Finite Size Scaling of Probability Distributions

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Motivation

- Large scale quantum simulation becomes increasingly accessible, but necessary shot totals can be a limiting factor
- How can we quantify the shot cost prior to using quantum devices?
- Is this data accessible from classical simulation?

- Rydberg Hamiltonian:

$$H = \frac{\Omega}{2}\sum_i (|g_i\rangle\langle r_i| + |r_i\rangle\langle g_i|) - \Delta\sum_i n_i + \sum_{i<j} V_{ij} n_i n_j$$

- Δ is the detuning, Ω is the Rabi frequency, V_{ij} is a van der Waals interaction potential, $|g\rangle \& |r\rangle$ are the ground and Rydberg states, respectively, and $V_{ij} = (\Omega R_b^6)/|r_i - r_j|^6$, where r_k are the positions of the Rydberg atoms and R_b is the Rydberg blockade radius. We use a Rydberg ladder configuration[1] with $\rho = 2$.
- Cumulative Probability:

$$\Sigma(p_\Lambda) = \sum_{\{n\}:p_{\{n\}}\leq p_\Lambda} p_{\{n\}}$$

[1] A. Kaufman, M. Asaduzzaman, Z. Ozzello, B. Senseman, J. Corona, and Y. Meurice, Diagnosing Device Performance in Rydberg-Ladder Gauge Simulators with Cumulative Probabilities and Filtered Mutual Information, (2025), arXiv:2507.14128 [quant-ph].

First Pass

- To motivate, we start with a given system's maximal probability, p_{max}
- For a quantum experiment, we want there to be enough shots such that $p_{max} > 1/N_{sh}$

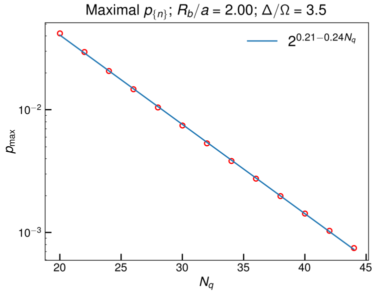

Figure: Scaling of p_{max} with N_q for Rydberg Ladder with $R_b/a = 2.00$

Cumulative Probability Distribution Curves

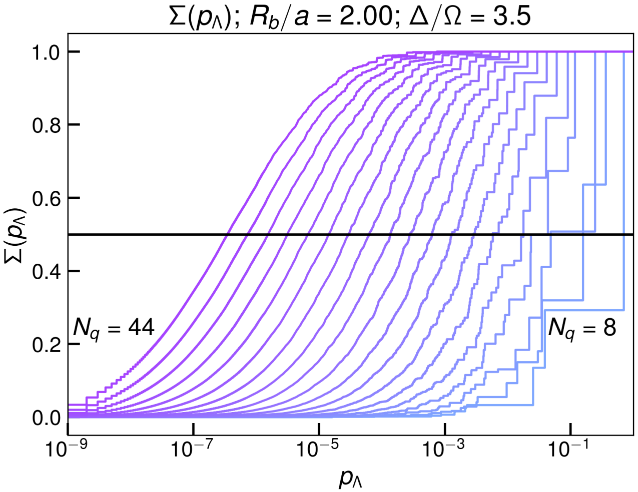

Figure: Cumulative Probability Distributions from 4-22 rungs.

Collapse

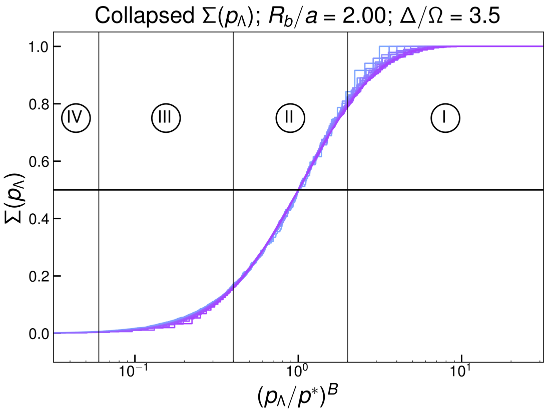

Figure: Collapsed $\Sigma(p_\Lambda)$ curves where $\Sigma(p^*) = 1/2$; 4 Regions shown.

- How can we describe the behavior of this curve?

Region I

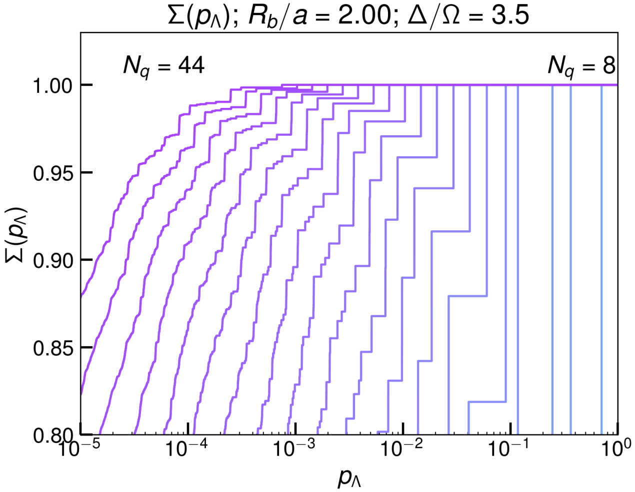

Figure: Region I focused

Region II

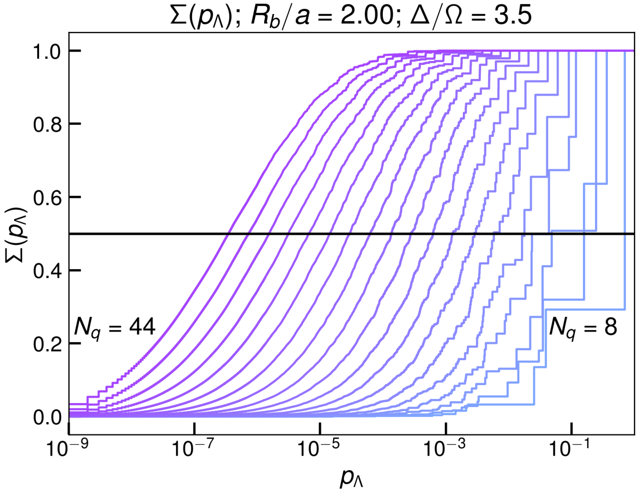

Figure: Region II zoomed

Fermi-Dirac Form

- We originally fit with:

$$\Sigma(p_\Lambda) \simeq \frac{(p_\Lambda/p^*)^\beta}{(1 + (p_\Lambda/p^*)^\beta)}$$

- Take $-ln(p_\Lambda) \to \epsilon$ and $-ln(p^*) \to \mu$. Then:

$$\Sigma \simeq \frac{1}{1 + e^{\beta(\epsilon-\mu)}}$$

- In Fermi-Dirac Statistics, there exists an ϵ such that the system is half-filled. For our cumulative probability distribution, $p^*_{1/2}$ serves this same purpose.
- Further discussion around this will be held later.

Region III

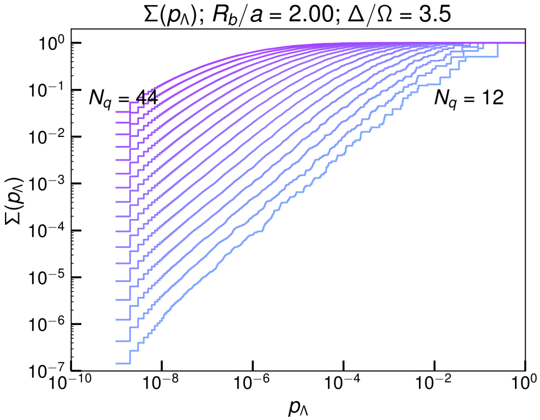

Figure: Cumulative Distributions in Region III

- Inspired by Region II, here we fit about $p^*_{0.01}$ to $\Sigma(p_\Lambda) \simeq A p_\Lambda^B$.

Region III Collapse

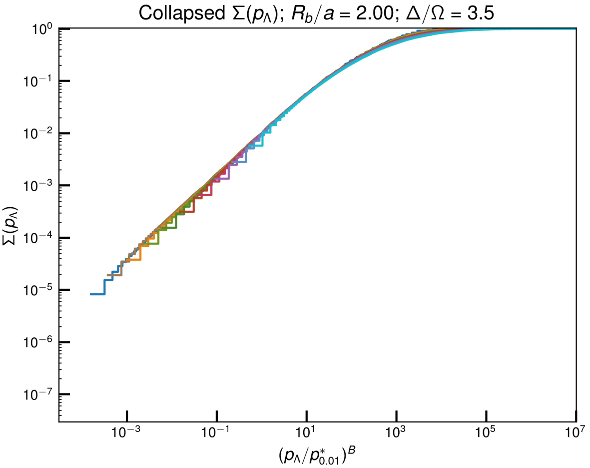

Figure: Collapse in Region III of Cumulative Distributions, around $p^*_{0.01}$

Error Estimate

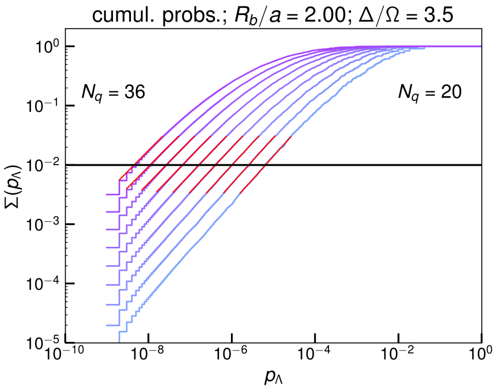

Figure: Cumulative Distributions in Region III

- Taking $\Sigma(p_\Lambda)$ to be your desired error rate, shot total can be estimated to achieve this goal.

Region IV

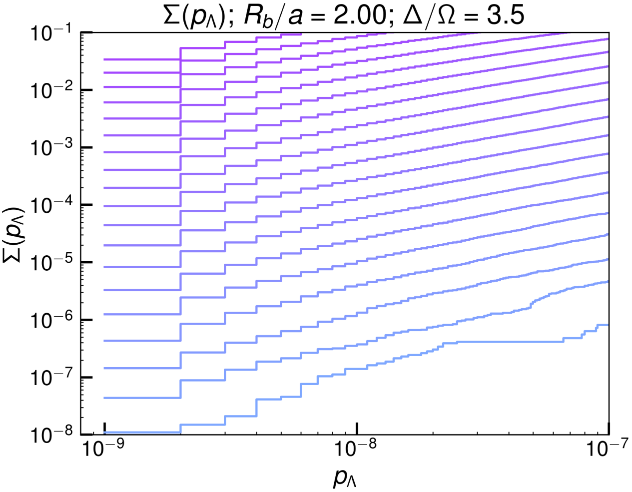

Figure: Region IV of Cumulative Distributions

Exponential Cost

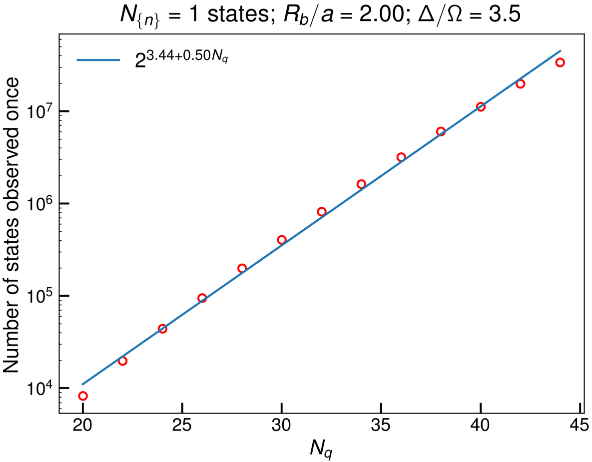

Figure: Scaling of Single Occurrence states with system size

Exponential Cost

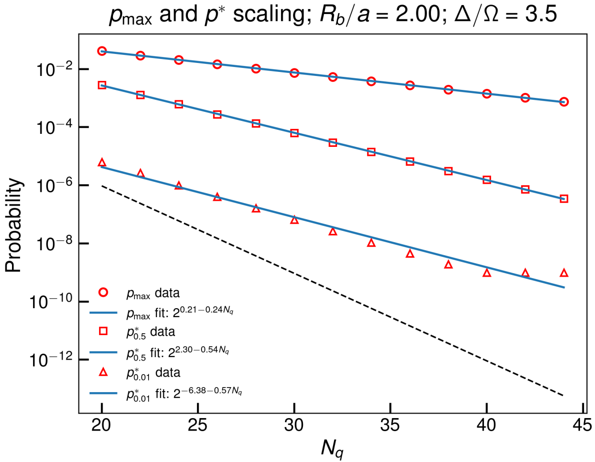

Figure: Scaling of p_{max}, $p^*_{0.5}$, and $p^*_{0.01}$ compared against 2^{-N_q}

Rydberg with $R_b/a = 3.00$

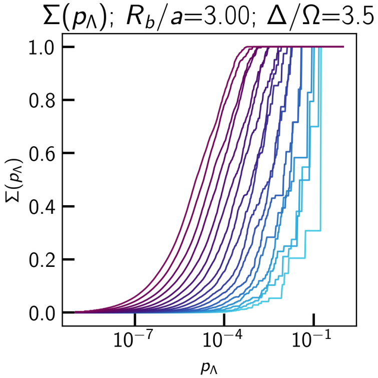

Figure: Cumulative Distributions at $R_b/a = 3.00$ from 12 to 44 atoms

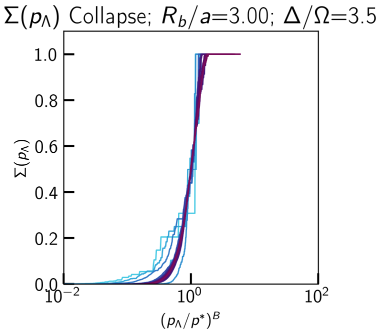

Figure: The Collapse of distributions at left.

Ising Model

- Shown $\Sigma(p_\Lambda)$ validity in different parameter ranges for one model, now we show for a different model.
- The Transverse-Field Ising Model (where we set $h = 1$):

$$H_{TFI} = -h\sum_{i=1}^{N_s}\sigma_i^x - J\sum_{i=1}^{N_s}\sigma_i^z\sigma_{i+1}^z$$

Ising with $J/h = 0.5$

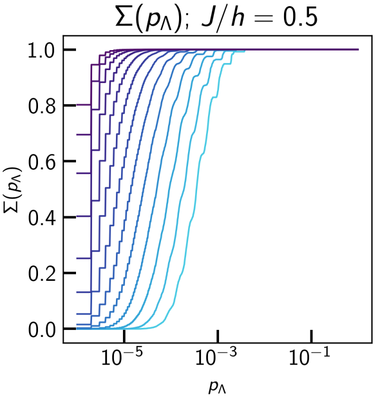

Figure: Cumulative Distributions at $J/h = 0.5$ from 12 to 24 sites

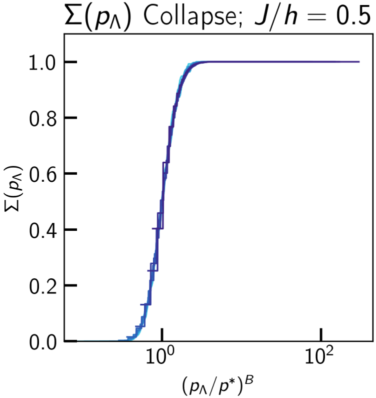

Figure: The Collapse of distributions at left.

Ising with $J/h = 1.0$

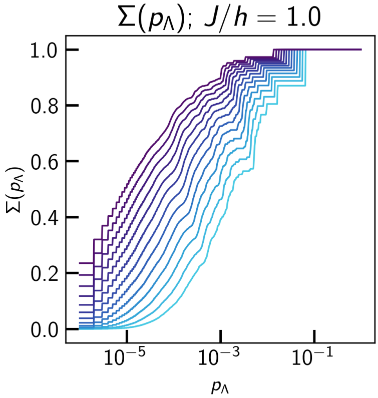

Figure: Cumulative Distributions at $J/h = 1.0$ from 12 to 24 sites

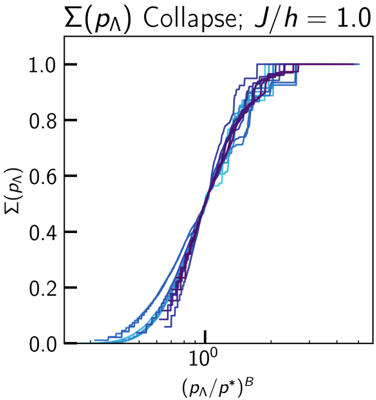

Figure: The Collapse of distributions at left.

Ising with $J/h = 1.5$

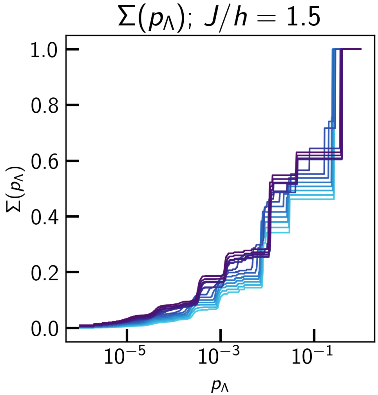

Figure: Cumulative Distributions at $J/h = 1.5$ from 12 to 24 sites

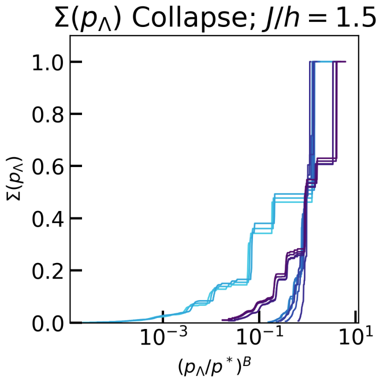

Figure: The Collapse of distributions at left.

Closing Comments

- We introduce using Cumulative Probability Distributions to determine large-scale behavior from small systems.
- We use these to bound error rates at various system sizes.
- We use different probabilistic quantities to quantify exponential costs with system size scaling.
- More in-depth characterization of additional parameter ranges and models is necessary.
- Read more here: **arXiv:2607.27013**
- Questions?
- Thank you!