The axial structure of the nucleon with the PACS10 superfine lattice

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For PACS collaboration

26 July, 2026, LATTICE 2026

Neutrino oscillation experiments

The interaction rate for lepton flavor f (**observable**)

$$N_f(E_{\mathrm{rec}}, L) \propto \sum_i \int \sigma_i(E_\nu)\Phi_f(E_\nu, L) g_{\sigma_i}(E_\nu, E_{\mathrm{rec}})\, dE_\nu$$

Reconstruction probability

Neutrino flux for oscillation measurements

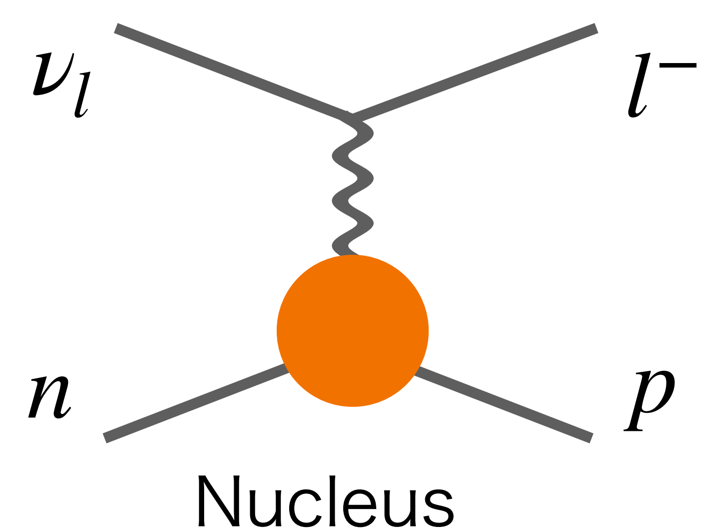
Cross section for the neutrino–nucleus.
The **charged-current quasi-elastic** (CCQE) scattering is the dominant process in the T2K energy range.

$$\frac{d\sigma}{dq^2}(E_\nu, q^2) = \frac{G_F^2|V_{ud}|^2}{2\pi}\frac{M_N^2}{E_\nu^2}\left[(\tau + r_l^2)A(\nu,q^2) - \frac{\nu}{M_N^2}B(\nu,q^2) + \frac{\nu^2}{M_N^4}\frac{C(\nu,q^2)}{1+\tau}\right]$$

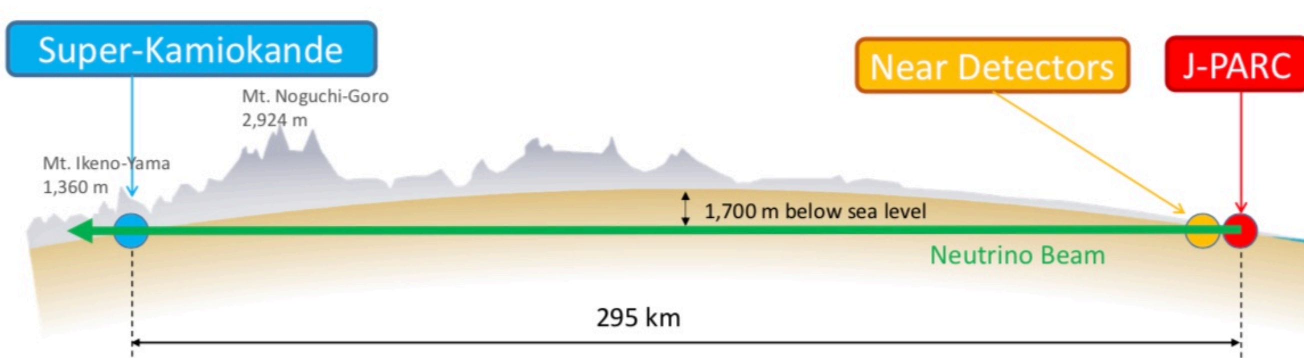
$$\begin{cases} A = \tau(G_M^v)^2 - (G_E^v)^2 + (1+\tau)F_A^2 - r_l^2\left[(G_M^v)^2 + F_A^2 + 4F_P(F_A - \tau F_P)\right] \\ B = 4\eta\tau G_M^v F_A \\ C = \tau(G_M^v)^2 + (G_E^v)^2 + (1+\tau)F_A^2 \end{cases}$$

F_A, F_P: Axial form factors

CCQE



Essential inputs for precise **neutrino oscillation** measurements.



https://t2k-experiment.org/t2k/

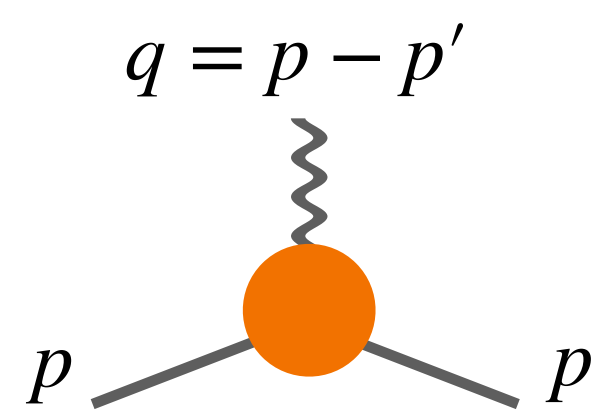
Nucleon matrix elements

For axial currents $A_\alpha(x) = \bar{u}(x)\gamma_5\gamma_\alpha d(x), \quad P_\alpha(x) = \bar{u}(x)\gamma_5 d(x)$

matrix elements are expressed using **form factors (FF)**:

$$\langle p(\mathbf{p}')\,|\,A_\alpha(x)\,|\,n(\mathbf{p})\rangle = \bar{u}_p(\mathbf{p}')\left(\gamma_\alpha\gamma_5 \underbrace{F_A(q^2)}_{\text{Axial FF}} + iq_\alpha\gamma_5 \underbrace{F_P(q^2)}_{\text{Induced-Pseudoscalar FF}}\right) u_n(\mathbf{p})e^{iq\cdot x}$$

$$\langle p(\mathbf{p}')\,|\,P(x)\,|\,n(\mathbf{p})\rangle = \bar{u}_p(\mathbf{p}')\left(\gamma_5 \underbrace{G_P(q^2)}_{\text{Pseudoscalar FF}}\right) u_n(\mathbf{p})e^{iq\cdot x}$$



Axial coupling: $g_A = F_A(0)$

Axial radius: $\langle r_A \rangle = -\dfrac{1}{F_A(0)} \dfrac{dF_A(q^2)}{dq^2}\bigg|_{q^2=0}$

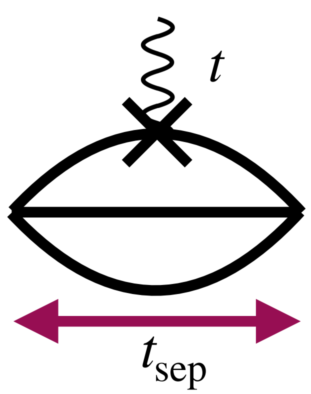
Physics at small q^2

Nucleon matrix elements

from LQCD

$$\frac{\langle N(t_{\rm sep},\mathbf{0})A_\alpha(t)\bar N(0,\mathbf{0})\rangle}{\langle N(t_{\rm sep},\mathbf{0})\bar N(0,\mathbf{0})\rangle} = \frac{\sum_{n,m}\langle 0|N|n\rangle\langle n|A_\alpha|m\rangle\langle m|\bar N|0\rangle e^{-E_n(t_{\rm sep}-t)-E_m t}}{\sum_n \langle 0|N|n\rangle\langle n|\bar N|0\rangle e^{-E_n t_{\rm sep}}}$$

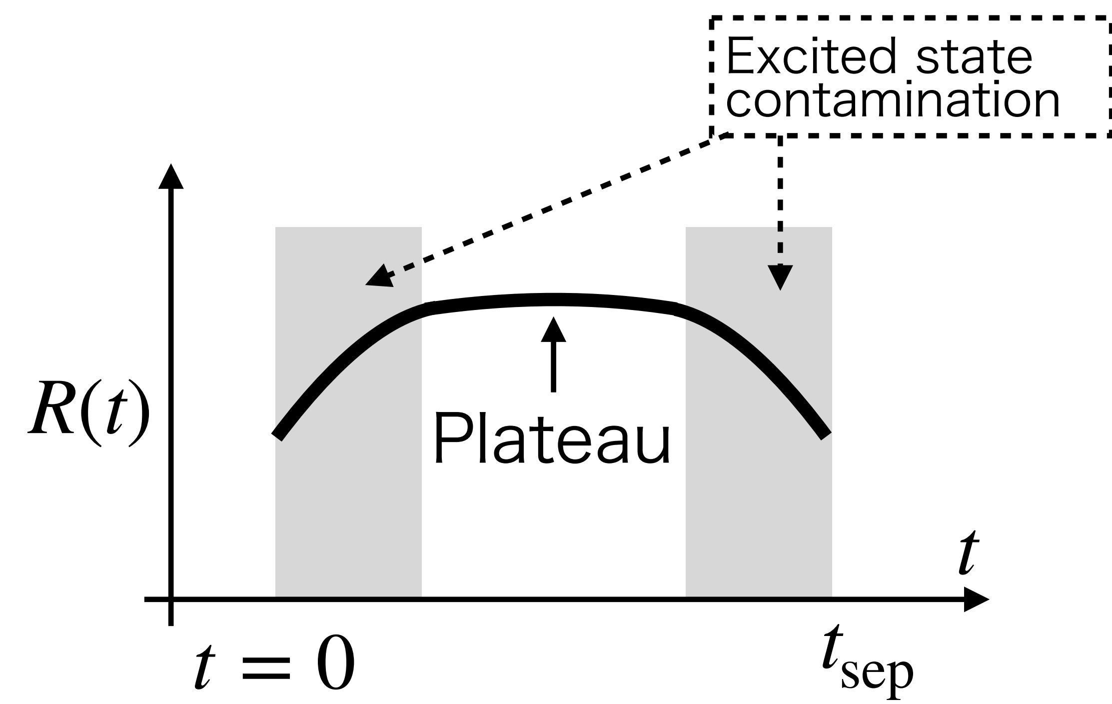
$$\longrightarrow \langle N(\mathbf{0})|A_\alpha|N(\mathbf{0})\rangle \qquad (t_{\rm sep} \gg t \gg 0)$$



The axial current operator: $A_\alpha(x) = \bar q(x)\gamma_5\gamma_\alpha q(x)$

The nucleon operator: $N(t,\mathbf{p})$

Proper ratios $R(t)$ of **two-point** and **three-point** functions yields **plateaux** corresponding to the matrix elements.



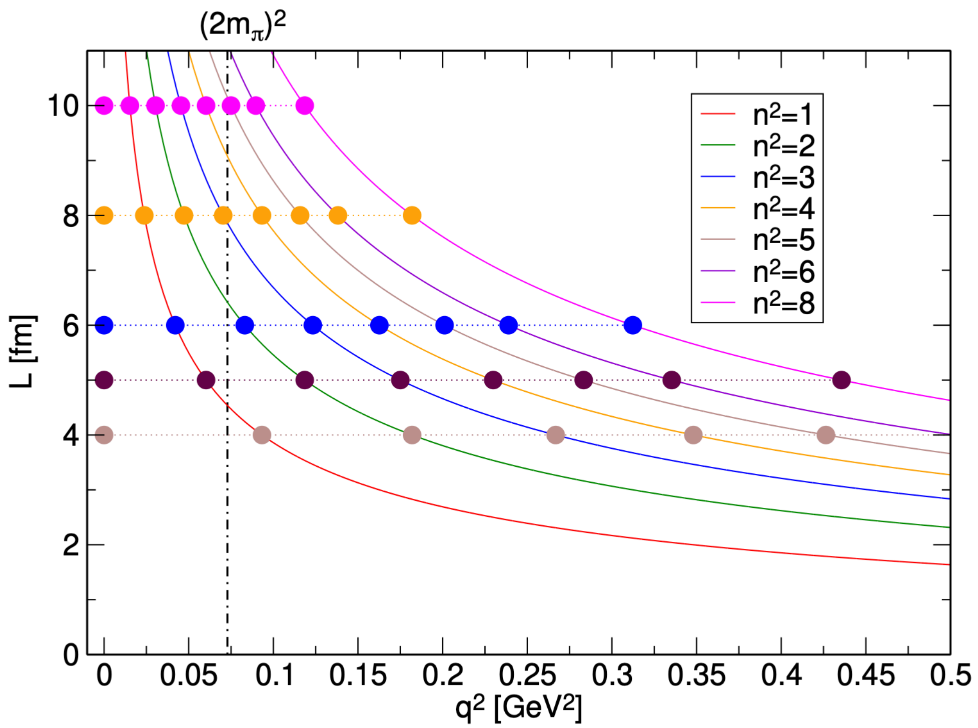
2+1 flavor

PACS10 gauge configurations

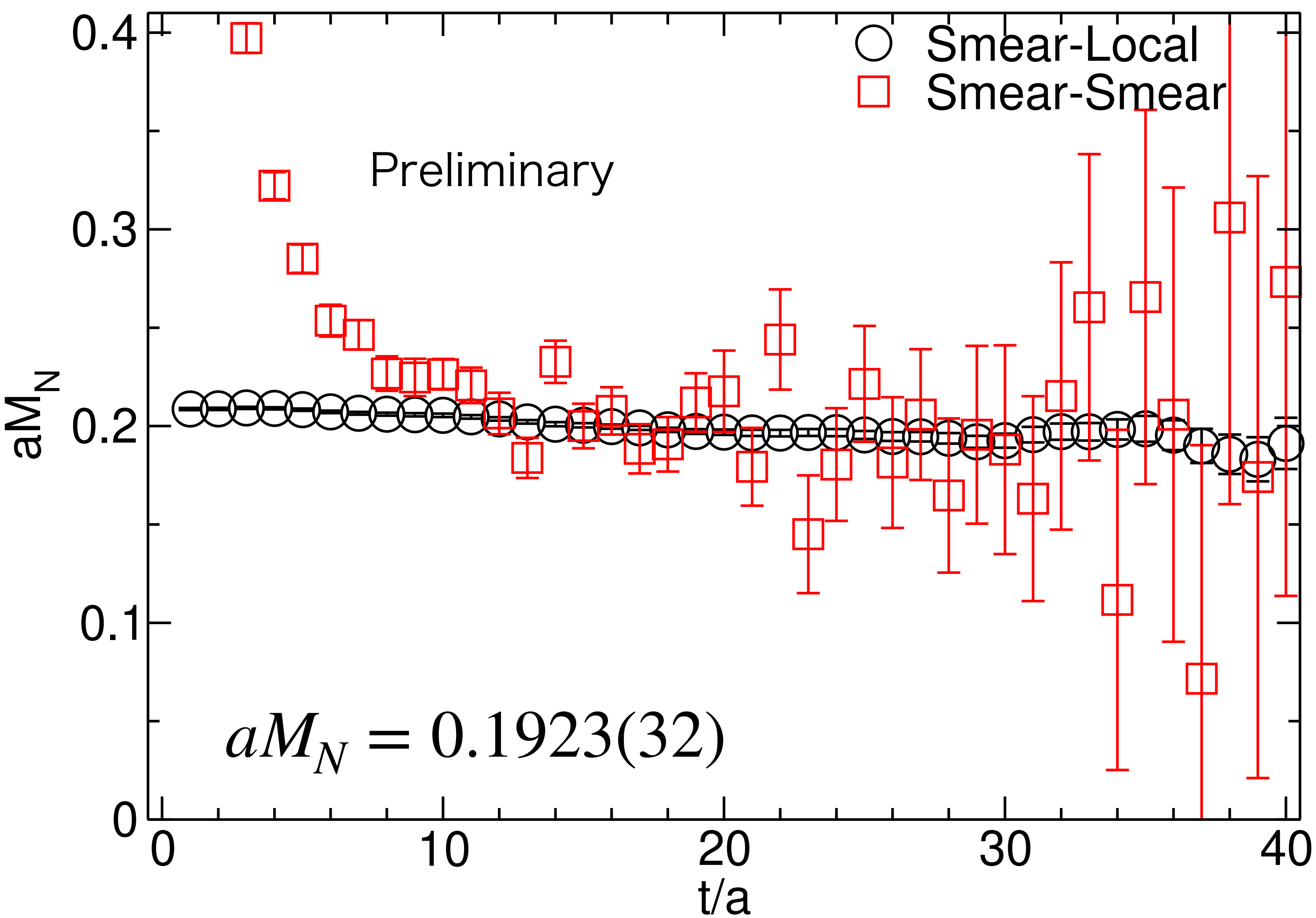
Lattice size	128^4	160^4	256^4 Main
Spatial volume	$(10.9\ \text{fm})^3$	$(10.1\ \text{fm})^3$	$(10.1\ \text{fm})^3$
Pion mass	135 MeV	138 MeV	142 MeV
Nucleon mass	0.935(11) GeV	0.947(3) GeV	0.959(36) GeV
Lattice spacing	0.09 fm (coarse)	0.06 fm (fine)	0.04 fm (super-fine)
$\|t_{\text{sink}} - t_{\text{src}}\|/a$	10, 12, 14, 16	13, 16, 19	20, 29
Renormalization	SF, RI-MOM/SMOM	SF	SF

- Volume is kept larger than $(10\ \text{fm})^3$

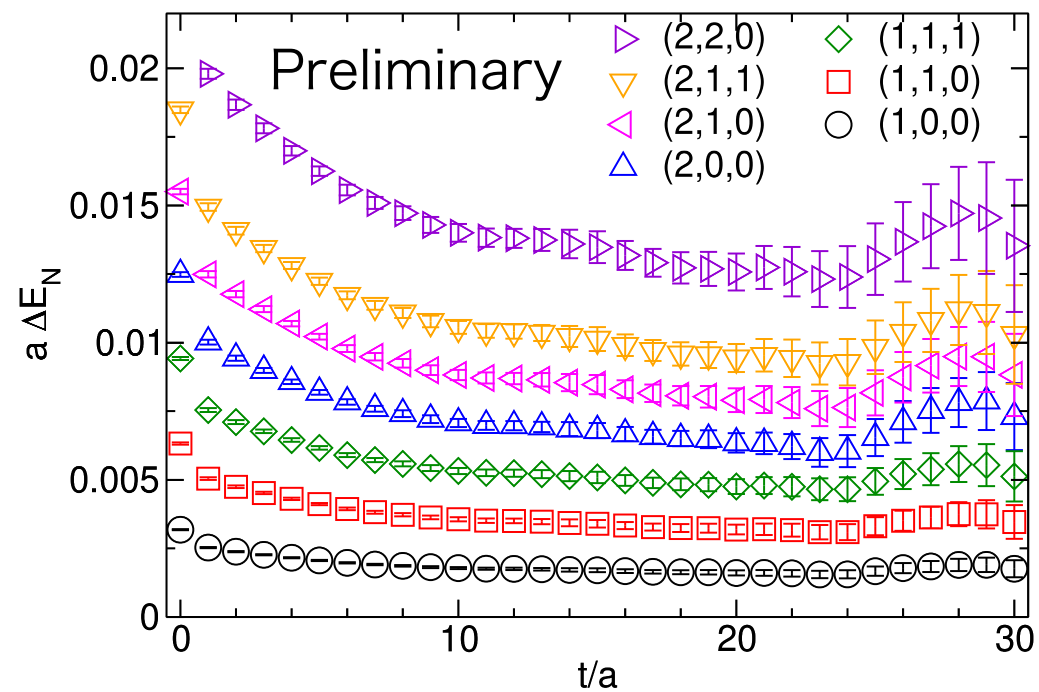
  High resolution at the low q^2 region
- Three lattice spacing → **continuum limit**
- Reproduce physical $m_\pi \approx 135$ MeV
- Statistical errors are improved by **All Mode Averaging** (AMA)



Effective energy plots (Preliminary)



Smear-Local

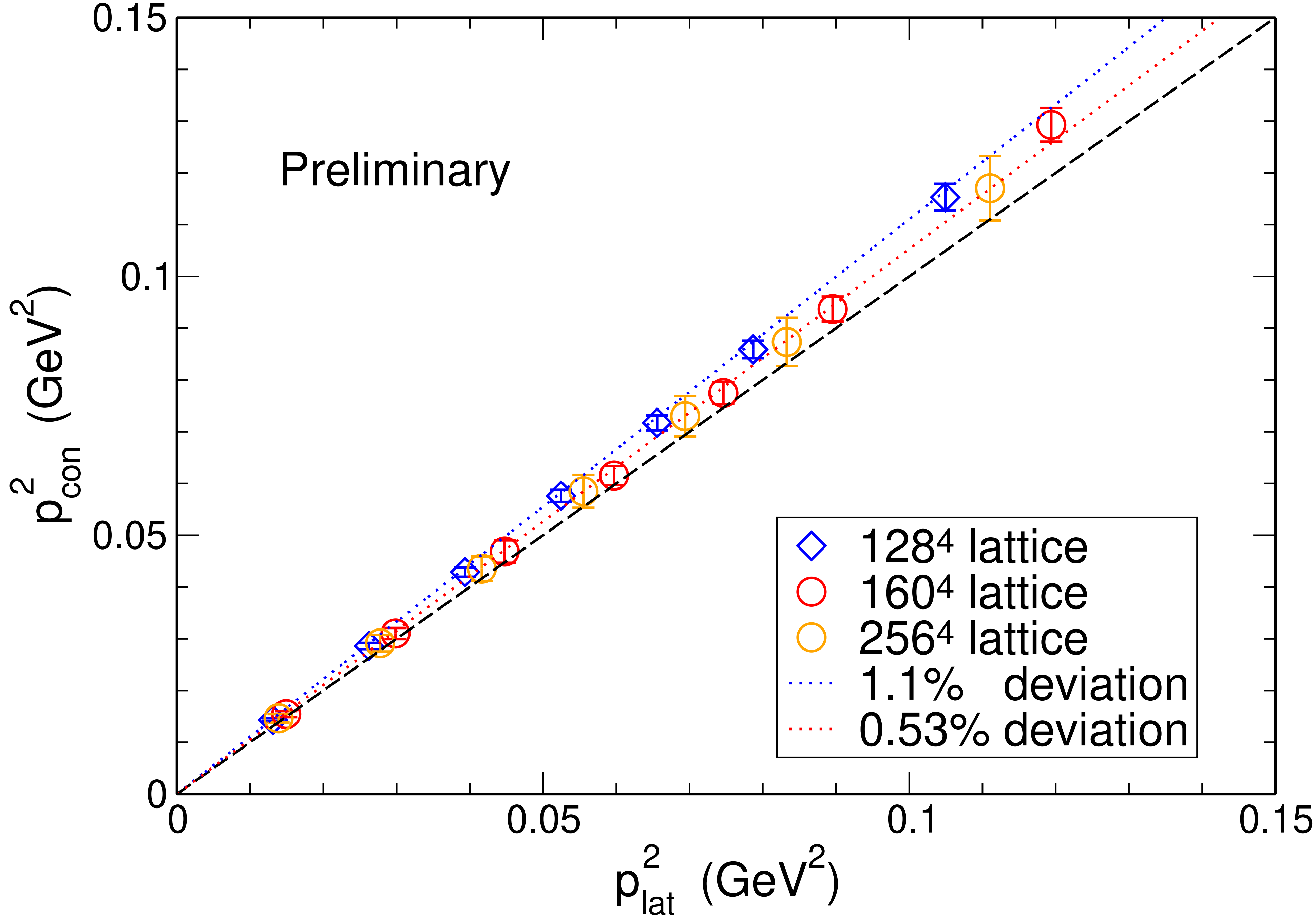


- The nucleon mass M_N from zero-momentum correlators.

- $\Delta E_N \equiv E(\mathbf{p}) - M_N$ from $\dfrac{C_{LS}^{2\mathrm{pt}}(t,\mathbf{p})}{C_{LS}^{2\mathrm{pt}}(t,\mathbf{0})} \to e^{-\Delta E_N t}$

Dispersion relation of nucleons

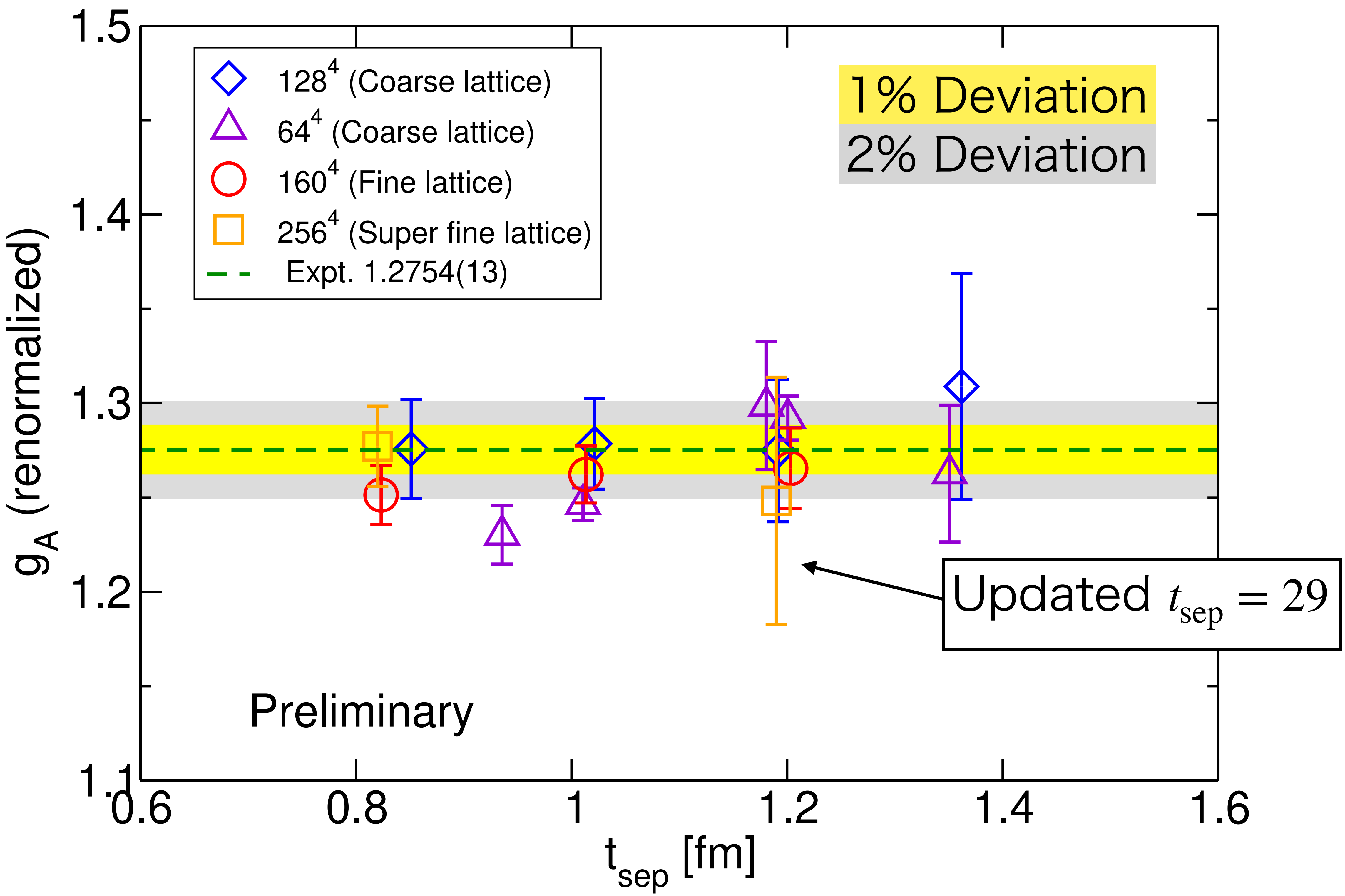
(Preliminary)



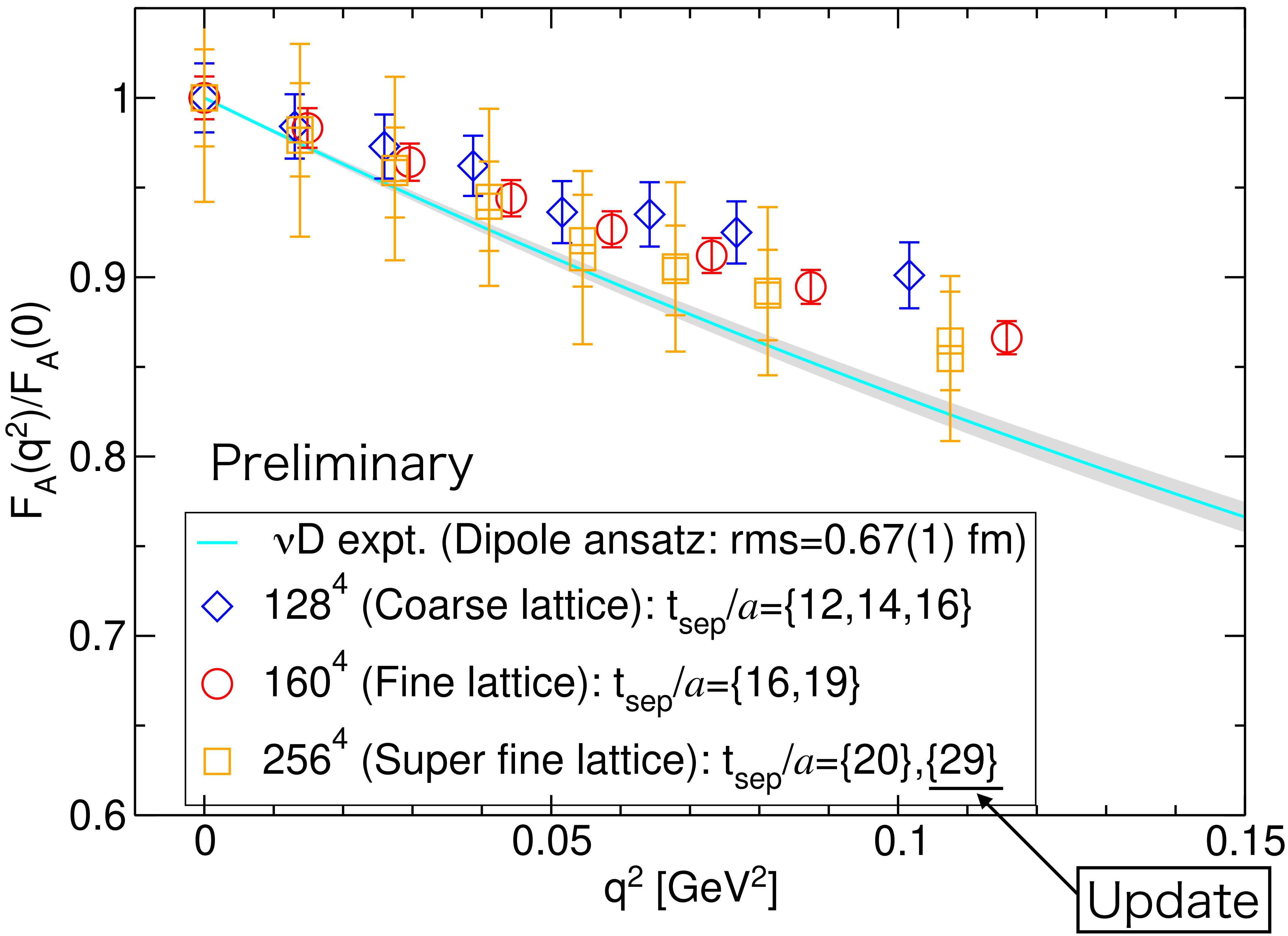
The continuum dispersion is reproduced on the simulation.

t_{sep} dependence of

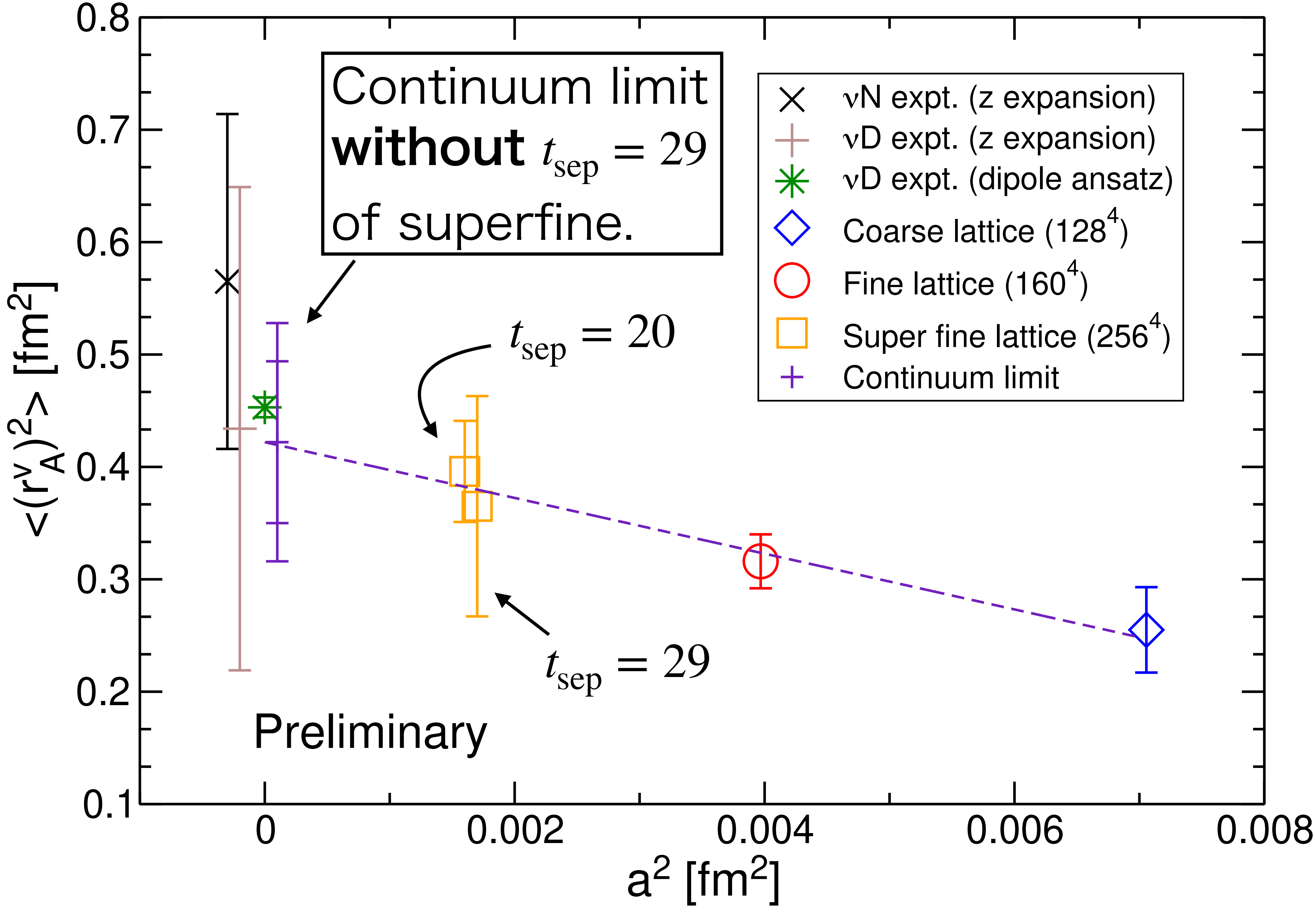
Axial vector coupling (Preliminary)



Axial form factor (Preliminary)



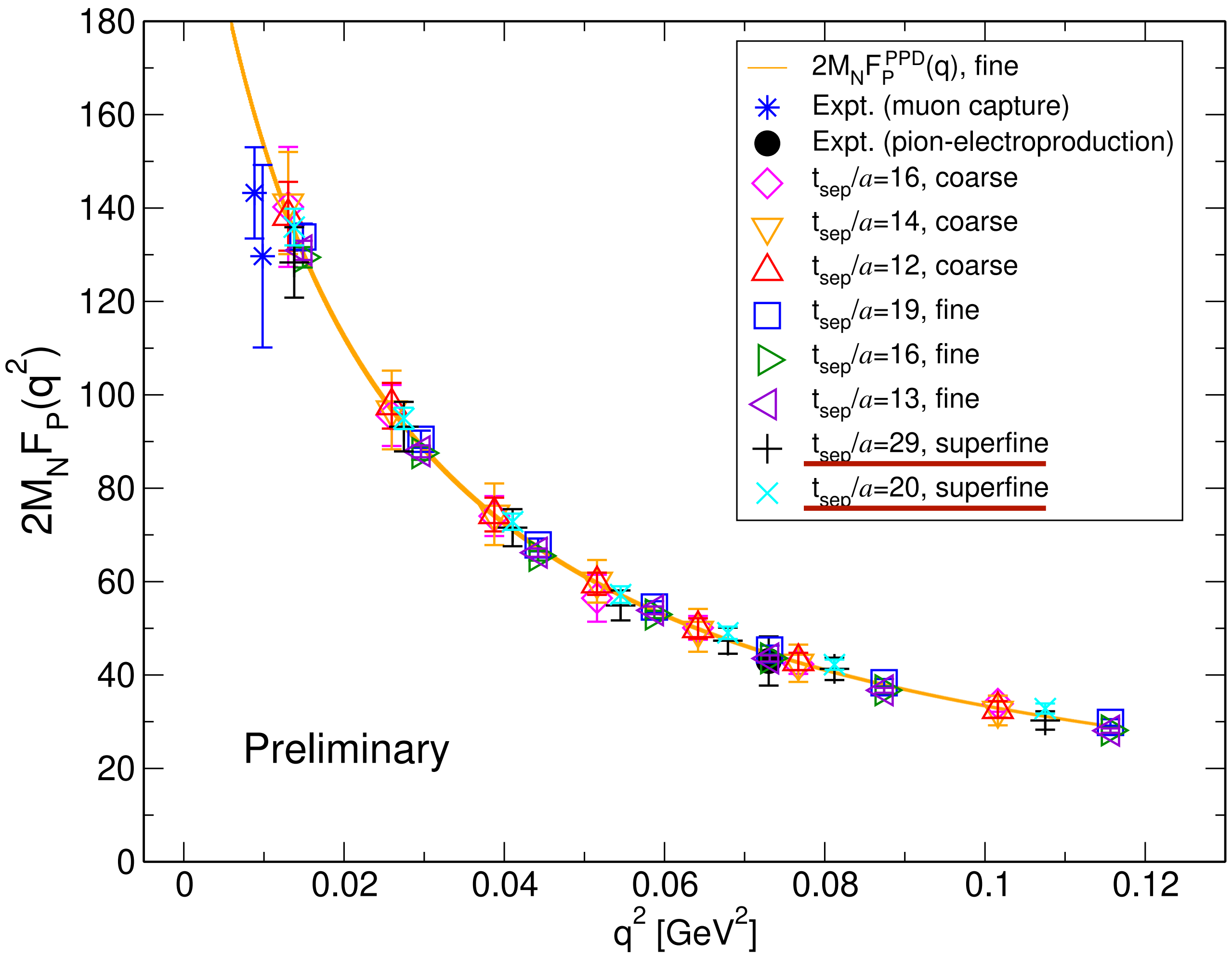
Axial radius $\langle (r_A^V)^2 \rangle$ (Preliminary)



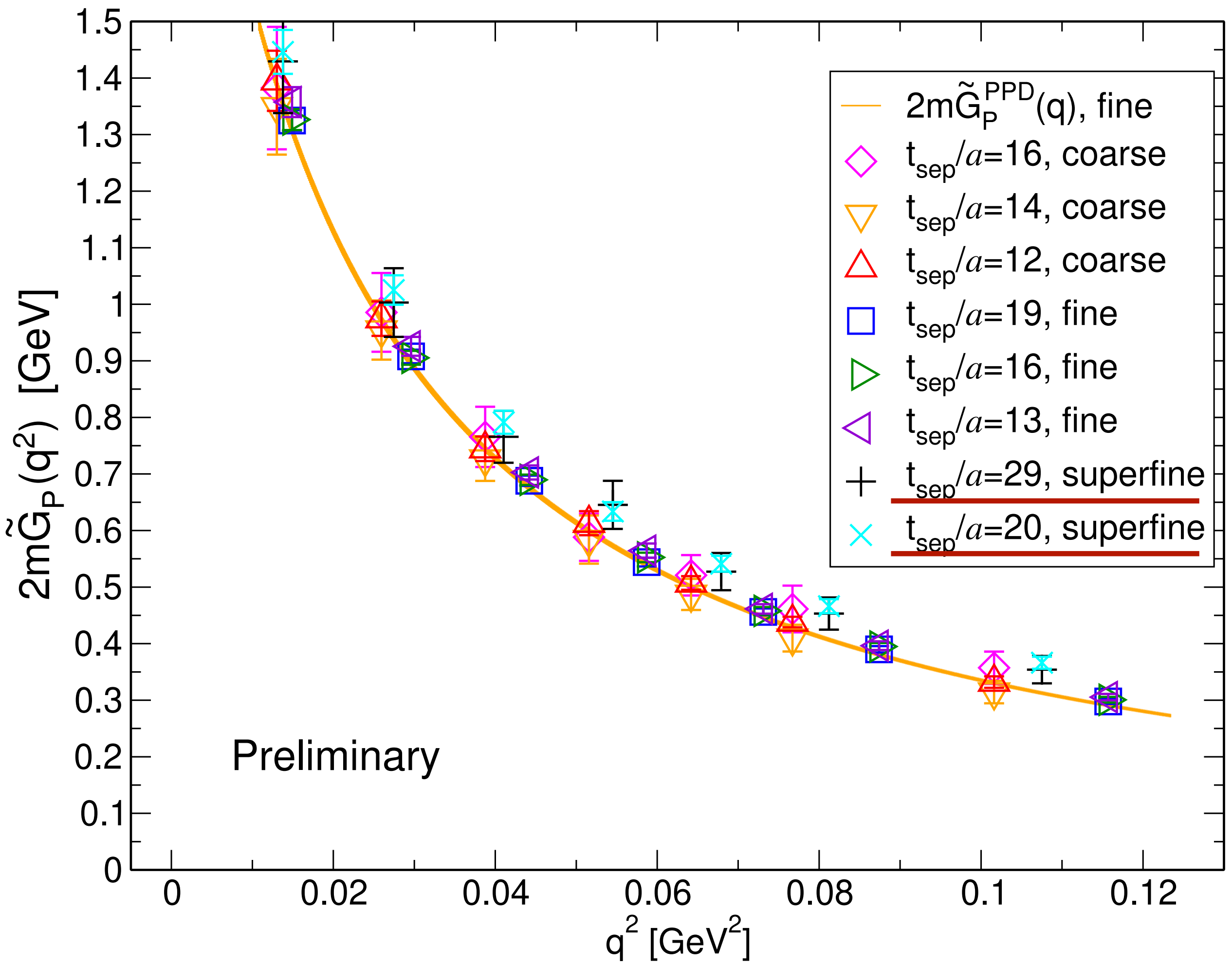
Pseudoscalar form factors

(Preliminary)

Induced Pseudoscalar



Pseudoscalar



Consistent with the pion-pole dominance (PPD) model.

Refer to PACS, Phys.Rev.D 112 (2025) 7, 074510.

Low energy relations

Generalized Goldberger-Treiman (GGT) relation

$$2M_N F_A(q^2) - q^2 F_P(q^2) = 2m_q G_P(q^2)$$

$$\rightarrow \quad R_1 + R_2 = 1 \qquad \left(R_1 \equiv \frac{q^2}{2M_N} \frac{\tilde{F}_P(q^2)}{\tilde{F}_A(q^2)}, \quad R_2 \equiv \frac{1}{2M_N} \frac{2m_{\mathrm{PCAC}} \tilde{G}_P(q^2)}{Z_A \tilde{F}_A(q^2)} \right)$$

Pion-Pole dominance (PPD) model

$$R_3 = 1, \quad R_4 = 1 \qquad \left(R_3 \equiv \frac{q^2 + M_\pi^2}{2M_N} \frac{\tilde{F}_P(q^2)}{\tilde{F}_A(q^2)}, \quad R_4 \equiv M_\pi^2 \frac{Z_A \tilde{F}_P(q^2)}{2m_{\mathrm{PCAC}} \tilde{G}_P(q^2)} \right)$$

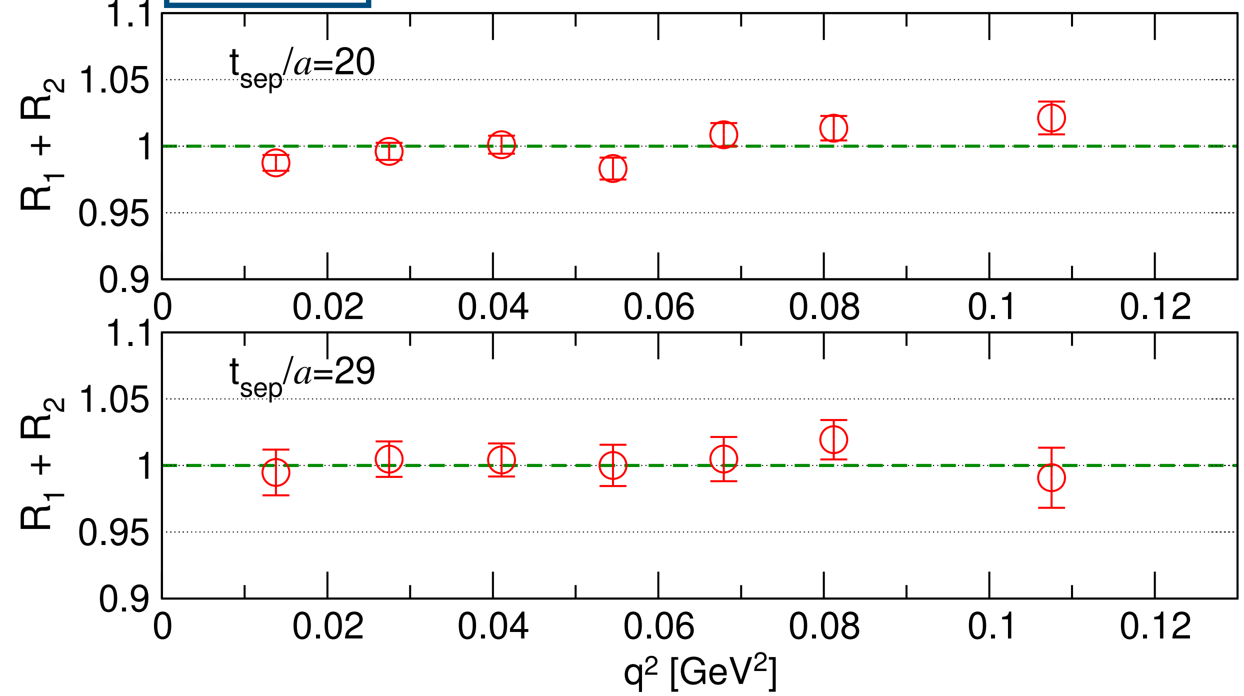
Numerically test the GGT relation and the PPD model.

Numerical check of

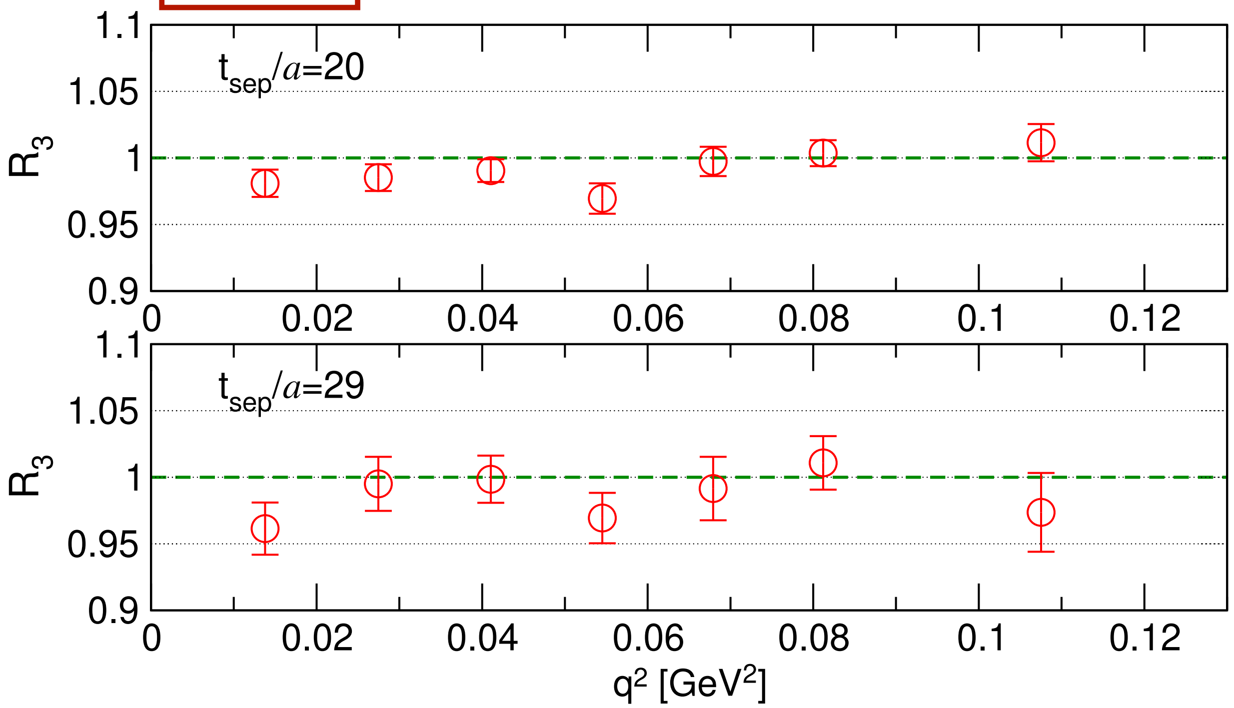
Low energy relations (Preliminary)

Our data are consistent with GGT relation (right) and PPD relations (bottom).

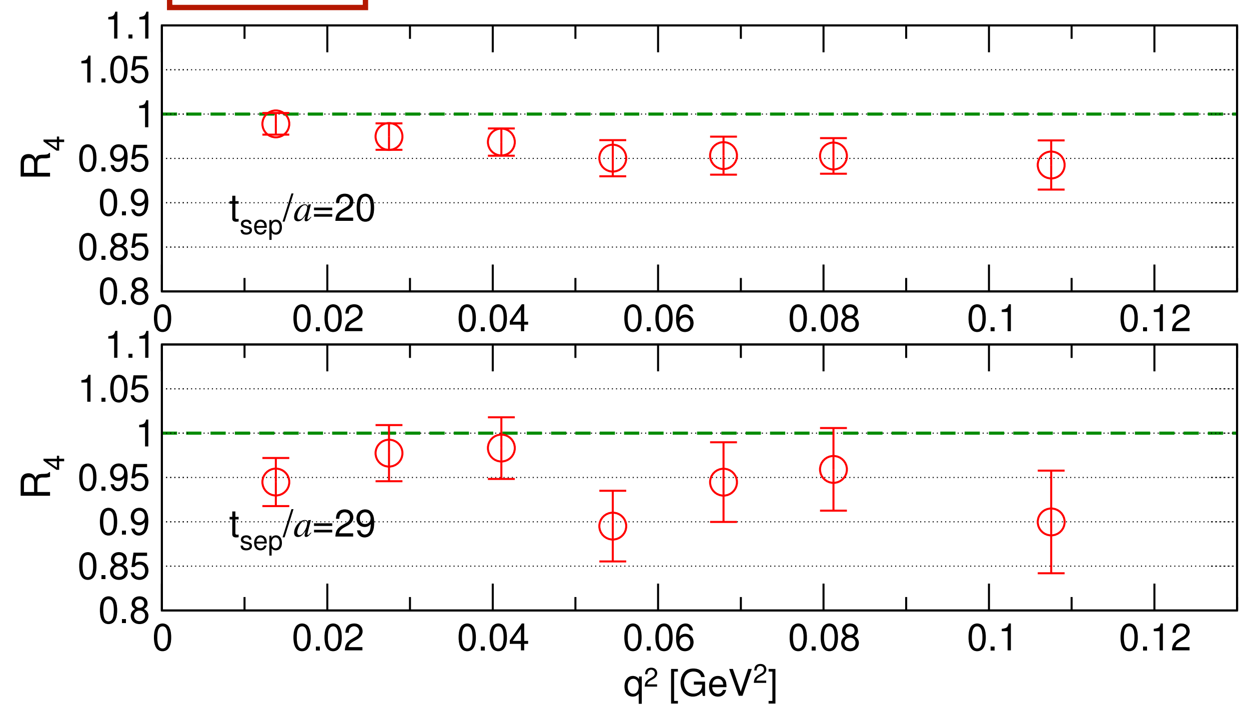
GGT



PPD



PPD



Summary

- PACS10 configurations have a **large volume** $(10\ \mathrm{fm})^4$ with the **physical pion mass.** $a = 0.09,\ 0.06,\ 0.04\ \mathrm{fm}$ are ready.
- Latest update for the **axial form factors** are shown.
- These are consistent with GGT relation and the PPD model.

Prospects

- Make sufficient measurements for $t_{\mathrm{sep}} = 29$.
- Take the continuum limit of the axial radius.
- Our continuum can be input for the T2K experiment.

Thank you for your attention!