Static energies in the Z_b tetraquark channel using novel BOEFT interpolators

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in collaboration with
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Motivation

- Explore the $I = 1$ hidden-heavy Z_b ($\bar{b}b\bar{q}q$) tetraquark channel
- $J^{PC} = 1^{+[-]}$
- Relevant for $Z_b(10610)$ and $Z_b'(10650)$ (near degenerate) exotic states discovered by Belle.
- Both states lie close to the $B\bar{B}^*$ and $B^*\bar{B}^*$ threshold, respectively

Belle, A. Bondar et al., [arXiv:1110.2251], Belle, A. Garmash et al., [arXiv:1403.0992]

- Controlled determination of whether the states observed in the Zb tetraquark channel correspond to shallow or deeply bound states, or to a near-threshold resonance
- In particular, use a new interpolator provided by Born-Oppenheimer Effective Field Theory (BOEFT), which is proposed to have good overlap with the tetraquark states.

M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

Background

- BOEFT exploits the energy hierarchy between heavy quarks and the light degrees of freedom (LDF) (light quarks and gluons) of QCD.

M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

- In the hidden-heavy sector, it provides an elegant alternative to the extremely challenging Lüscher's method by facilitating the extraction of the static energies for infinitely heavy quarks in the presence of LDF on the lattice.
- BOEFT gives, in addition to the spectrum, physical insights into the binding mechanisms of QCD in the form of the potentials. It can probe QCD at both short-distance and large-distance scales.
- Existing lattice QCD calculations on string breaking and hybrid static potentials are compatible with the BOEFT framework.

[Juge, Kuti, Morningstar 2003]; [Bali, Neff, Duessel, Lippert, Schilling, 2005]; [Capitani, Philipsen, Reisinger, Riehl, Wagner 2019]; [Höllwieser, Knechtli, Korzec, Peardon, Struckmeier, Urrea-Niño, 2025]

Background

- The first step is to calculate static energies with infinitely heavy quarks on the lattice.
- The heavy quarks are static and supply a color source.
- Different configurations of the LDF (light quarks and gluons) identify different static energies characterised by the quantum numbers of the LDF $\implies$ BO quantum numbers, Λ_η^σ, which are the representations of the cylindrical symmetry group.

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- Different configurations of the LDF (light quarks and gluons) identify different static energies characterised by the quantum numbers of the LDF $\implies$ BO quantum numbers, Λ_η^σ, which are the representations of the cylindrical symmetry group.
- The second step is to plug these static energies into the coupled Schrödinger equations in the framework of BOEFT to extract the spectrum of XYZ states comprising two heavy quarks or one heavy quark-antiquark pair.

M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

Experimental observation and BOEFT converging

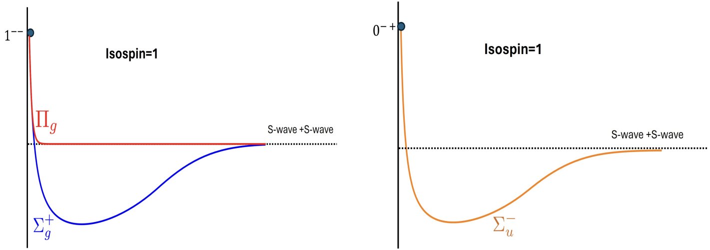
- Experimental Observation:
  - $Z_b'(10650)$ lies very close to $B^*\bar{B}^*$ threshold
  - $Z_b(10610)$ lies very close to $B\bar{B}^*$ threshold
  - Suppression of Z_b' decay to $B\bar{B}^*$
- If the physical Z_1 and Z_2 tetraquarks obtained as a result of solving the coupled Schrödinger equations are close, then Z_b and Z_b' with $J^{PC} = 1^{+-}$ are equal superpositions of Z_1 and Z_2 with 1^{--} and 0^{-+} as LDF quantum numbers respectively:

$$|Z_b'\rangle = \frac{1}{\sqrt{2}}\Bigg(\underbrace{|S_{b\bar{b}} = 0\rangle|K^{PC} = 1^{--}\rangle}_{Z_1} - \underbrace{|S_{b\bar{b}} = 1\rangle|K^{PC} = 0^{-+}\rangle}_{Z_2}\Bigg)$$

$$|Z_b\rangle = \frac{1}{\sqrt{2}}\Bigg(|S_{b\bar{b}} = 0\rangle|K^{PC} = 1^{--}\rangle + |S_{b\bar{b}} = 1\rangle|K^{PC} = 0^{-+}\rangle\Bigg)$$

- The experimental observation suggests that the LDF (adjoint mesons) associated with Z_1 and Z_2 are degenerate, a result supported by lattice calculations. Sharma, Urrea-Niño, et. al., [arXiv:2603.29581 [hep-lat]]

We investigate the Z_2 tetraquark (Σ_u^-) channel



M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

- Only one BO potential (Σ_u^-) associated with the adjoint meson, $k^{PC} = 0^{-+}$, corresponding to $Z_2 \implies$ **Relatively simple**
- Two BO potentials (Σ_g^+, Π_g) in the Z_1 tetraquark channel associated with the adjoint meson, $k^{PC} = 1^{--}$.

Ingredients needed for the lattice calculation

- A pair of static quark-antiquark ($\bar{Q}(0)Q(r)$) separated by a distance r and a pair of light quark-antiquark ($\bar{q}q$).
- Some freedom to choose the spatial configuration of $\bar{q}q$.

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- Static limit:
  - Static quarks do not propagate spatially.
  - $\mathcal{L} = Q^\dagger(i\partial_0 + gA_0)Q$ where Q is two component field.
  - On lattice, the static propagator contains a phase which is just a temporal Wilson time.
    $S_Q(x,y) = \theta(x_0 - y_0)\delta(\vec{x} - \vec{y})U(x,y)P_+$, where $P_+ = \frac{1}{2}(1+\gamma_0)$. Kurth and Sommer, arXiv:hep-lat/0007002
  - Dependence on the heavy quark mass gets traded for a lattice self-energy which diverges as $a \to 0$.
    $E_{\text{stat}} \sim E_{\text{self}} + \mathcal{O}(a^0) \sim \frac{1}{a}e^{(1)}g_0^2 + ...$ Morte, Shindler, Sommer, arXiv:hep-lat/0506008
  - To improve the signal, we use HYP2 smearing on temporal Wilson lines, which is known to provide the largest reduction in $e^{(1)}$. Donnellan, Knechtli, Leder, Sommer, arXiv:1012.3037 [hep-lat]; Hasenfratz and Knechtli, arXiv:hep-lat/0103029

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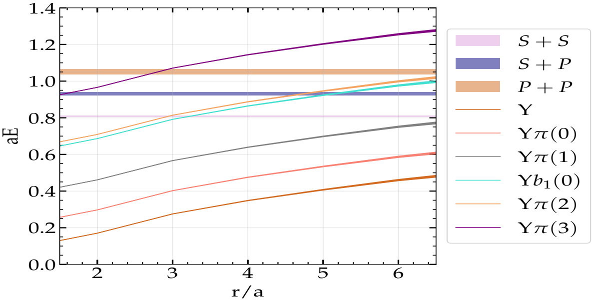
- Analysis was done on 255 gauge configurations part of a CLS ensemble with $a = 0.08636$ fm, $N_f = 2 + 1$, $m_\pi = 290$ MeV, $24^3 \times 128$ lattices with open boundary conditions in time.
- We added 15 levels of 3D HYP smearing to the spatial links.

Type of states to expect in the Σ_u^- channel



- The experimental tetraquark states lie close to the $S + S$ threshold.
- However, there is a tower of non-interacting QCD states that lie below this threshold, particularly at smaller values of r, and should be included in the basis, and their number increases with the volume.

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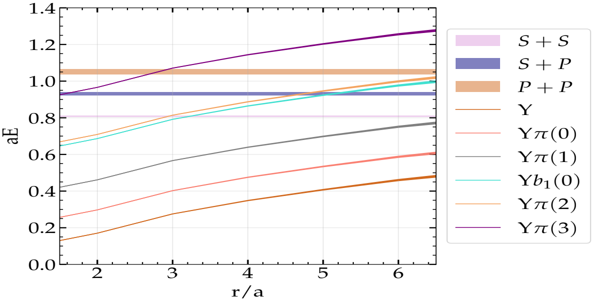

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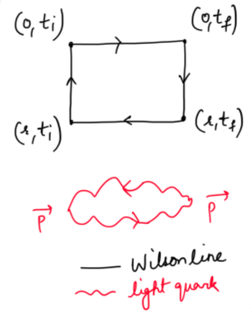
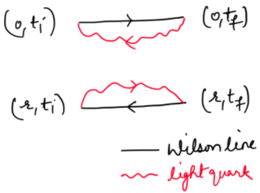
- However, there is a tower of non-interacting QCD states that lie below this threshold, particularly at smaller values of r, and should be included in the basis, and their number increases with the volume.
- We expect to show that the lowest-lying tetraquark state can be disentangled only with O_{BO} without worrying about the low-lying QCD states.

Diagonal correlators of traditionally used interpolators



- Operator class (i): $O^{\Upsilon\pi(\vec{p})}$-like
- $[\bar{b}(0)U\gamma_z P_+ b(r)][\bar{q}\gamma_5 q]_{\vec{p}}$
- $[\bar{q}\Gamma' q]_{\vec{p}} \equiv \frac{1}{V}\sum_{\vec{x}} \bar{q}(\vec{x})\Gamma' q(\vec{x})e^{i\vec{p}\vec{x}}$
- Momentum is in units of $2\pi/L$.
- Spatial Wilson lines are used to make the operator gauge invariant.



- Operator class (ii): $O^{B\bar{B}^*}$-like
- $\Gamma^1_{BA}\Gamma^2_{CD}\bar{b}^a_C(0)q^a_A(0)\bar{q}^b_B(r)b^b_D(r)$
- Spin Fierz transformations needed to contract color (a, b) indices similar to spin (A, B, C, D).
- Γ^1 and Γ^2 determine the quantum numbers.

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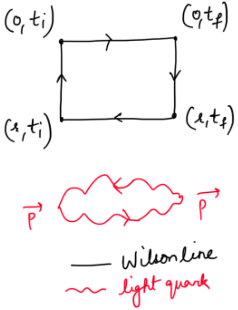
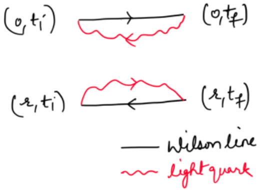
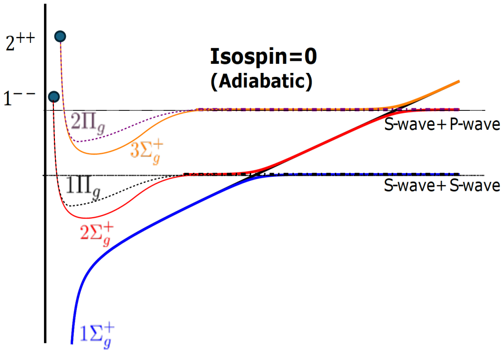


- Operator class (ii): $O^{B\bar{B}^*}$-like



- So far, inclusion of class (i) and class (ii) operators in the GEVP basis provided as output only the low-lying $\Upsilon\pi(\vec{p})$-type non-interacting levels at small r values ($\lesssim 0.5$ fm). [Peters, Bicudo, Cichy, Wagner , arXiv: 0905.2160 [hep-lat]]; [Prelovsek, Bahtiyar, Petkovic , arXiv:1912.02656 [hep-lat]]; [Sadl,Prelovsek, [arXiv:2109.08560 [hep-lat]]

Novel tetraquark interpolator provided by BOEFT



M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

- As $r \to \infty$, states with 3 and $\bar{3}$ sources should become pairs of a $3-$hadron and a $\bar{3}-$hadron (or a pair of static-light mesons).
- As $r \to 0$, the 3 and $\bar{3}$ sources reduce to a linear combination of $1-$source and $8-$source.

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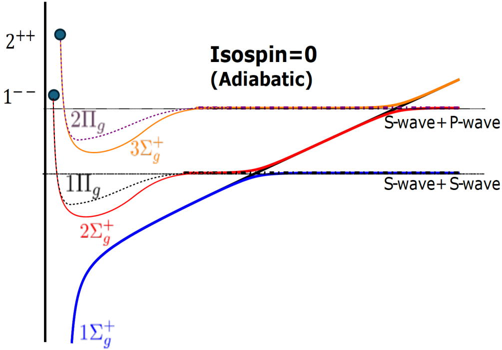

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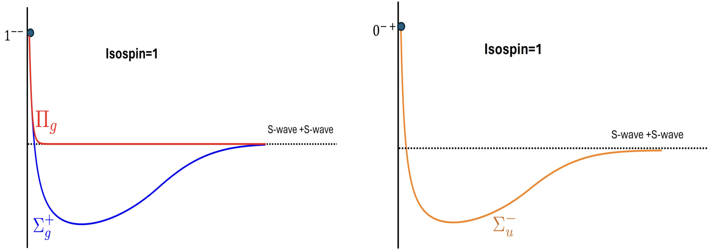
- For $Q\bar{Q}$ (Σ_g^+), BO potentials should smoothly connect $1-$potential at small r to $(3+\bar{3})-$potential at large r.
- Rest $(3+\bar{3})-$potentials at large r should connect to $8-$potential at small r.

Novel tetraquark interpolator: Operator class (iii)



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- $O_{BO} = \bar{Q}(0)\Gamma_1\, U(0, r/2)\, T^a\, \underbrace{\bar{q}(r/2)\Gamma_2 T^a q(r/2)}_{\text{adjoint meson}\equiv O^a_{AM}}\, U(r/2, r)Q(r)$

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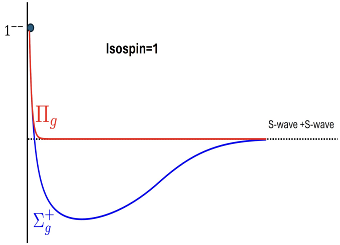

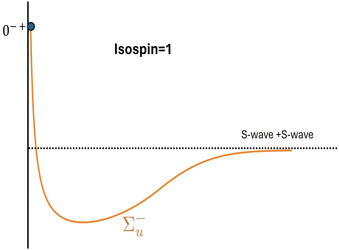

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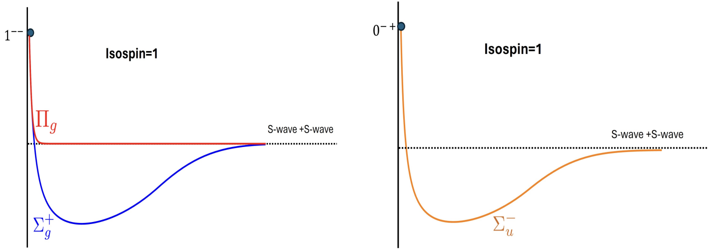
- For short distances, at the leading order, $\langle O_{BO}|\bar{Q}Q\rangle$ will result in a Tr $(T^a) = 0$.

Novel tetraquark interpolator: Operator class (iii)



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- At each r, ideally, $E_{O_{BO}}$ can be extracted from the effective mass plateaus of the correlator $\langle O_{BO}(t)|O^\dagger_{BO}(0)\rangle$

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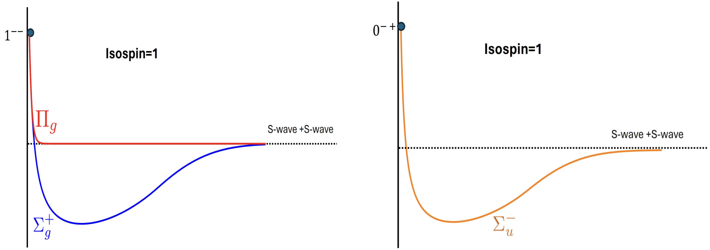

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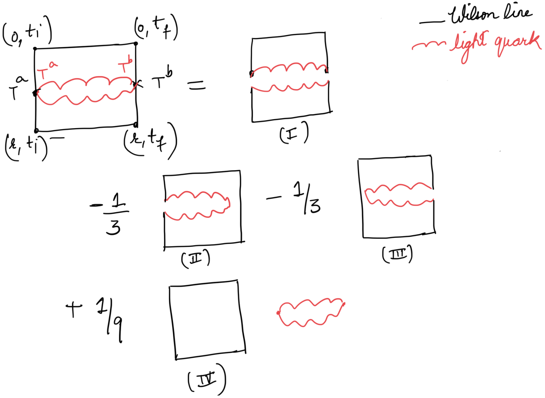
- For $r \to 0$, $E_{O_{BO}} = V_8 + \Lambda_{AM} + \mathcal{O}(r^2)$, where Λ_{AM} is the adjoint meson mass.

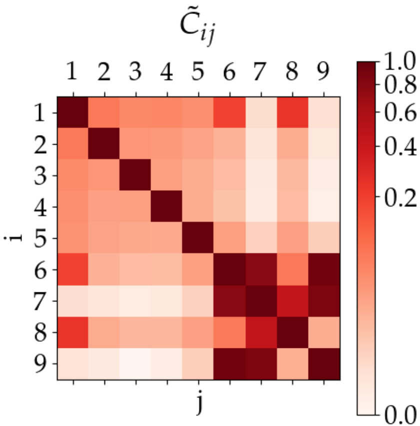
Novel tetraquark interpolator: Operator class (iii)

- The novel tetraquark interpolator is proposed because it has zero overlap with quarkonium and pion type states.
- Use $2\, T^a_{ik} T^a_{jl} = \delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ik}\delta_{jl}$ to express its diagonal correlator in terms of fundamental Wilson lines.



Time averaged correlation matrix

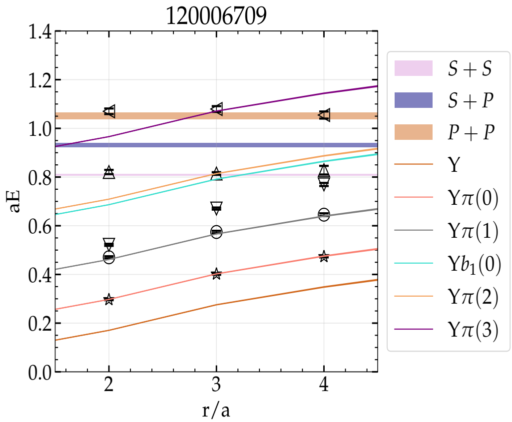
- We constructed a 9×9 correlation matrix, with
  - Class (i)
    - Op1,2,3,4: $\Upsilon\pi(0,1,2,3)$
    - Op5: $\Upsilon b_1(0)$
  - Class (ii)
    - Op6: $B\bar{B}^*$ (S+S)
    - Op8: $B_0^*\bar{B}_1^*$ (P+P)
  - Class (iii)
    - Op7: O_{BO}(S+S and P+P)
    - Op9: O_{BO}(S+S)
- $\tilde{C}_{ij}(r) = \dfrac{|\sum_{t=1}^{n_t} C_{ij}(r,t)|}{\sqrt{|\sum_{t=1}^{n_t} C_{ii}(r,t)||\sum_{t=1}^{n_t} C_{jj}(r,t)|}}$



$r/a = 2$

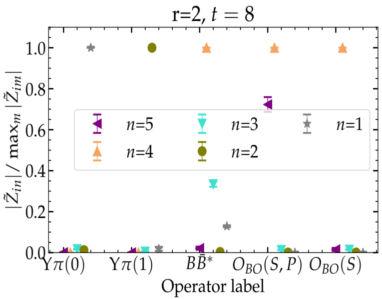
GEVP on 5x5 correlation matrix

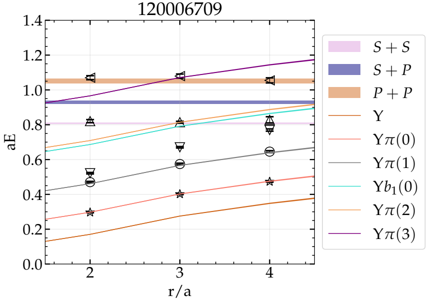
- Class (i)
  - Op1: $\Upsilon\pi(0)$
  - Op2 $\Upsilon\pi(1)$
- Class (ii)
  - Op6: $B\bar{B}^*$ (S+S)
- Class (iii)
  - Op7: O_{BO}(S+S and P+P)
  - Op9: O_{BO}(S+S)
- Colours represent non-interacting energies and black markers show lattice results.



- 4 out of static energies ($n = 1, 2, 4, 5$) can be understood using the non-interacting curves. $n = 3$ (the downward triangles) is not so trivial.

Overlaps at $r/a = 2$





- $\tilde{Z}_{in}(t) = \dfrac{e^{E_n t/2}}{|(u^n(t), C(t)u^n(t))|^{\frac{1}{2}}} C_{ij}(t) u_j^n(t)$
- $n = 1$ and $n = 2$ are singlet states. ✓
- $n = 3$ has some overlap with $i = 3$ ($B\bar{B}^*$). **Singlet state?**
- $n = 4$ has max overlap with both O_{BO} and $B\bar{B}^*$. **Tetraquark state**
- O_{BO}-type operators do not have overlap with singlet eigenenergies. ✓

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- This state has overlap with $B\bar{B}^*$ operator which couples with all types of pionic excitations.
- $n = 3$ has large excited state contamination.
- Our GEVP basis is incomplete. We are yet to include all the lowest-lying $\Upsilon\pi(\vec{p})$ operators.
- As an example: our operator for $\Upsilon\pi(\vec{p} = 1)$ only has longitiudnal momentum profile. The static quark-antiquark pair is separated along the z direction.
  $O_2 = O^{\Upsilon\pi(1)} \propto [\bar{b}(0)U\gamma_z P_+ b(r)]\ \left([\bar{q}\gamma_5 q]_{\vec{p}=\vec{e}_z} + [\bar{q}\gamma_5 q]_{\vec{p}=-\vec{e}_z}\right)$
- A transverse momentum profile can also have the same quantum numbers.

How to interpret $n = 3$?

- Upon adding a static quark axis, the symmetry group changes from O_h to D_{4h}.
- For O_h, momentum profiles $(\pm n, 0, 0)$, $(0, \pm n, 0)$, $(0, 0, \pm n)$, have different little groups (C_{4x}, C_{4y}, C_{4z}), but these little groups are equivalent as they are related by C_{4v} rotations $\implies$ All profiles belong to the same momentum star. Moore and Fleming [arXiv:hep-lat/0507018v3]

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- When the static quark axis is fixed along the z-direction, the little group of $(0, 0, \pm n)$ *i.e.,* C_{4z}) is no longer equivalent to the little groups of $(\pm n, 0, 0)$, $(0, \pm n, 0)$ which are C_{2x} and C_{2y}, respectively $\implies$ two different momentum stars.

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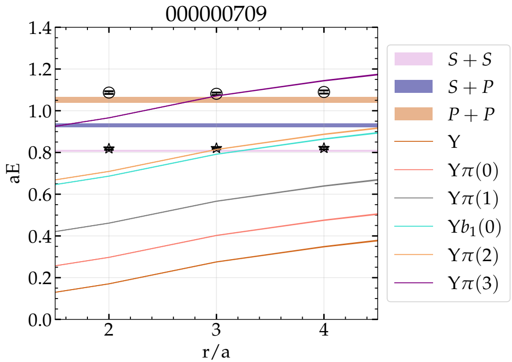
- Following operator also transforms as Σ_u^-:
  $[\bar{b}(0) U \gamma_z P_+ b(r)] \left([\bar{q}\gamma_5 q]_{\vec{p}=\vec{e}_x} + [\bar{q}\gamma_5 q]_{\vec{p}=-\vec{e}_x} + [\bar{q}\gamma_5 q]_{\vec{p}=\vec{e}_y} + [\bar{q}\gamma_5 q]_{\vec{p}=-\vec{e}_y} \right)$
- And there are more ...

Good news: GEVP on 2x2 correlation matrix

- Class (iii)
  - Op7: O_{BO}(S+S and P+P)
  - Op9: O_{BO}(S+S)
- Colours represent non-interacting energies and black markers show lattice results.



- Due to the decoupling of the octet and singlet type energies, we can only use O_{BO} type operators to obtain the lowest-lying tetraquark state.
- The states are stable with respect to the variation of the GEVP basis.

- Conclusions
  - BOEFT provides interpolators that do not couple with Quarkonium plus pion-type states.
  - Overlaps extracted using GEVP provide evidence for the decoupling of octet and singlet systems.
  - This decoupling can be exploited to extract the spectrum of all XYZ exotics using interpolators provided by BOEFT.
- Outlook
  - Add all the singlet-type operators and compute the full GEVP spectrum at short distances $\implies$ show decoupling of octet and singlet systems rigorously.
  - Improve the signal by more self-averaging.
  - Go to smaller lattice spacings to capture the deviation of the tetraquark state from the S-wave+S-wave threshold.
  - Calculate static energies in the Z_1 tetraquark channel.