TENSOR RENORMALIZATION GROUP STUDY OF COLD AND DENSE TWO- AND THREE-COLOR QCD IN THE STRONG COUPLING LIMIT

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1. Introduction
2. Strong Coupling limit of SU(2) case
3. Strong Coupling limit of SU(3) case
4. Summary

1 Introduction

2 Strong Coupling limit of SU(2) case

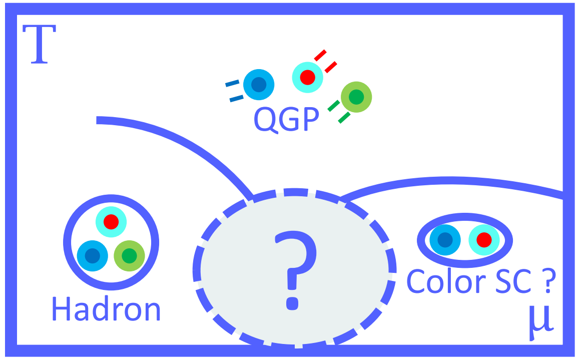
3 Strong Coupling limit of SU(3) case

4 Summary

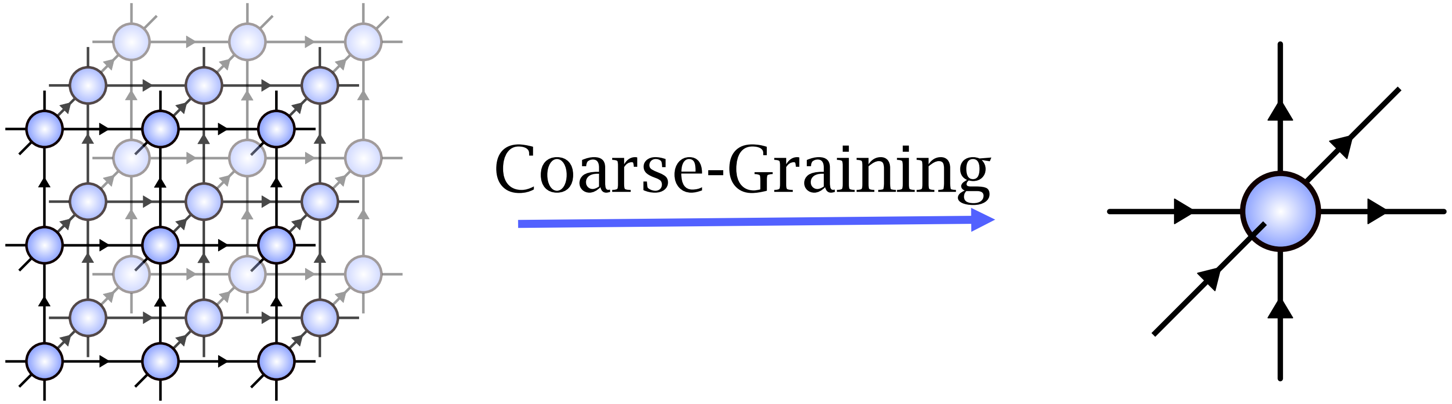
BACKGROUND

GOAL : First-principles calculations of finite density QCD

- **Finite density** is hot topic
- MC simulations are hindered by sign problem
- Tensor Network is next candidate
- We use Tensor Renormalization Group for finite density QCD



$$Z(T,\mu,m) =$$

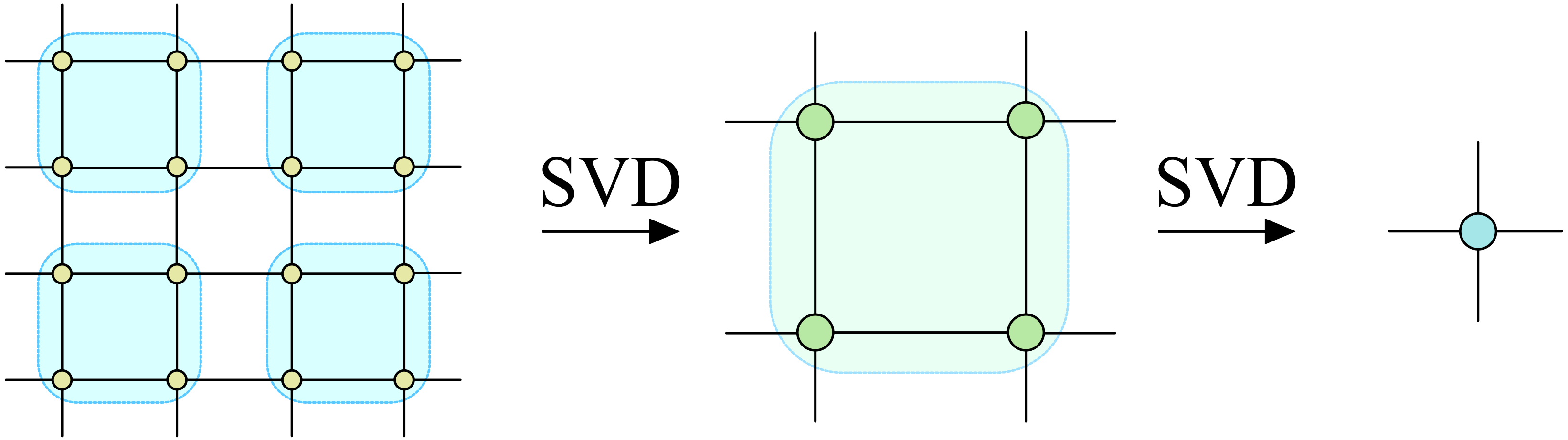


ABOUT TRG

[M. Levin and C. P. Nave, (2007).] [Xie et al, (2012).]

- Tensor Renormalization Group : A computational method based on SVD
    - Rewrite partition function Z as a tensor network & approximate it
    - Thermodynamic limit
    - **NO sign problem !** → For finite density QCD ?

$$Z =$$



STRONG COUPLING LIMIT

S. Akiyama and D. Kadoh, JHEP. **2021**, 188 (2021).

$$S[U,\chi,\bar{\chi}] = \cancel{\underbrace{\beta\sum_n\left[1-\frac{1}{N}\sum \cdots_{\mu\nu}(n)\right]}_{\text{Gauge action}}} + \underbrace{\sum_n\left[\frac{1}{2}\sum_{\nu=1}^{4}\eta_\nu(n)\left(e^{\mu\delta_{\nu,4}}\bar{\chi}(n)U_\nu(n)\chi(n+\hat{\nu}) - e^{-\mu\delta_{\nu,4}}\bar{\chi}(n+\hat{\nu})U_\nu^\dagger(n)\chi(n)\right) + m\,\bar{\chi}(n)\chi(n)\right]}_{\text{Fermion action (staggered, finite }\mu)}.$$

- Strong coupling limit : β=0
- All link variables at hopping term can be integrated out

$$\int_{SU(N)} dU\, U_{i_1j_1}\cdots U_{i_kj_k}\, U^\dagger_{j'_1i'_1}\cdots U^\dagger_{j'_ki'_k} = \sum_{\sigma,\tau\in S_k}\left(\prod_{r=1}^{k}\delta_{i_r,i'_{\sigma(r)}}\right)\left(\prod_{r=1}^{k}\delta_{j_r,j'_{\tau(r)}}\right)\mathrm{Wg}_N(\sigma^{-1}\tau).$$

- Fermion partition function can be simulated using Grassmann TN
- We consider both SU(2) and SU(3) case

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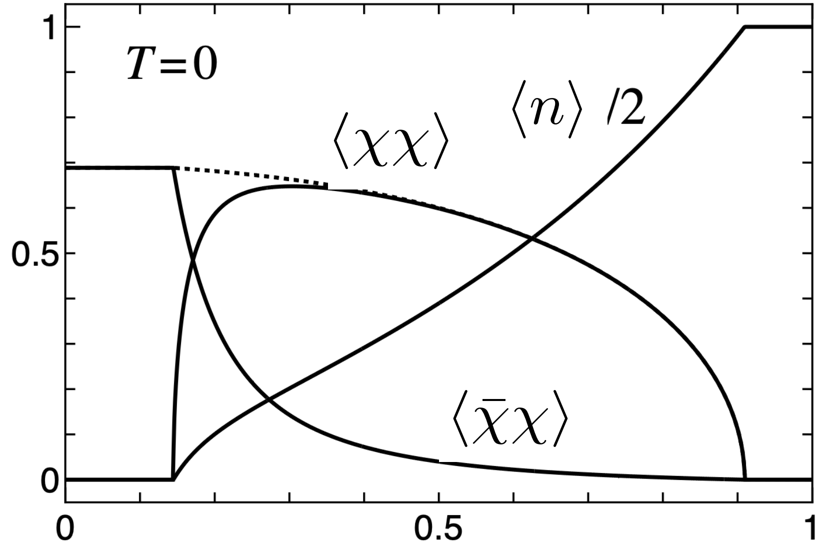
STRONG COUPLING QC$_2$D AT ZERO TEMPERATURE

$$S[U,\chi,\bar\chi] = \underbrace{\cancel{\beta\sum_n\left[1-\frac{1}{N}\sum \ldots P_{\mu\nu}(n)\right]}}_{\text{Gauge action}} + \underbrace{\sum_n\left[\frac{1}{2}\sum_{\nu=1}^{4}\eta_\nu(n)\left(e^{\mu\delta_{\nu,4}}\bar\chi(n)U_\nu(n)\chi(n+\hat\nu) - e^{-\mu\delta_{\nu,4}}\bar\chi(n+\hat\nu)U_\nu^\dagger(n)\chi(n)\right) + m\,\bar\chi(n)\chi(n)\right]}_{\text{Fermion action (staggered, finite }\mu)}.$$

- Consider strong coupling limit ($\beta \to 0$)
- MF prediction at $T = 0$ (Cold & dense)
  - low μ : $\langle\bar\chi\chi\rangle \neq 0,\ \langle\chi\chi\rangle = 0$
  - At $\mu_{\rm low} \approx m_\pi/2$: $\langle\chi\chi\rangle \neq 0$
  - Higher μ : $\langle\bar\chi\chi\rangle = 0,\ \langle\chi\chi\rangle = 0,\ \langle n\rangle = 2$

$\langle n\rangle$: number density
$\langle\bar\chi\chi\rangle$: chiral cond.
$\langle\chi\chi\rangle$: diquark cond.



Y. Nishida, K. Fukushima, and T. Hatsuda (2004).

NUMERICAL SETUP I

[1] S. Akiyama, et al. (2021).

- Apply Grassmann-ATRG[1] method to 4D Strong coupling QC_2D
- Employ multiple GPU parallelization [Y. Sugimoto and S. Sasaki (2025). 2507.21909]

-- Setup --

- parameter : $V = 1024^4$, $T = 0$, $m = 1.0$, $D = 55$

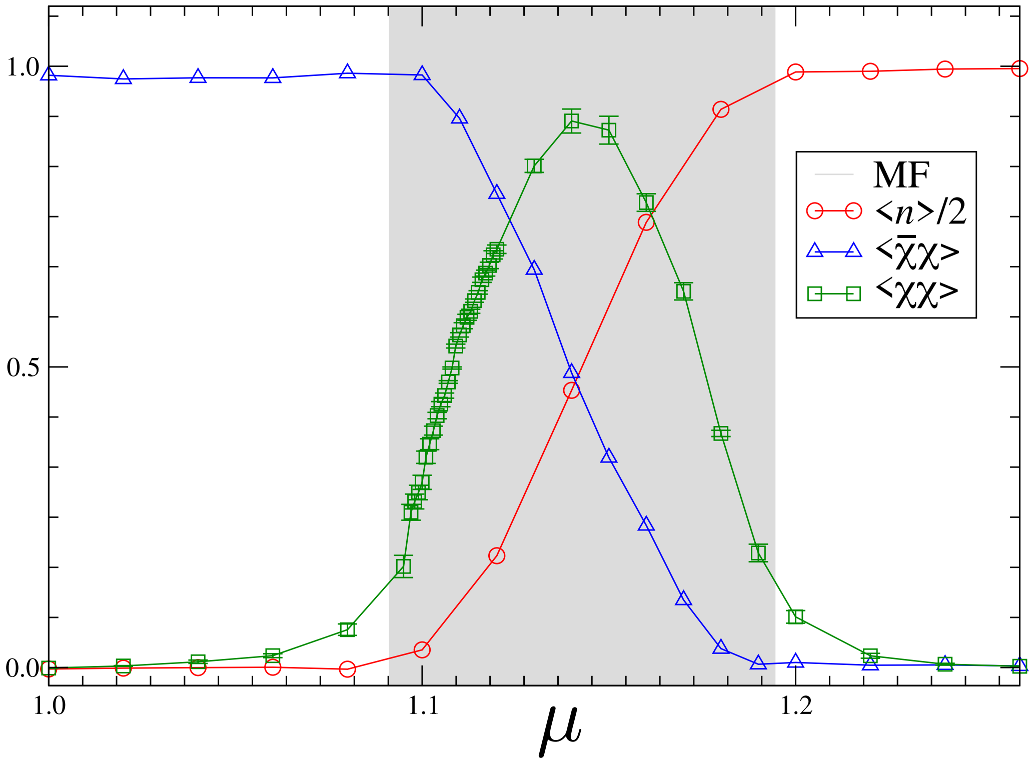
  D : bond dimensions (# of retained singular values)

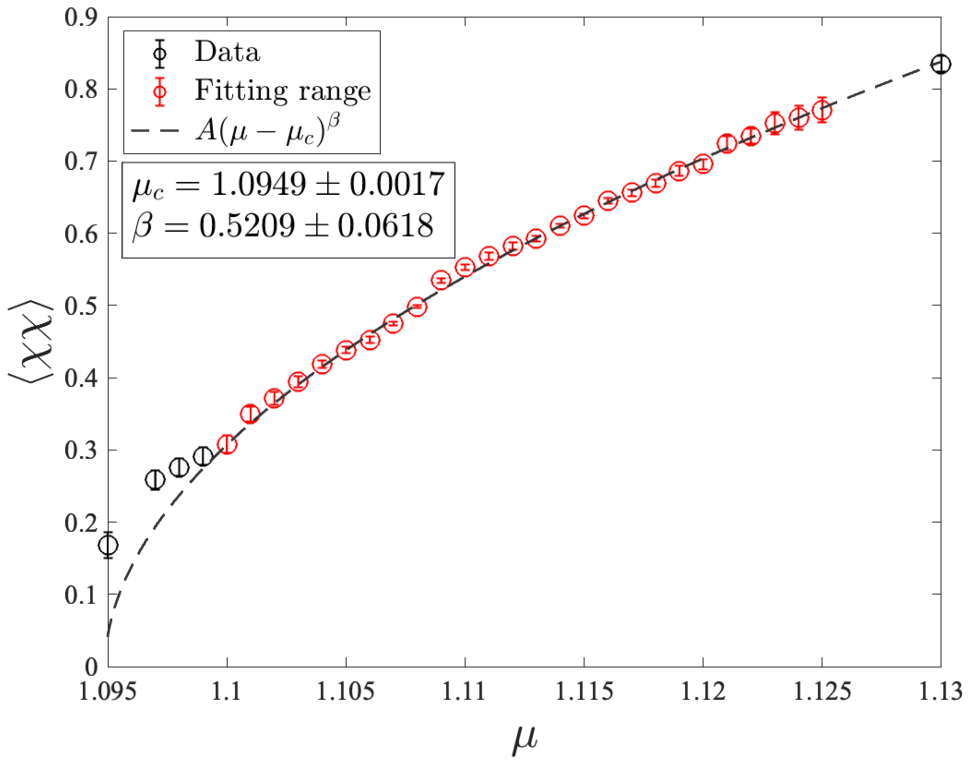
- Evaluate free energy : $f(\mu, m) = -\frac{1}{V} \ln Z(\mu, m)$

$\rightarrow$ Calculate $\langle n \rangle, \langle \bar{\chi}\chi \rangle, \langle \chi\chi \rangle$

NUMERICAL RESULT

- Physical quantities behaves consistent with mean-field theory
- Fitting as power law for $\langle \chi\chi \rangle = A(\mu - \mu_c)^{\beta}$ gives $\beta = 0.521 \pm 0.062 \sim 0.5$(MF)





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STRONG COUPLING QCD

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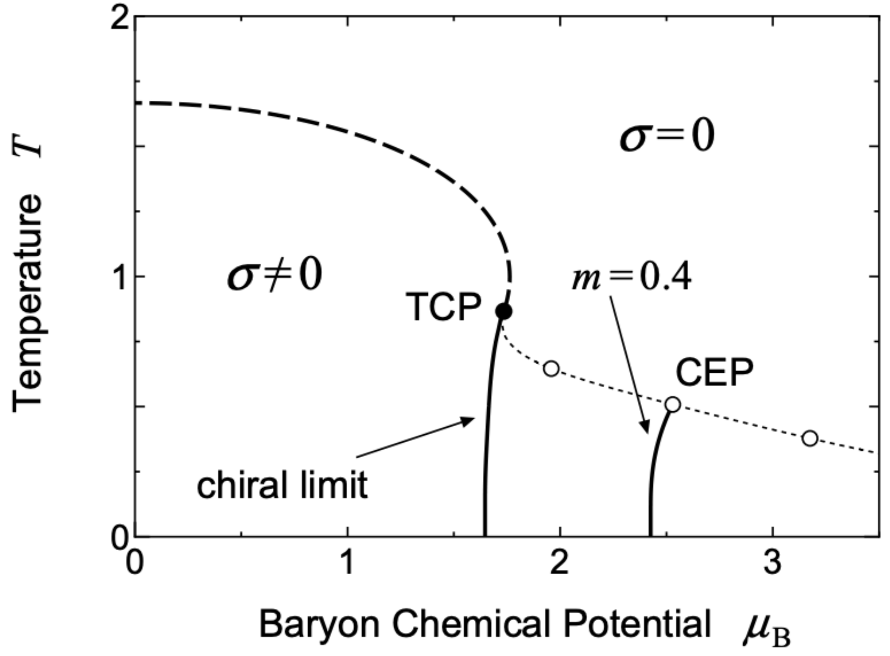
- Consider strong coupling limit ($\beta \to 0$)
  - No dynamical gluons
- Chiral & Nuclear transition
  - CEP at finite T & mass
- 1-st order transition at $T = 0$

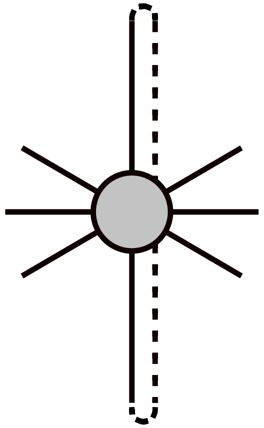


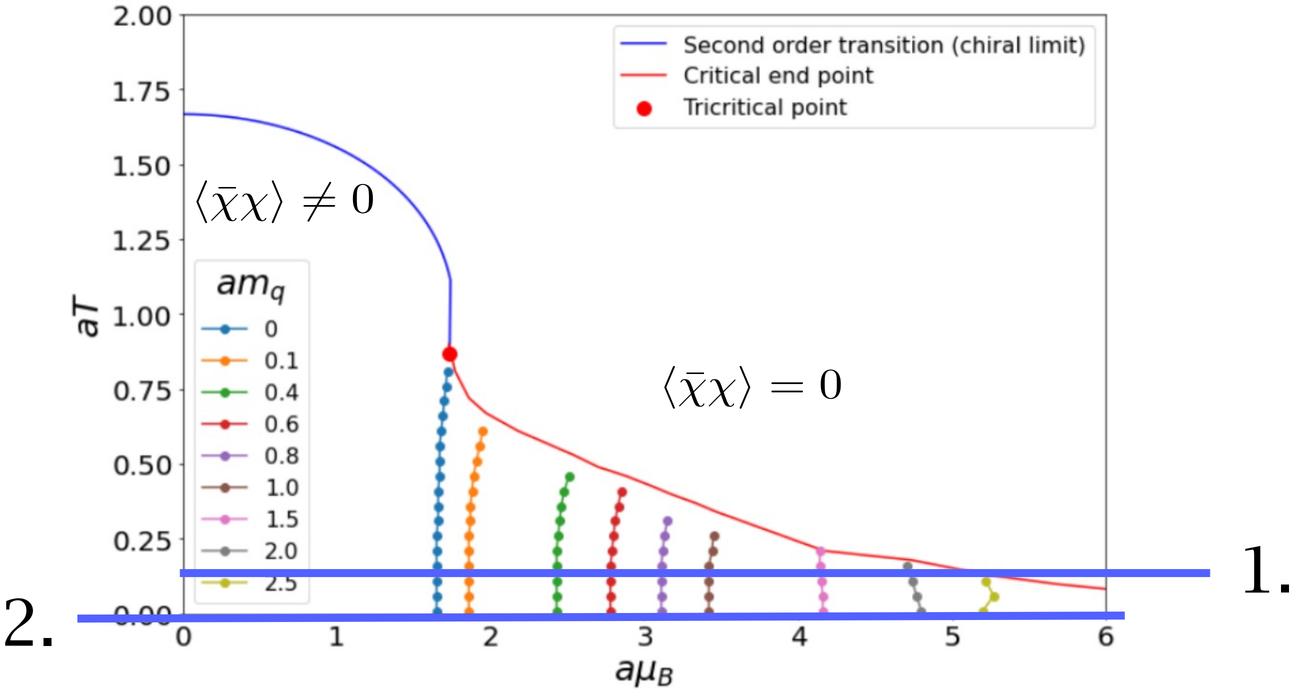
Phys. Rev. D **69**, 094501 (2004).

TARGET

[1] S. Akiyama, et al. (2021).

- Two setup with Grassmann-ATRG [1]
  1. $L_\tau = 8$: Coarse-graining along τ direction → 3D GATRG
  2. $L_\tau = 1024$ (zero temperature) : 4D GATRG

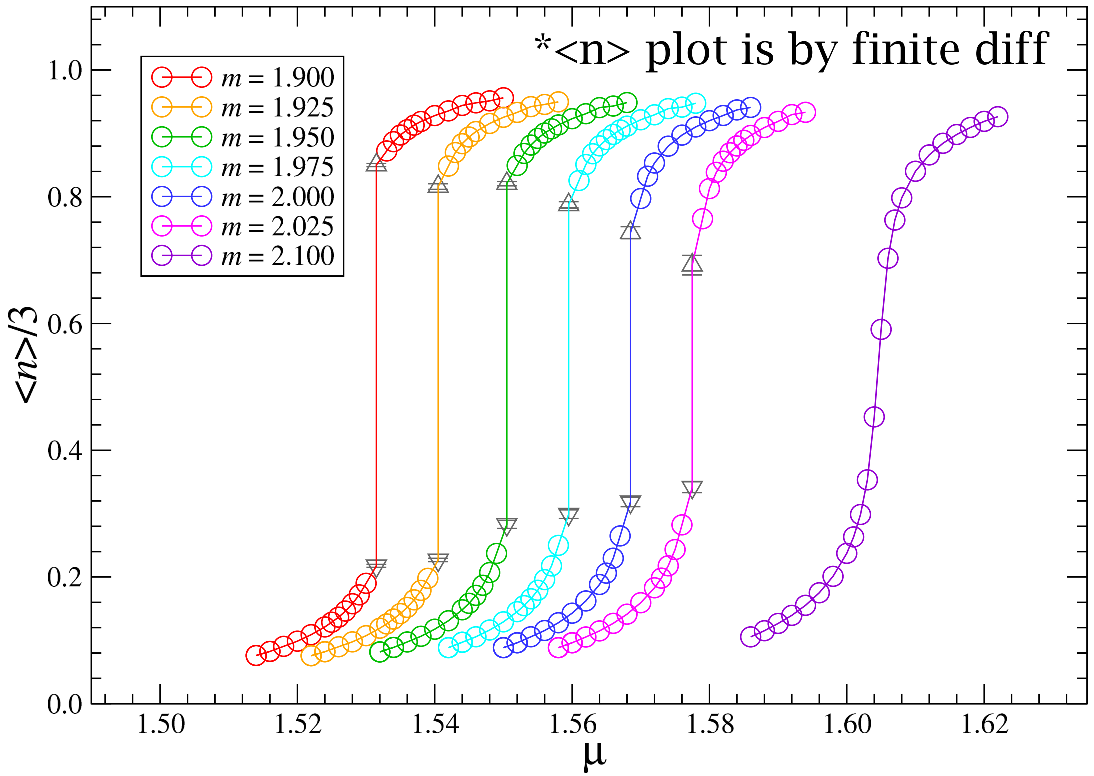


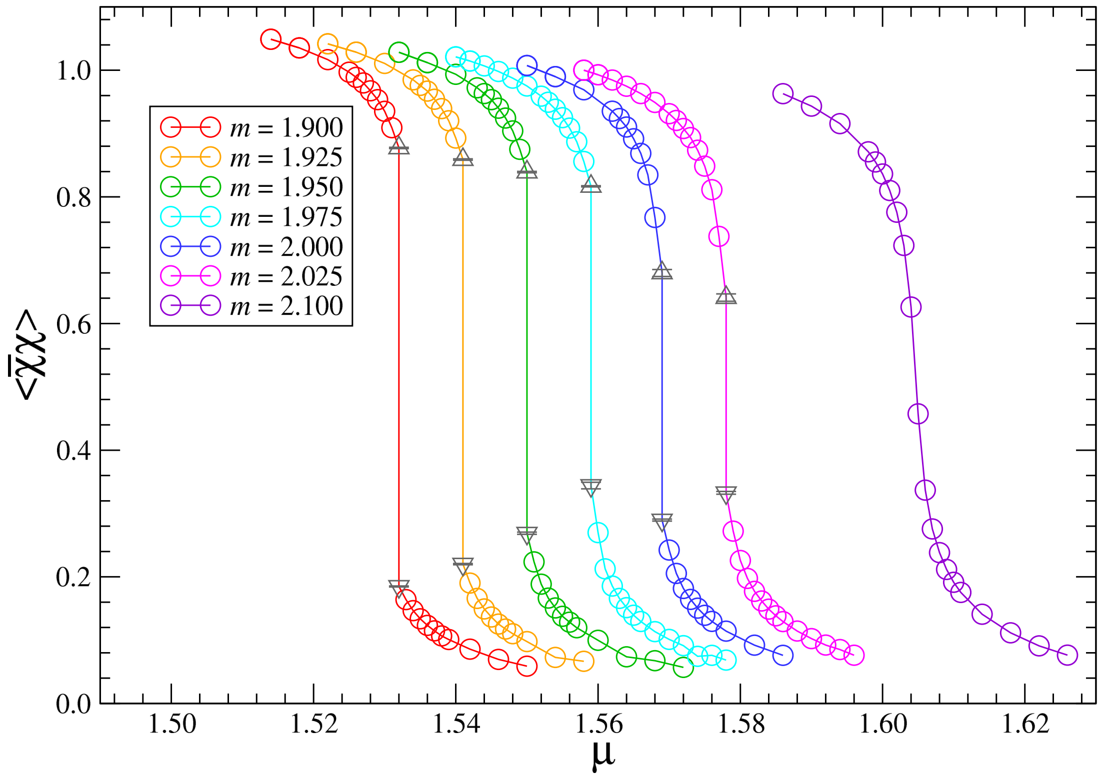


MF prediction [Phys. Rev. D 107, 094514 (2023)]

1. FINITE TEMPERATURE

- Investigate $\langle n \rangle$ and $\langle \bar{\chi}\chi \rangle$ at $D_{\mathrm{4d}} = 55, D_{\mathrm{3d}} = 120$
- Increasing m, gaps become small

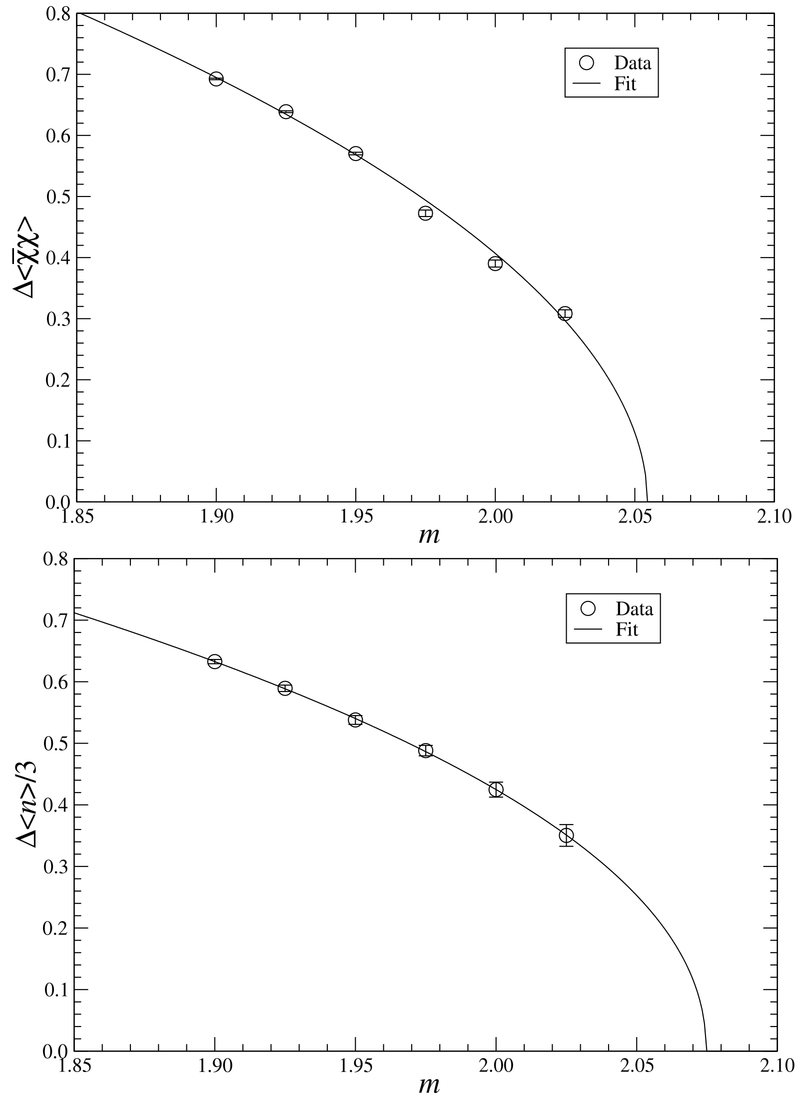




1. FINITE TEMPERATURE

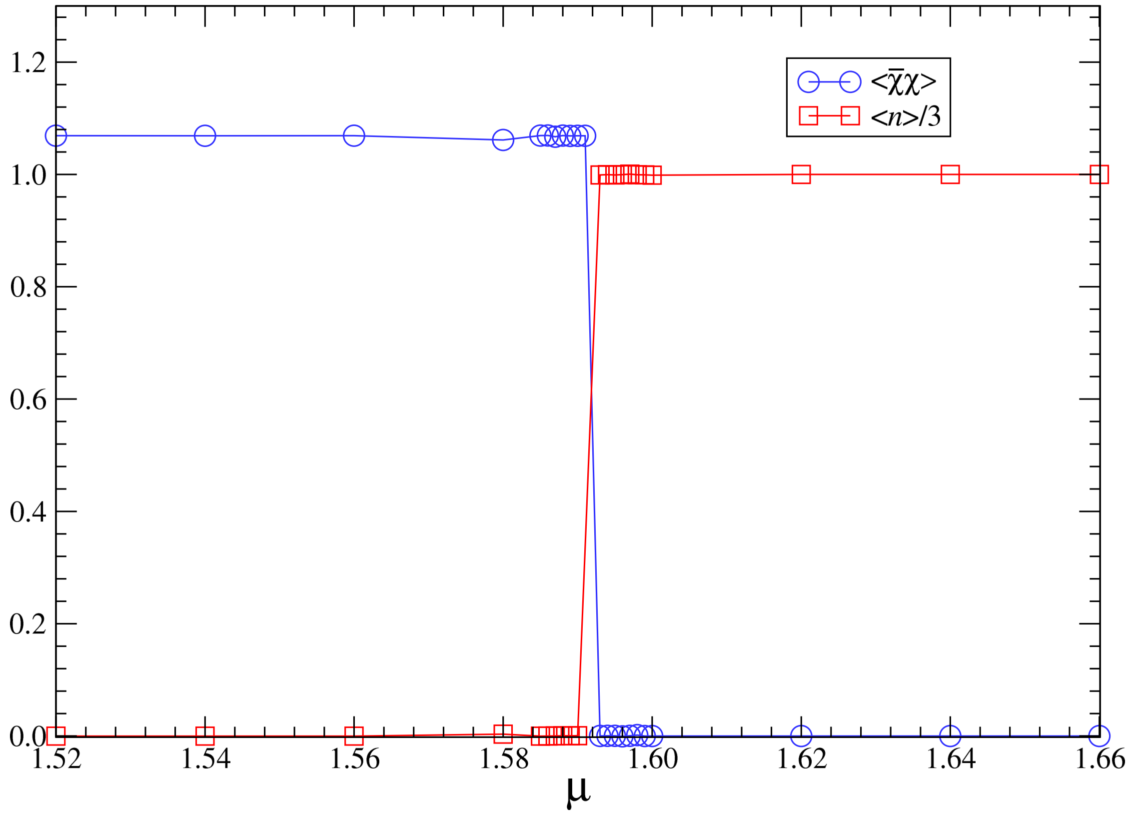
[1] Phys. Rev. D 107, 094514 (2023)
[2] Phys. Rev. D 69, 094501 (2004).

- Fit of critical exponent β and critical mass
  - $\Delta\langle n\rangle \quad : \; m_c^n = 2.075(23), \beta_n = 0.47(10)$
  - $\Delta\langle\bar{\chi}\chi\rangle : m_c^\chi = 2.0545(34), \beta_\chi = 0.514(20)$
- m_c is consistent with each other
- deviation from previous research
  - MC in dual & reweighting =1.7(1)
  - Mean field ~ 2.4



2. ZERO TEMPERATURE

- Investigate $\langle n \rangle$ and $\langle \bar{\chi}\chi \rangle$ at $D_{\rm 4d} = 55, m = 2.07$
- Clear 1-st order transition is observed → expected from MF



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SUMMARY & FUTURE WORK

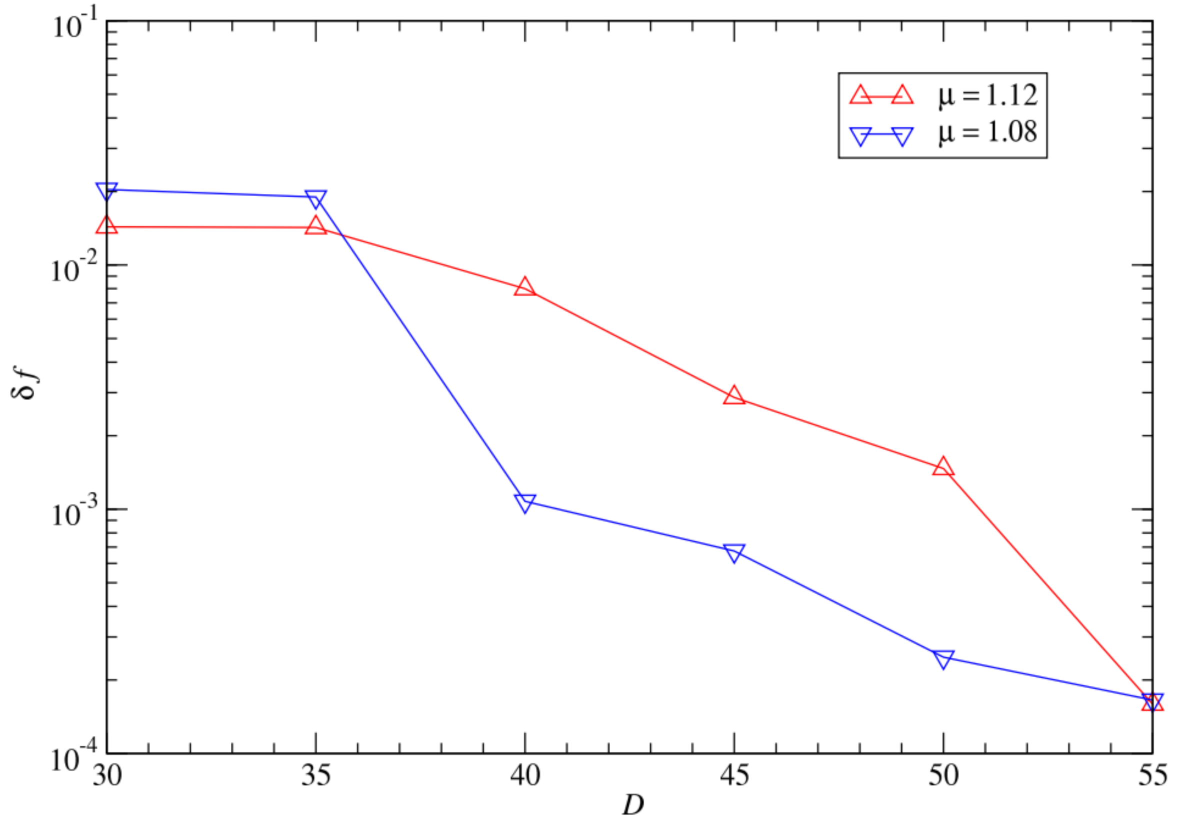
Summary

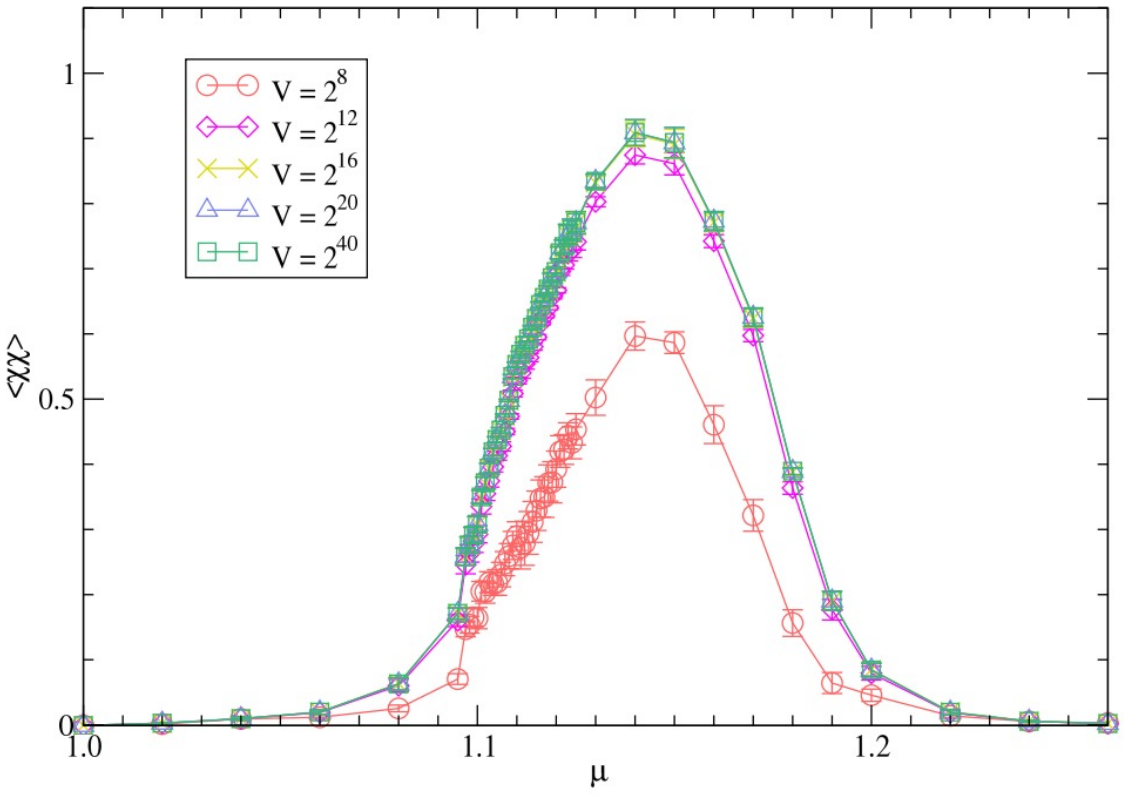
- Performed (3+1)d finite density QCD calculations at strong coupling limit
- SU(2) : T=0 calculation & estimated critical exponents
- SU(3) : Observed 1-st order phase transition at zero temperature

Future work

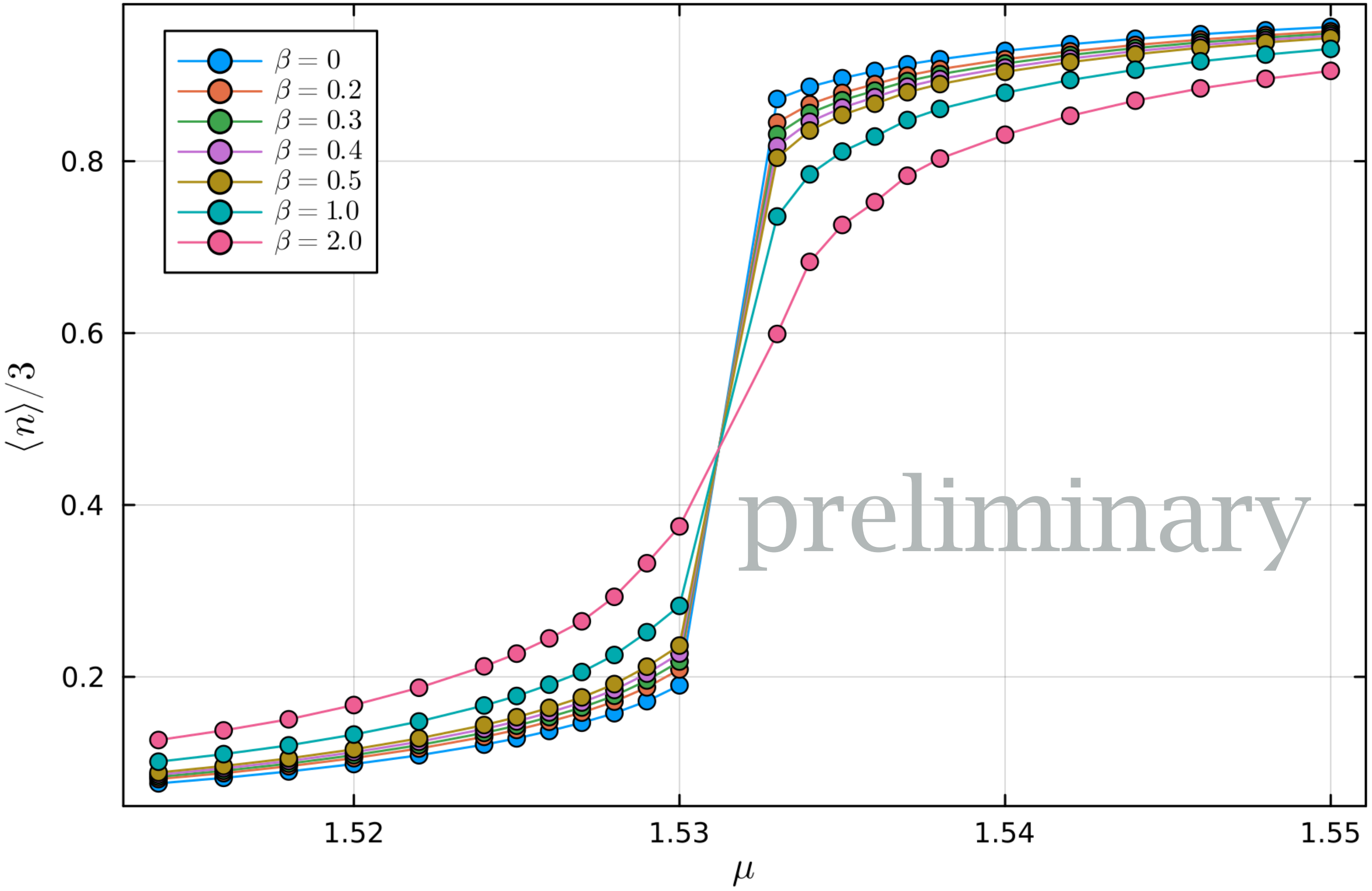
- finite β in (3+1)d in strong coupling regime
- Wilson fermion

- Convergence for SU2

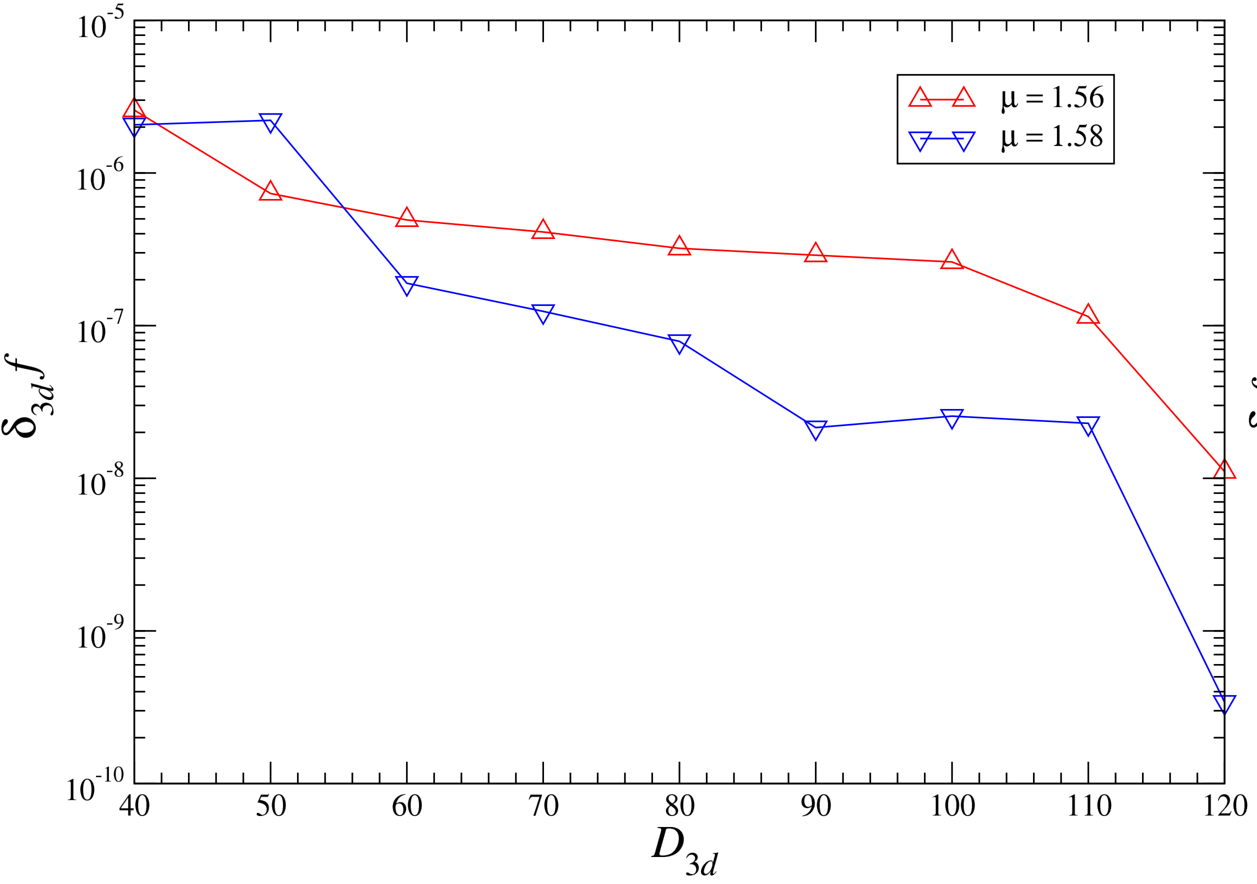


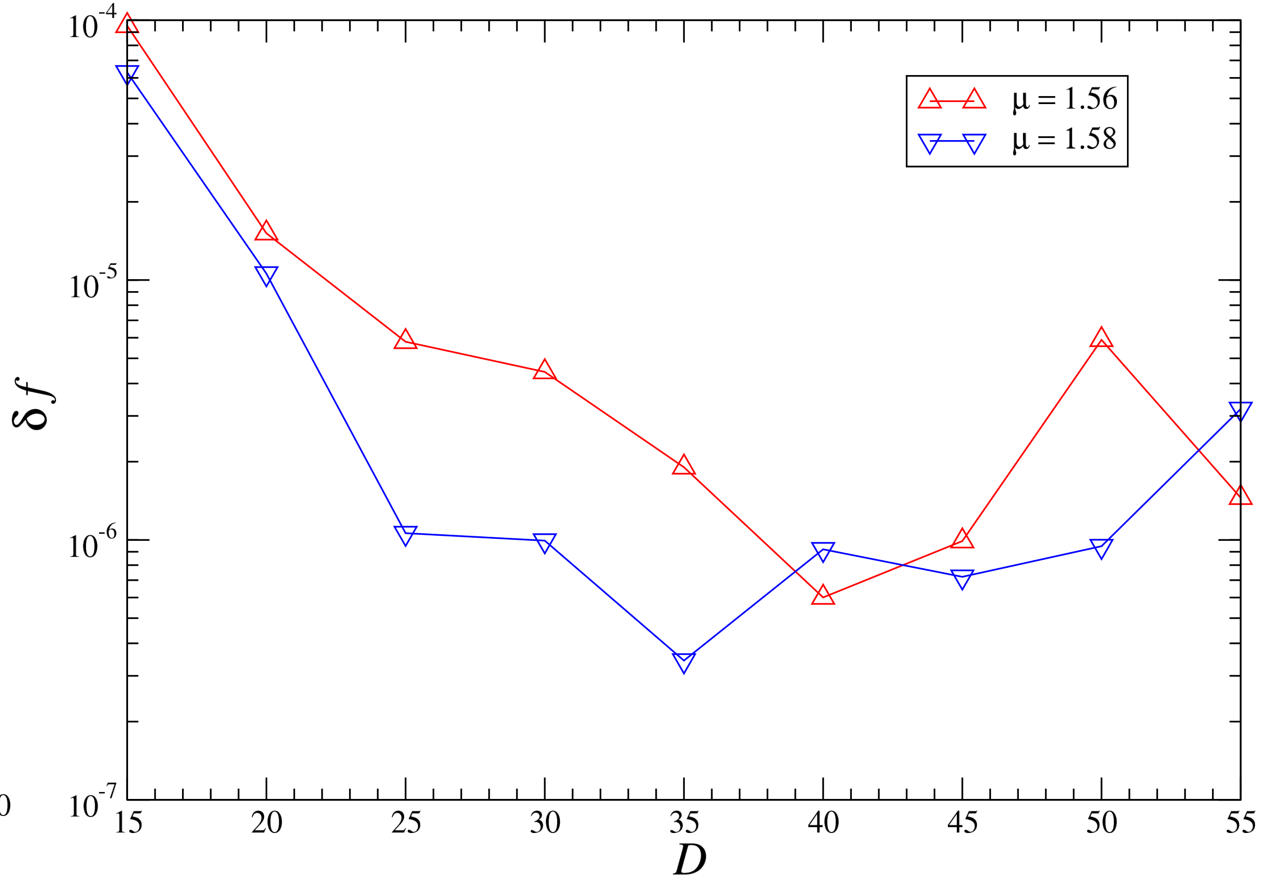


- O(β) correction makes gaps smaller
- At some βc(m), gap vanishes



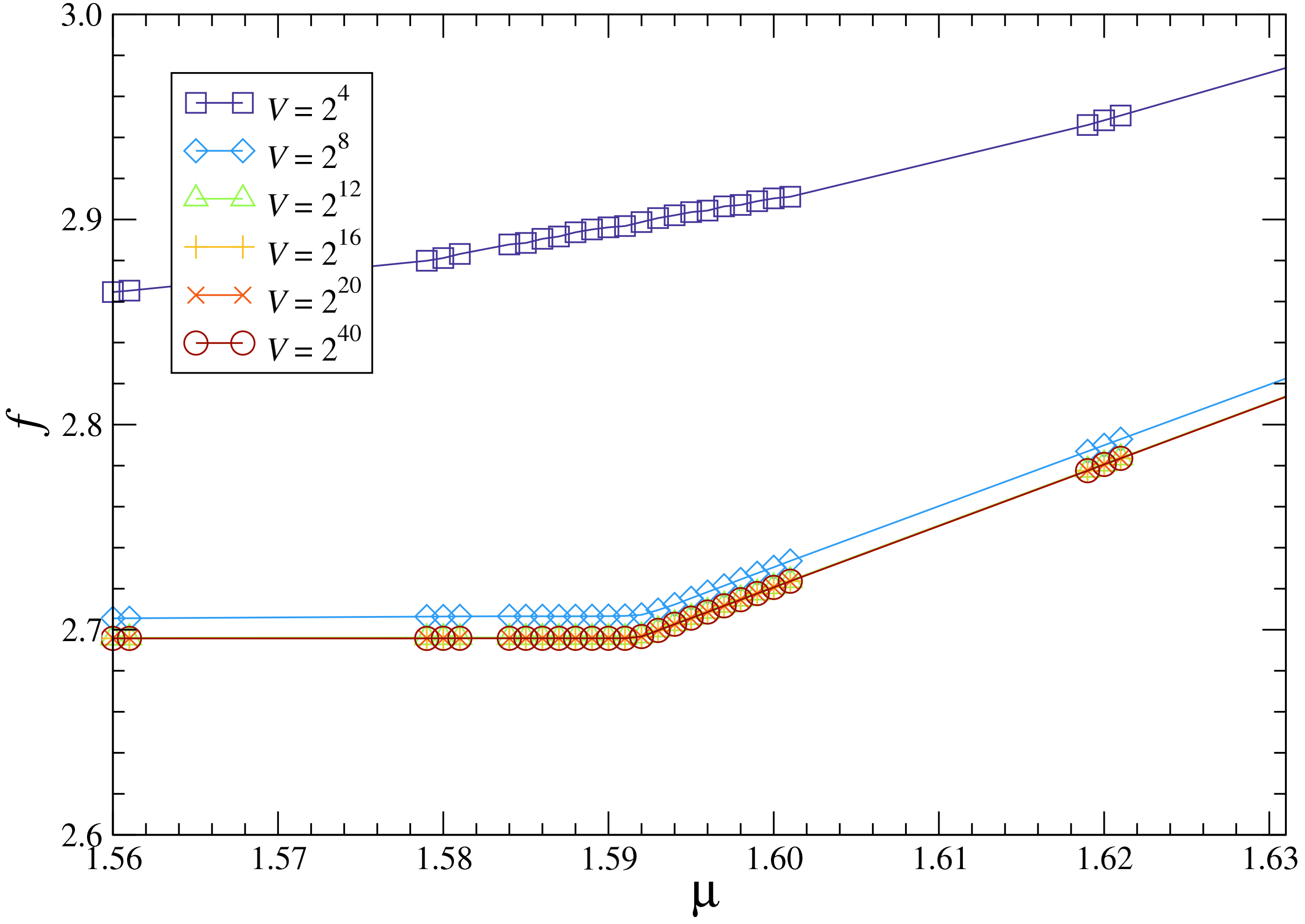
- Convergence for SU3





well converged

Free energy at zero temperature



Free energy at finite temperature