

PRECISION DETERMINATION OF B_c AND B_c^* MESON DECAY CONSTANTS FROM LATTICE QCD



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WORK WITH

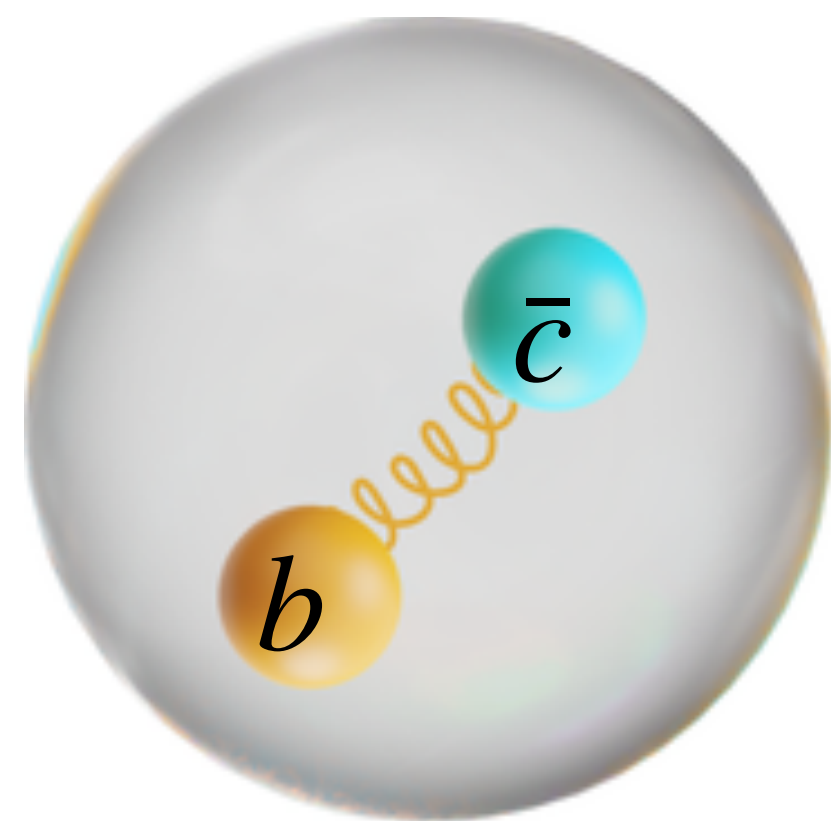


**DEBSUBHRA
CHAKRABORTY**



**NILMANI
MATHUR**

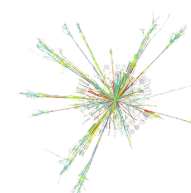
B_c meson



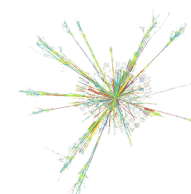
Only meson composed of two different heavy quarks



Unique laboratory for two-scale heavy-quark dynamics



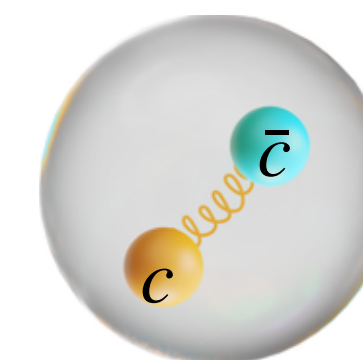
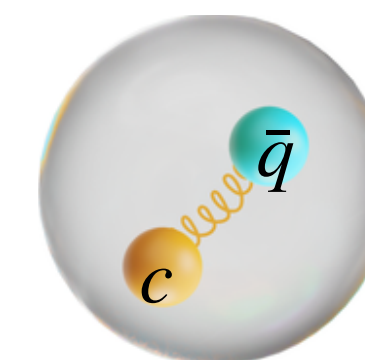
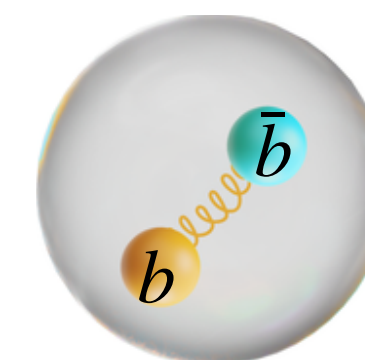
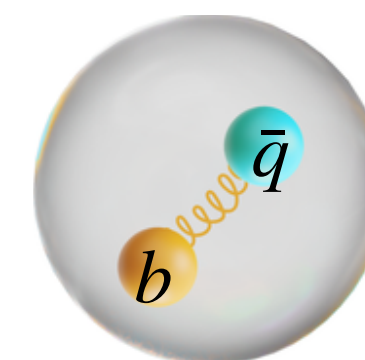
Large B_c samples at the LHC (More in future LHC, FCC)



Precision studies of leptonic decays becoming feasible

- Purely leptonic decay, $\bar{B}_c \rightarrow l\bar{\nu}$, all hadronic effects encoded in a single decay constant
- Useful in accurate determinations of $|V_{cb}|$ (next slide)

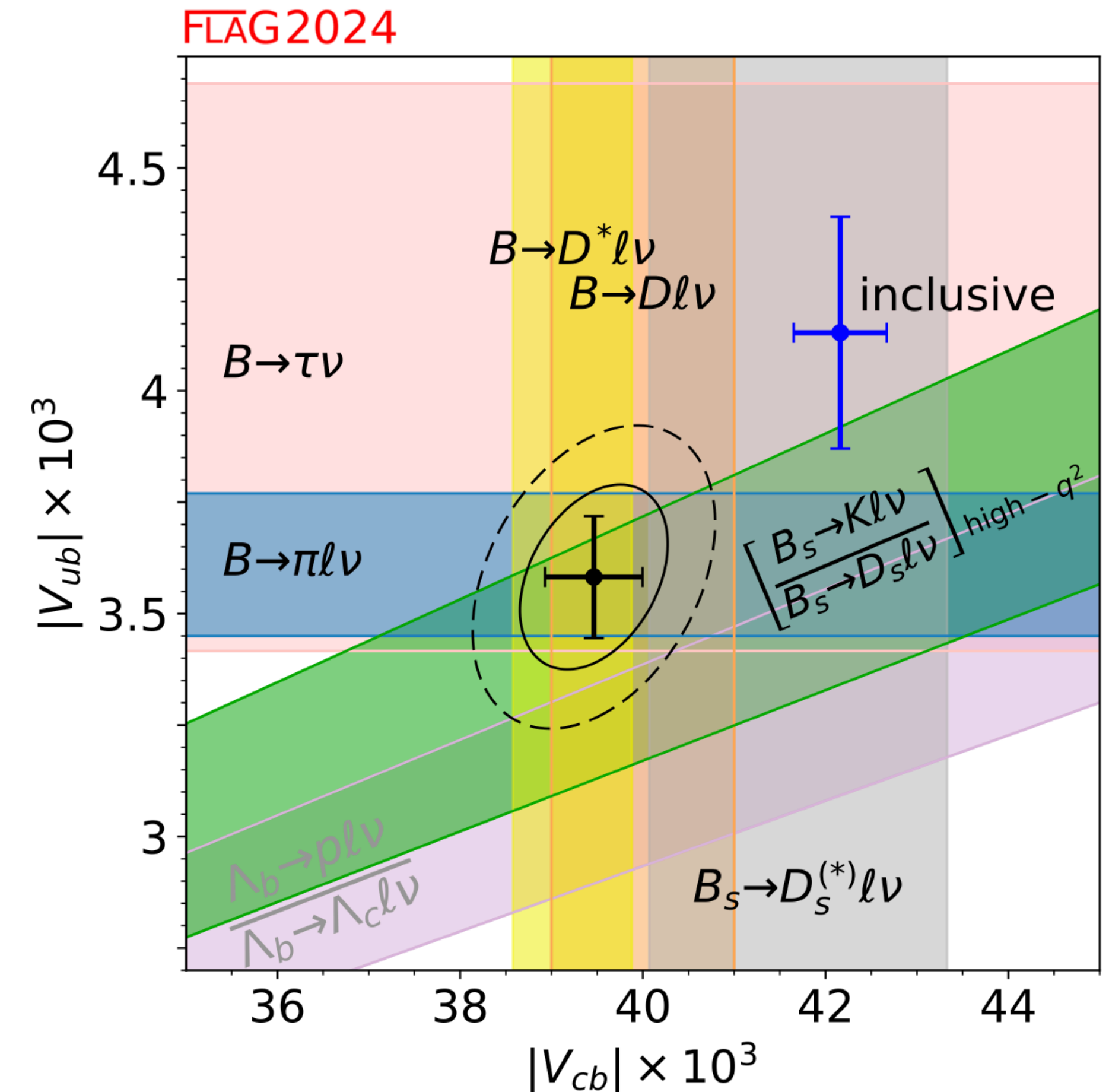
Combines features of heavy-light mesons and heavy quarkonia
A unique heavy-heavy system connecting precision flavour physics and lattice QCD



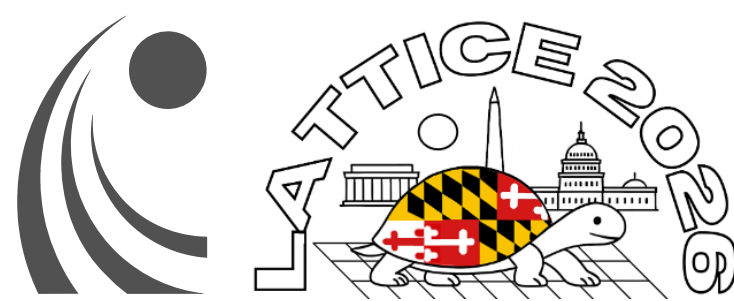
Why f_{B_c} ?

A precise determination of $|V_{cb}|$ is a central goal of precision flavor physics.

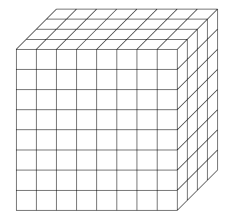
- Determined from inclusive and exclusive semileptonic B-meson decays
- A longstanding $2 - 3\sigma$ tension exists between the two determinations
- Exclusive determinations require precise hadronic form factors, making lattice QCD an essential theoretical input.
- A precise f_{B_c} strengthens unitarity bounds, leading to improved $B \rightarrow D^{(*)}$ form factors and a more precise exclusive determination of $|V_{cb}|$



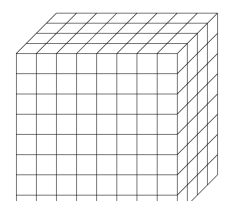
Our Setup



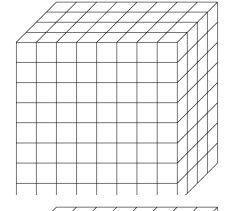
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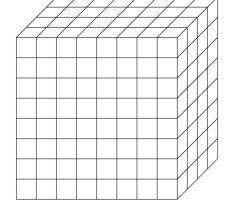
$N_f = 2 + 1 + 1$ MILC HISQ ensemble, $96^3 \times 288$ lattice, $a \sim 0.0327$ fm



HISQ action for valence quarks as well



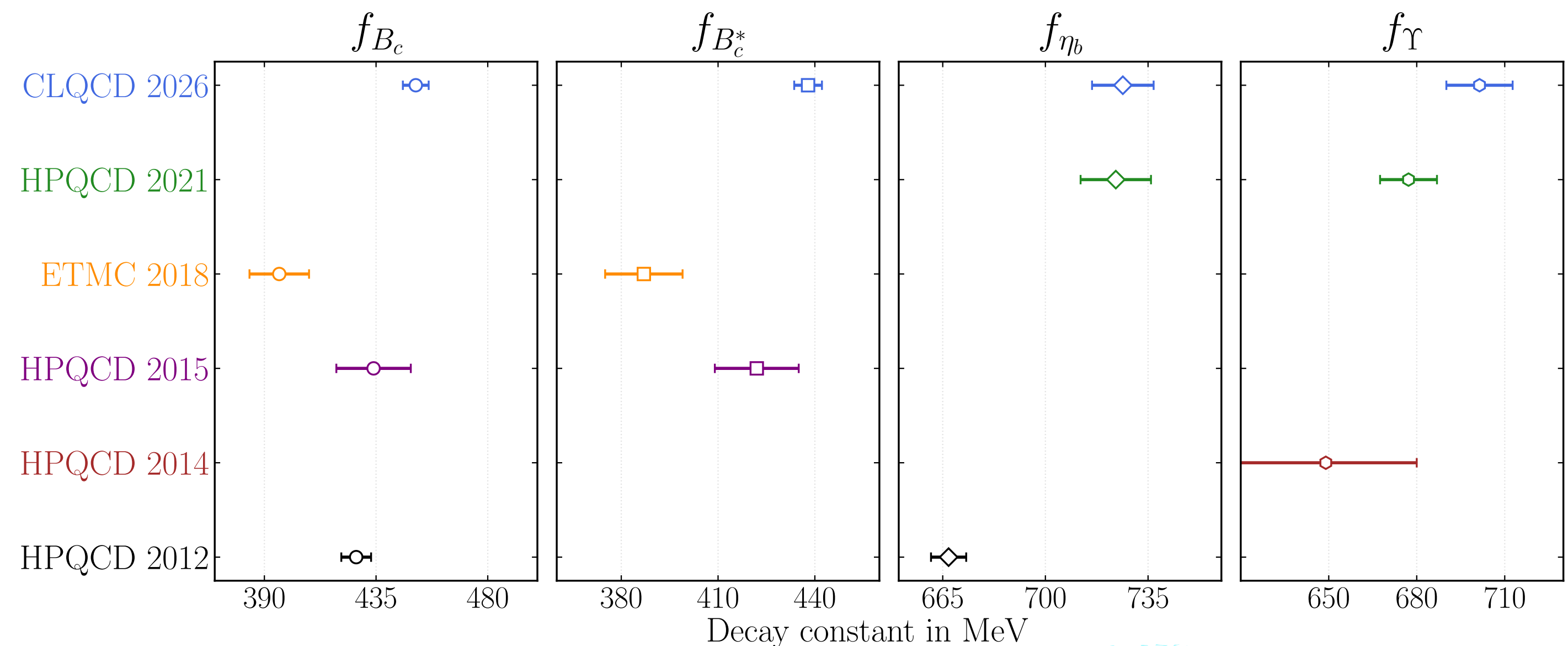
Charm tuned to physical spin-averaged charmonium, bottom tuned to physical Υ



400 gauge configurations (so far), random color wall source to point sink setup

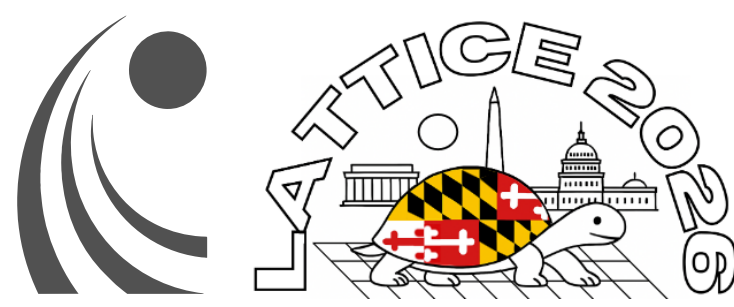
Targets of the work:

- Precise determination of f_{B_c} and $f_{B_c^*}$
- Results for tensor decay constant $f_{B_c^*}^T$
- Excited states using TGEVP (later part of the talk)

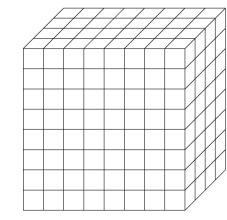


Lattice results so far

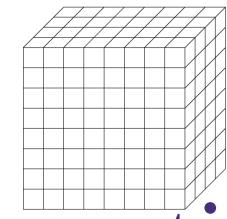
Why HISQ?



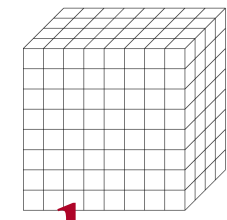
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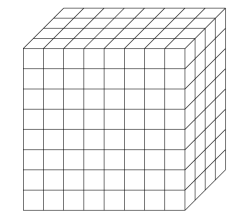
Ultrafine lattice spacing ($a \sim 0.0327 \text{ fm}$) allows a fully relativistic treatment of the bottom quark, $am_b = 0.6223$, (also $am_c = 0.137$)



Unlike coarser lattices, which often require specialized heavy-quark actions (e.g., NRQCD or RHQA), our ultrafine lattice allows the bottom quark to be simulated directly with the HISQ action



Highly Improved Staggered Quark (HISQ) action significantly reduces heavy-quark discretization errors through higher-order improvement.



Staggered fermions retain four continuum tastes per quark flavor, with meson operators classified by their spin–taste structure

$$\begin{array}{c} \bar{\psi} \Gamma \psi \\ \downarrow \\ \bar{\chi} (\Gamma_S \otimes \Gamma_T) \chi \end{array}$$

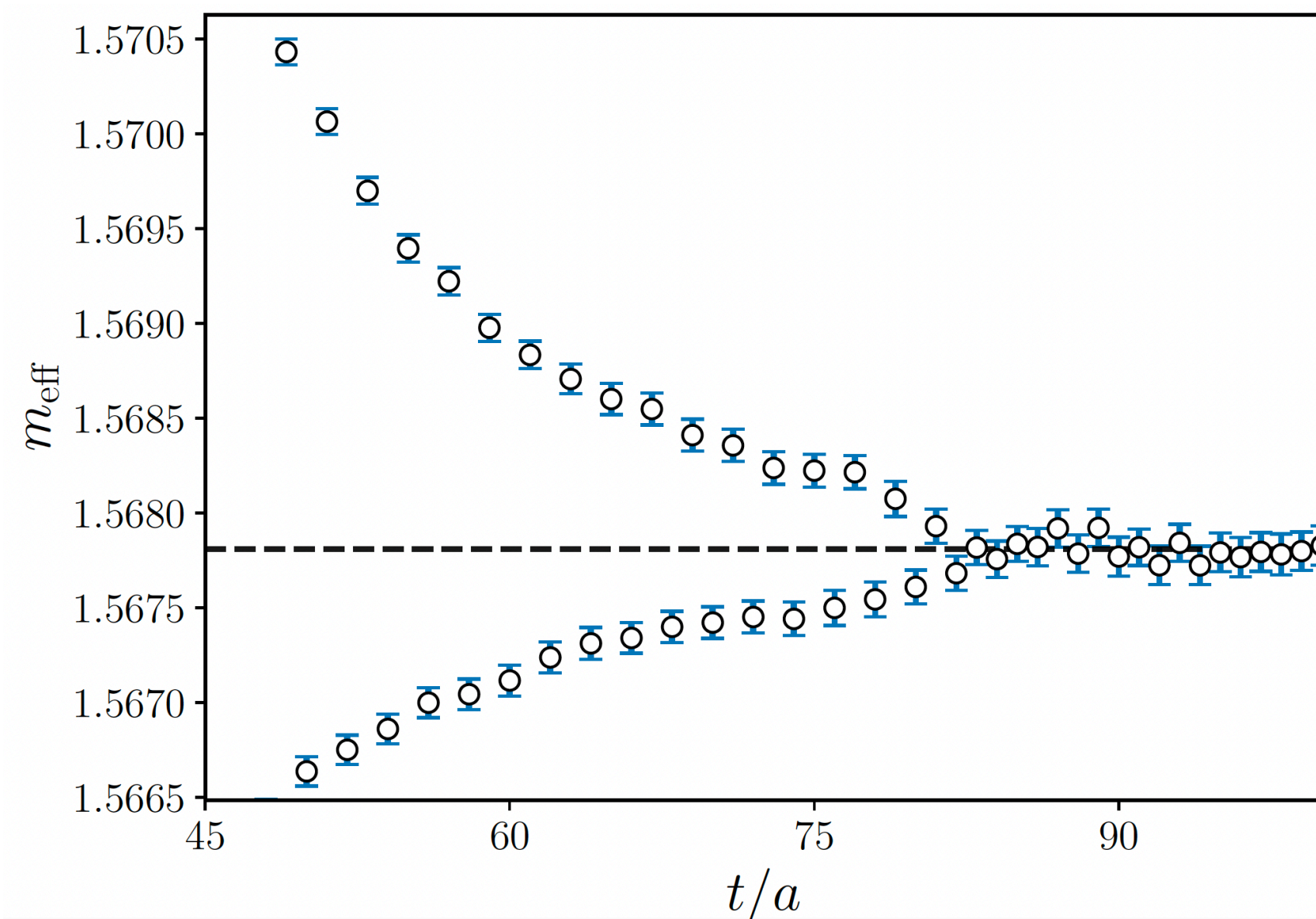
- Taste splitting effects suppressed (HISQ), ultrafine lattice spacing taste splitting expected to be small
- No mixed action effects, enables high-precision lattice calculations with controlled discretization errors

Oscillating Correlators

- ▶ Staggered meson operators couple to both the desired state and its opposite-parity partner
- ▶ Consequently, the two-point correlation function contains oscillating contributions

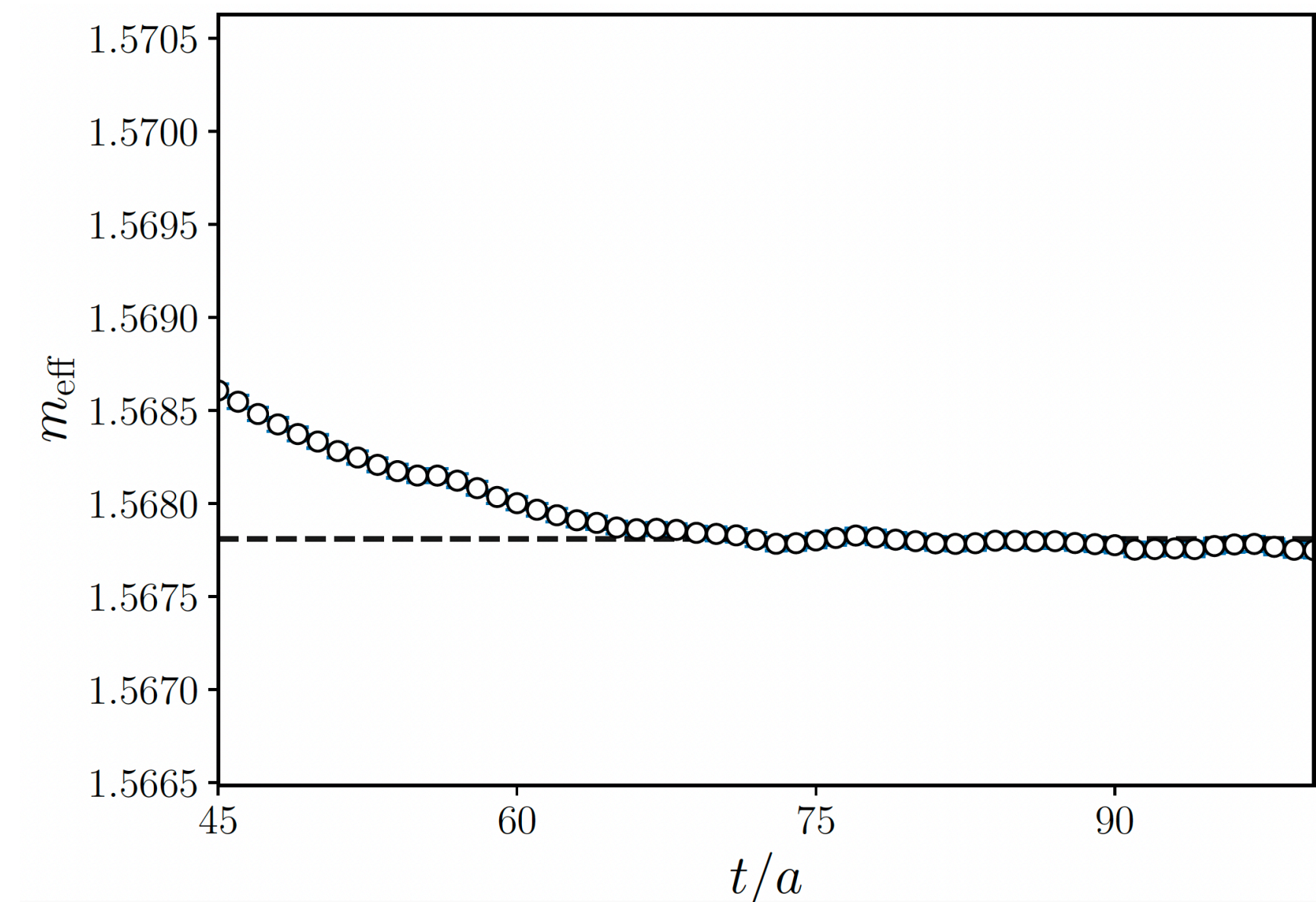
$$C(t) = \sum_n A_n \left(\exp(-E_n t) + \exp(-E_n(T - t)) \right) - (-1)^t \sum_n B_n \left(\exp(-E_n^o t) + \exp(-E_n^o(T - t)) \right)$$

Oscillating contributions produce an even-odd time-slice pattern



A three-point smoothing is applied to reduce these oscillations

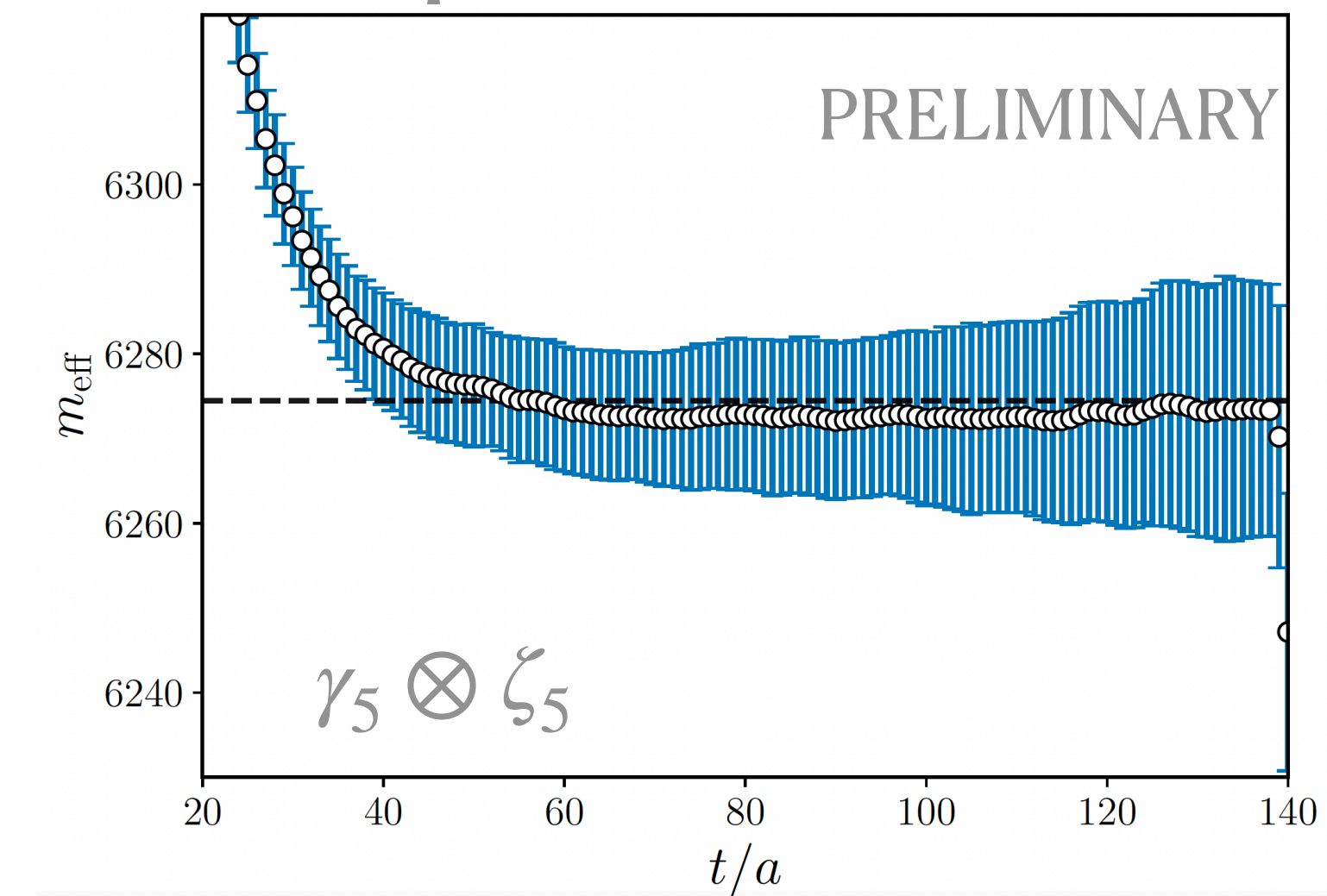
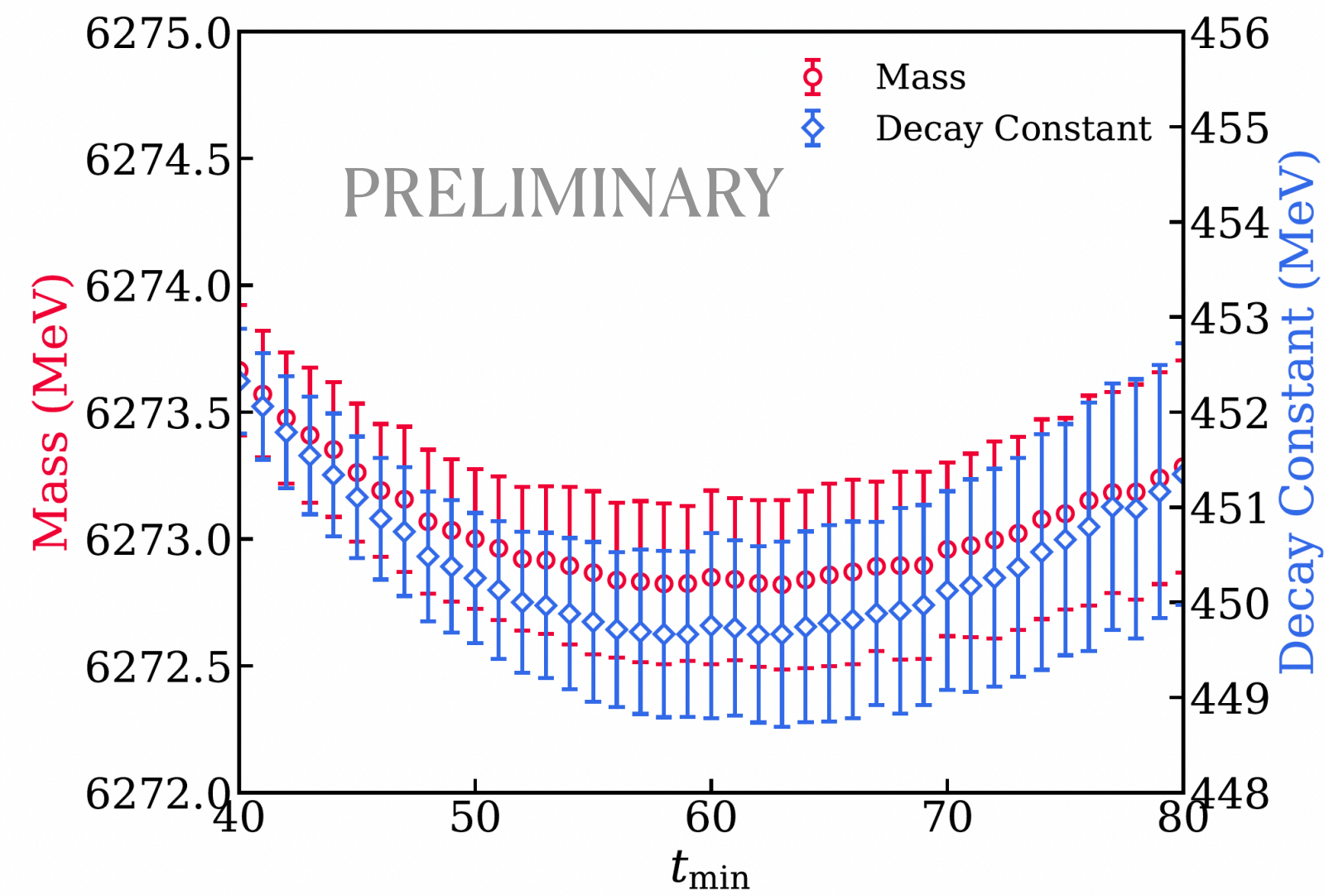
$$C(t) = \sqrt{C(t) \sqrt{C(t-1) C(t+1)}}$$



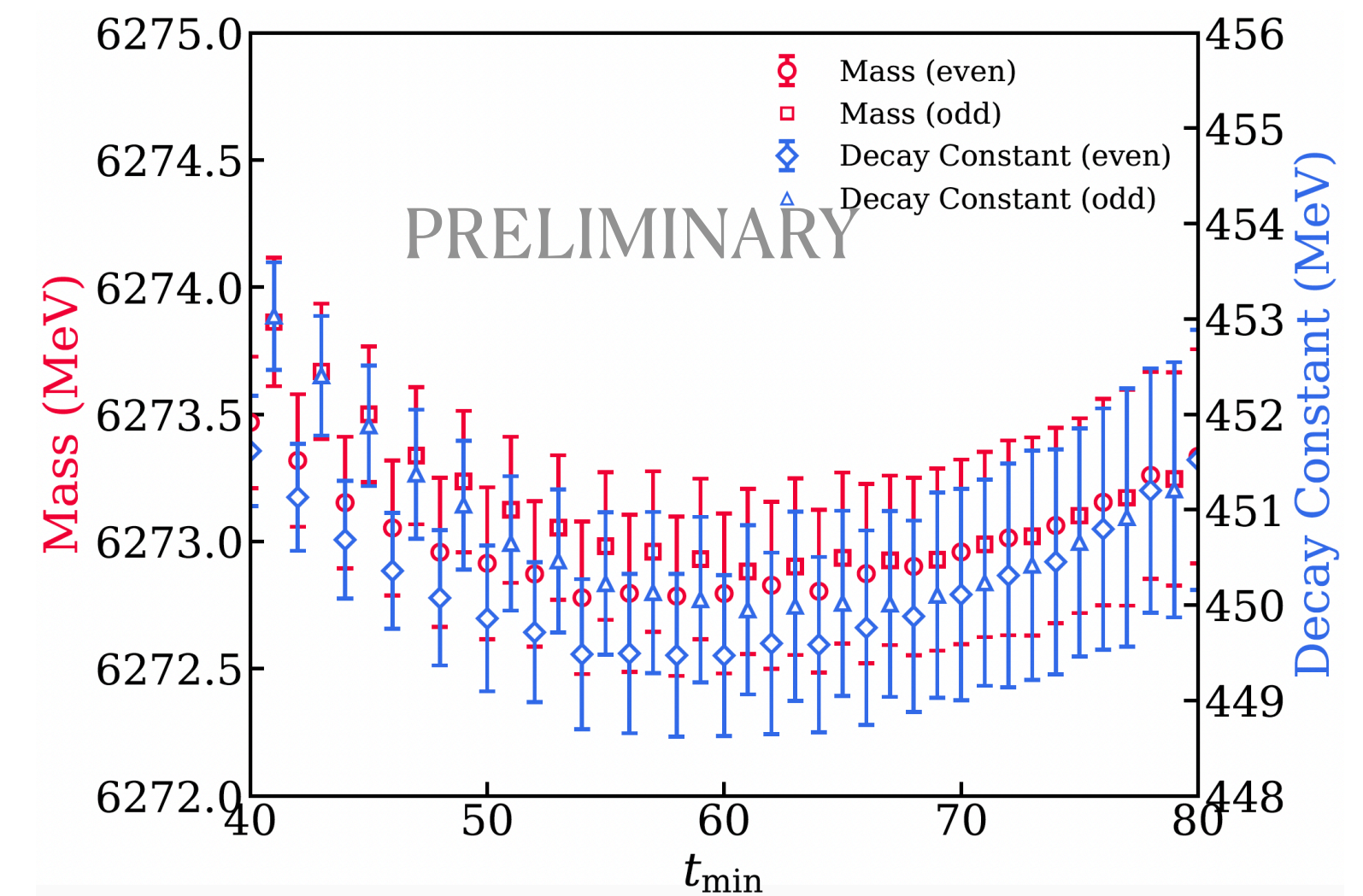
B_c signal

Effective mass constructed from smoothed correlators and shown without bootstrap/jackknife uncertainties to highlight the signal quality

$$f_{ps} = \left(m_{q_1} + m_{q_2} \right) \sqrt{\frac{2A}{E^3}}$$

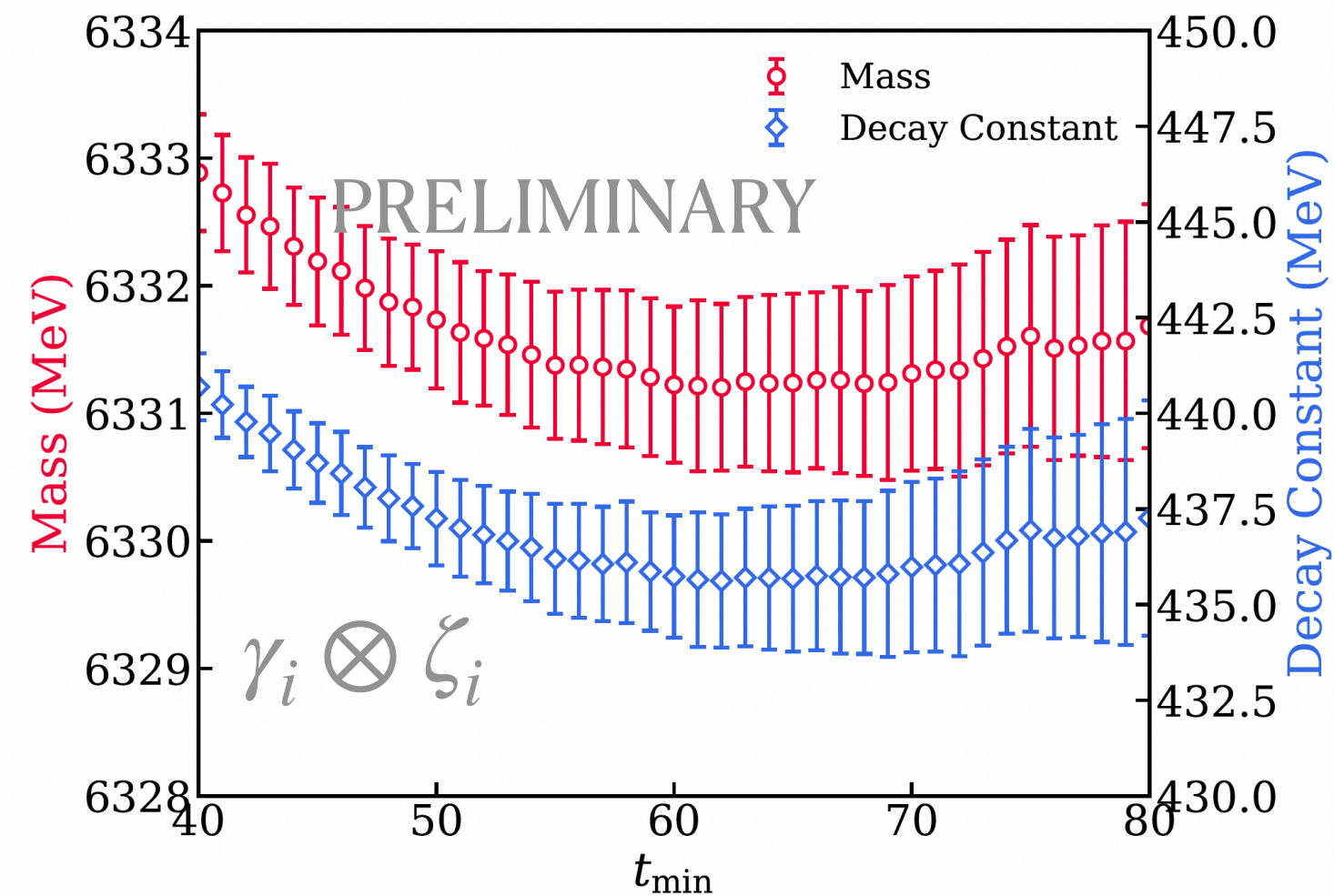


A naive check to verify that no information is lost through smoothing



Oscillating ansatz becomes increasingly noisy at large Euclidean times, single exponential fits to smooth data

B_c^* signal

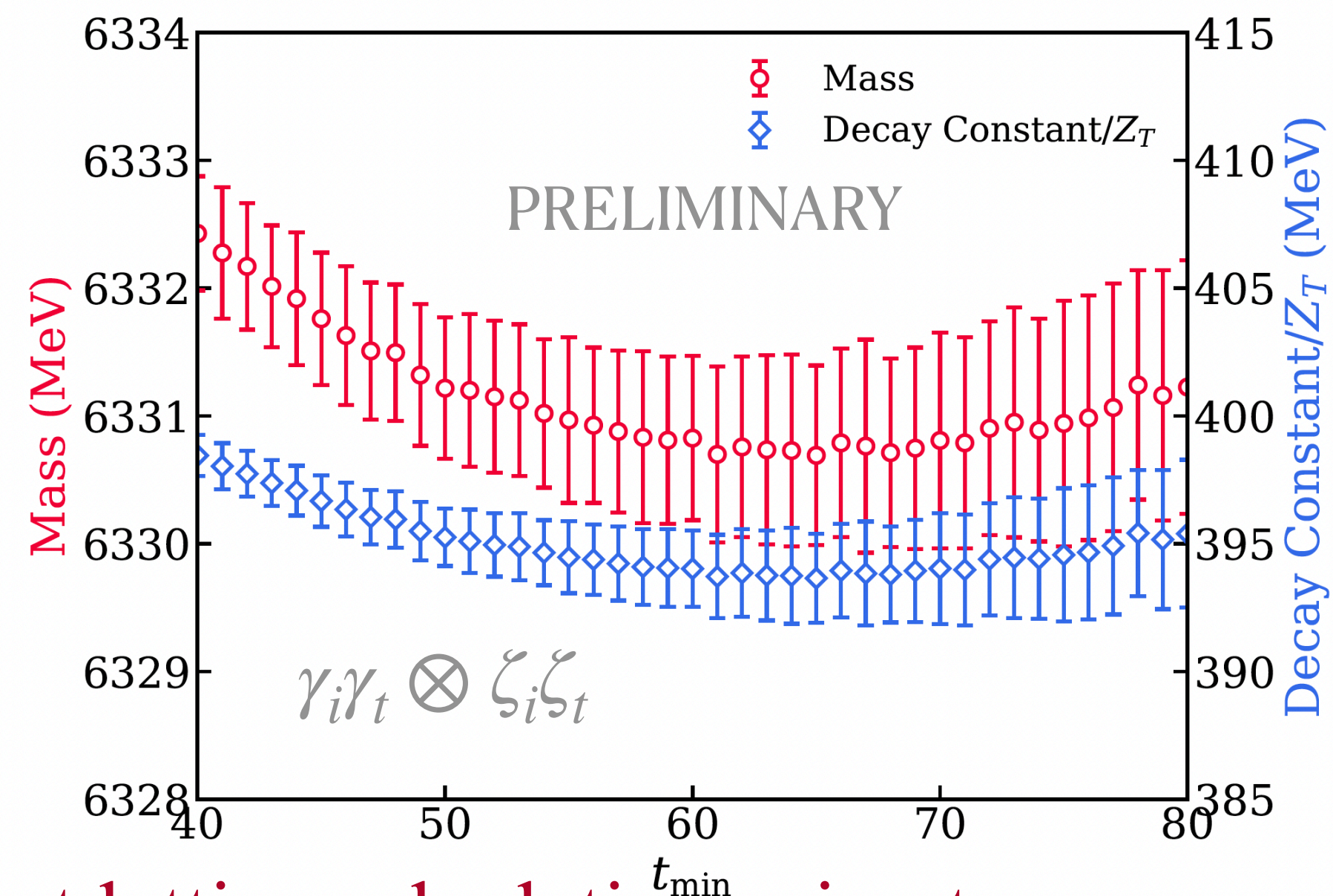


$$f_V = Z_V \sqrt{\frac{2A}{E}}$$

Z_V : PRD 102, 054511 (2020) HPQCD

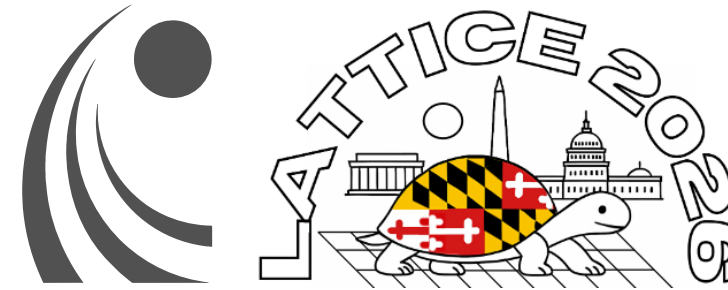
We are currently investigating the determination of Z_V

- Preliminary $B_c^* - B_c$ splitting (through ratio): 59.07(30) MeV
- We also report B_c^* using tensor interpolating operators
- Tensor currents, absent at tree level in the Standard Model, provide a sensitive probe of physics beyond the Standard Model



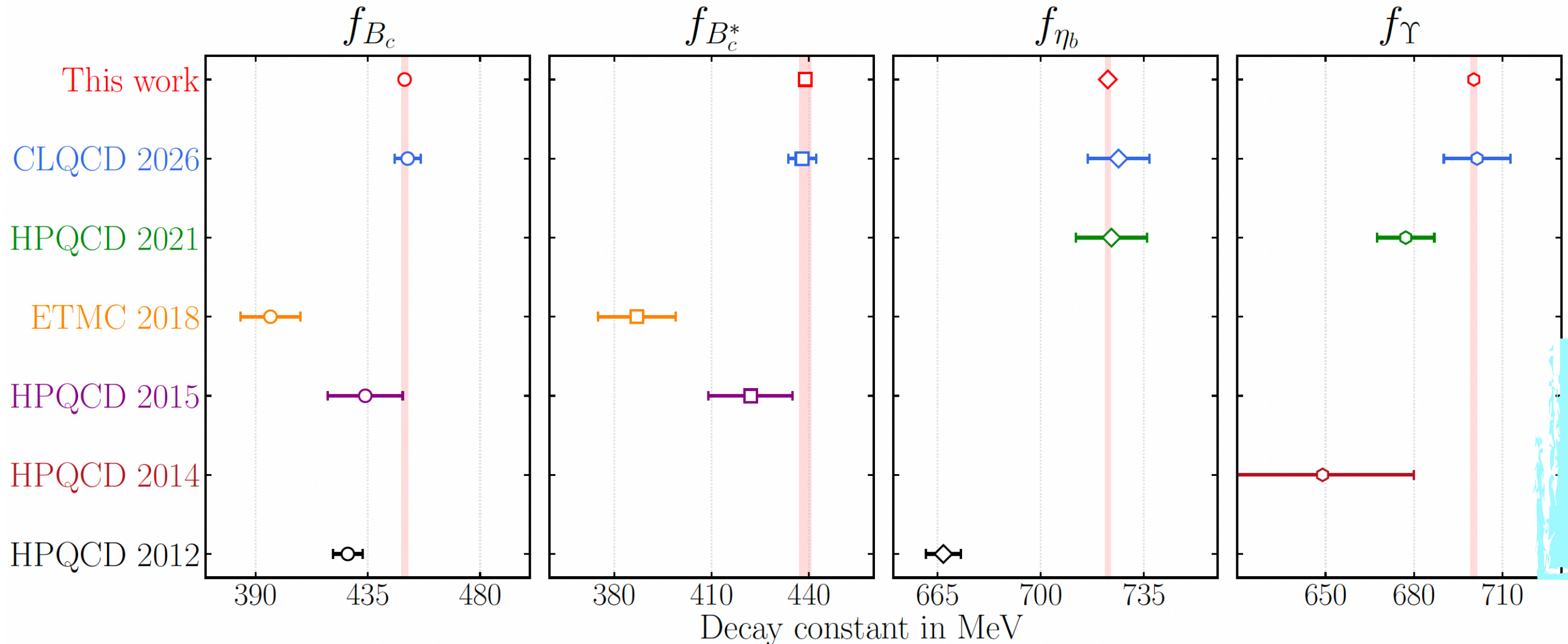
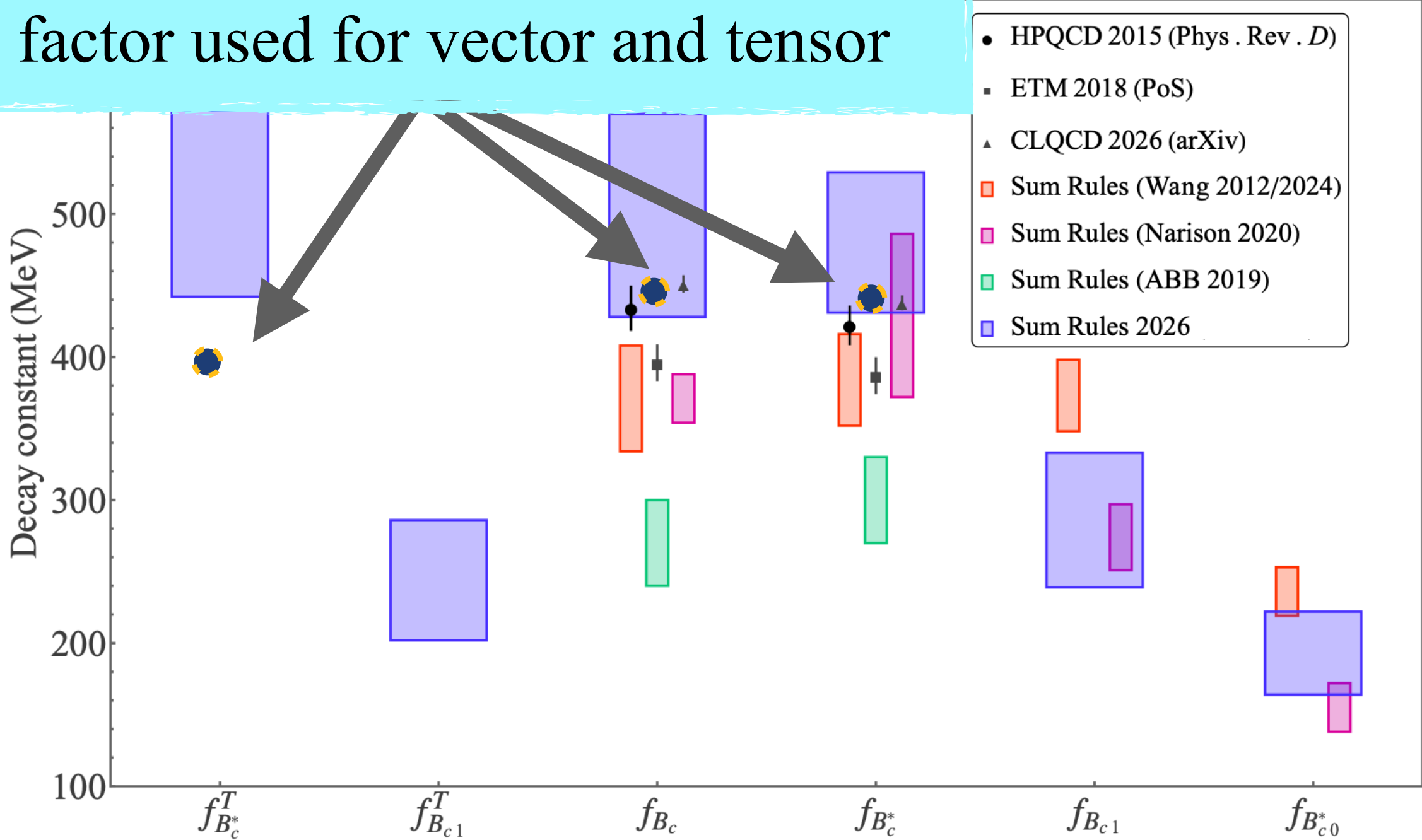
First lattice calculation using tensor operators;
currently investigating renormalisation factor

Results



Our data points (*enlarged for better visibility*)
No renormalisation factor used for vector and tensor

Hadron	Mass [in MeV]	f/Z [in MeV]
η_c	2983.55(30)	396.41(81)
η_b	9404.07(14)	722.76(68)
B_c	6272.85(34)	449.76(97)
B_c^*	6331.22(61)	438.82(161)
B_c^* (tensor)	6330.82(64)	394.02(150)
Υ	9458.14(18)	700.30(79)
Υ (tensor)	9458.27(17)	646.47(70)



A recent pheno calculation
arXiv:2606.25733[hep-ph]
Reina, Gubernari, Wüthrich

No renormalisation factor used
here for B_c^* , Υ

No systematics yet!!

Extracting More ...



So far, we have focused on single-exponential fits to extract the ground-state properties



However, correlation functions contain richer spectral information



Can be accessed using advanced spectral extraction techniques

Using these methods, we



get a validation of our ground states numbers



can validate the impact of excited-state contamination



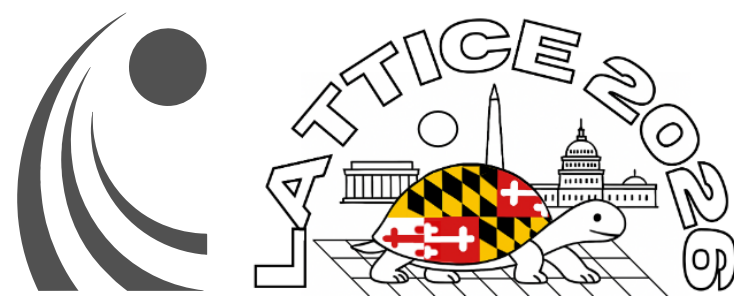
get faster convergence and improved numerical stability



We use TGEVP for further analysis

- Lanczos, **Wagman**
(PRL 134 (2025) 24, 241901)
- Prony GEVM, **Ostmeyer, Sen, Urbach**
(EPJA 61, 26 (2025))
- Transfer Matrix GEVP, **Chakraborty, Sood, Radhakrishnan, Mathur**
(PRD 112 (2025) 7, 074506)

TGEVP Setup



Truncated Krylov Subspace $\mathcal{K}_m$

$$\mathcal{K}_m \equiv \{ |\chi\rangle, \hat{\mathcal{T}} |\chi\rangle, \hat{\mathcal{T}}^2 |\chi\rangle, \dots, \hat{\mathcal{T}}^m |\chi\rangle \},$$

for $2m - 1 \leq n_T$

Transfer Matrix T_{ij}^m

$$T_{ij}^m = \langle v_i | \hat{\mathcal{T}} | v_j \rangle = C(i + j + 1) / \sqrt{C(2i) \times C(2j)}$$

One time step ahead, encodes transfer matrix action on Krylov basis

Overlap Matrix V_{ij}^m

$$V_{ij}^m = \langle v_i | v_j \rangle = C(i + j) / \sqrt{C(2i) \times C(2j)}$$

Encodes overlaps between the Krylov basis states using correlator values

Solve the Generalised Eigenvalue Problem

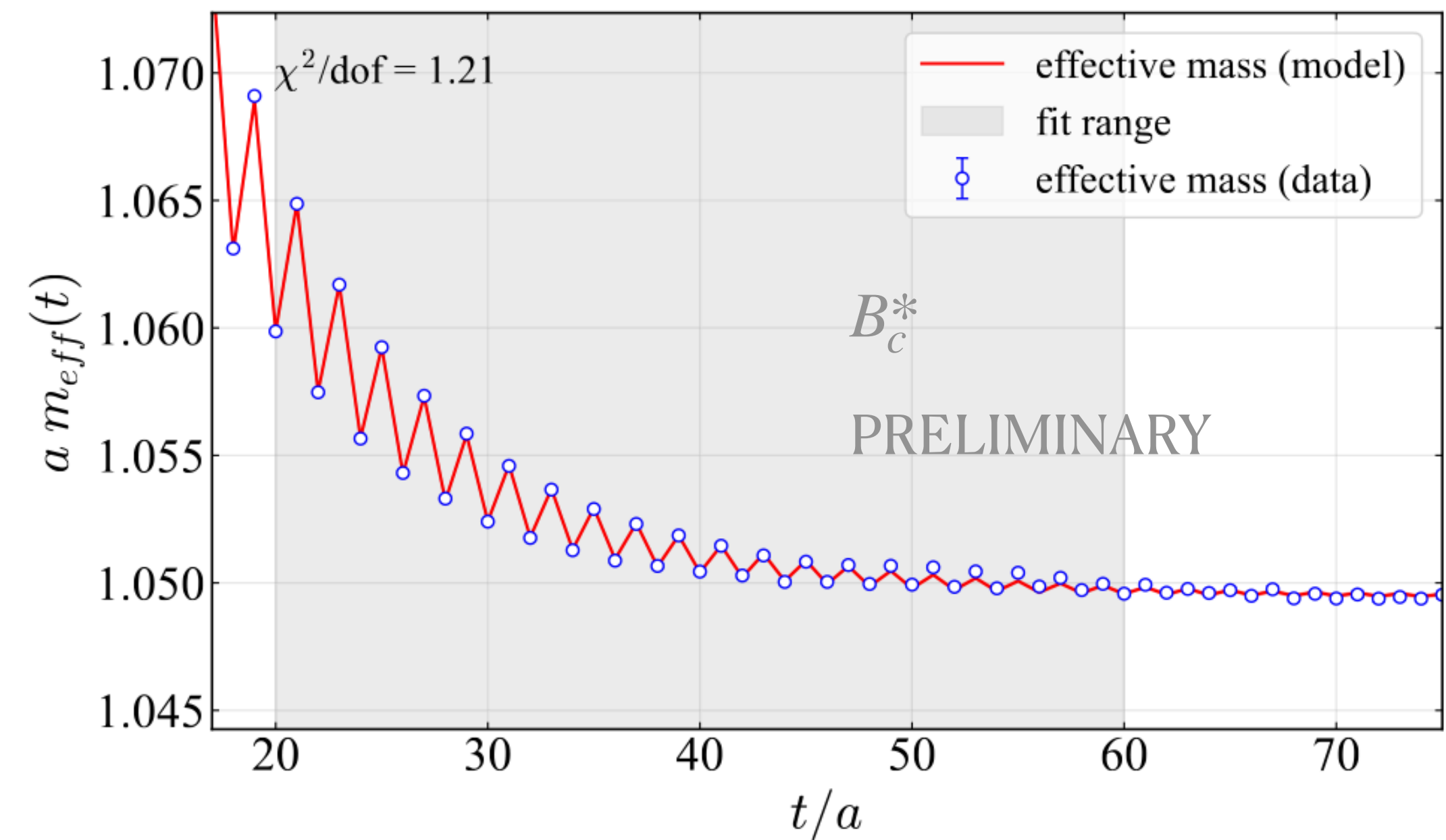
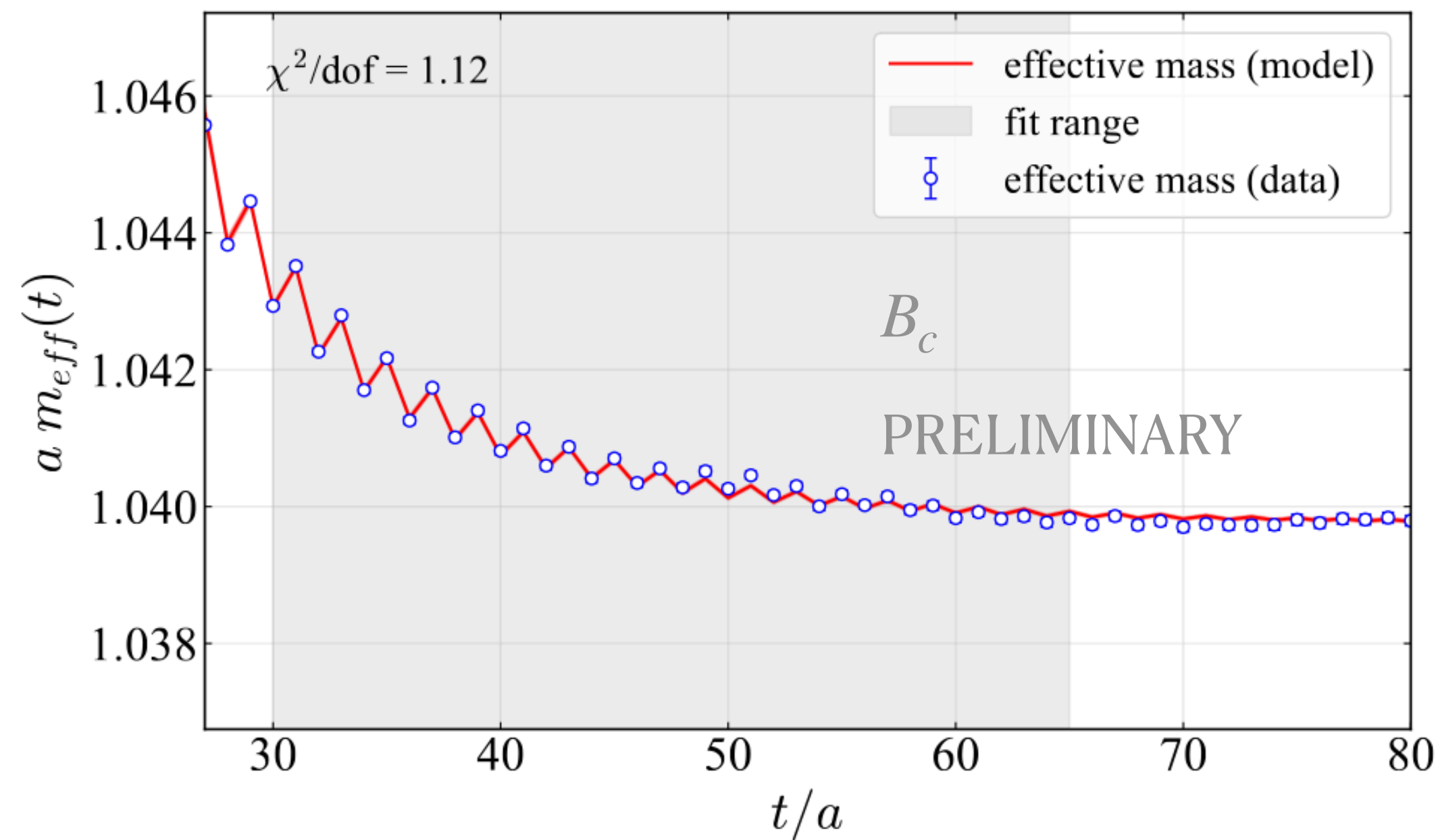
$$T^m x_n^m = \lambda_n^m V^m x_n^m \quad E_n^m = -\log \lambda_n^m$$

T^m and V^m are positive definite $\rightarrow \lambda_n$'s are real and positive $0 < \lambda_n^m \leq 1$

TGEVP Results

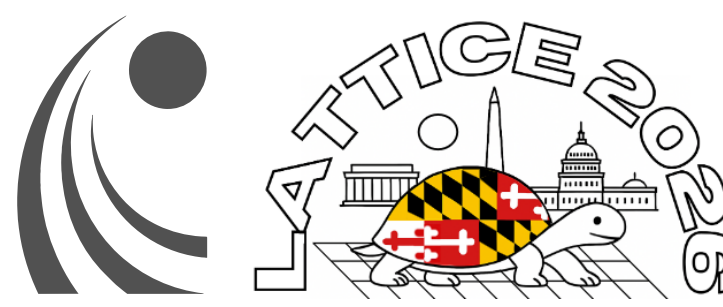
$$C(t) = \sum_n A_n \left(\exp(-E_n t) + \exp(-E_n(T - t)) \right) - (-1)^t \sum_n B_n \left(\exp(-E_n^o t) + \exp(-E_n^o(T - t)) \right)$$

Ansatz incorporates four non-oscillating and three oscillating states



Ground-state information obtained from the late-time region is used to constrain fits at early times, where excited-state contributions are significant

TGEVP Results



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Hadron	State	Mass [in MeV]	f/Z [in MeV]
η_c	1S	$2983.83 \left({}^{+0.17}_{-0.17} \right)$	$395.91 \left({}^{+0.51}_{-0.59} \right)$
	2S	$3665 \left({}^{+55}_{-60} \right)$	—
η_b	1S	$9406.77 \left({}^{+0.10}_{-0.09} \right)$	$723.07 \left({}^{+0.40}_{-0.41} \right)$
	2S	$9989 \left({}^{+47}_{-49} \right)$	—
B_c	1S	$6274.40 \left({}^{+0.20}_{-0.19} \right)$	$449.47 \left({}^{+0.54}_{-0.58} \right)$
	1P (0^+)	$6708 \left({}^{+11}_{-14} \right)$	—
	2S	$6854 \left({}^{+63}_{-58} \right)$	—
B_c^*	1S	$6332.67 \left({}^{+0.38}_{-0.39} \right)$	$438.81 \left({}^{+1.11}_{-1.12} \right)$
	1P (1^+)	$6745 \left({}^{+15}_{-16} \right)$	—
B_c^* (tensor)	1S	$6332.47 \left({}^{+0.43}_{-0.44} \right)$	$394.15 \left({}^{+1.08}_{-1.04} \right)$
	1P (1^+)	$6744 \left({}^{+15}_{-17} \right)$	—
Υ	1S	$9460.88 \left({}^{+0.11}_{-0.11} \right)$	$700.63 \left({}^{+0.51}_{-0.54} \right)$
	1P (χ_{b1})	$9897.4 \left({}^{+8.4}_{-7.9} \right)$	—
	2S	$10016 \left({}^{+32}_{-32} \right)$	$465 \left({}^{+45}_{-47} \right)$
Υ (tensor)	1S	$9460.98 \left({}^{+0.11}_{-0.11} \right)$	$646.94 \left({}^{+0.45}_{-0.47} \right)$
	1P (χ_{b1})	$9901.6 \left({}^{+8.2}_{-9.1} \right)$	—
	2S	$10001 \left({}^{+31}_{-33} \right)$	—

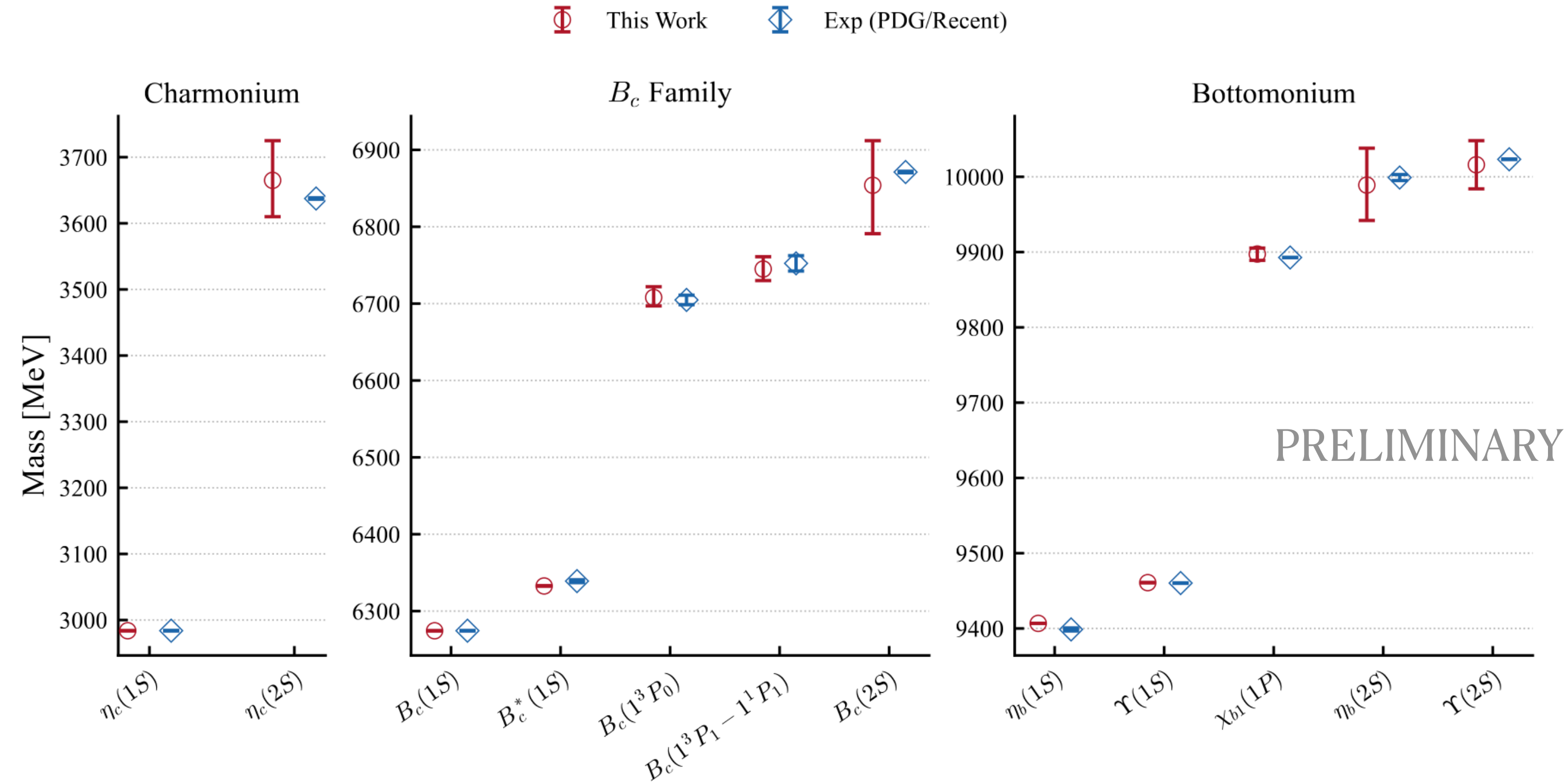
TGEVP numbers

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Single exponential fits

1-2 MeV difference in some cases
 t_{min} dependence
 TGEVP more clean?.....

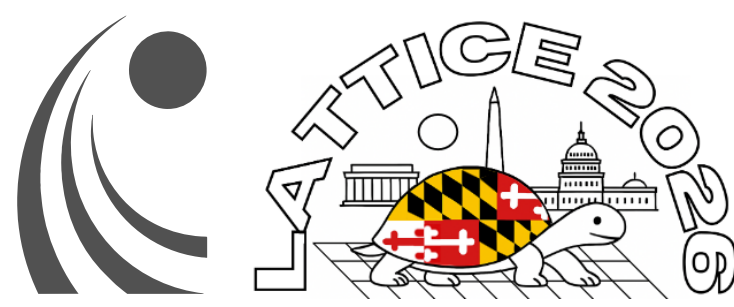
Summary TGEVP



► Excited-state decay constants:
work in progress (more statistics
and systematics underway)

- Excited-state spectrum is well established in lattice QCD
- Single-operator TGEVP provides a complementary cross-check of excited-state contamination and extraction

Summary



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- ☒ Performed a high-precision lattice QCD calculation of the B_c meson family decay constants using fully relativistic $N_f = 2 + 1 + 1$ HISQ ensembles
- ☒ Obtained preliminary results:
$$f_{B_c} = 449.76(97)(?) \text{ MeV}, f_{B_c^*}/Z = 438.82(161)(?) \text{ MeV}, f_{B_c^*}^T/Z = 394.02(150)(?) \text{ MeV}$$
- ☒ Implemented TGEVP to strengthen extraction of ground-state and enable a systematic study of excited-state properties
- ☐ To increase statistics by analyzing more configurations; investigate renormalisation factors
- ☐ A complete systematic error budget for ground- and excited-state B_c spectroscopy

Thanks to the MILC Collaboration, especially Steven Gottlieb, for providing the configurations

THANK YOU



Navdeep Singh Dhindsa

30/07/26



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