# $B_s \to Kl\nu$ semileptonic form factors from lattice QCD with domain-wall heavy quarks

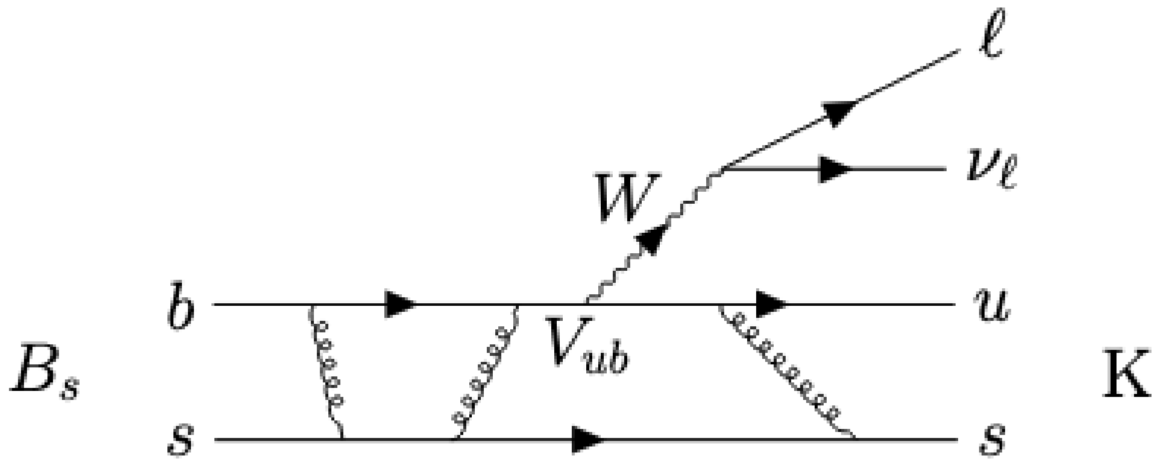
**Junya Tokoro**[a]

Hidenori Fukaya[a], Shoji Hashimoto[b,c], Takashi Kaneko[b,c]
(JLQCD collaboration)

[a] The University of Osaka, [b] KEK, [c] SOKENDAI

# Motivation

We study the process
$B_s \to Kl\nu$
to determine $|V_{ub}|$




This is an independent exclusive approach to $|V_{ub}|$ from $B \to \pi l\nu$

The absence of valence $\pi$ may lead to better controlled chiral extrapolation

LHCb combines $B_s \to K\mu\nu$ and $B_s \to D_s\mu\nu$ to determine $|V_{ub}/V_{cb}|$
Lattice QCD analysis for both processes are essential

# Setup

2+1 flavor Mobius Domain Wall Fermion (+ relativistic valence b quark)

| $a$ [fm] | $\beta$ | $L^3 \times N_t \times L_s$ | $N_{\mathrm{cfg}}$ | $am_l$ | $am_s$ | $am_Q$ |
|---|---|---|---|---|---|---|
| 0.080 | 4.17 | $32^3 \times 64 \times 12$ | 100 | 0.0190 | 0.0400 | 0.44037, 0.55046, 0.68808 |

$a^{-1} = 2.453[\mathrm{GeV}]$ $\quad$ $m_{res} \cong 1[\mathrm{MeV}]$

$M_\pi \cong 500$[MeV], $m_Q < m_b$ → Both chiral and heavy quark extrapolations are needed
→Ensembles at finer lattice spacing, $a = 0.055, 0.044$[fm] are available

The heavy quark masses are restricted to $am_Q < 0.7$ to control discretization errors
→ In $B \to \pi l \nu$(203.04938), $B \to D^* l \nu$(2306.05657) analysis, these errors were few%

For 3pt functions, There are 5 types of "source-sink separations"
→We use 2 types to check the ground state saturation in this report

# Form factors

We can calculate the form factors from the matrix elements, $\langle K(\vec{p}_K)|(\bar{u}\gamma_\mu b)(\vec{q},t)|B_s(\vec{0})\rangle$

$$f_\parallel(q^2) = \frac{\langle K(\vec{p}_K)|(\bar{u}\gamma_0 b)(\vec{q},t)|B_s(\vec{0})\rangle}{\sqrt{2M_{B_s}}}, \qquad f_\perp(q^2) = \frac{\langle K(\vec{p}_K)|(\bar{u}\gamma_i b)(\vec{q},t)|B_s(\vec{0})\rangle}{p_{K\,i}\sqrt{2M_{B_s}}}$$

We consider 2 methods to extract these matrix elements

- Direct method:
Extract the ground-state matrix element directly by fitting the 3pt functions

- Ratio method:
Construct ratios of 2pt and 3pt functions to cancel time dependence
and the current renormalization factors

# 2pt function

2pt function :

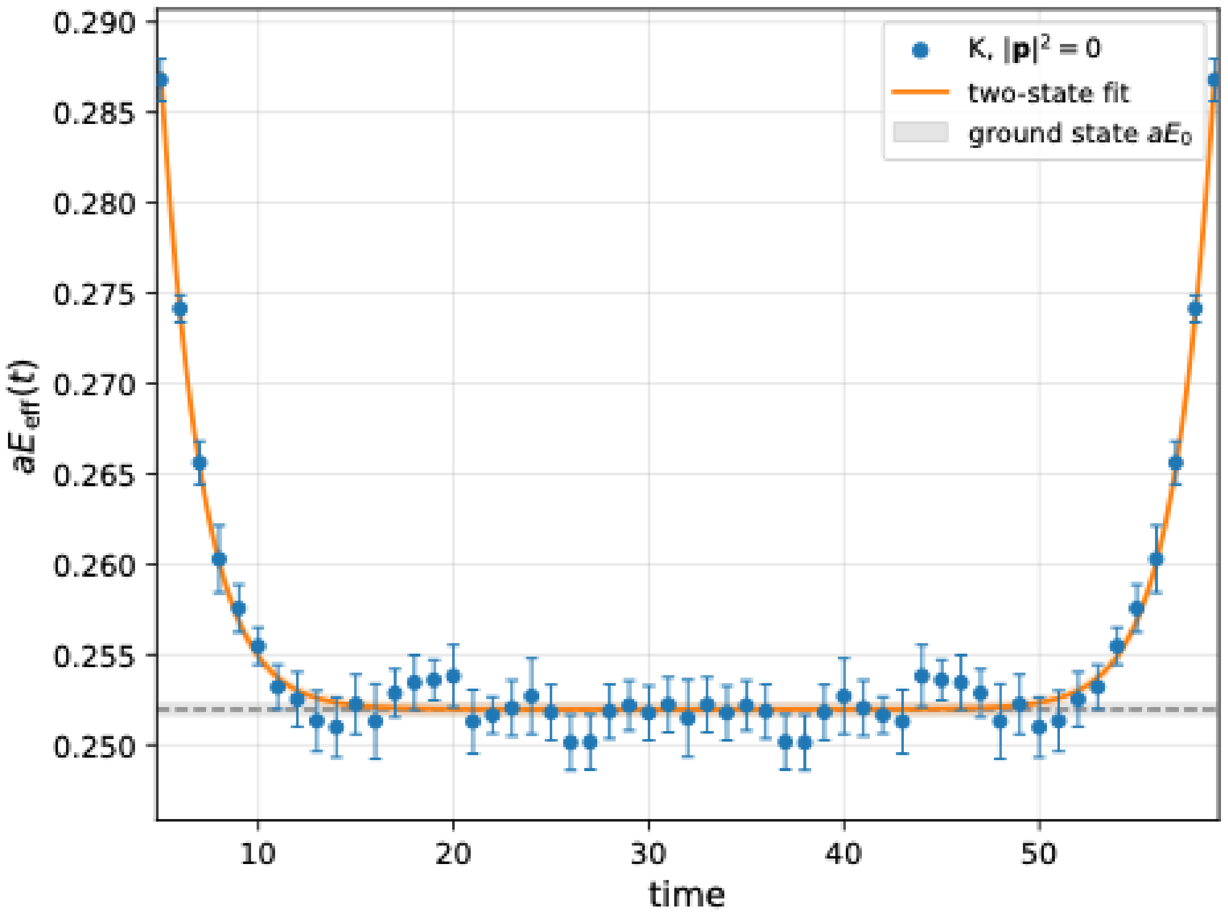
$$C_2^M(t,\vec{p}) = \sum_x \langle 0 | O_M(\vec{x},t) O_M^\dagger(\vec{0},0) | 0 \rangle \, e^{-i\vec{p}\vec{x}}$$
$$= \sum_{n=0} A_n^M(\vec{p}) \left( e^{-E_{p,n}^M t} + e^{-E_{p,n}^M (N_t - t)} \right)$$

$N_t$ is lattice size(time)
Periodic boundary condition

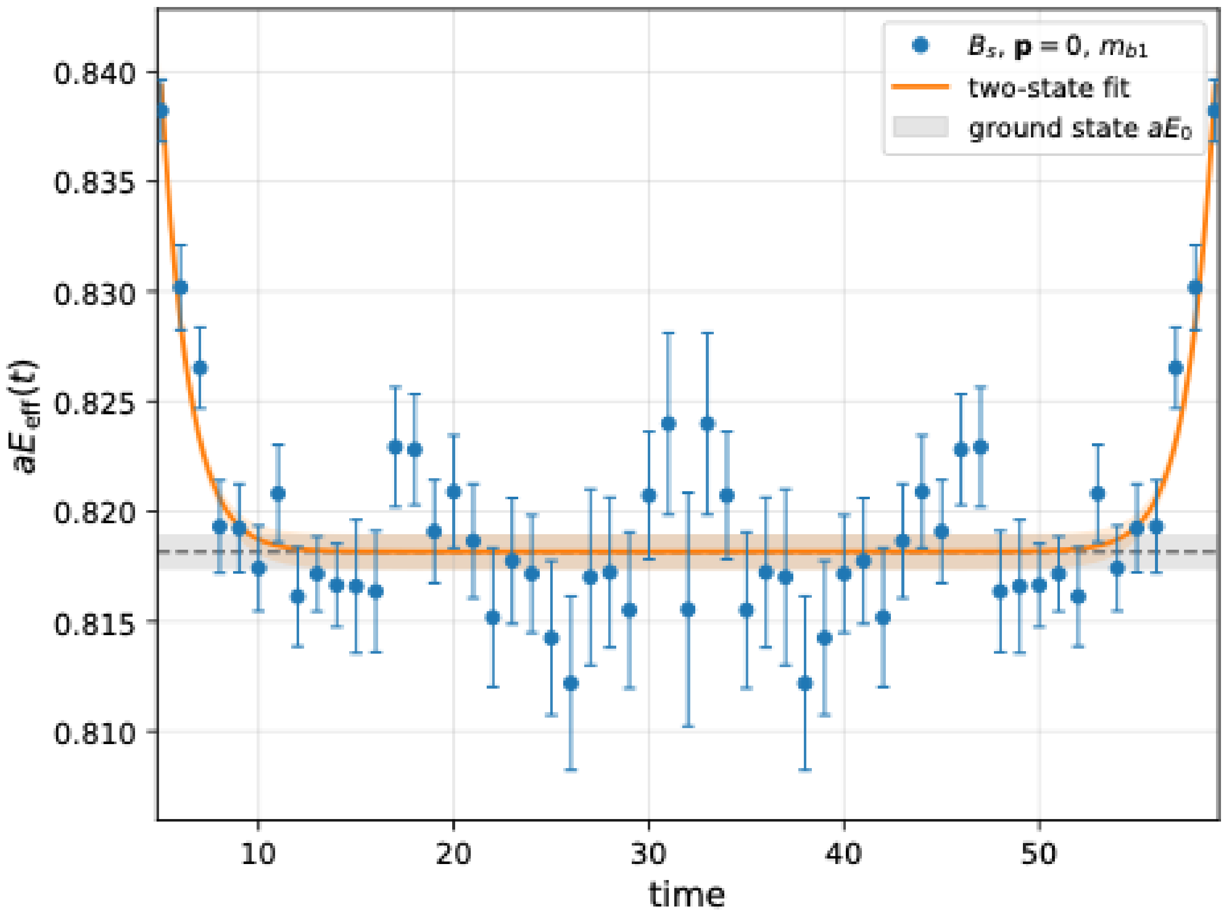
We use the fitting form, $\sum_{n=0}^{n=1} A_n^M(\vec{p}) \left( e^{-E_{p,n}^M t} + e^{-E_{p,n}^M (N_t - t)} \right)$ to obtain $A_n^M$ and $E_{p,n}^M$ parameters

$E_{p,n}^M$ are used as input to the fits of 3pt functions and ratios, $A_n^M$ are used to extract current matrix elements

# Plots of effective mass




$K$, momentum $p_k = 0$




$B_s$, momentum $p_{B_s} = 0$

The two-state fit captures the excited-state contributions away from the plateau region

# 3pt function

## 3pt function :

$$C_{3,\mu}^{K\to B_s}(t,T,\vec{p}) = \sum_{x,y}\langle 0|O_{B_s}(\vec{x},T)\big(\bar{u}\gamma_\mu b\big)(\vec{y},t)O_K^\dagger\big(\vec{0},0\big)|0\rangle\, e^{-i\vec{p}(\vec{x}-\vec{y})}$$

$$= \sum_{n,m}\sqrt{A_n^K(\vec{p})}\,\sqrt{A_m^{B_s}(\vec{0})}\,\frac{\langle B_{sm}(\vec{0})|\big(\bar{u}\gamma_\mu b\big)(\vec{q},t)|K_n(\vec{p}_K)\rangle}{\sqrt{4E_{p,n}^K E_{0,m}^{B_s}}}\, e^{-E_{p,n}^K t} e^{-E_{0,m}^{B_s}(T-t)}$$

T: source-sink separation

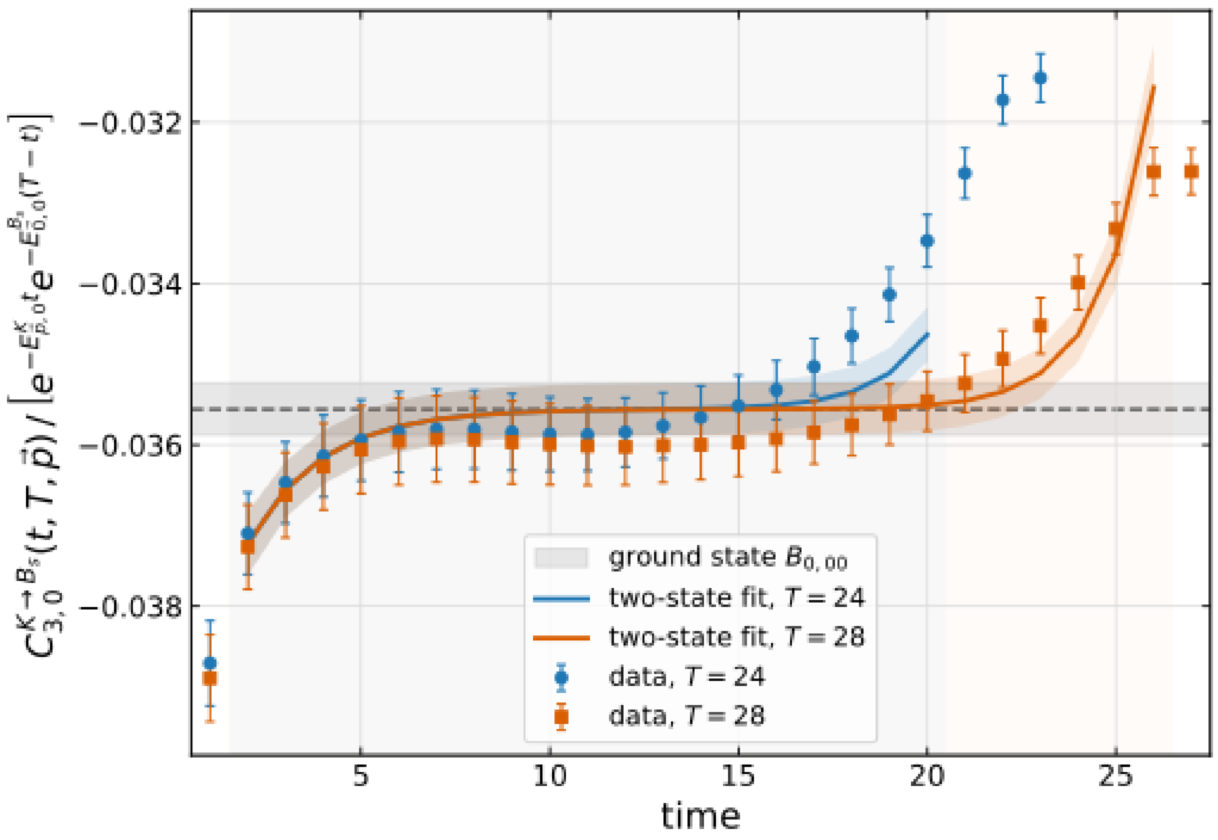
## Fitting form:

Simultaneous fit
$T$= 24, 28

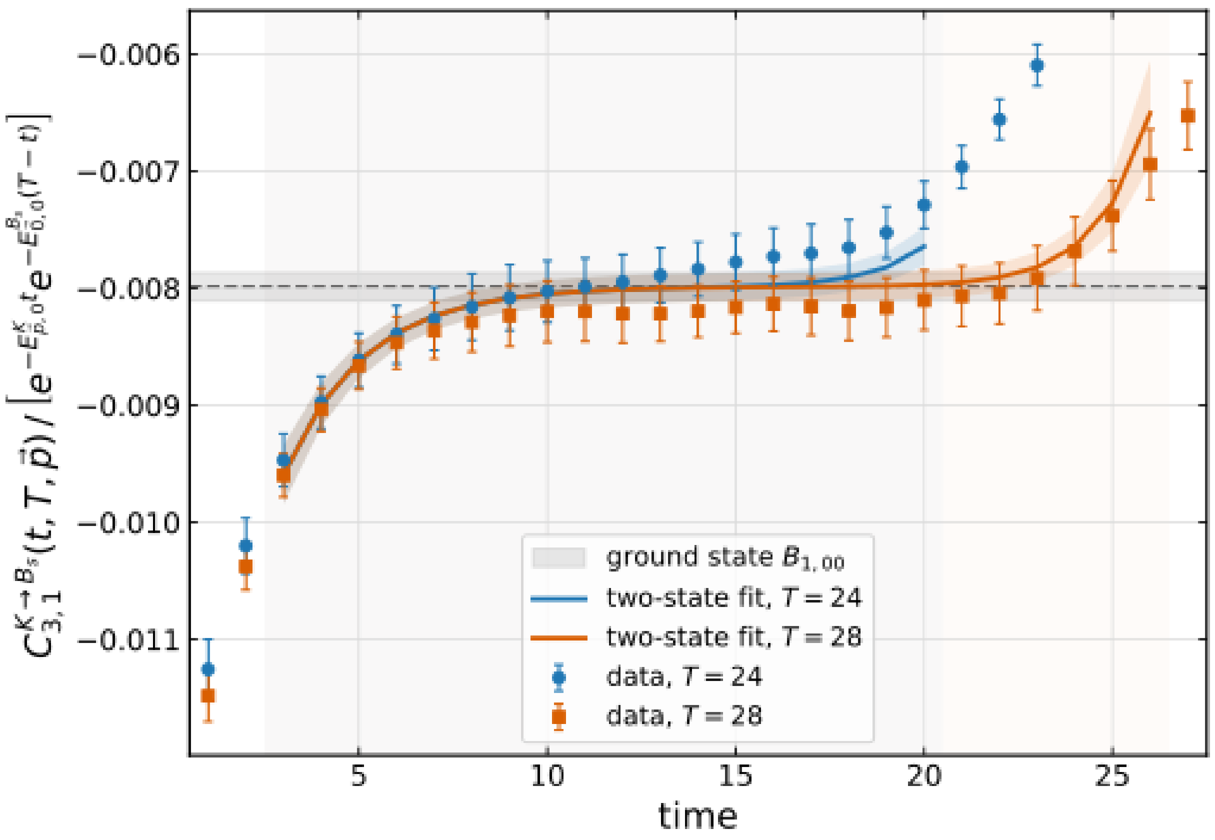
$$B_{\mu,00}e^{-E_{p,0}^K t}e^{-E_{0,0}^{B_s}(T-t)} + B_{\mu,10}e^{-E_{p,1}^K t}e^{-E_{0,0}^{B_s}(T-t)} + B_{\mu,01}e^{-E_{p,0}^K t}e^{-E_{0,1}^{B_s}(T-t)}$$

Combining $B_{\mu,00}$ with $E$ and $A$ gives the matrix element, $\langle K(\vec{p}_K)|\big(\bar{u}\gamma_\mu b\big)(\vec{q},t)|B_s\big(\vec{0}\big)\rangle$

# 3pt function fit




$$C_{3,0}^{K\to B_s}(t,T,\vec{0})/e^{-E^{K}_{p,0}t}e^{-E^{B_s}_{0,0}(T-t)}$$




$$C_{3,1}^{K\to B_s}(t,T,(1,0,0)\,)/e^{-E^{K}_{p,0}t}e^{-E^{B_s}_{0,0}(T-t)}$$

The consistency between the $T=24,28$ supports the identification of the ground-state contribution

# Double ratio

We consider "Double ratio", $R_D$

$$R_D \equiv \frac{C_{3,0}^{B_s\to K}(t,T,\vec{0})\,C_{3,0}^{K\to B_s}(t,T,\vec{0})}{C_{3,0}^{B_s\to B_s}(t,T,\vec{0})\,C_{3,0}^{K\to K}(t,T,\vec{0})}$$

$$\sim \frac{1}{2E_{0,0}^{K}}\frac{\left|\langle K(\vec{0})|(\bar{u}\boldsymbol{\gamma_0}b)(\vec{0},0)|B_s(\vec{0})\rangle\right|^2}{2E_{0,0}^{B_s}}$$

In the ground-state limit,
time dependence cancel between
the numerator and denominator

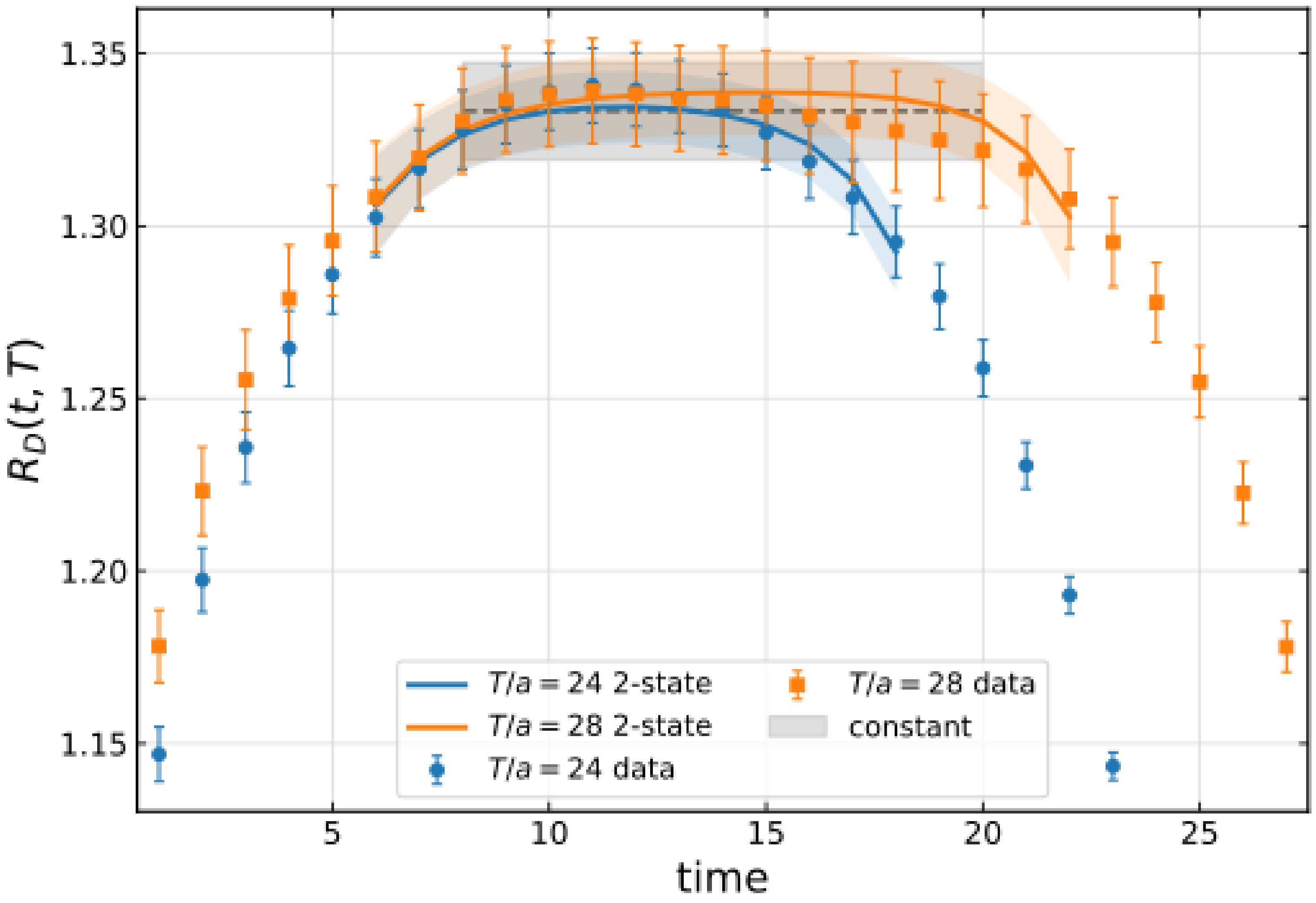
Fitting function:

Simultaneous fit
$T$= 24, 28

$$D\left(1+E\left(e^{-\Delta E_{0,0}^{K}t}+e^{-\Delta E_{0,0}^{K}(T-t)}\right)+F\left(e^{-\Delta E_{0,0}^{B_s}t}+e^{-\Delta E_{0,0}^{B_s}(T-t)}\right)\right)$$

Combining $D$ with $E_{0,0}^{M}$ gives the matrix element, $\langle K(\vec{p}_K)|(\bar{u}\gamma_0 b)(\vec{q},t)|B_s(\vec{0})\rangle$

# Double ratio fit




Within the fit ranges, time dependence associated with excited-state contamination is captured

For comparison, we also perform the constant fit with a narrower time range

## $R_0$ ratio

$$R_0(\vec{p}) \equiv \frac{C_{3,0}^{B_s\to K}(t,T,\vec{p})}{C_2^K(T-t,\vec{p})} \frac{C_2^K(T-t,\vec{0})}{C_{3,0}^{B_s\to K}(t,T,\vec{0})}$$

$$\sim f_\parallel(q^2) \frac{1}{f_\parallel(q_{max}^2)}$$

used to extract $f_\parallel$ at nonzero recoil

We have obtained $f_\parallel(q_{max}^2)$ from $R_D$ fit

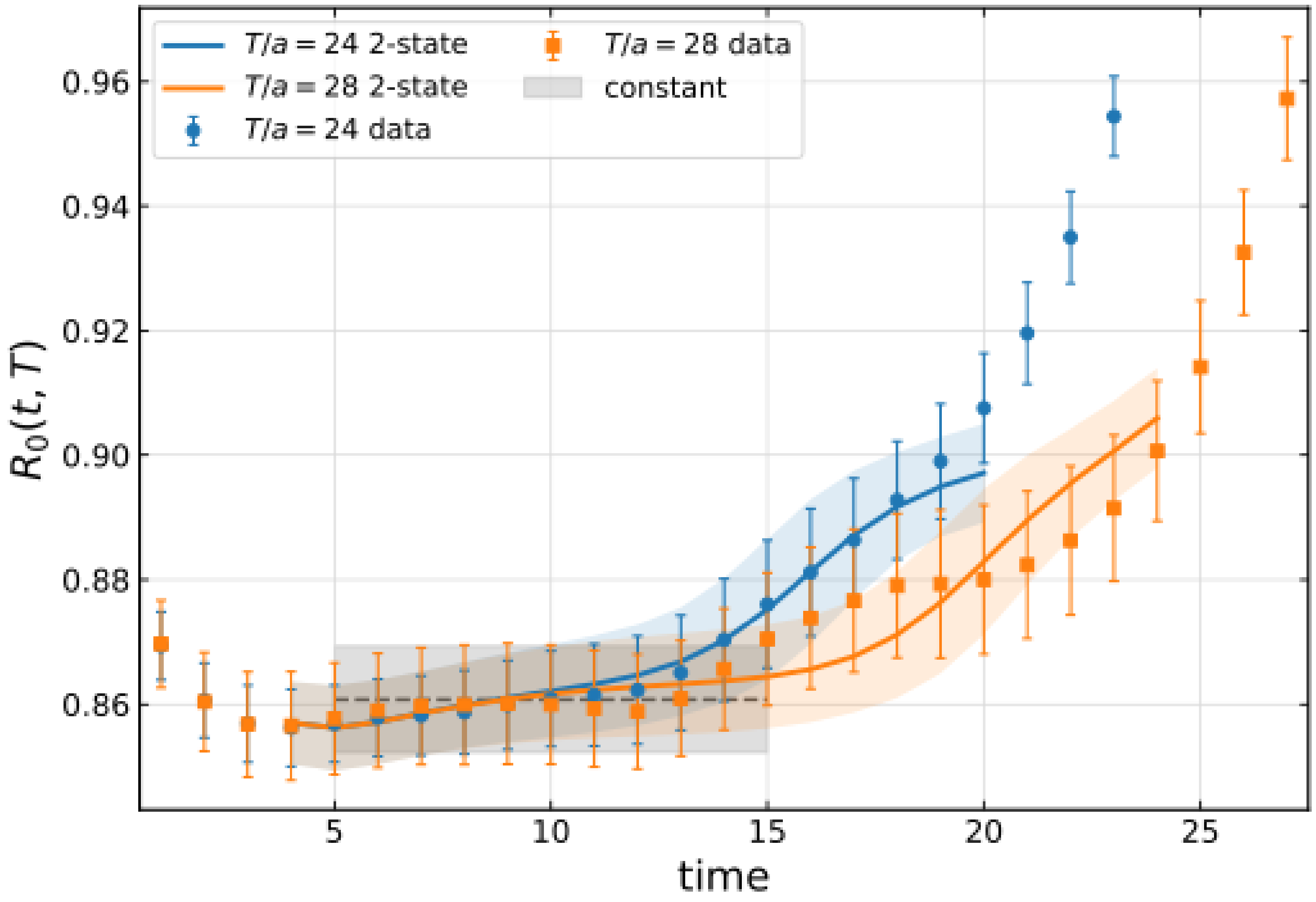
Fitting function:

$$G\left(1 + He^{-\Delta E_{p,0}^K t} + Ie^{-\Delta E_{0,0}^{B_s}(T-t)}\right)$$

Simultaneous fit
$T$= 24, 28

Combining $G$ with $f_\parallel(q_{max}^2)$ gives the form factor, $f_\parallel(q^2)$

## $R_0$ ratio fit




Within the fit ranges, time dependence associated with excited-state contamination is captured

## $R_i$ ratio

$$R_i(\vec{p}) \equiv \frac{C_{3,i}^{B_s \to K}(t, T, \vec{p})}{C_{3,0}^{B_s \to K}(t, T, \vec{p})}$$

$$\sim f_\perp(q^2) \frac{p_i}{f_\parallel(q^2)}$$

used to extract $f_\perp$ at nonzero recoil

We have obtained $f_\parallel(q^2)$ from $R_D$ , $R_0$ fit

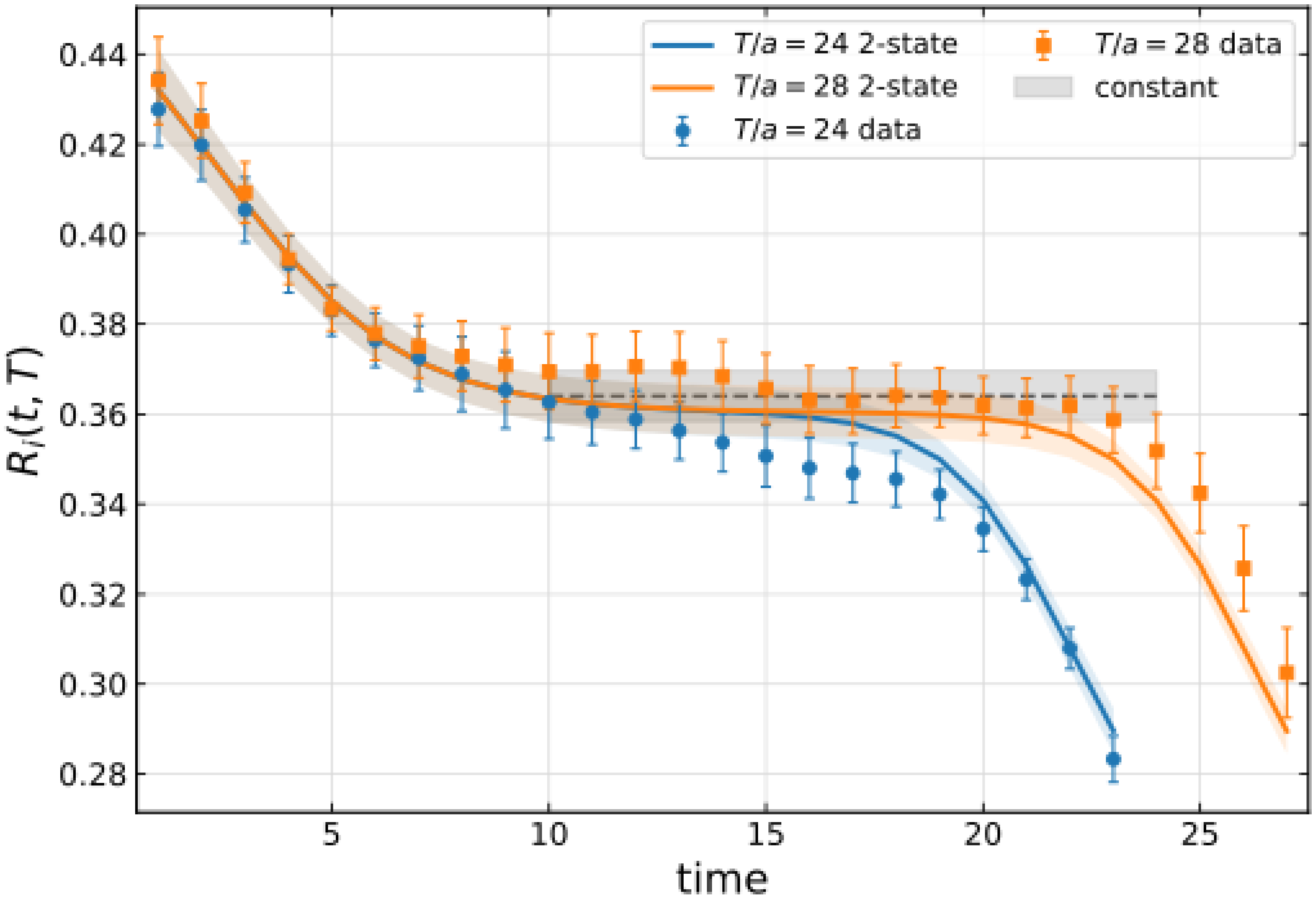
Fitting function:

$$J\left(1 + K e^{-\Delta E_{p,0}^{K} t} + L e^{-\Delta E_{0,0}^{B_s}(T-t)}\right)$$

Simultaneous fit
$T$= 24, 28

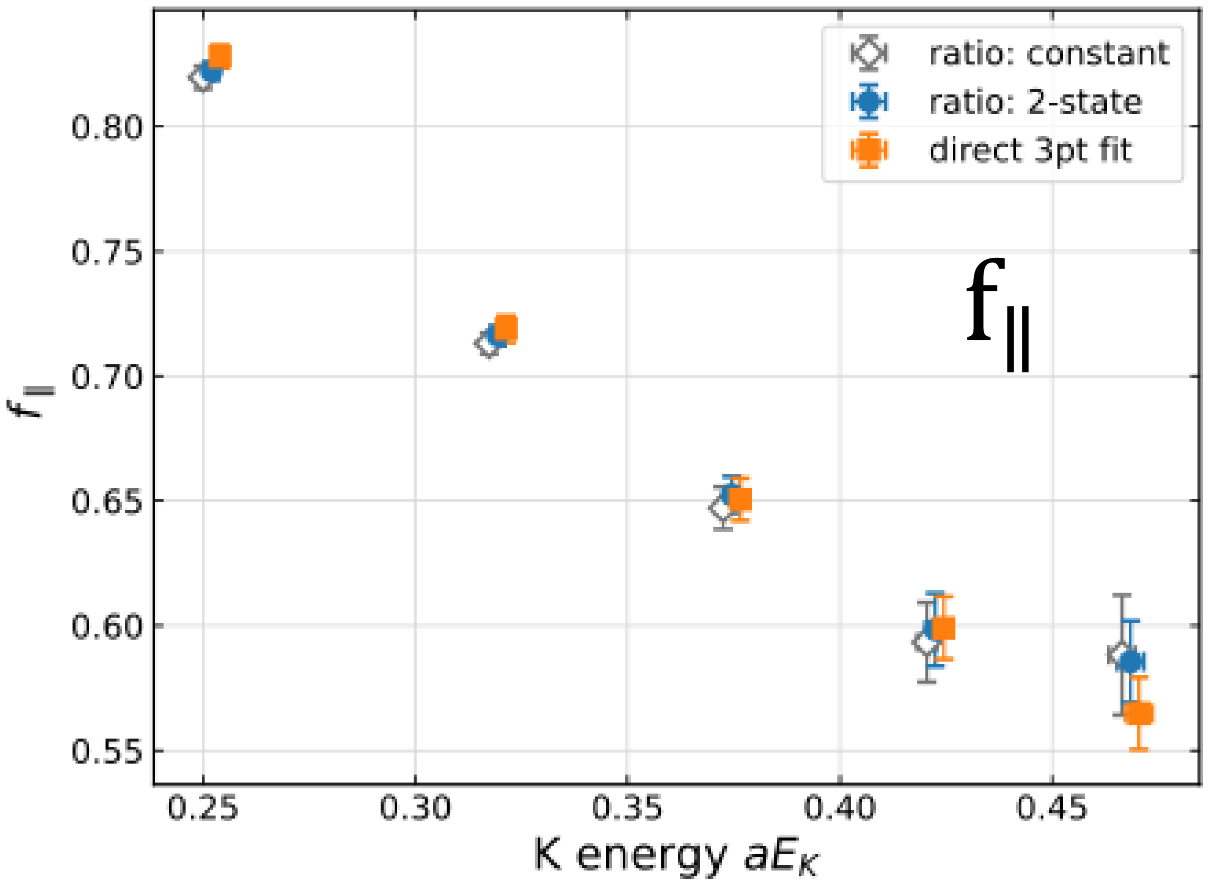
Combining $J$ with $f_\parallel(q^2)$ gives the form factor, $f_\perp(q^2)$

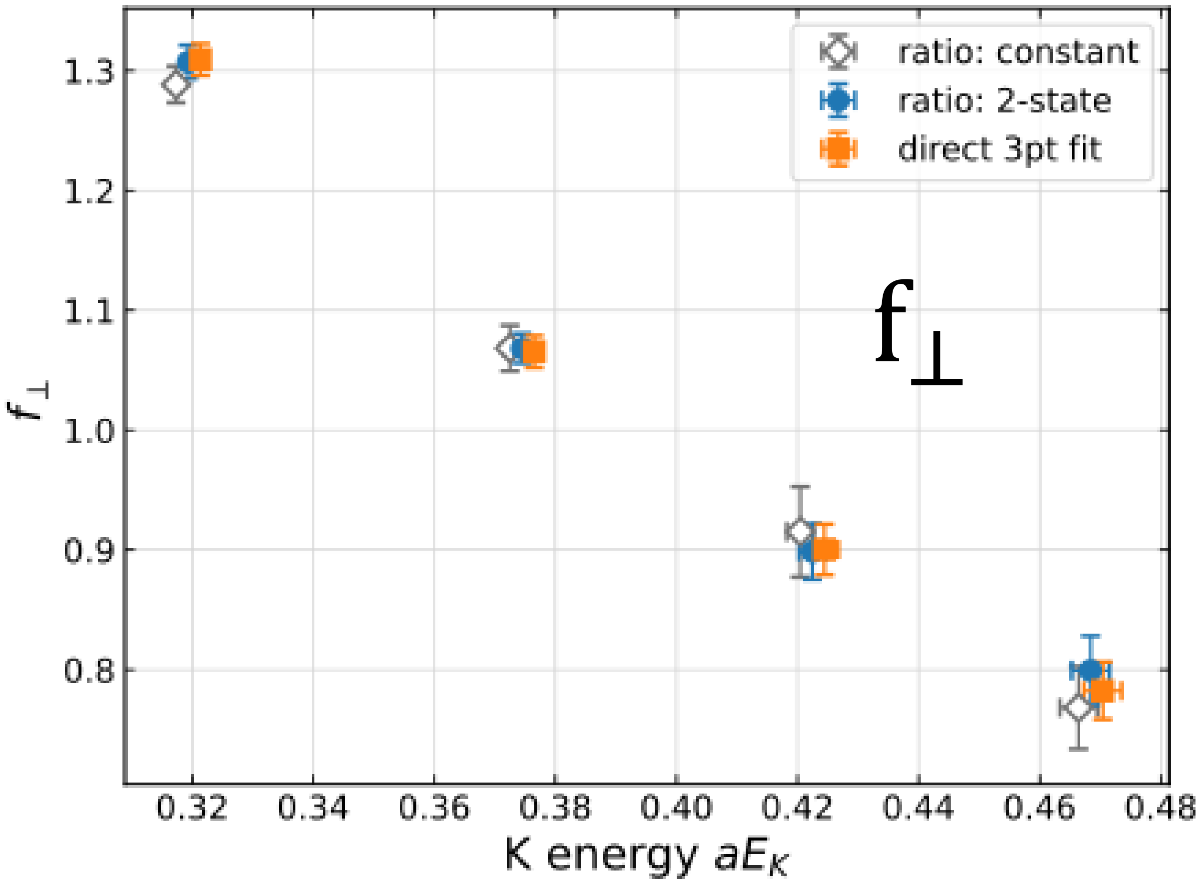
# $R_i$ ratio fit




Within the fit ranges, time dependence associated with excited-state contamination is captured

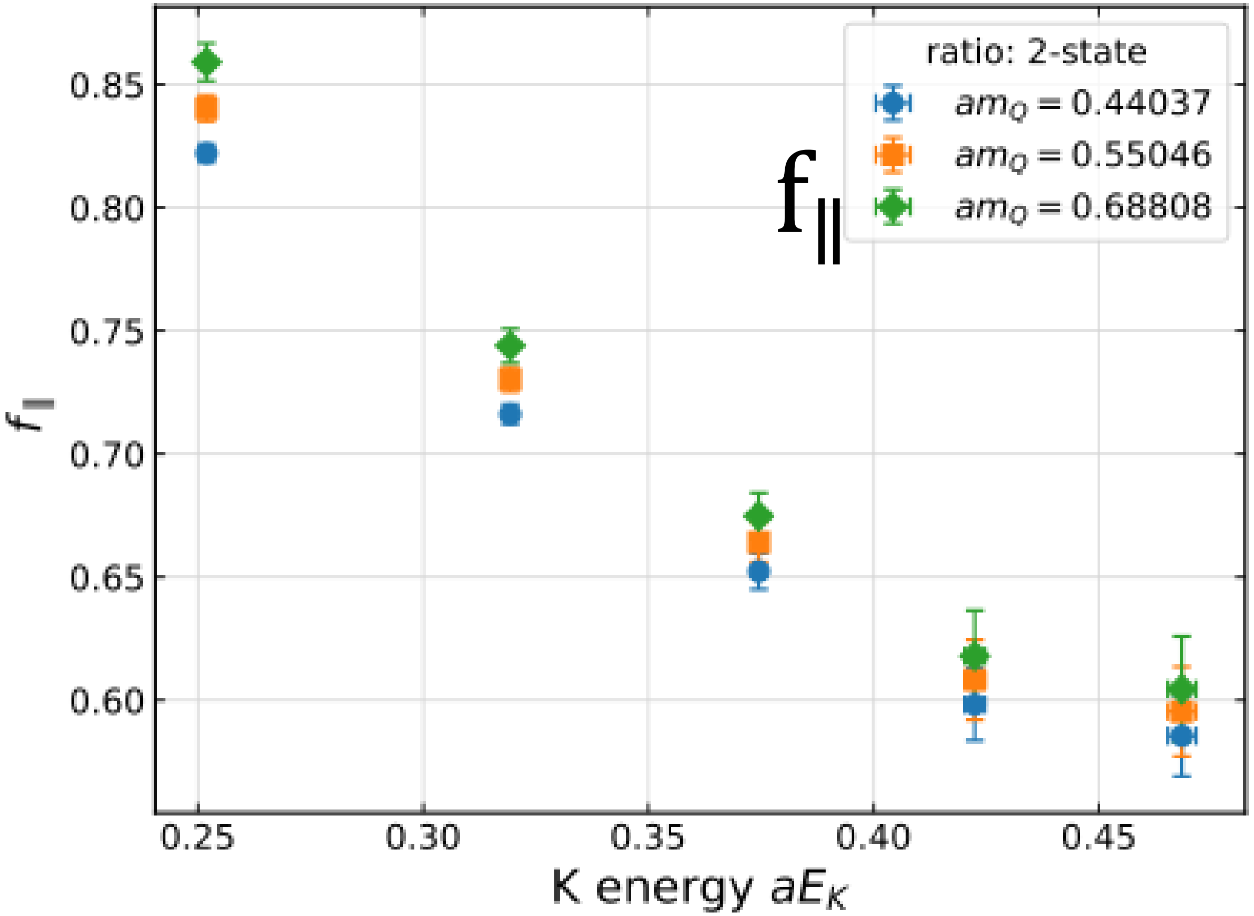
# Form-factor results ($am_Q = 0.44037$)

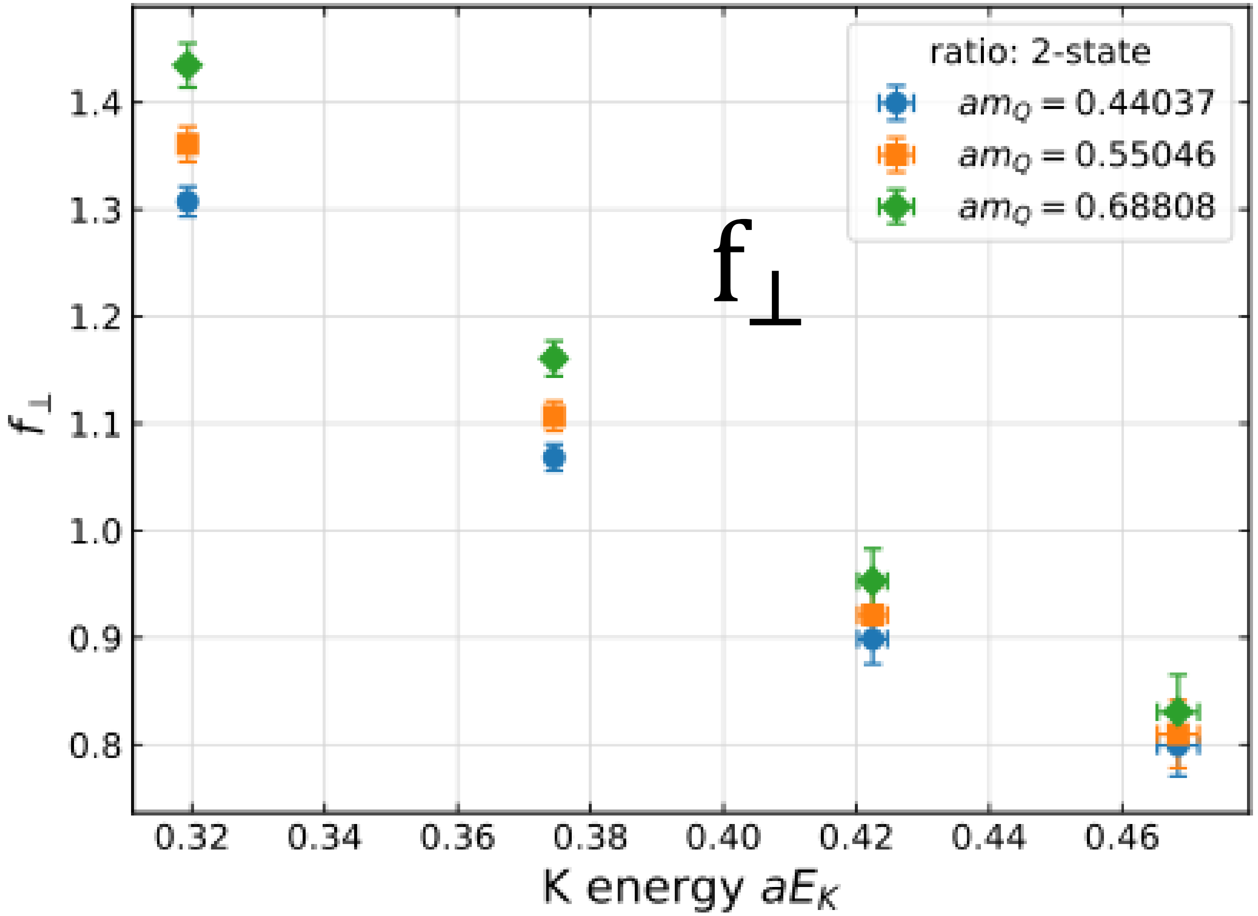



The form factors obtained from the direct and ratio methods are consistent within $1\sigma$ at all momenta

# Heavy quark mass dependence




The form factors show a mild and smooth dependence on the heavy-quark mass

## Summary

- JLQCD has started a study of the $B_s \to Kl\nu$ decay using relativistic domain-wall heavy quarks

- we reported preliminary results on a single ensemble ($a^{-1} = 2.453[\mathrm{GeV}]$, $M_\pi \cong 500[\mathrm{MeV}]$)

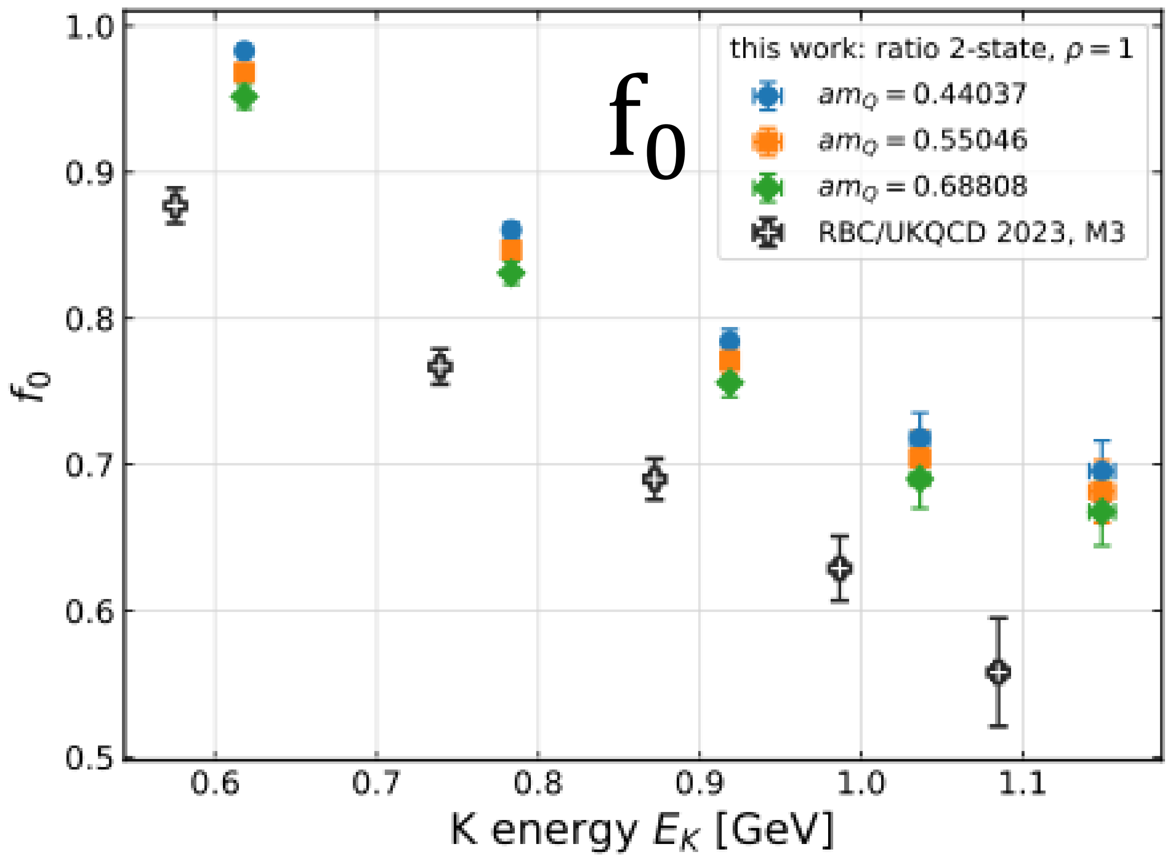
- Consistent form factor results (within $1\sigma$) were obtained from two methods with different treatments of excited-state contributions

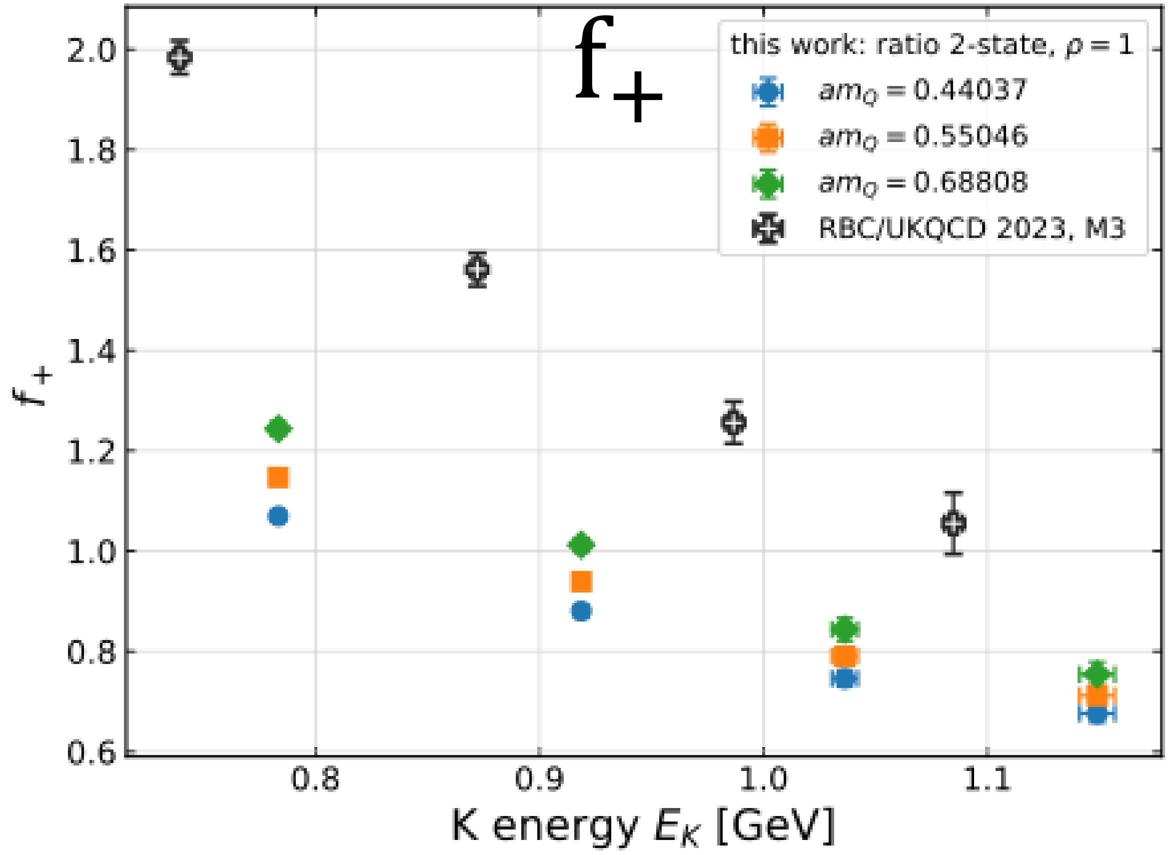
- The analysis will be extended to other parameter sets, followed by an extrapolation to the physical point to determine the form factors

# Heavy quark mass dependence

RBC/UKQCD, Phys. Rev. D 107, 114512 (2023), arXiv:2303.11280, Table III. $M3$ ensemble

$M_\pi \cong 411[MeV],\ a^{-1} \cong 2.3833, m_b$:physical




For comparison with RBC/UKQCD, we convert $f_\parallel$ and $f_\perp$ to $f_0$ and $f_+$

The heavy quark mass dependence trends toward the RBC/UKQCD results