Hadron structure from Hamiltonian lattice gauge theory with tensor networks

Manuel Schneider
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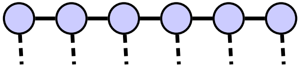

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Center for Theoretical and Computational Physics

[arXiv:2504.07508]
[arXiv:2409.16996]

The 43rd International Symposium on Lattice Field Theory
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Collaborators

Mari Carmen Bañuls

Krzysztof Cichy

C.-J. David Lin

Outline

Parton Distribution Functions

- PDF: probability of constituent with momentum fraction ξ

[EIC]

Parton Distribution Functions

- PDF: probability of constituent with momentum fraction ξ
- factorization at large energy $Q^2 = -q^2$, e.g. DIS:

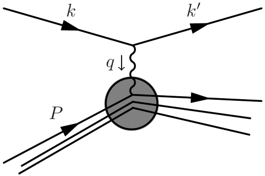
$$\sigma(\xi, Q^2) = \hat{\sigma}(\xi, Q^2) \otimes f(\xi)$$

experiment

perturbative

non-perturbative

[EIC]



Deep Inelastic Scattering
[Schwartz 2014]

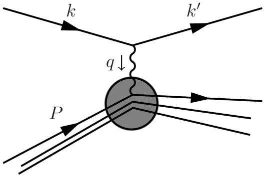
Parton Distribution Functions

- PDF: probability of constituent with momentum fraction ξ
- factorization at large energy $Q^2 = -q^2$, e.g. DIS:

$$\underbrace{\sigma(\xi, Q^2)}_{\text{experiment}} = \underbrace{\hat{\sigma}(\xi, Q^2)}_{\text{perturbative}} \otimes \underbrace{f(\xi)}_{\text{non-perturbative}}$$

$$\boxed{f_\Psi(\xi) = \int dz^- e^{-i\xi P^+ z^-} \langle P | \bar{\psi}(z^-)\gamma^+ W(z^- \leftarrow 0)\psi(0) | P \rangle}$$

[EIC]



Deep Inelastic Scattering
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Parton Distribution Functions

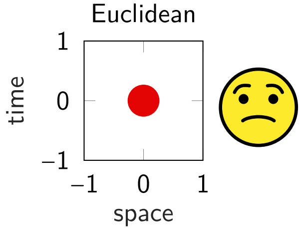
- PDF: probability of constituent with momentum fraction ξ
- factorization at large energy $Q^2 = -q^2$, e.g. DIS:

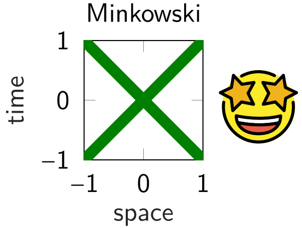
$$\sigma(\xi, Q^2) = \hat{\sigma}(\xi, Q^2) \otimes f(\xi)$$

experiment — perturbative — non-perturbative

$$\boxed{f_\Psi(\xi) = \int dz^- e^{-i\xi P^+ z^-} \langle P | \bar{\psi}(z^-)\gamma^+ W(z^- \leftarrow 0)\psi(0) | P \rangle}$$

- z^-: lightcone coordinate
- lattice QCD in Euclidean space: lightcone → point
- Hamiltonian formalism: lightcone in Minkowski space → tensor network states/quantum devices





Outline

Tensor Networks

- generic state scales exponentially

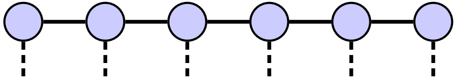
$$|\psi\rangle = \sum_{s_1, s_2, \ldots, s_N} \Psi^{s_1 s_2 \ldots s_N} |s_1\rangle \otimes |s_2\rangle \otimes \ldots \otimes |s_N\rangle$$

Tensor Networks

- generic state scales exponentially
- tensor network state as ansatz
- 1d: matrix product state (MPS)

$$|\psi\rangle = \sum_{s_1,s_2,\dots,s_N} \Psi^{s_1 s_2 \dots s_N} |s_1\rangle \otimes |s_2\rangle \otimes \dots \otimes |s_N\rangle$$

$$\Psi^{s_1 s_2 \dots s_N} = \sum_{\{i_x\}} A^{1,s_1}_{i_1} \cdot A^{2,s_2}_{i_1,i_2} \cdot A^{3,s_3}_{i_2,i_3} \dots A^{N,s_N}_{i_{N-1}}$$

$=$ 

Tensor Networks

- generic state scales exponentially
- tensor network state as ansatz
- 1d: matrix product state (MPS)
- truncation to bond dimenion D
- polynomial resource scaling

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$$\Psi^{s_1 s_2 \ldots s_N} \approx \sum_{\{i_x\}=1}^{D} A^{1,s_1}_{i_1} \cdot A^{2,s_2}_{i_1,i_2} \cdot A^{3,s_3}_{i_2,i_3} \ldots A^{N,s_N}_{i_{N-1}}$$

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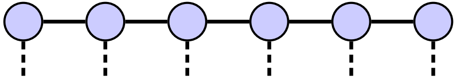

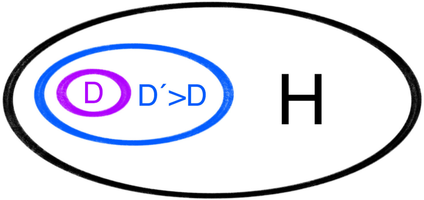

Tensor Networks

- generic state scales exponentially
- tensor network state as ansatz
- 1d: matrix product state (MPS)
- truncation to bond dimenion D
- polynomial resource scaling
- good approximation for ground states and low excited states [Hastings 2007]
- no sampling, fundamentally different systematics compared to Monte Carlo

$$|\psi\rangle = \sum_{s_1, s_2, \ldots, s_N} \Psi^{s_1 s_2 \ldots s_N} |s_1\rangle \otimes |s_2\rangle \otimes \ldots \otimes |s_N\rangle$$

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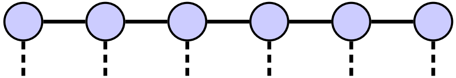
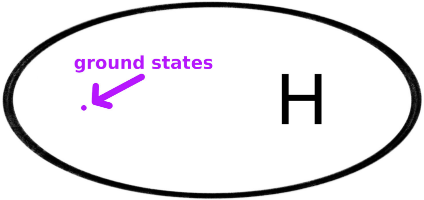
$=$ 



Outline

Schwinger Model [Schwinger 1962; Hamer et al. 1997]

QED in 1+1 dimensions

$$\mathcal{H} = -i\bar{\Psi}\gamma^1(\partial_1 - igA_1)\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2$$

$$H = x\sum_{n=0}^{N-2}\left[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right] + \frac{\mu}{2}\sum_{n=0}^{N-1}\left[1+(-1)^n\sigma_n^z\right] + \sum_{n=0}^{N-2}\left[\frac{1}{2}\sum_{k=0}^{n}\left((-1)^k + \sigma_k^z + 2q_k\right)\right]^2$$

- scattering → PDF [Dai et al. 1995]
- matrix elements

$$\langle h|\,\sigma^+(z^-)\,W_{z^-\leftarrow 0}\,\sigma^-(0)\,|h\rangle$$

Schwinger Model [Schwinger 1962; Hamer et al. 1997]

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- scattering → PDF [Dai et al. 1995]
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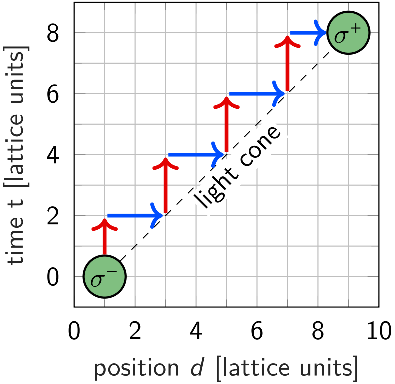
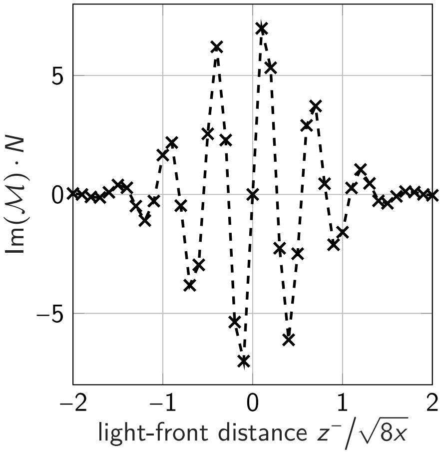
$$\langle h|\,\sigma^+(z^-)\,W_{z^-\leftarrow 0}\,\sigma^-(0)\,|h\rangle$$

- lightcone → time- and space-like steps
- time evolution: $e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{eo}}e^{-i\delta\tau H_{oe}}e^{-i\delta\tau H_L}\right)^{N_\tau}$
- spatial evolution: change electric field along the path →move static charges



Outline

Results



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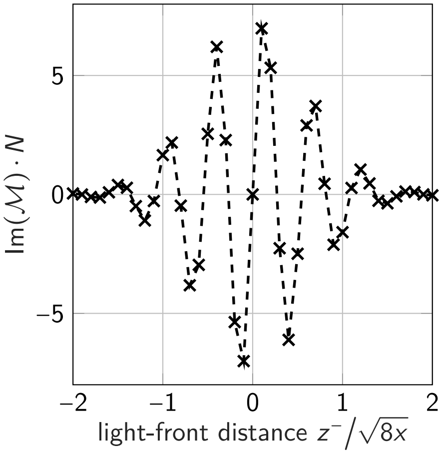
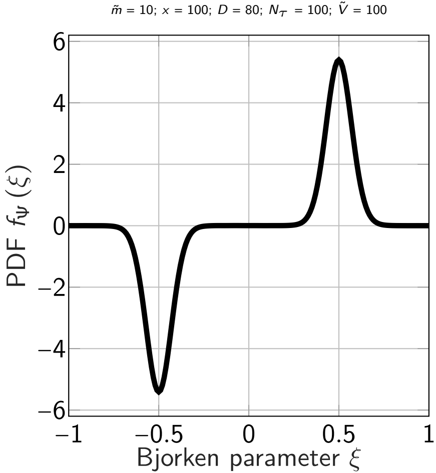
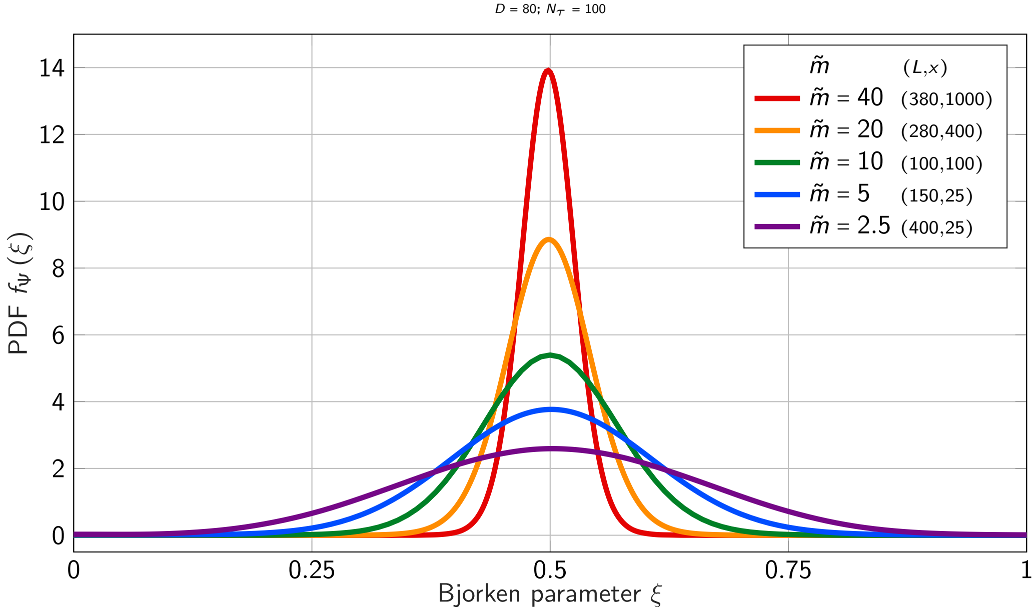


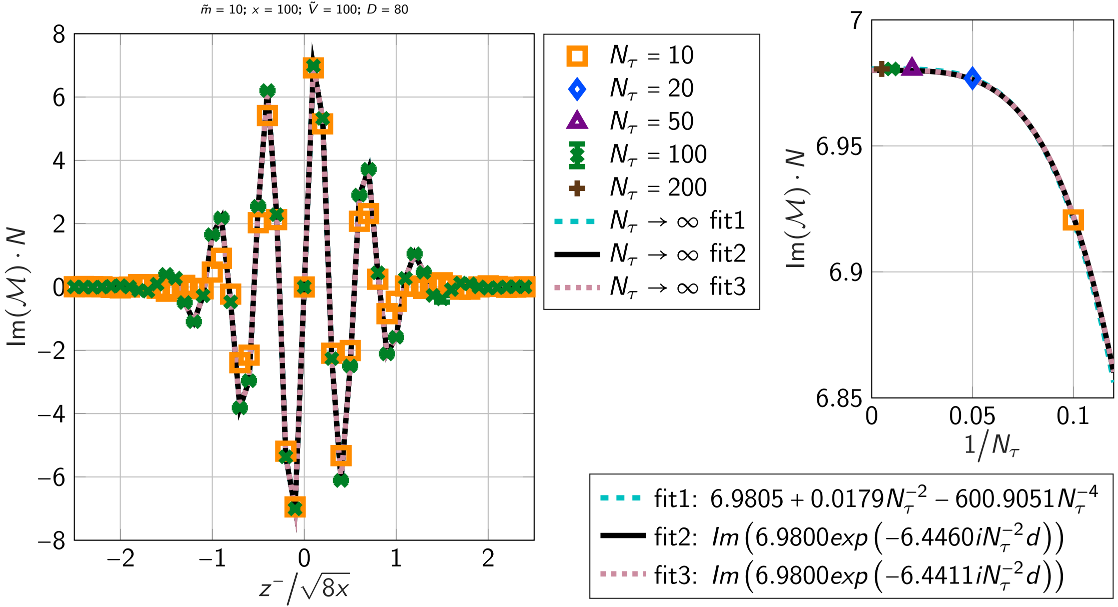
$\xrightarrow{}$
FT



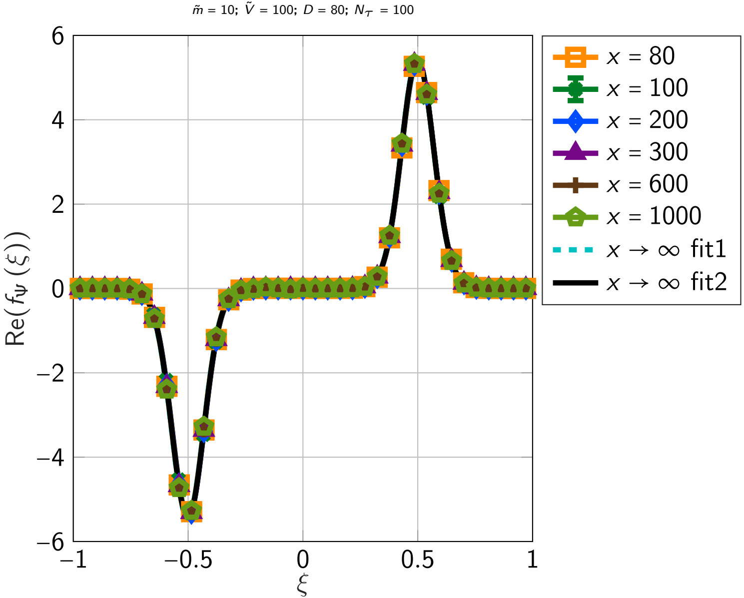
Mass dependence

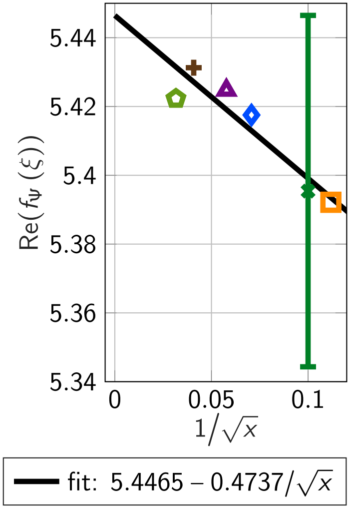


Results: N_τ-dependence

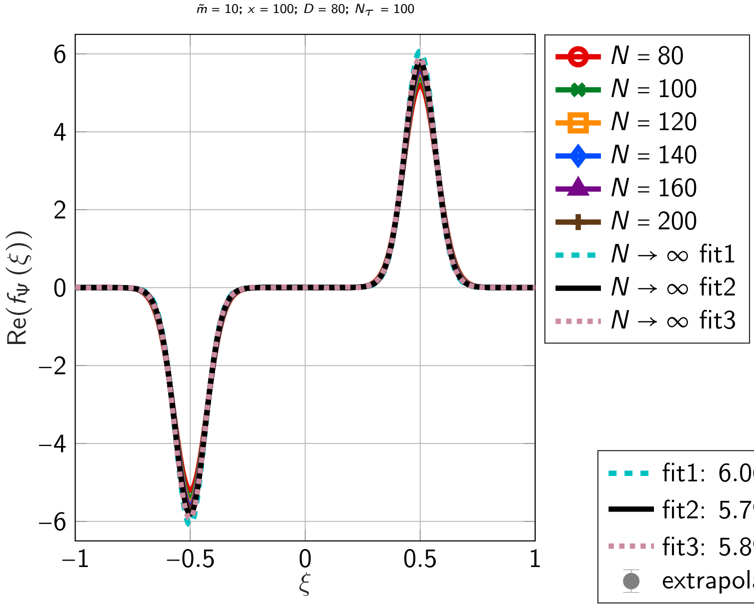


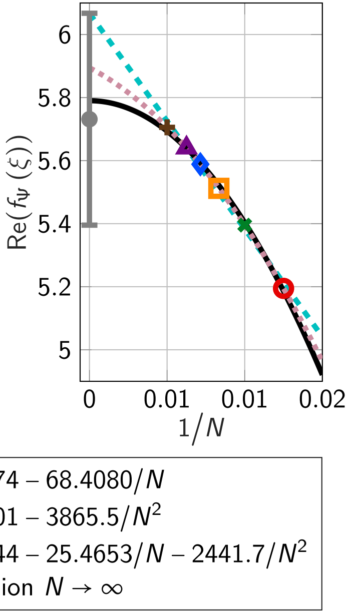
Results: x-dependence



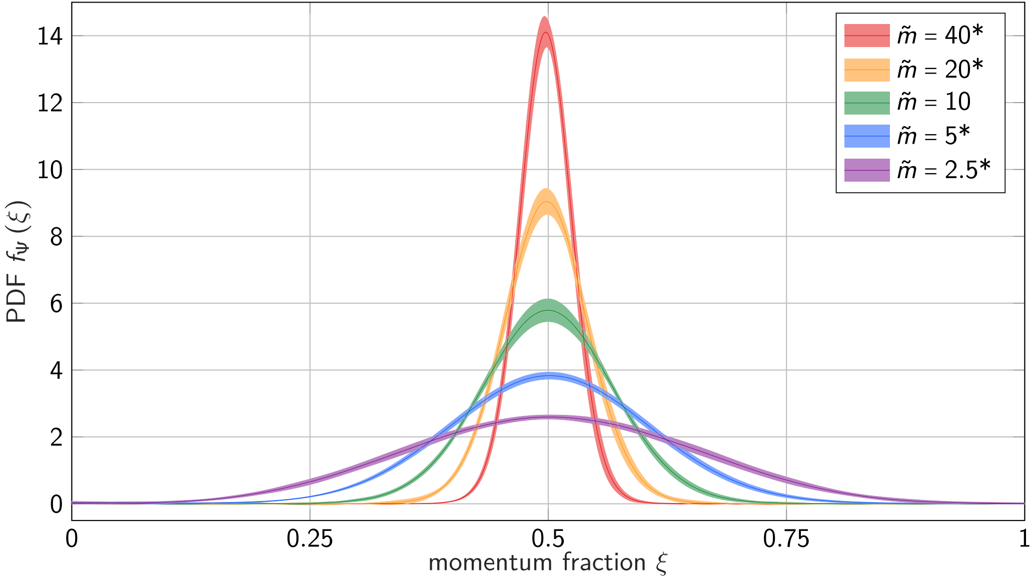


Results: N-dependence



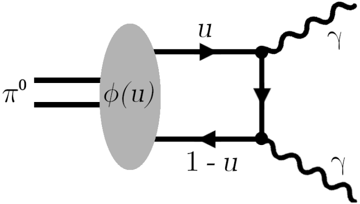


Results: PDF [*preliminary]



Lightcone Distribution Amplitude (LCDA) [preliminary]

LCDA: decay or hadronization



$$\pi^0 \to q(u)\bar{q}(1-u) \to \gamma\gamma$$

$$\boxed{if\phi(u) = \int dz^- e^{iuP^+z^-} \left\langle 0 \middle| \bar{\psi}(z^-)\gamma^+\gamma_5 W(z^- \leftarrow 0)\psi(0) \middle| P \right\rangle}$$

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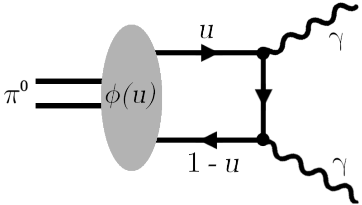

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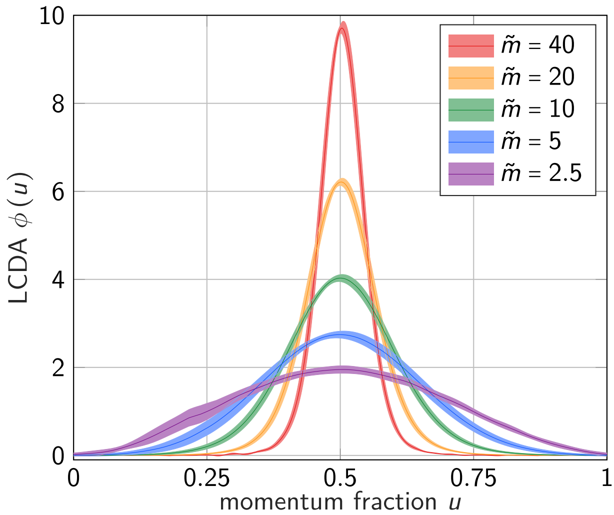


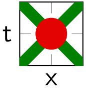
$$\boxed{if\phi(u) = \int dz^- e^{iuP^+z^-} \langle 0 | \bar{\psi}(z^-)\gamma^+\gamma_5 W(z^- \leftarrow 0)\psi(0) | P \rangle}$$

Outline

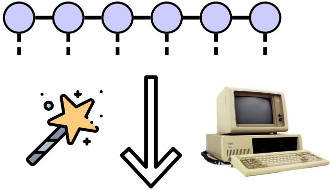
Summary

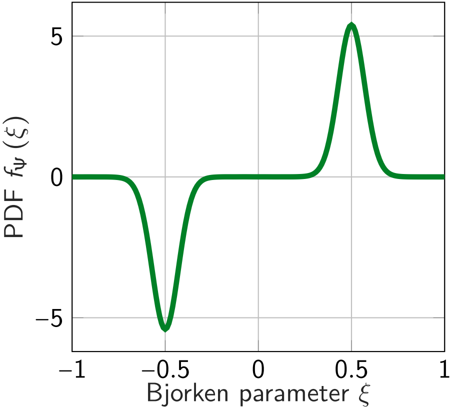
Summary:

- PDF → universal structure of hadrons
- Euclidean space: lightcone → point



- ⇒ use tensor network states / quantum devices
- Schwinger model:
  fermion- and anti-fermion-PDF of the vector meson





Outlook:

- further lightcone observables
- same analysis for QCD ☺

[arXiv:2504.07508]
[arXiv:2409.16996]

[1] M. C. Bañuls, K. Cichy, C.-J. D. Lin, and M. Schneider, "Parton distribution functions in the schwinger model from tensor network states," Phys. Rev. D **113**, L011502 (2026) doi:10.1103/lcyh-rxf3.

[2] M. Schneider, M. C. Bañuls, K. Cichy, and C.-J. D. Lin, "Parton Distribution Functions in the Schwinger Model with Tensor Networks," in Proceedings of the 41st international symposium on lattice field theory — pos(lattice2024), Vol. 466 (2025), p. 024, doi:10.22323/1.466.0024.

[3] M. D. Schwartz, *Quantum Field Theory and the Standard Model*, (Cambridge University Press, Mar. 2014), ISBN: 978-1-107-03473-0, 978-1-107-03473-0, doi:10.1017/9781139540940.

[4] M. B. Hastings, "An area law for one-dimensional quantum systems," Journal of Statistical Mechanics: Theory & Exp. **2007**, 08024 (2007) doi:10.1088/1742-5468/2007/08/P08024.

[5] M. C. Bañuls, K. Cichy, J. I. Cirac, and K. Jansen, "The mass spectrum of the schwinger model with matrix product states," JHEP **2013**, 158 (2013) doi:10.1007/JHEP11(2013)158.

[6] J. Dai, J. Hughes, and J. Liu, "Perturbative analysis of the massless schwinger model," Phys. Rev. D **51**, 5209–5215 (1995) doi:10.1103/PhysRevD.51.5209.

[7] J. Schwinger, "Gauge invariance and mass. ii," Phys. Rev. **128**, 2425–2429 (1962) doi:10.1103/PhysRev.128.2425.

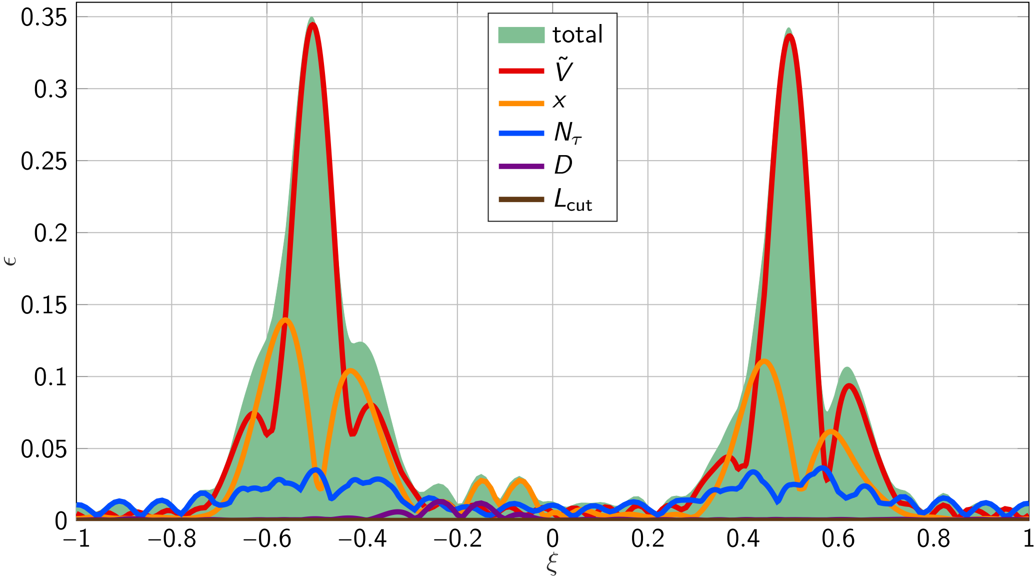
[8] C. J. Hamer, Z. Weihong, and J. Oitmaa, "Series expansions for the massive schwinger model in hamiltonian lattice theory," Phys. Rev. D **56**, 55–67 (1997) doi:10.1103/PhysRevD.56.55.

[9] Y. Mo and R. J. Perry, "Basis function calculations for the massive schwinger model in the light-front tamm-dancoff approximation," Journal of Computational Physics **108**, 159–174 (1993) doi:10.1006/jcph.1993.1171.

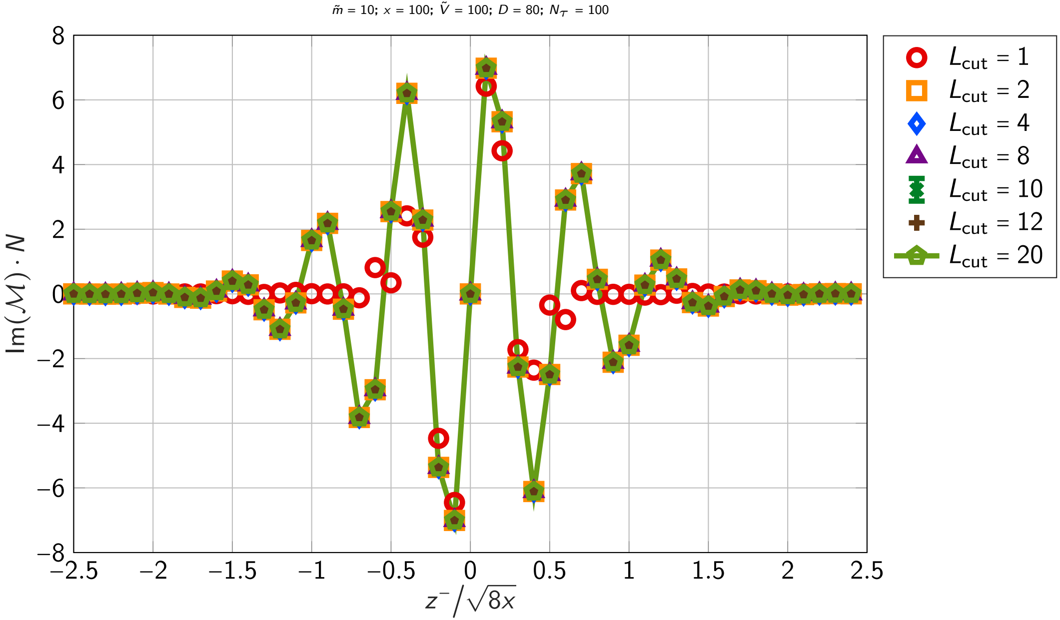
[10] Further image sources, EIC, www.computerhistory.org/timeline/1981, https://openmoji.org, www.flaticon.com/free-icons/search.

Outline

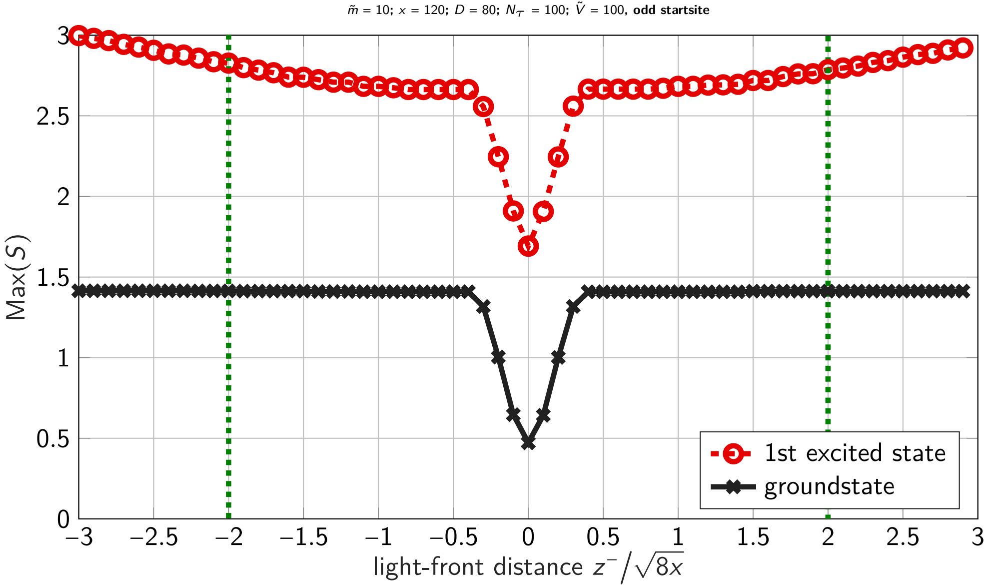
Contributions to error



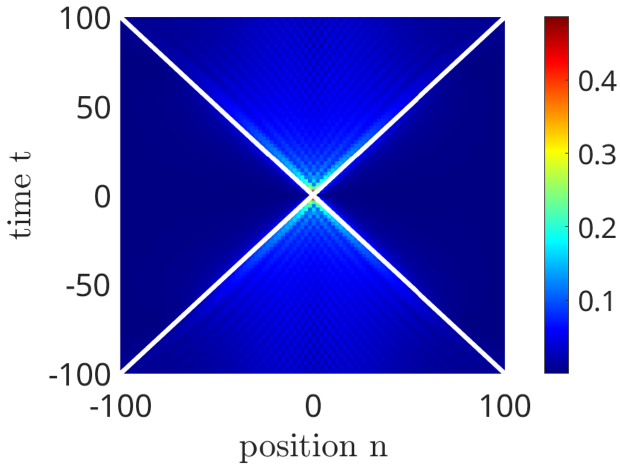
L_{cut}-dependence: truncation of electric field



Results: Entanglement entropy



Light-cone structure



$$\langle P|\, e^{iHt} \prod_{k<n} (i\sigma_k^z)\, \sigma_n^+ e^{-iH_0 t} \prod_{k'<0} (-i\sigma_{k'}^z)\, \sigma_0^- \,|P\rangle$$

- even-to-even matrix element
- calculated to each site at each timeslice
- static charge fixed at orign

Spin formulation of the Schwinger Model

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - gA\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho$$

$$\mathcal{H} = -i\bar{\Psi}\gamma^1(\partial_1 - igA_1)\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2 + A_0\left(g\Psi^\dagger\Psi + \rho - \partial_1 E\right)$$

Legendre transformation
$(E = F_{01})$

Spin formulation of the Schwinger Model

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - g\not{A} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho$$

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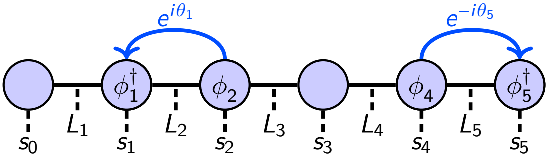
temporal Gauge: $A_0 \equiv 0$

Spin formulation of the Schwinger Model



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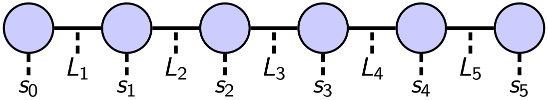
$$H = -\frac{i}{2a}\sum_n\left(\phi_n^\dagger e^{i\theta_n}\phi_{n+1} - \phi_{n+1}^\dagger e^{-i\theta_n}\phi_n\right) + m\sum_n(-1)^n\phi_n^\dagger\phi_n + \frac{ag^2}{2}\sum_n L_n^2$$

staggered fermions
$(\theta = agA_1, gL = E)$

$$\phi_n \sim \begin{cases}\Psi_{\text{upper}}(x) & \text{if } n \text{ even}\\ \Psi_{\text{lower}}(x) & \text{if } n \text{ odd,}\end{cases}$$

Spin formulation of the Schwinger Model



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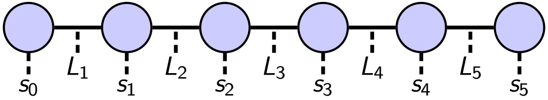
decoupling

$$\phi_n \to \prod_{k<n}\left(e^{-i\theta_k}\right)\phi_n$$

Spin formulation of the Schwinger Model



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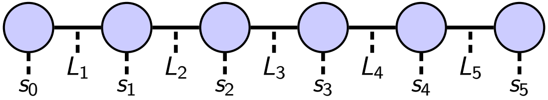
$$H = \frac{1}{2a}\sum_n(\sigma_n^+\sigma_{n+1}^- + \sigma_{n+1}^-\sigma_n^+) + \frac{m}{2}\sum_n[1 + (-1)^n\sigma_n^z] + \frac{ag^2}{2}\sum_n L_n^2$$

Jordan-Wigner transformation

$$\hat{\phi}_n = \prod_{k<n}(i\sigma_k^z)\,\sigma_n^-$$

Spin formulation of the Schwinger Model



$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - g\not{A} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho$$

$$\mathcal{H} = -i\bar{\Psi}\gamma^1(\partial_1 - igA_1)\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2 \xcancel{+ A_0\left(g\Psi^\dagger\Psi + \rho - \partial_1 E\right)}$$

$$H = -\frac{i}{2a}\sum_n \left(\phi_n^\dagger \xcancel{e^{i\theta_n}}\phi_{n+1} - \phi_{n+1}^\dagger \xcancel{e^{-i\theta_n}}\phi_n\right) + m\sum_n (-1)^n \phi_n^\dagger\phi_n + \frac{ag^2}{2}\sum_n L_n^2$$

$$H = \frac{1}{2a}\sum_n (\sigma_n^+\sigma_{n+1}^- + \sigma_{n+1}^-\sigma_n^+) + \frac{m}{2}\sum_n [1 + (-1)^n\sigma_n^z] + \frac{ag^2}{2}\sum_n L_n^2$$

Gauss's law:
$\left(x = \frac{1}{a^2g^2}, \mu = \frac{2m}{ag^2}\right)$

$$L_n - L_{n-1} = Q_n = \frac{1}{2}\left[(-1)^n + \sigma_n^z\right] + q_n$$

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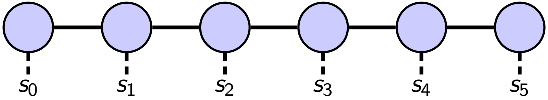

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$$H = x\sum_{n=0}^{N-2}[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+] + \frac{\mu}{2}\sum_{n=0}^{N-1}[1 + (-1)^n\sigma_n^z] + \sum_{n=0}^{N-2}\left[\sum_{k=0}^{n} Q_k\right]^2$$

Gauss's law:

$\left(x = \frac{1}{a^2g^2}, \mu = \frac{2m}{ag^2}\right)$

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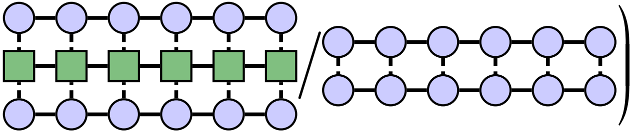
Efficient Tensor Network operations

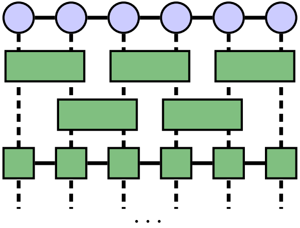
- Find groundstate and excited states

$$\min\left(E = \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \right.$$

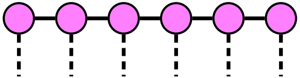


- Apply operators / time evolution

$$\hat{O}\,|\Psi\rangle =$$

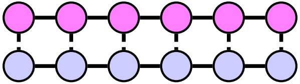


$$\longrightarrow |\Phi\rangle =$$



- Calculate overlap

$$\langle \Psi | \Phi \rangle =$$



Time evolution with MPS

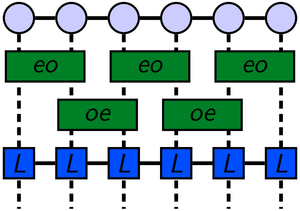
$$H = x\sum_{n=0}^{N-2}\left[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right] + \frac{\mu}{2}\sum_{n=0}^{N-1}\left[1+(-1)^n\sigma_n^z\right] + \sum_{n=0}^{N-2}\left[\frac{1}{2}\sum_{k=0}^{n}\left((-1)^k+\sigma_k^z+2q_k\right)\right]^2$$

Suzuki-Trotter decomposition:

$$e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{eo}}e^{-i\delta\tau H_{oe}}e^{-i\delta\tau H_L}\right)^{N_\tau}$$

$$e^{-i\delta\tau H_{eo}} = \prod_{n=0,2,\dots} e^{-i\delta\tau x\left[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right]}$$

$$e^{-i\delta\tau H_{oe}} = \prod_{n=1,3,\dots} e^{-i\delta\tau x\left[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right]}$$



$e^{-i\delta\tau H_L}$: MPO with indices $L_n \in [-n, n]$
→ truncate to L_{cut}
Optimize to get new MPS