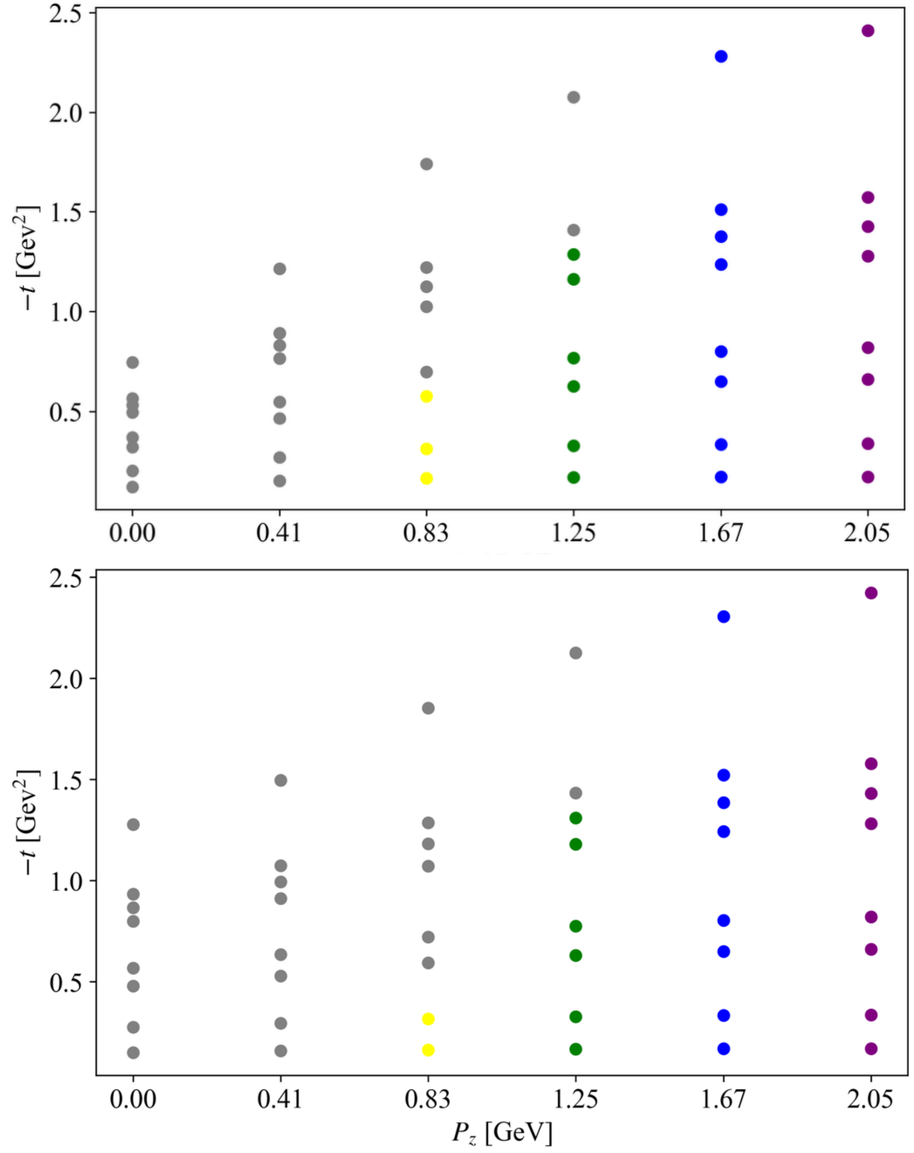
ξ=0 calculations

P_3 [GeV]	0	±0.41	±0.83	±1.24	±1.66	±2.07
t_s/a	12	12	12	12	10	10
N_{confs}	1,198	1,198	1,198	1,198	1,198	1,198
N^π_{src}	1	8	8	$8 \to 64$	$24 \to 100$	$200 \to 400$
N^K_{src}	1	8	8	8	24	200
N^π_{tot}	1,198	9,584	9,584	9,584	57,504	383,360
N^K_{tot}	1,198	9,584	9,584	9,584	28,752	239,600

Increased statistics compared to PDFs

- $-t$ different for each momentum transfer even at P_3 fixed.
- $-t$ depends on mass

$$-t = \mathbf{\Delta}^2 - (E(p_f) - E(p_i))^2$$



$-t/P_3^2$ defines reliability of calculation

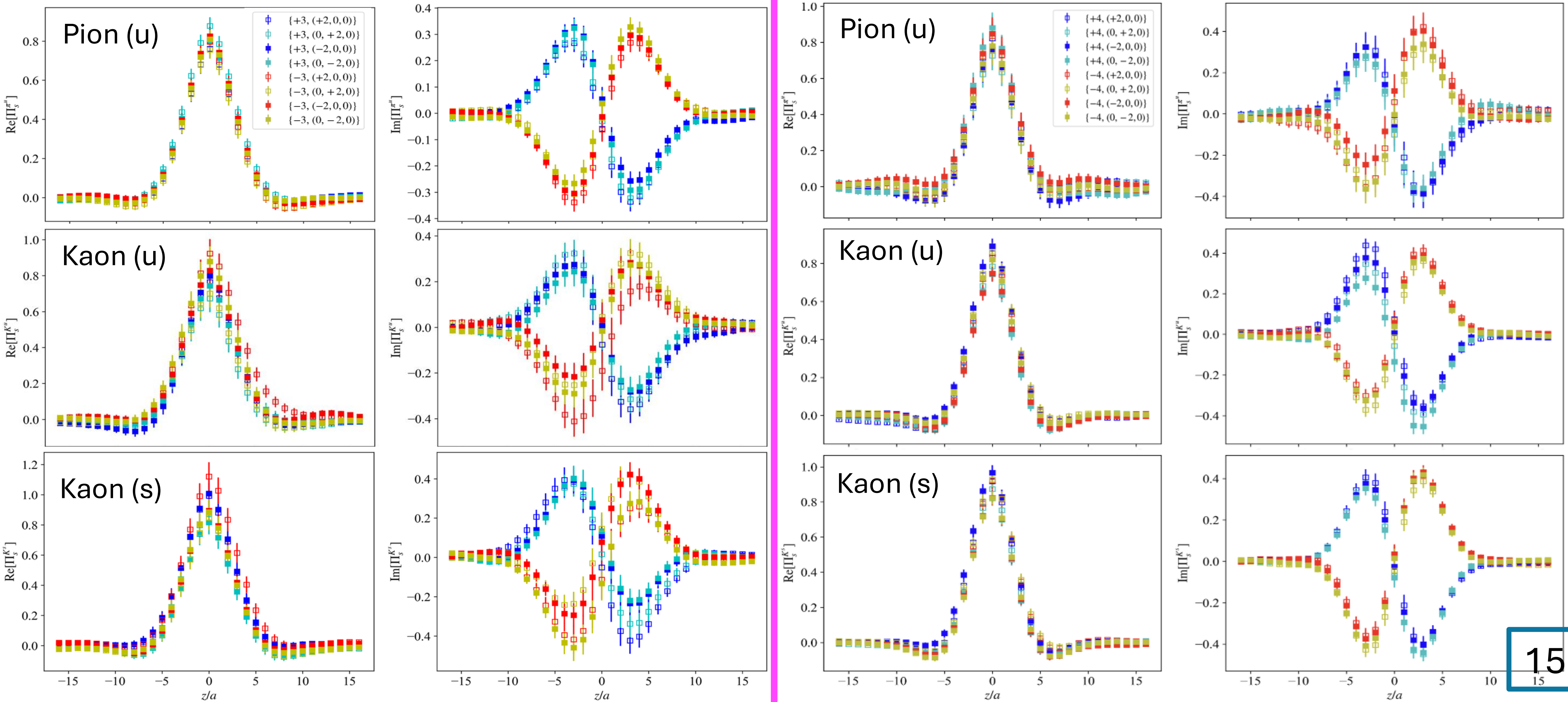
(Selected) Matrix Elements for γ_0

$P_3 = 1.25\ GeV$

Statistics:9,584

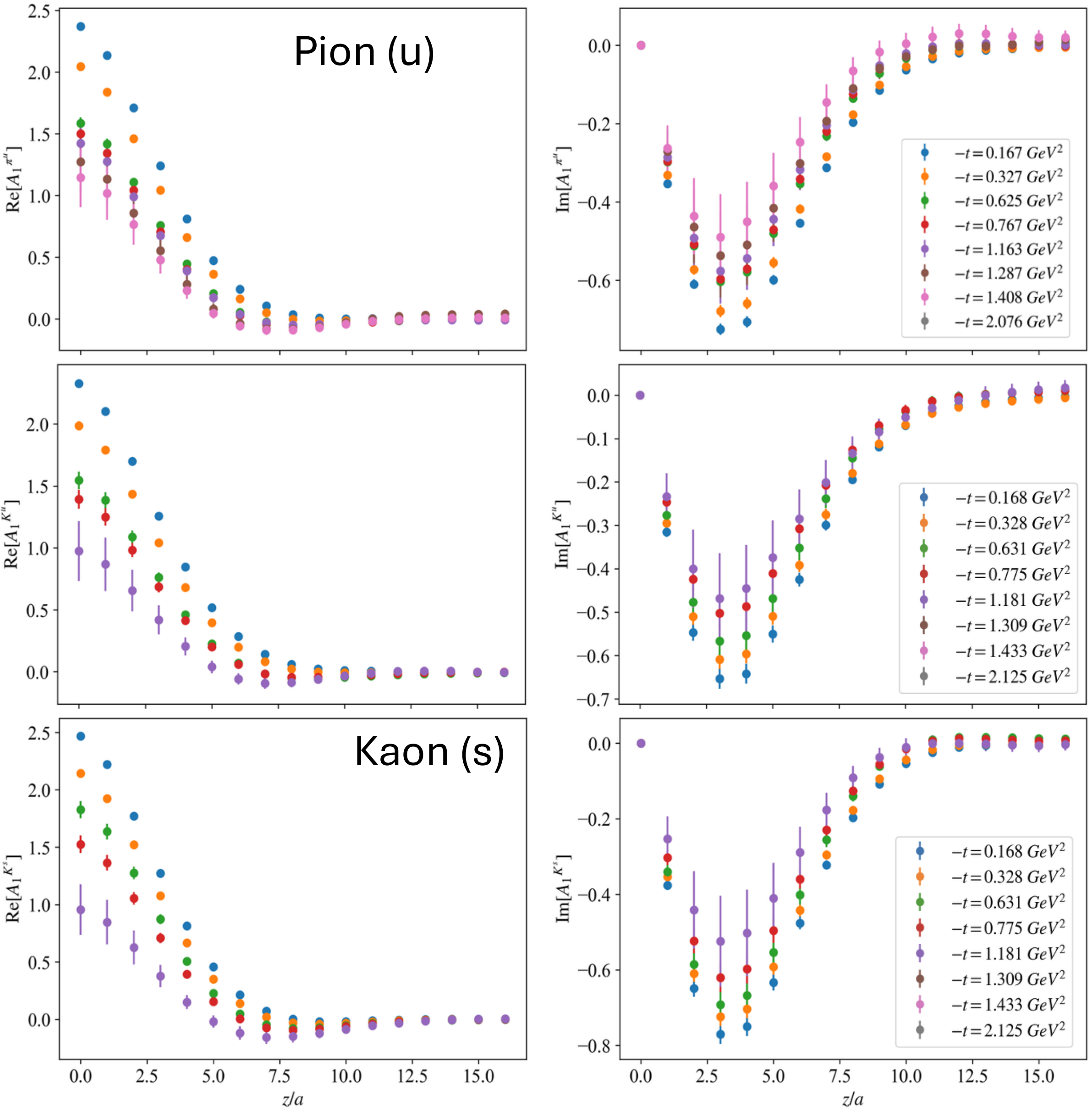
$P_3 = 1.67\ GeV$

Statistics:
Pion=57,504
Kaon=28,752

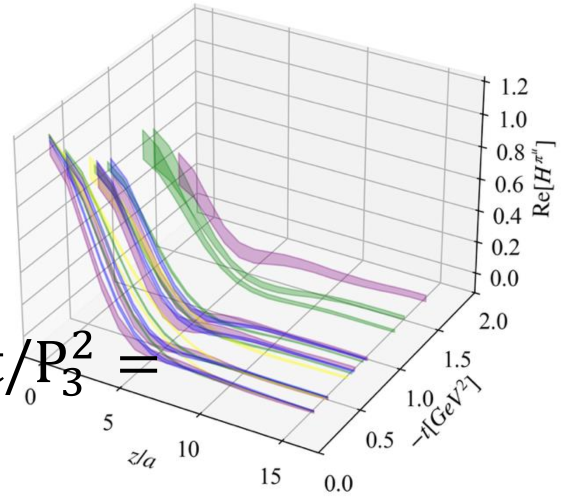


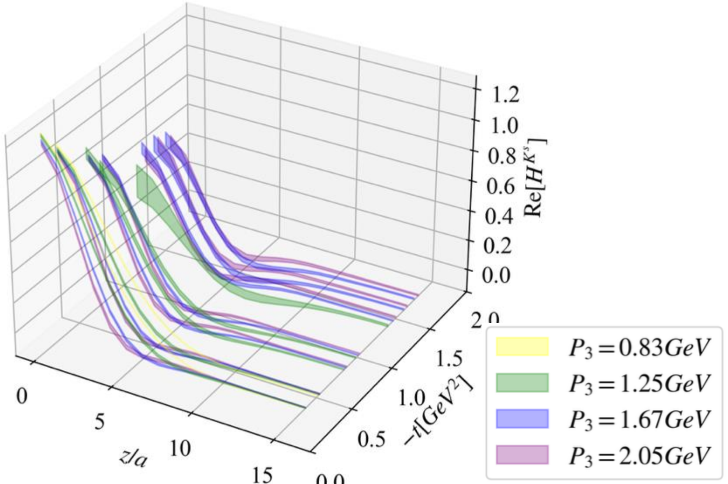
Bare Amplitude A_1

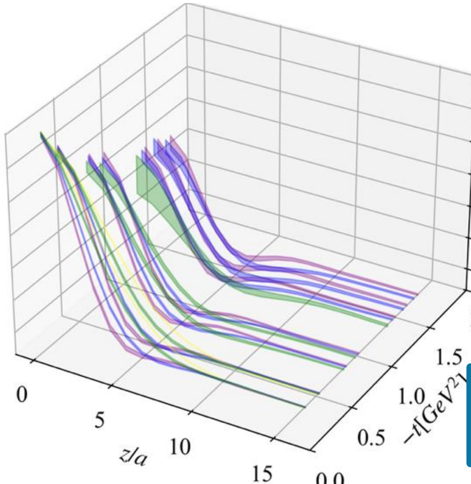
$(P_3 = 1.25\ GeV|all - t)$



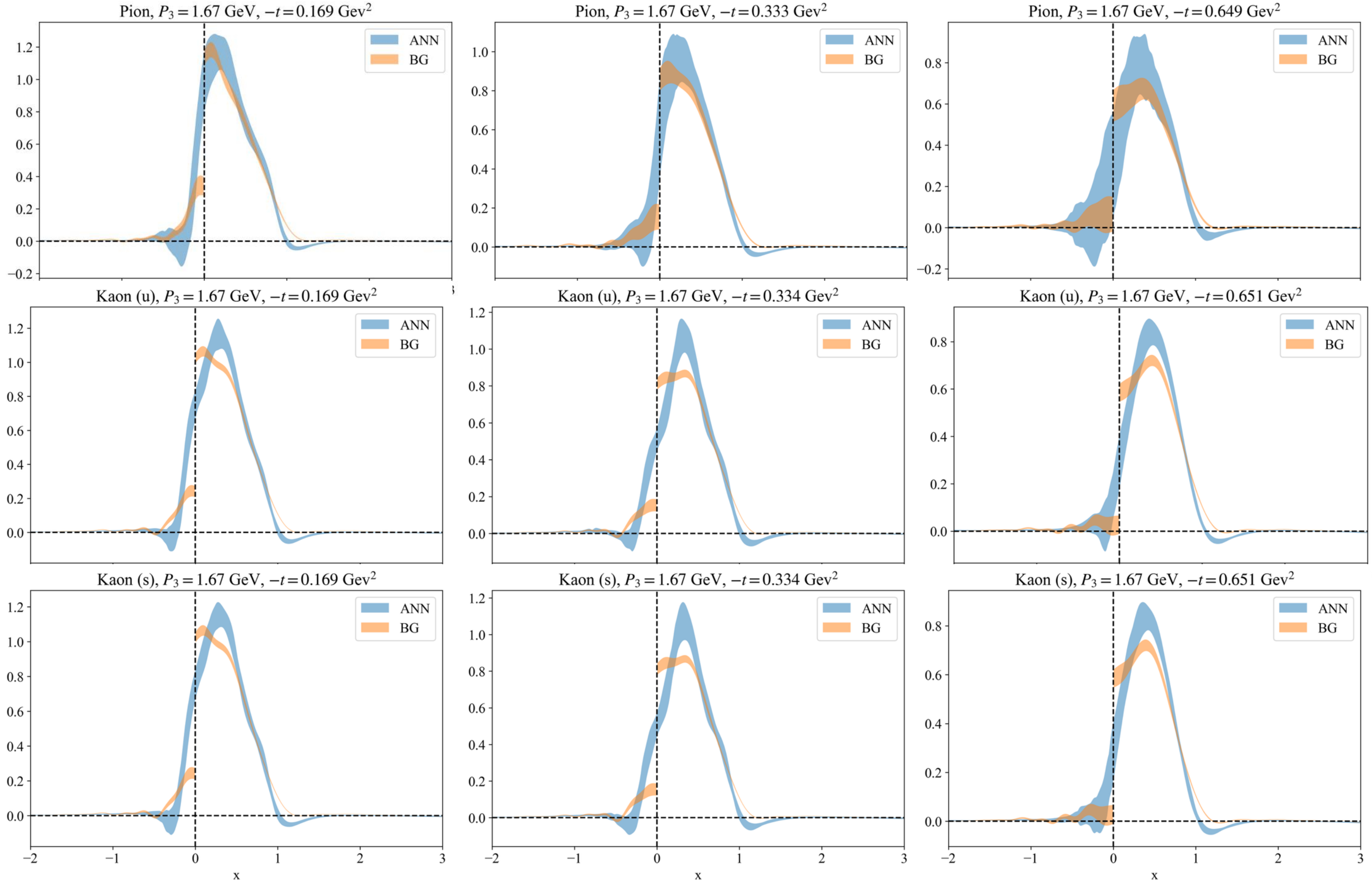
- Good signal quality for most values of $-$ t, but large values unreliable
- Similar qualitative behavior for pion and kaon
- $A_3(\xi = 0) = 0$
- $H(\xi = 0) = 0.5\ A_1(\xi = 0)$
- Focus on results for $-\mathrm{t}/\mathrm{P}_3^2 = 0.85$





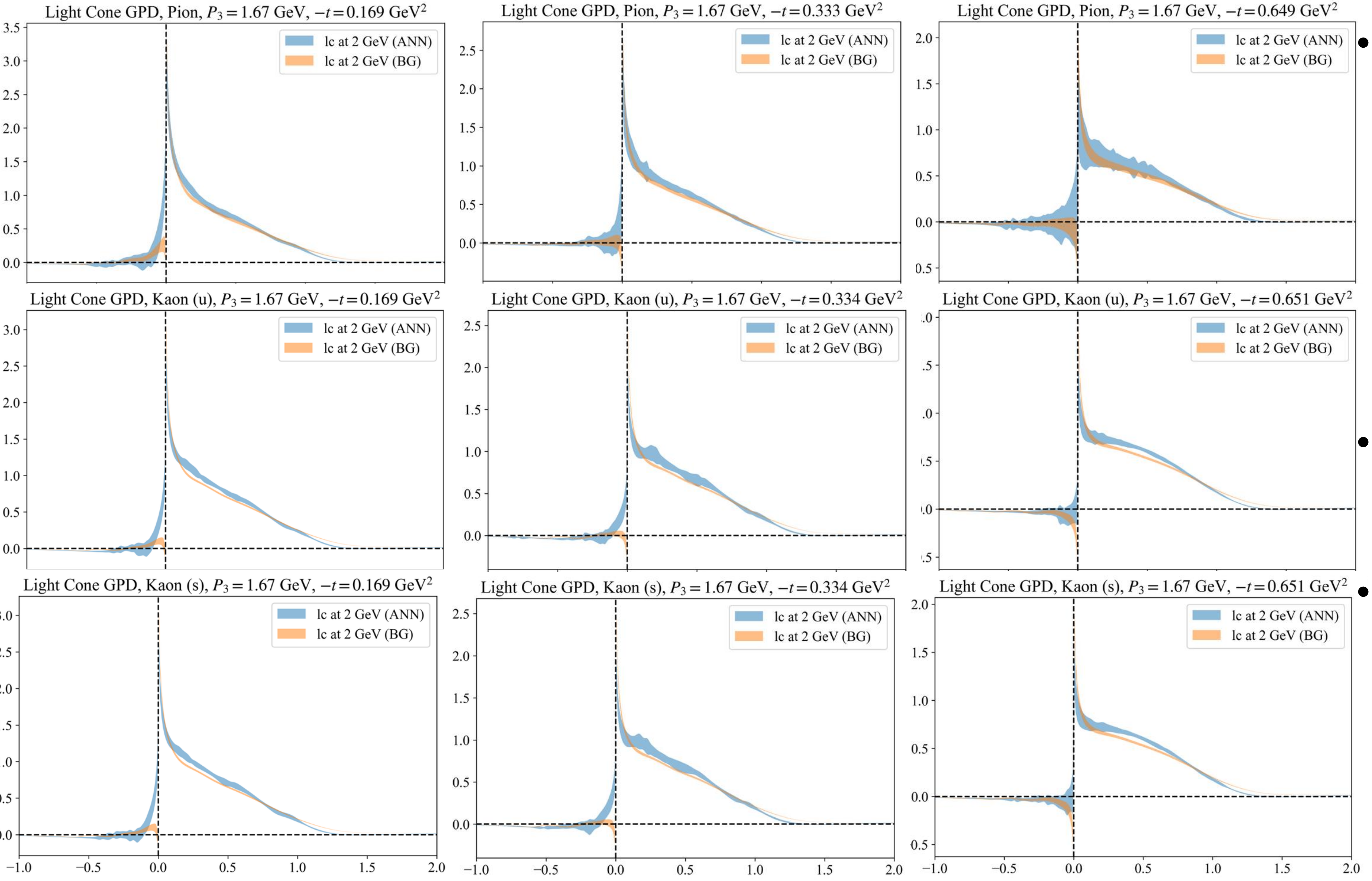


Quasi-GPDs in x-space



- Same trend for other $-\mathrm{t}$, P_3 combinations
- BG and ANN share similarities in intermediate-x region
- Reconstruction method has impact to the light-cone distribution

Light-cone GPDs



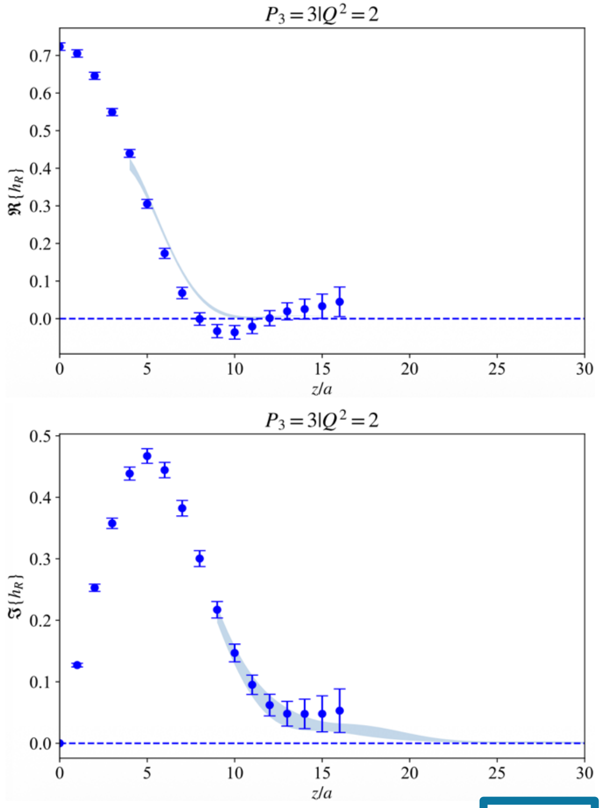
- BG and ANN differences in quasi-GPDs appear as small differences in intermediate-x region
- Anti-quark region not reliable
- Small- and large-x suffer from increased uncertainties

Summary and Future Work

Summary and Future Work

- Extracting pion and kaon GPDs for a wide kinematic range is possible due to Lorentz-invariant parametrization utilizing asymmetric kinematic frames. This opens a practical path toward pion and kaon tomography from lattice QCD

- Improvements in renormalization (hybrid) and x-space reconstruction (neural networks), and exploration of large-z behavior



Follow-up work

- Parametrization of –t dependence and impact-parameter space distributions
- GPDs from Short Distance Factorization on existing data
- Nonzero skewness
- Twist-3 GPDs
- Mellin moments via OPE on these nonlocal operators

[Miller et al., arXiv:2606.28102], also slides of J. Miller

Thank you!

Please let me know if you have any questions

<u>Collaborators:</u> Minhuan Chu, Krzysztof Cichy, Martha Constantinou, and Joshua Miller

<u>Acknowledgements:</u>
- ASCR Leadership Computing Challenge (ALCC)
- DOE CSGF
- NERSC
- QGT Collaboration
- Temple University

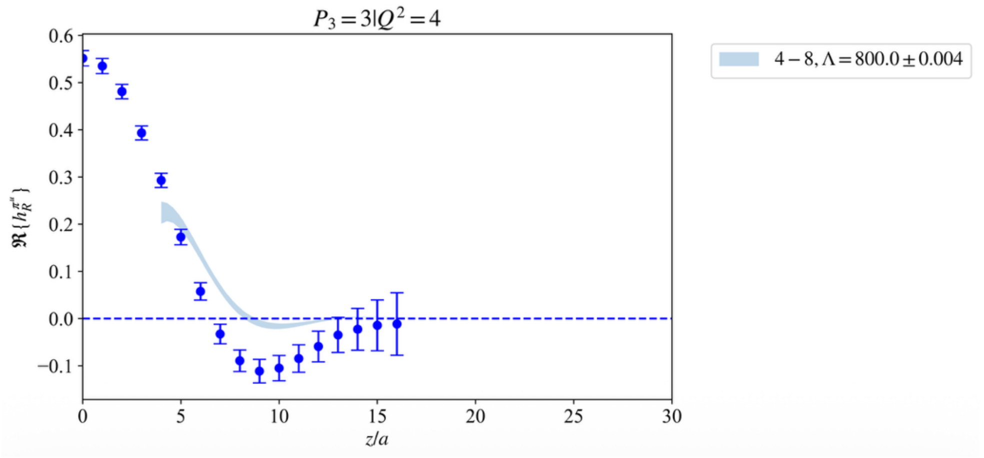
Backup

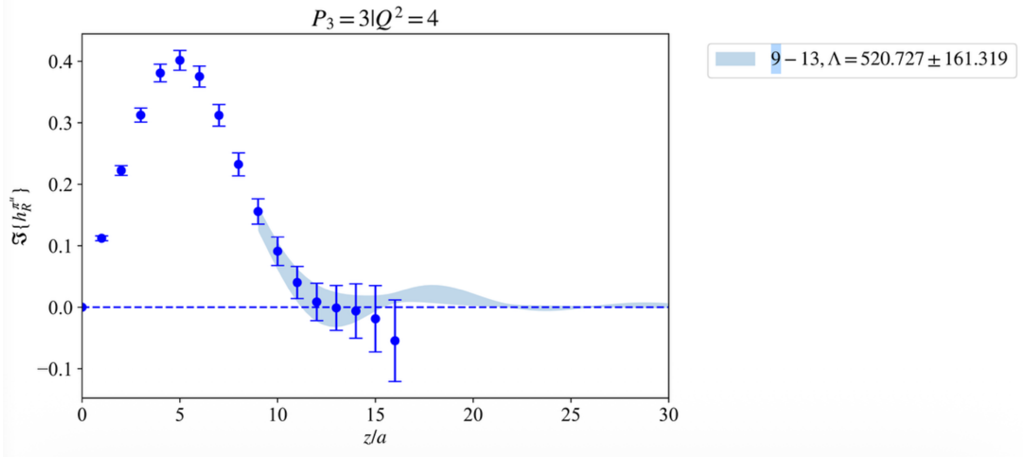
Conversion from lattice units to physical

```
In[1]:= P3 = {0, 1, 2, 3, 4, 5};

In[2]:= Delta = {{1, 0}, {1, 1}, {2, 0}, {2, 1}, {2, 2}, {3, 0}, {3, 1}, {4, 0}};

In[3]:= (* -t : *)

In[4]:= (* Pion *)

In[12]:= Table[Table[{StringJoin[StringJoin["P3=", ToString[P3[[i]]]], " ", StringJoin["D=", ToString[Delta[[j, 1]]], ToString[Delta[[j, 2]]]]],
      SetAccuracy[
        (0.197 / 0.0934) ^2
         ((2 Pi / 32) ^2 (Delta[[j, 1]] ^2 + Delta[[j, 2]] ^2)  - (Sqrt[0.125 ^2 + (2 Pi / 32) ^2 ( P3[[i]] ^2)] - Sqrt[0.125 ^2 + (2 Pi / 32) ^2 (Delta[[j, 1]] ^2 + Delta[[j, 2]] ^2 + P3[[i]] ^2)]) ^2), 4]} ,
    {j, 1, 8}] , {i, 1, 6}]

Out[12]= {{{P3=0 D=10, 0.120}, {P3=0 D=11, 0.200}, {P3=0 D=20, 0.319}, {P3=0 D=21, 0.369}, {P3=0 D=22, 0.494}, {P3=0 D=30, 0.531}, {P3=0 D=31, 0.565}, {P3=0 D=40, 0.745}},
  {{P3=1 D=10, 0.149}, {P3=1 D=11, 0.268}, {P3=1 D=20, 0.463}, {P3=1 D=21, 0.547}, {P3=1 D=22, 0.765}, {P3=1 D=30, 0.830}, {P3=1 D=31, 0.891}, {P3=1 D=40, 1.214}},
  {{P3=2 D=10, 0.163}, {P3=2 D=11, 0.311}, {P3=2 D=20, 0.576}, {P3=2 D=21, 0.697}, {P3=2 D=22, 1.025}, {P3=2 D=30, 1.125}, {P3=2 D=31, 1.221}, {P3=2 D=40, 1.741}},
  {{P3=3 D=10, 0.167}, {P3=3 D=11, 0.327}, {P3=3 D=20, 0.625}, {P3=3 D=21, 0.767}, {P3=3 D=22, 1.163}, {P3=3 D=30, 1.287}, {P3=3 D=31, 1.408}, {P3=3 D=40, 2.076}},
  {{P3=4 D=10, 0.169}, {P3=4 D=11, 0.333}, {P3=4 D=20, 0.649}, {P3=4 D=21, 0.801}, {P3=4 D=22, 1.236}, {P3=4 D=30, 1.376}, {P3=4 D=31, 1.512}, {P3=4 D=40, 2.282}},
  {{P3=5 D=10, 0.170}, {P3=5 D=11, 0.337}, {P3=5 D=20, 0.661}, {P3=5 D=21, 0.819}, {P3=5 D=22, 1.278}, {P3=5 D=30, 1.427}, {P3=5 D=31, 1.573}, {P3=5 D=40, 2.411}}}

In[6]:= (* Kaon *)

In[11]:= Table[Table[{StringJoin[StringJoin["P3=", ToString[P3[[i]]]], " ", StringJoin["D=", ToString[Delta[[j, 1]]], ToString[Delta[[j, 2]]]]],
      SetAccuracy[
        (0.197 / 0.0934) ^2
         ((2 Pi / 32) ^2 (Delta[[j, 1]] ^2 + Delta[[j, 2]] ^2)  - (Sqrt[0.25 ^2 + (2 Pi / 32) ^2 ( P3[[i]] ^2)] - Sqrt[0.25 ^2 + (2 Pi / 32) ^2 (Delta[[j, 1]] ^2 + Delta[[j, 2]] ^2 + P3[[i]] ^2)]) ^2), 4]} ,
    {j, 1, 8}] , {i, 1, 6}]

Out[11]= {{{P3=0 D=10, 0.151}, {P3=0 D=11, 0.275}, {P3=0 D=20, 0.479}, {P3=0 D=21, 0.568}, {P3=0 D=22, 0.799}, {P3=0 D=30, 0.867}, {P3=0 D=31, 0.933}, {P3=0 D=40, 1.277}},
  {{P3=1 D=10, 0.158}, {P3=1 D=11, 0.295}, {P3=1 D=20, 0.530}, {P3=1 D=21, 0.634}, {P3=1 D=22, 0.911}, {P3=1 D=30, 0.994}, {P3=1 D=31, 1.074}, {P3=1 D=40, 1.497}},
  {{P3=2 D=10, 0.164}, {P3=2 D=11, 0.317}, {P3=2 D=20, 0.594}, {P3=2 D=21, 0.722}, {P3=2 D=22, 1.073}, {P3=2 D=30, 1.182}, {P3=2 D=31, 1.286}, {P3=2 D=40, 1.853}},
  {{P3=3 D=10, 0.168}, {P3=3 D=11, 0.328}, {P3=3 D=20, 0.631}, {P3=3 D=21, 0.775}, {P3=3 D=22, 1.181}, {P3=3 D=30, 1.309}, {P3=3 D=31, 1.433}, {P3=3 D=40, 2.125}},
  {{P3=4 D=10, 0.169}, {P3=4 D=11, 0.334}, {P3=4 D=20, 0.651}, {P3=4 D=21, 0.804}, {P3=4 D=22, 1.244}, {P3=4 D=30, 1.385}, {P3=4 D=31, 1.523}, {P3=4 D=40, 2.305}},
  {{P3=5 D=10, 0.170}, {P3=5 D=11, 0.337}, {P3=5 D=20, 0.662}, {P3=5 D=21, 0.821}, {P3=5 D=22, 1.282}, {P3=5 D=30, 1.431}, {P3=5 D=31, 1.579}, {P3=5 D=40, 2.423}}}
```

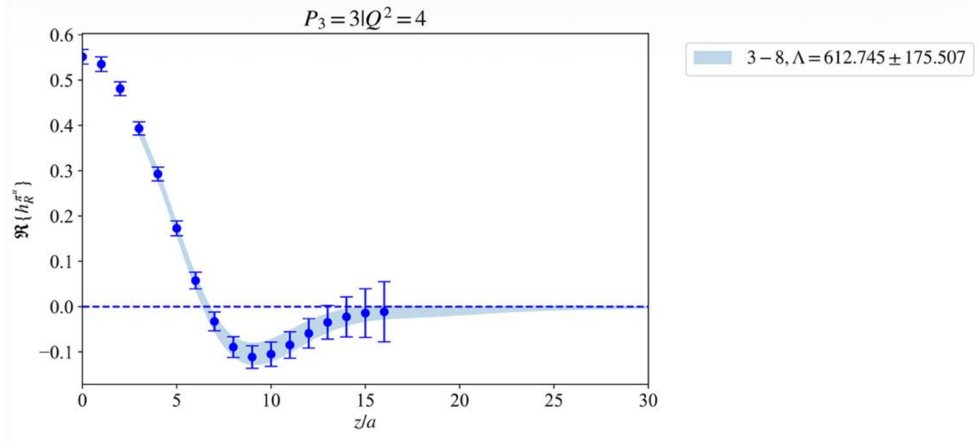
LA and NLA Fits: $P_3 = 3| - t = 4$ (Pion)

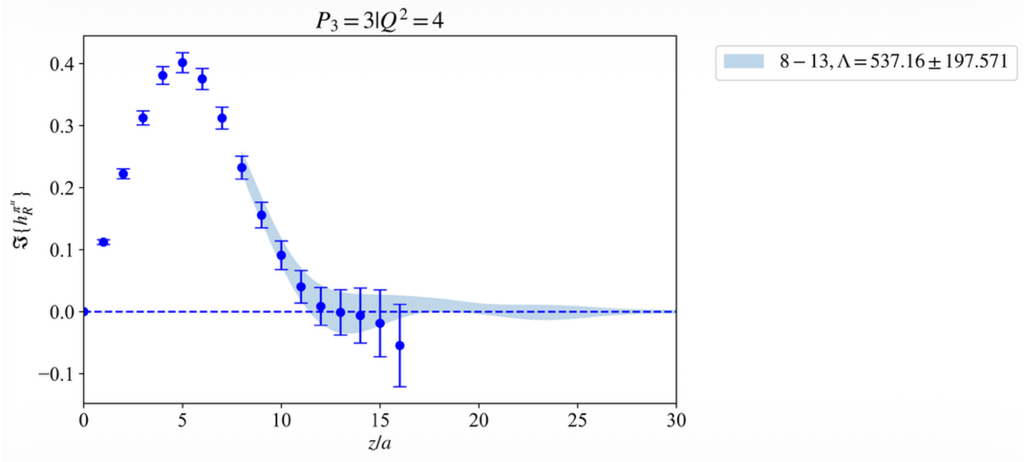
LA:





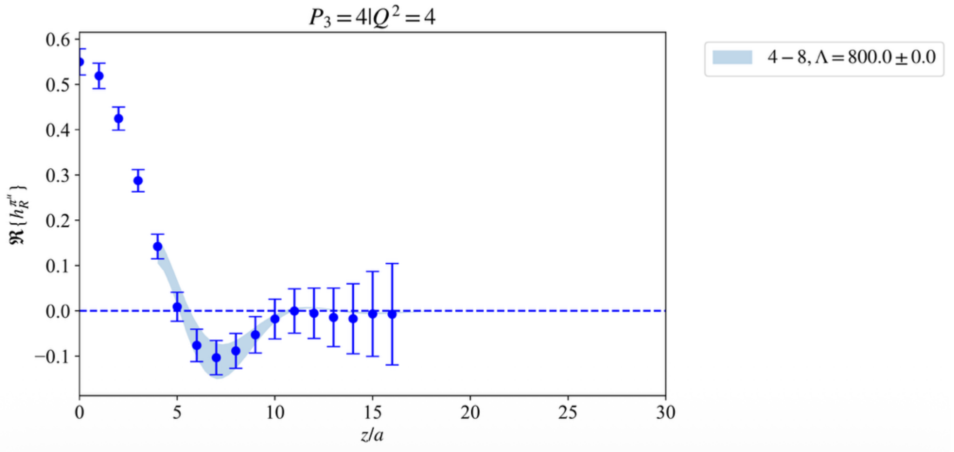
NLA:

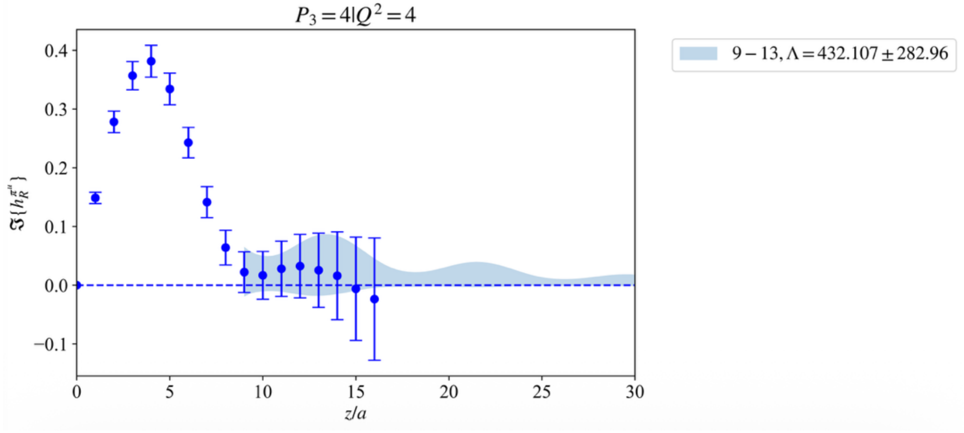




LA and NLA Fits: $P_3 = 4| - t = 4$ (Pion)

LA:





NLA: