Tensor-Network Study of Confining Flux Tubes and Effective Strings in $(2+1)$D Compact $U(1)$ Gauge Theory

Flux profiles, spectra and entanglement

Alessio Negro, Carsten Urbach, Lorenzo Verzichelli

University of Bonn
Helmholtz Institute für Strahlen- und Kernphysik

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Why?

- Understanding Hamiltonian EST physics.
- Finer couplings without autocorrelation or statistical fluctuations.
- Entanglement measures and real-time evolution.

Variational uncertainties, truncation and finite-volume systematics.

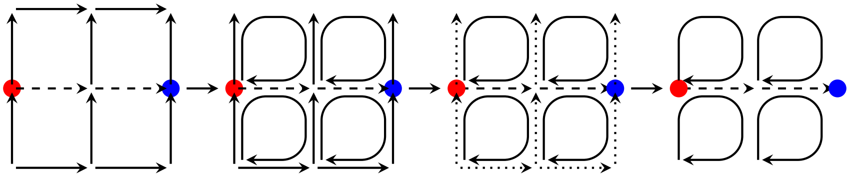
Hamiltonian and operators

Dual Hamiltonian

$$\hat{H}_{\text{dual}} = \frac{g^2}{2}\sum_{\mathbf{x},i}(\hat{L}_{\mathbf{x}} - \hat{L}_{\mathbf{x}+a\mathbf{i}} + n_{ij}^{(\gamma)})^2 + \frac{1}{2g^2}\sum_{\mathbf{x}}(2 - \hat{P}_{\mathbf{x}} - \hat{P}_{\mathbf{x}}^{\dagger})$$

Gauss's law

Forbids the dynamics of the remaining gauge links.



Plaquette basis [2510.27611] [2510.18594]

- Solve the one-plaquette problem:

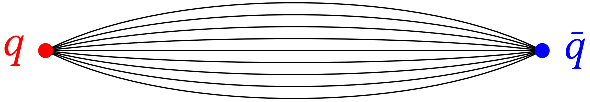
$$\hat{H}_{\mathrm{dual}} = -2g^2 \frac{\mathrm{d}^2}{\mathrm{d}\varphi_P^2} + \frac{1}{g^2}(1 - \cos\varphi_P), \qquad \hat{H}_{\mathrm{dual}}\psi_n = E_n \psi_n.$$

- Digitise by retaining the lowest Λ eigenstates.
- Project each local operator into this basis:

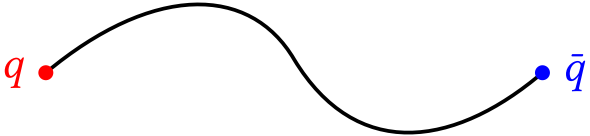
$$(\mathcal{O}_\Lambda)_{mn} = \langle \psi_m | \hat{\mathcal{O}} | \psi_n \rangle , \quad m, n = 0, \ldots, \Lambda - 1.$$

Flux tube as a string

- Chromo-electric and chromo-magnetic fields are confined inside a thin flux tube.



- The fluctuations of the string generate a finite width of the flux tube.



- **Effective string theory** (EST) describes long confining strings.

Hamiltonian EST

- The **Nambu–Gotō** EST predicts

$$V_{\mathrm{NG}}(R) = V_0 + \sigma R \sqrt{1 - \frac{\pi v_s}{12\sigma R^2}}\,.$$

- **Lüscher** term potential:

$$V_{\text{Lüscher}}(R) = V_0 + \sigma R - \frac{\pi v_s}{24R}\,.$$

v_s: worldsheet velocity, c_b: bulk light speed, v_s/c_b: $\widehat{H}$-rescaling invariant.

Measurement of $\Delta E^2_{\mathrm{ws}}(\hat{k}) = m^2_{\mathrm{ws}} + c_b^2 \hat{k}^2$ is still needed.

Effective strings in compact U(1)

- For compact $U(1)$ [2412.01313], where $m_{0^{--}}/\sqrt{\sigma} \to 0$ as $g^2 \to 0$:

$$E_0^{\text{closed}}(R) = \sigma R - \frac{\pi \nu_s}{6R} + \frac{c_b}{\pi R}\,\mathrm{Li}_2(e^{-\mu R}) + \cdots,$$
$$V_{\text{open}}(R) = V_0 + \sigma R - \frac{\pi \nu_s}{24R} + \frac{c_b}{4\pi R}\,\mathrm{Li}_2(e^{-\mu R}) + \cdots.$$

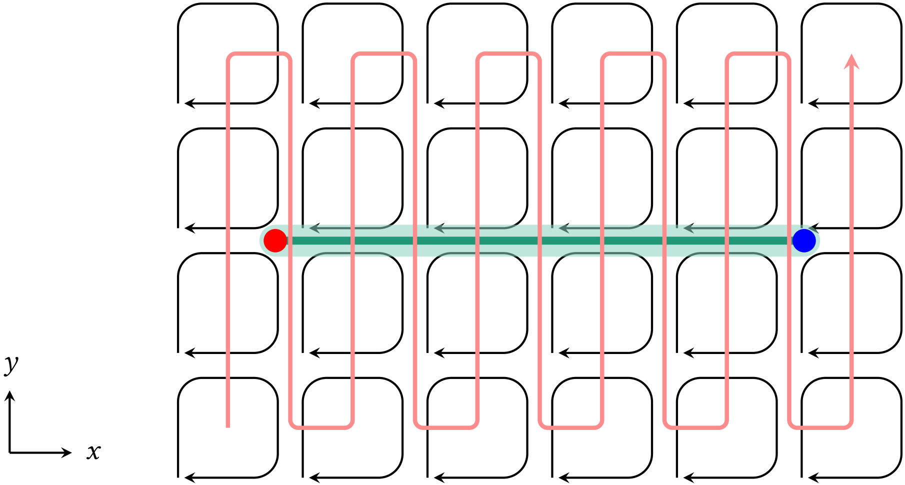
  The massive correction is suppressed for $\mu R \gg 1$.

- Profile of the flux tube [2601.19520]:

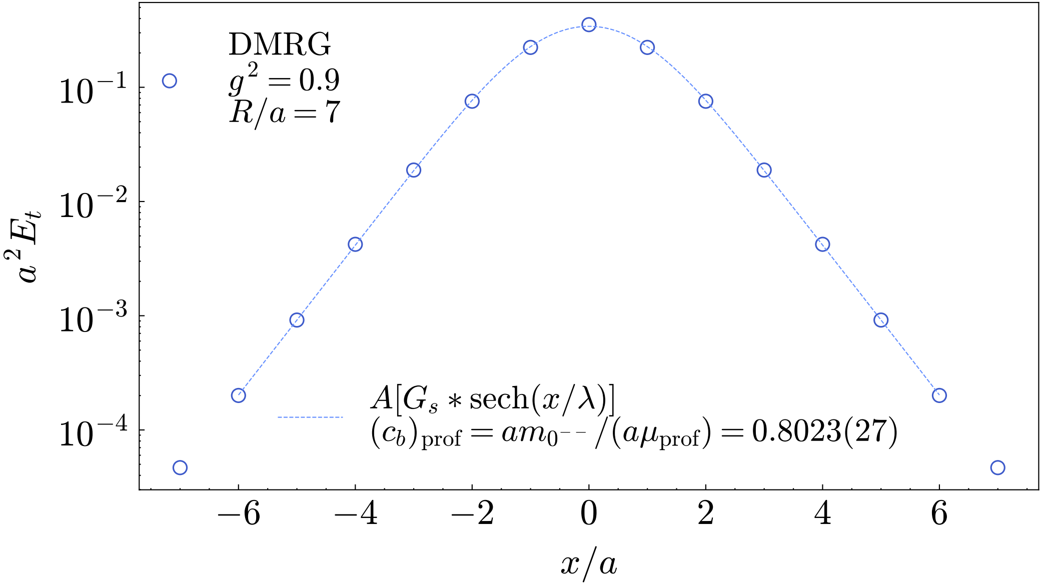
$$\langle F_{01}(y)\rangle = \frac{e^2}{\sqrt{2\pi^3}\,\lambda s}\int_{-\infty}^{\infty} dt\, e^{-t^2/(2s^2)}\,\mathrm{sech}\left(\frac{y-t}{\lambda}\right),$$
$$\lambda = \mu_{\text{prof}}^{-1}, \qquad s^2 \propto \log(MR).$$

Matrix product state

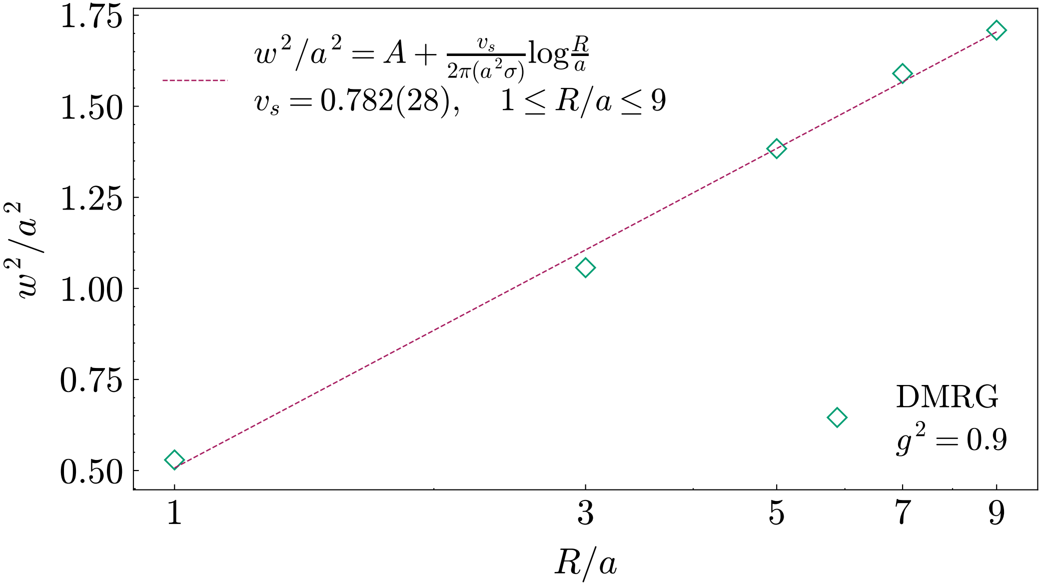
$$i_{\text{MPS}} = x(N_y - 1) + y$$



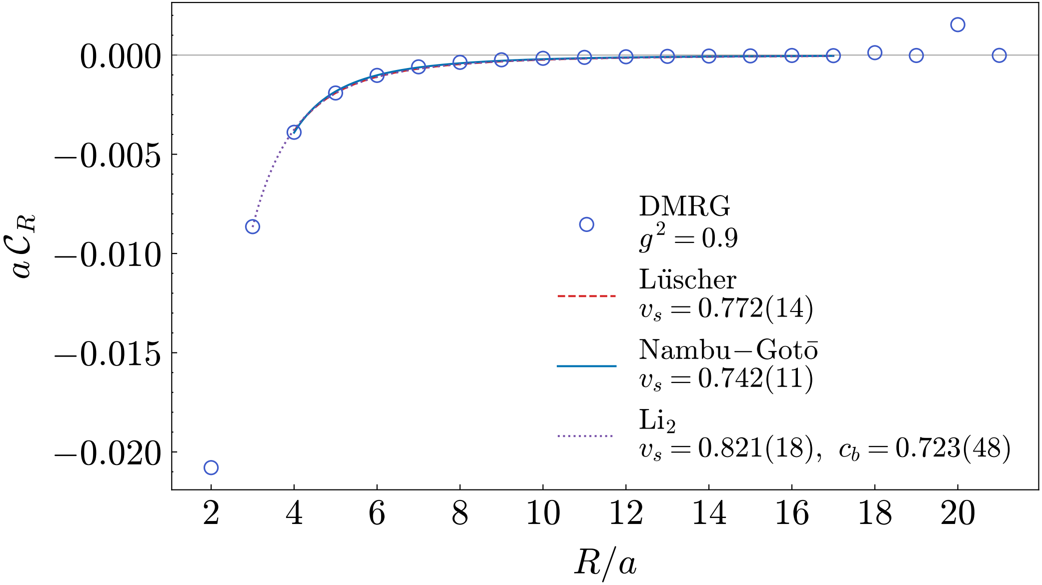
Transverse flux tube profile



Logarithmic broadening and roughening [2601.19520]

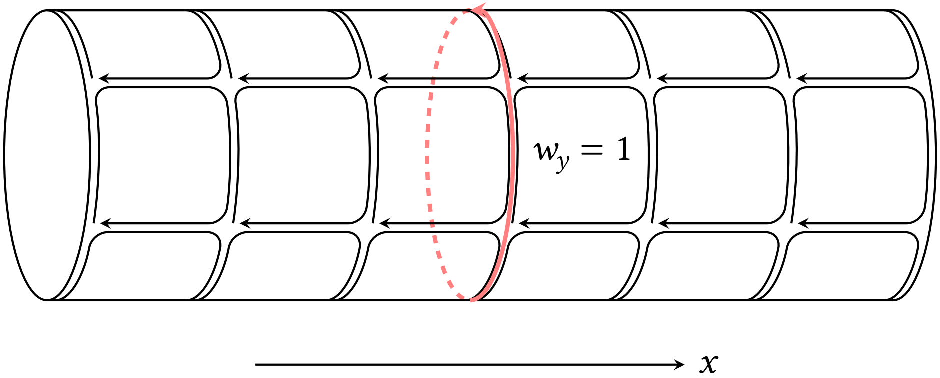


Open string potential

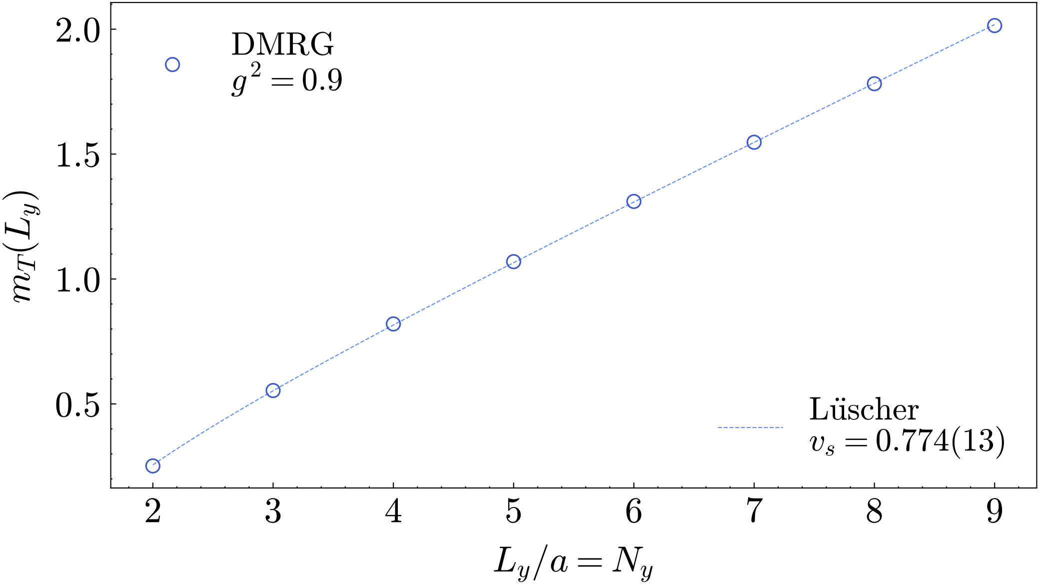


Torelon winding sector

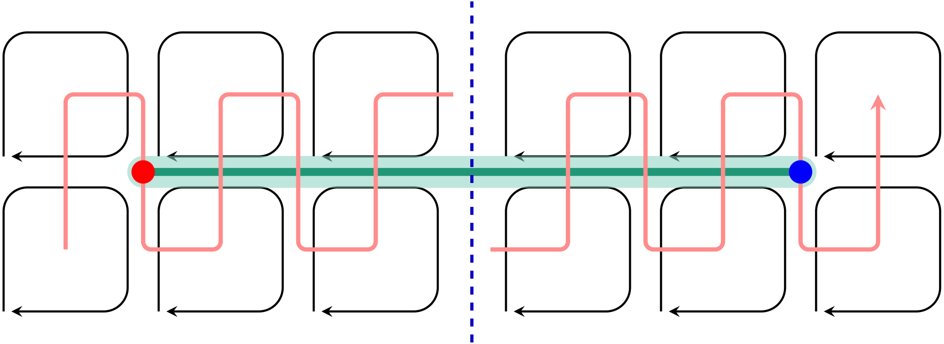
$$m_T = E_0(w_y = 1) - E_0(w_y = 0) = \sigma L_y - \pi v_s/(6L_y) + \cdots$$



Torelon potential

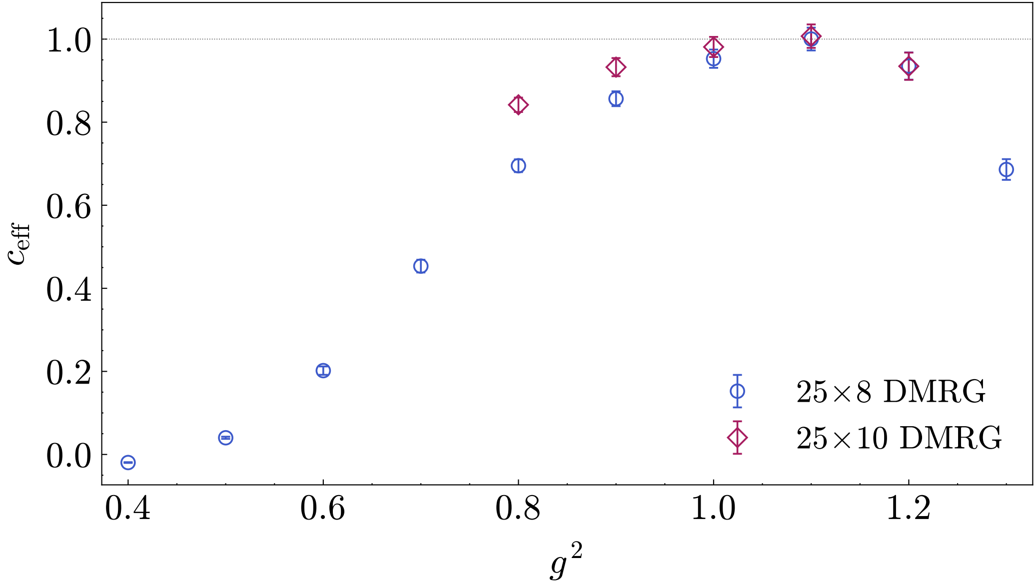


Flux tube entanglement entropy



- Long strings: one massless transverse mode $\Longrightarrow c = 1$ $(1+1)$D CFT.
- Measure at the mid-string cut: $\Delta S_{q\bar{q}}(R) = S_{q\bar{q}}(R) - S_{\text{vac}}$.
- For an open string, fit $\Delta S_{q\bar{q}}(R) = A + \dfrac{c_{\text{eff}}}{6} \ln(R/a)$ to obtain c_{eff}.

Entanglement entropy coefficient



Conclusions and outlook

- **Conclusions:** Hamiltonian tensor networks access flux-tube profiles, string spectra and entanglement.
- **Outlook:** Real-time evolution and further entanglement measures.

Thank you

Questions and discussion