Confinement and String Breaking in the Compact Abelian Higgs Model

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2607.???? (forthcoming)

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Motivation

Event Generators

- Form a bridge between theoretical models and experimental data
- Integrate many different effects, including nonperturbative ones

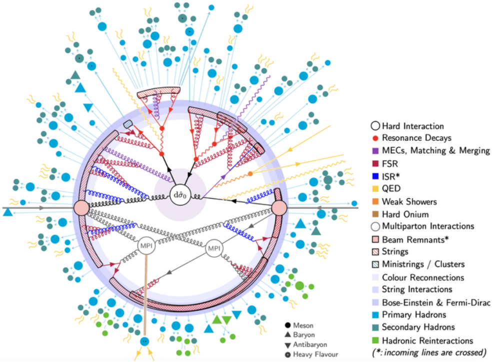


Figure: Diagram of top quark pair production in PYTHIA 8[1].

[1]Bierlich, C., *SciPost Physics Codebases*, **8** (2022)

PYTHIA's Tuned Parameters[2]

HAD Parameters	Monash 13	(Default)	Comment
String breaks: p_T and z distributions			
`StringPT:sigma`	0.335	0.304	Soft p_T in string breaks (in GeV)
`StringPT:enhancedFraction`	0.01	0.01	Fraction of breakups with enhanced p_T
`StringPT:enhancedWidth`	2.0	2.0	Enhancement factor
`StringZ:aLund`	0.68	0.3	Lund FF a (hard fragmentation supp)
`StringZ:bLund`	0.98	0.8	Lund FF b (soft fragmentation supp)
`StringZ:aExtraSquark`	0.0	0.0	Extra a when picking up an s quark
`StringZ:aExtraDiquark`	0.97	0.50	Extra a when picking up a diquark
`StringZ:rFactC`	1.32	1.00	Lund–Bowler c-quark parameter
`StringZ:rFactB`	0.855	0.67	Lund–Bowler b-quark parameter
Flavour composition: mesons			
`StringFlav:ProbStoUD`	0.217	0.19	Strangeness-to-UD ratio
`StringFlav:mesonUDvector`	0.5	0.62	Light-flavour vector suppression
`StringFlav:mesonSvector`	0.55	0.725	Strange vector suppression
`StringFlav:mesonCvector`	0.88	1.06	Charm vector suppression
`StringFlav:mesonBvector`	2.2	3.0	Bottom vector suppression
`StringFlav:etaSup`	0.60	0.63	Suppression of eta mesons
`StringFlav:etaPrimeSup`	0.12	0.12	Suppression of eta$'$ mesons
Flavour composition: baryons			
`StringFlav:probQQtoQ`	0.081	0.09	Diquark rate (for baryon production)
`StringFlav:probSQtoQQ`	0.915	1.000	Strange-diquark suppression
`StringFlav:probQQ1toQQ0`	0.0275	0.027	Vector diquark suppression
`StringFlav:decupletSup`	1.0	1.0	Spin-3/2 baryon suppression
`StringFlav:suppressLeadingB`	off	off	Optional leading-baryon suppression
`StringFlav:popcornSpair`	0.9	0.5	
`StringFlav:popcornSmeson`	0.5	0.5	

[2] Mrenna, S., "Hadronization Models in Event Generation" iQuS Workshop, April 2026

A Simple Confining Lattice Model: Spin-1 Compact Abelian-Higgs Model

The Model: Spin-1 Truncated CAHM

Low-energy effective theory of 1+1D scalar electrodynamics on a chain of qutrit sites [3]:

$$H_{CAHM} = \frac{U}{2}\sum_i (L_i^z)^2 + \frac{Y}{2}\sum_{\langle ij\rangle}(L_i^z - L_j^z)^2 - \frac{X}{\sqrt{2}}\sum_i L_i^x \tag{1}$$

- $L^z\,|m\rangle = m\,|m\rangle$ plays the role of the electric field
- U: energy density of the field ($U/2$ per excited link)
- Y: energy per domain wall — effective "matter" mass/coupling, set by Gauss's law
- X (set to 1): only off-diagonal term — drives string motion *and* string breaking

[3] Meurice, Y., *PRD* **104** 094513 (2021)

Confinement: the High-Y Limit

- Domain walls (changes in m) cost $U/2$ per unit separation $\Rightarrow$ linear confining potential
- As $Y \to \infty$: two-domain-wall sector $\cong$ 1D lattice string problem
- Restricting Hamiltonian to that sector gives an exactly solvable effective string Hamiltonian, $(i, j) =$ endpoint sites:

$$\hat{H}_s = Y + \sum_{i<j}^{L} \frac{U}{2}(j-i)\left|i,j\right\rangle\left\langle i,j\right| - \frac{X}{2}\Big(\left|i+1,j\right\rangle\left\langle i,j\right| + \left|i,j+1\right\rangle\left\langle i,j\right| + \text{h.c.}\Big)$$

- Diagonalizes into string-meson momentum eigenstates $|\Psi_{k,\ell}\rangle$ built from Bessel functions $\mathcal{J}_{r-\nu_{k,\ell}}(4X\cos k/U)$ [4]
- These get mixed and dressed by virtual domain-wall pairs once Y is finite

[4]Surace, F. and Lerose, A., *New J. Phys.* **23** 062001 (2021)

Probing the Spectrum: a Local Chemical Potential

Standard approach: insert external charges, minimize the field energy in their presence.

Our approach:

$$H' = H_{CAHM} - \mu\, L^z_{\{s\}} \tag{2}$$

- Applied to a set of sites $\{s\}$ (e.g. two sites at separation L) $\rightarrow$ drives the local field toward $m = 1$
- Analogous to external charges, but μ is *tunable*, unlike fixed charge insertion

Results

String Formation and Breaking

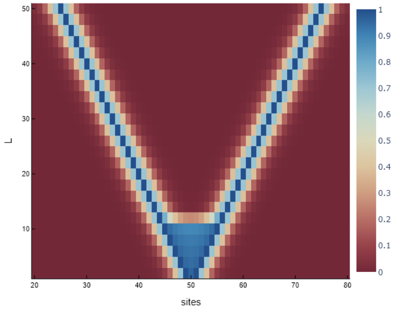


Figure: $\langle L^z \rangle$ across minimum energy states with external charges at distance L

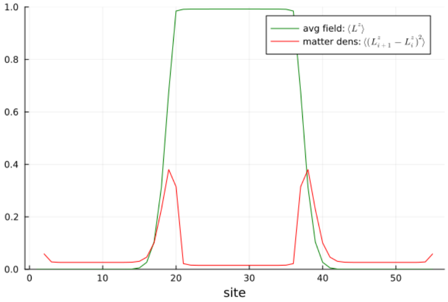
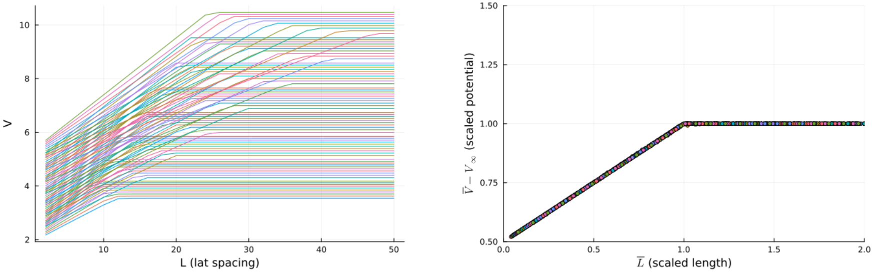


Figure: $\langle L^z \rangle$ and $\langle (\Delta L^z)^2 \rangle$ of optimized string state

- Almost constant-flux string up to $L \approx 10$
- Beyond that: string *breaks* into two independent, localized mesons
- Direct visualization of the breaking transition

String Potential and Universal Collapse



- $V(L)$ for many choices of U, Y; all display an initial linear slope σ up to some length L^*
- $\sigma > U/2$: string is dressed with virtual domain-wall pairs
- Rescaling: all (U, Y) collapse onto one curve

$$\overline{V} = V/(2\sigma L^*), \quad \overline{L} = L/L^*$$

Real-Time Evolution – String Breaking

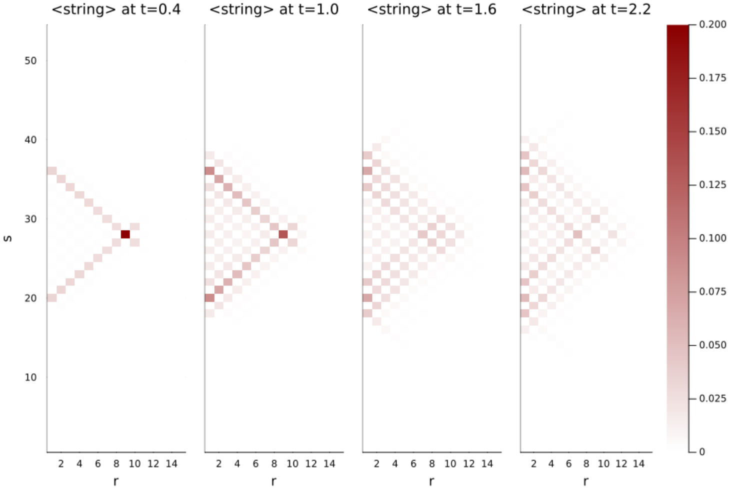


Figure: Time-evolution of configuration strings with $Y = 2.5$, $U = 0.25$; Initial state is a tensor product of electric flux sites, var(H)=$4.5Y$

Real-Time Evolution – String Breaking

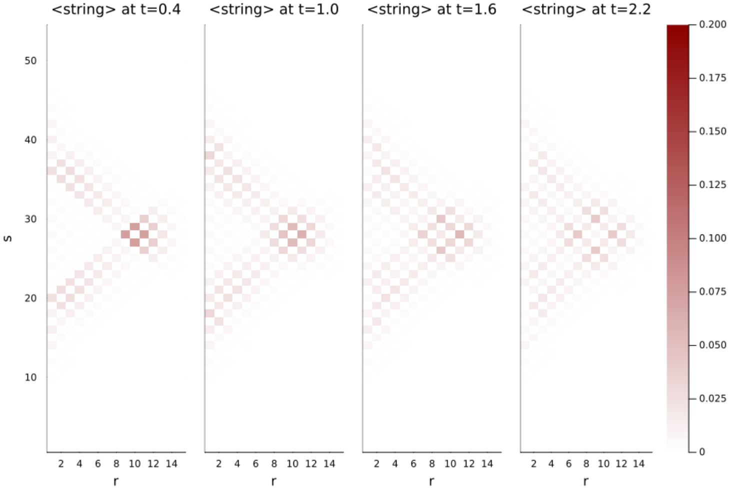


Figure: Time-evolution of configuration strings with $Y = 2.5$, $U = 0.25$;
Initial state is a DMRG string state, var(H)=0.2Y

Conclusion

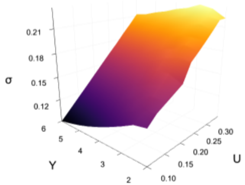
- Spin-1 truncated CAHM: a minimal model hosting genuine string-meson phenomenology
- Exact diagonalization + DMRG agree on a consistent linear-potential picture
- Real-time evolution permits direct observation of string breaking

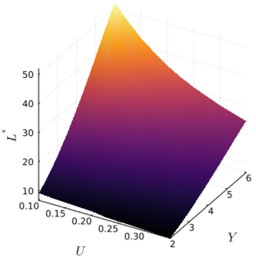
Outlook

Use adiabatically-prepared strings as initial states for *real-time* string-breaking dynamics — permitting PYTHIA-style parametrization of hadronization in simple quantum systems.

IOWA

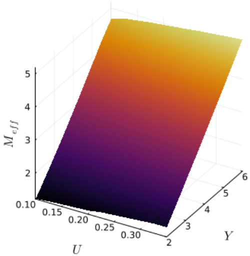
String Tension and Breaking Length





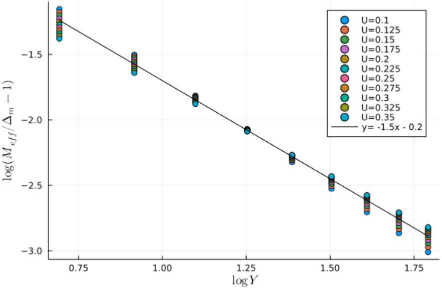
σ and L^* extracted from the fitted potential, over the full (U, Y) scan.

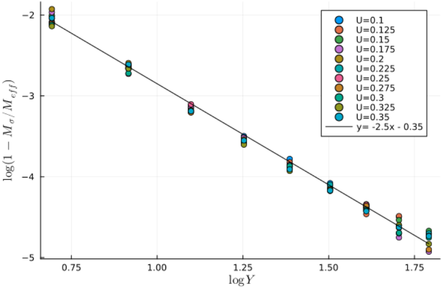
Localized Meson Mass Scale



- M_{eff}: vacuum-subtracted DMRG energy for μ on a *single* site
- Quite similar to the finite-volume mass gap Δ_m over the parameter range
- $M_{\mathit{eff}}/\Delta_m \approx 1 + 1.22\, Y^{-1.5}$
- M_{eff} linear in $Y + U/2$ — the expanded $(L^z)^2$ coefficient

Comparing Mass Scales



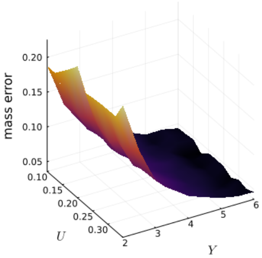


- $M_{eff}/\Delta_m \approx 1 + 1.22\, Y^{-1.5}$ $\quad M_\sigma/M_{eff} \approx 1 - 1.41\, Y^{-2.5}$
- Linear fit $M = a(Y + U/2) + b(Y - U/2) + c$: $a \approx 1.6$–1.7 for all three mass measures (Δ_m, M_{eff}, $M_\sigma \equiv \sigma L^*$)

Testing the Semiclassical Picture

Semiclassical expectation: a string breaks once its energy equals the cost of two separated mesons,

$$M_{eff} + \sigma L^* = 2M_{eff} \quad \Longleftrightarrow \quad M_{error} \equiv M_{eff} - \sigma L^* \to 0 \tag{3}$$



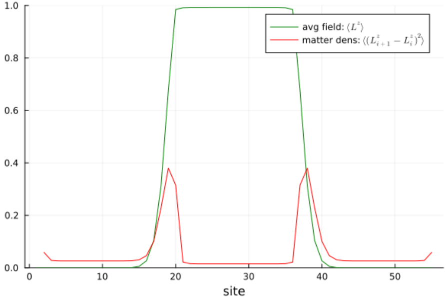
- M_{error} falls off as $\sim Y^{-1.45}$, essentially independent of U
- 12% deviation at $Y = 2.0 \rightarrow < 1\%$ at $Y = 6.0$
- Stiffer strings (larger Y) behave more classically

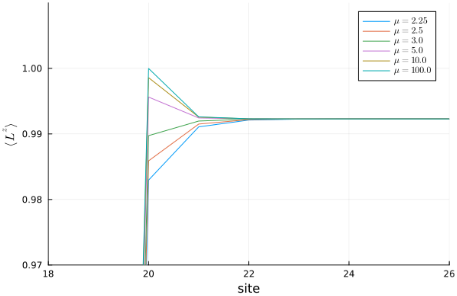
DMRG Setup

- Chains of $\mathcal{O}(100)$ sites, μ applied to two sites separated by distance L
- Ground state of H' found via DMRG for a scan over $U \in (0.1, \ldots, 0.35)$, $Y \in (2.0, \ldots, 6.0)$
- Two competing minima:
  - **String state**: constant electric flux between the two sites
  - **Broken state**: localized excitations pinned at the two sites, flux vanishes in between
- Which one wins depends on L relative to a breaking length L^*

Adiabatic Removal of the Chemical Potential

- Concern: is the string state part of the spectrum, or just an artifact of pinning the field with a large μ?
- Fix: ramp μ down slowly across successive DMRG rounds — broken state is a distant local minimum, so the string survives





Example: $U = 0.25$, $Y = 6.0$, $L = 16$; μ lowered $100 \rightarrow 2.25$ (minimum value preserving the string) $\rightarrow$ vacuum-subtracted energy shifts by only 1.5%; the "horned" edge relaxes into a natural taper.