Renormalizing the two-photon contribution to $K_L \rightarrow \mu^+ \mu^-$ decay

The 43th International Symposium on Lattice Field Theory

July 28, 2026

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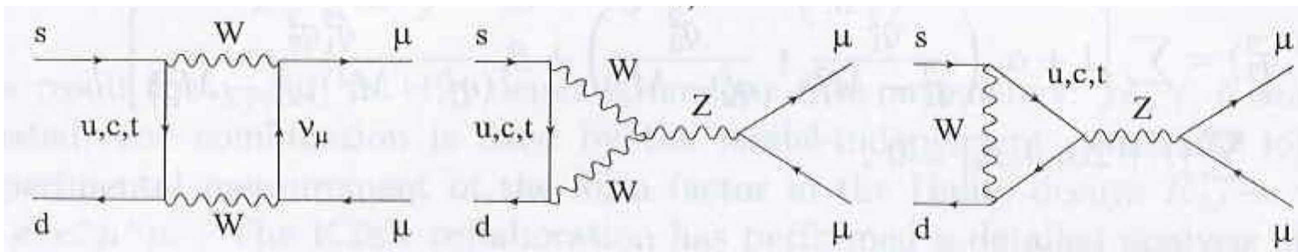
Outline

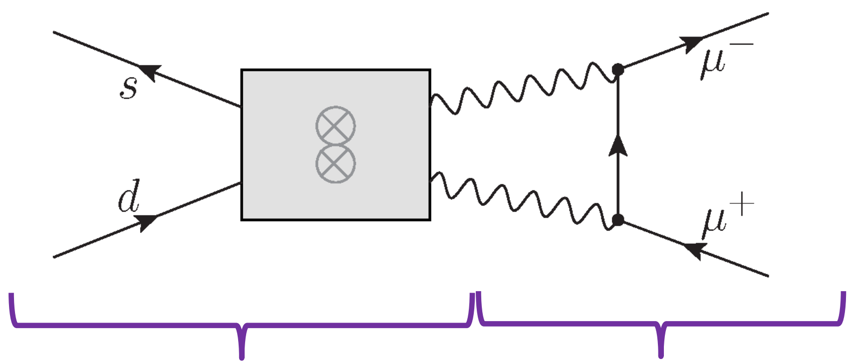
- Physics of $K_L \to \mu^+ \mu^-$
- $N_f = 3$ weak interactions without GIM
- Renormalizing two photon contribution to $K_L \to \mu^+ \mu$
  - Content of recent paper: 2607.14077 [hep-lat]
  - Quark contractions and divergent sub-diagrams
  - Required counter terms
- Determining the counter terms: Next talk, Ceran Hu

Physics of $K_L \rightarrow \mu^+ \mu^-$

- 2^{nd}-order-weak, ``strangeness changing neutral current'' – test of the standard model at high energies and one loop



- Obscured by 2 photon $O(\alpha_{\mathrm{EM}}{}^2 G_F)$ process:



- Compute using lattice QCD
- 1.6% expt + 10% lattice → 13% new SM test

Euclidean
(real)

Wick rotate
(complex)

Three-flavor calculation

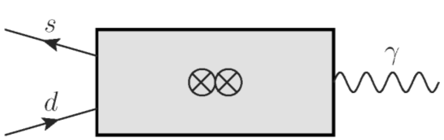
- Three-flavor calculation w/o renormalization
  - 4 → 3 flavors: Active charm too heavy for current resources
  - 24^3 x 64, 1/a=1.023 GeV, physical mass calculation

    Phys.Rev.D 113 (2026) 7, 074523: 2509.04346 [hep-lat]

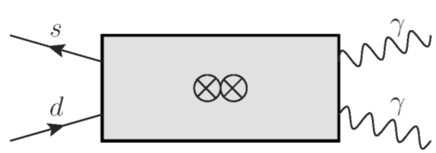
    (P. Boyle, E-H Chao, N. Christ, C. Hu, L. Jin and Y. Zhao)

	$\mathrm{Re}\mathcal{A} \times 10^{-4}$ [MeV^3]	$\mathrm{Im}\mathcal{A} \times 10^{-4}$ [MeV^3]
LD2γ	$5.27(1.30)(0.17)$	$16.61(2.96)(0.38)$
SD$\mathrm{G_F^2}$	$-2.47(18)$	—
exp.	$\pm 1.53(14)$	$\pm 7.10(3)$

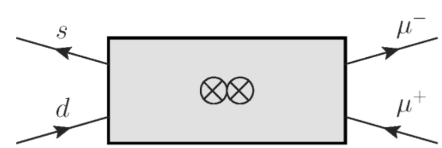
- Without GIM cancellation, three-flavor weak interaction requires 3-6 LECs determined from four-flavor theory:



Class – A

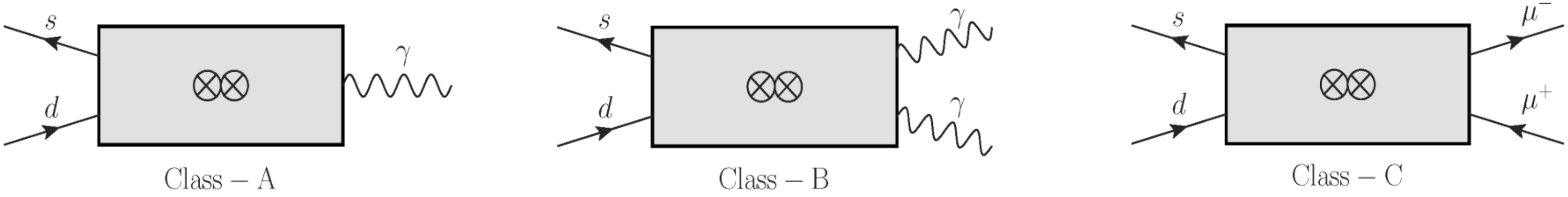


Class – B



Class – C

Limited four-flavor calculation is needed

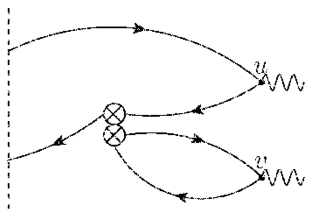


- Fortunately, local counter terms can be determined using unphysical heavy quarks and small volumes
- This allows a practical four-flavor $K_L \rightarrow \mu^+ \mu^-$ calculation to determine counter terms
- Work with two ensembles:
  - 24ID 24^3x64 1/a=1.023 GeV (N_f = 3)
  - 32IF 32^3x64 1/a=3.1 GeV (“N_f = 4”)

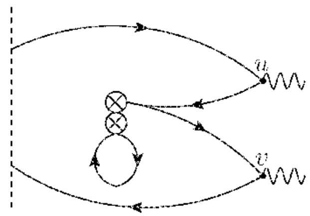
Used for full $K_L \rightarrow \mu^+ \mu^-$ calculation

Quark Contractions & sub-diagrams

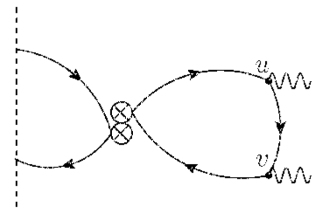
- Lattice calculation: five quark line topologies:



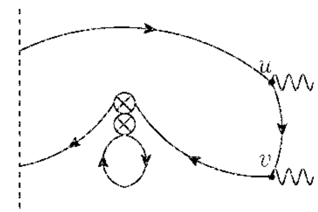
Type − 1



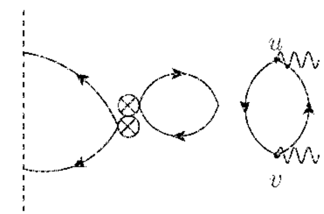
Type − 2



Type − 3

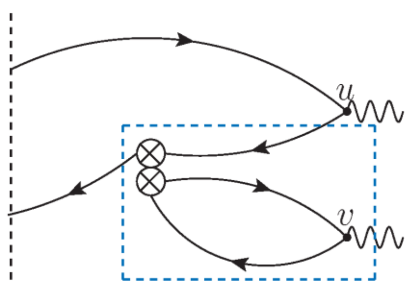


Type − 4

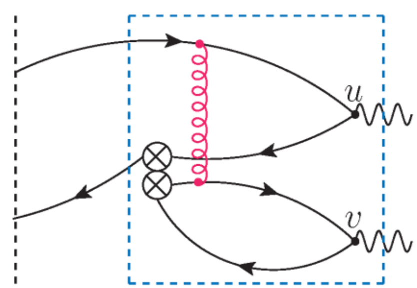


Type − 5

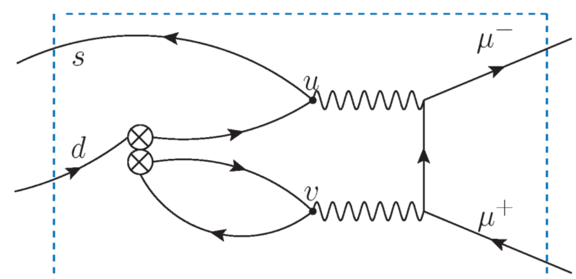
- Identify divergent sub-diagrams: Type-1

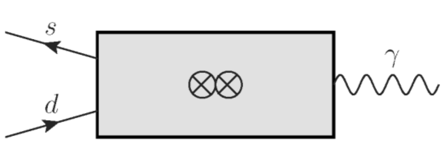


Type−1 − Class−A

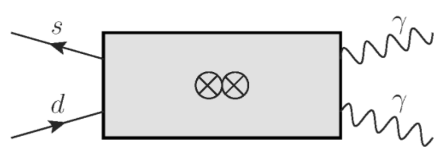


Type−1 − Class−B

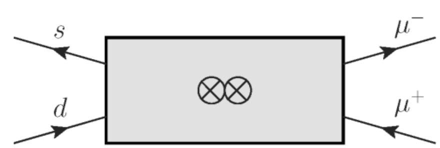




Class − A



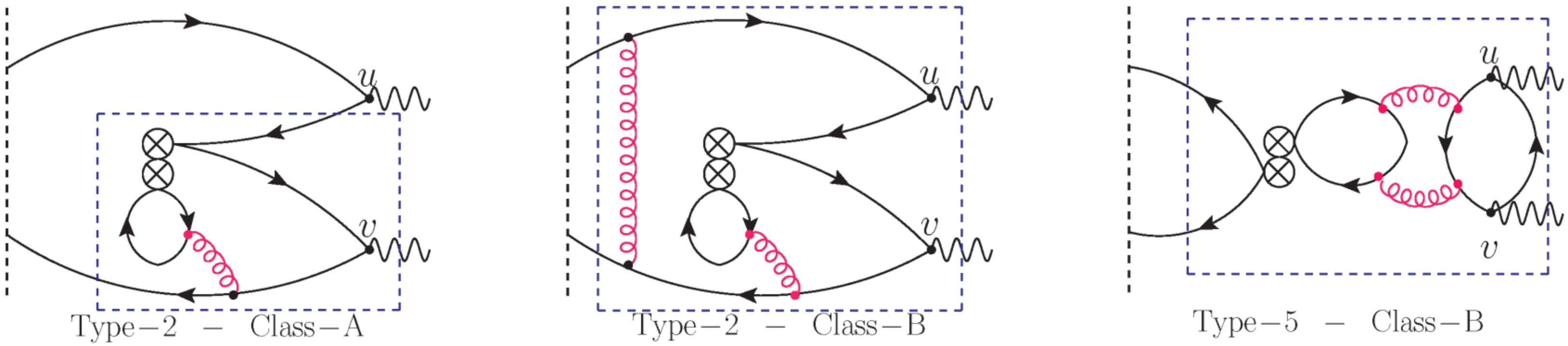
Class − B



Class − C

Quark Contractions & sub-diagrams

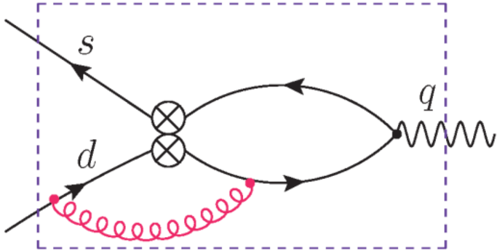
- Identify divergent sub-diagrams: Type-2 & 5



- Order in $\alpha_s(m_c)$ at which divergent sub-diagrams appear:
- Dropping all operators in H_W except Q_1 and Q_2 works to order $[\alpha_s(m_c)]^0$ and $|V_{ts}V^*_{td}/(V_{us}V^*_{ud})|^0$

contraction	sub-diag A	sub-diag B	sub-diag C
Type-1	0	1	0
Type-2	1	2	1
Type-3	X	0	0
Type-4	1	1	1
Type-5	X	2	2

Class-A counter terms – Conserved Current



- Extra power of q_μ reduces degree of divergence to +1.
- Three possible counter terms

$$Q_1^A = \left(\bar{s}\gamma^\nu d + \bar{d}\gamma^\nu s\right)\partial^\mu F^{\mu\nu}$$

Leading order in α_s

$$Q_2^A = \frac{1}{2}\left\{\bar{s}\gamma^\nu\gamma^5(\overleftarrow{D}^\mu - \overrightarrow{D}^\mu)d + \bar{d}\gamma^\nu\gamma^5(\overleftarrow{D}^\mu - \overrightarrow{D}^\mu)s\right\}\widetilde{F}^{\mu\nu}$$

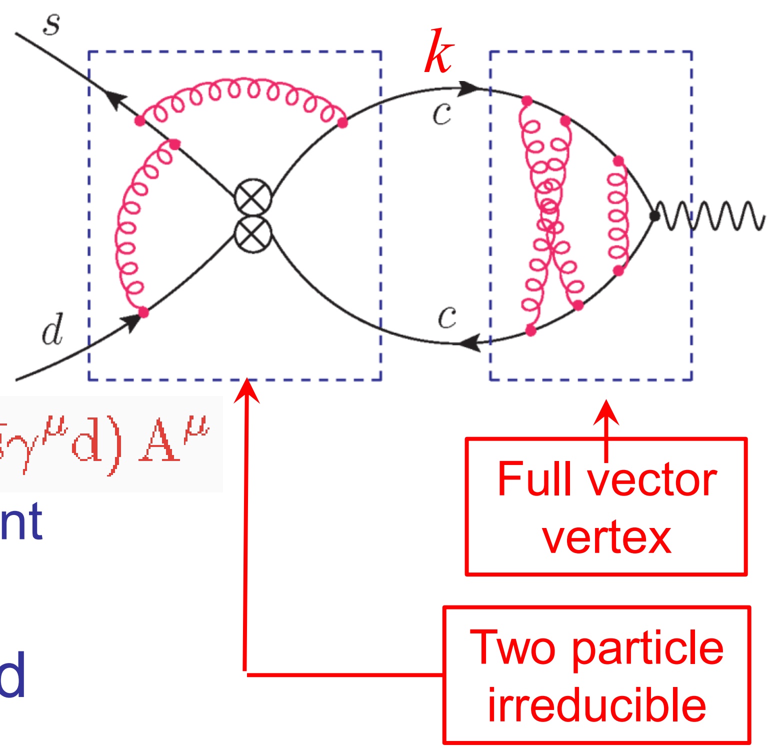
$$Q_3^A = (m_d + m_s)\left(\bar{s}\sigma^{\mu\nu}d + \bar{d}\sigma^{\mu\nu}s\right)F^{\mu\nu},$$

  – Only $Q_1{}^A$ appears at order $[\alpha_s(m_c)]^0$

- Determine 3 coefficients
  – Match 4-point functions between 3- and 4-flavor ensembles
  – Non-perturbative, Landau gauge-fixed, RI/MOM
  – Use $m_l = m_s$ to reduce physical volume and inversion cost

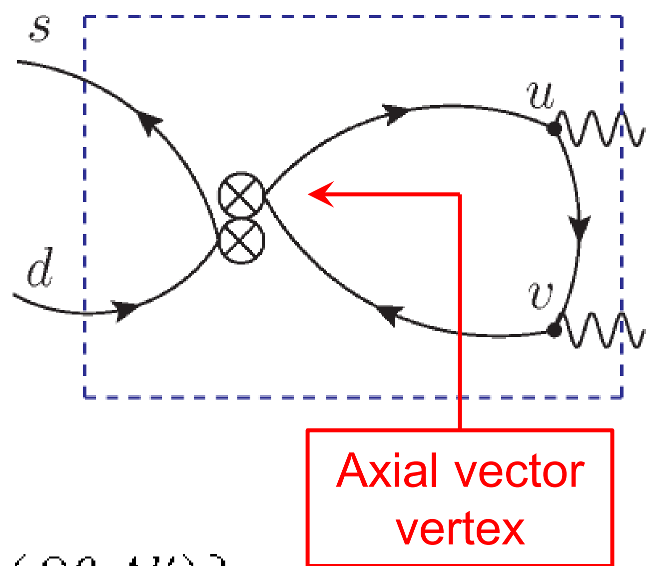
Class-A counter terms – non-conserved current

- Each *k* region requires different counter terms
- 0.1/a ≤ *k* ≤ 4/*a* Non-conserved current
  - Needs 1 counter term: $O_4^A = m_{c'}^2 \, (\bar{s}\gamma^\mu d)\, A^\mu$
  - Coef. determined by imposing current conservation
- 0 ≤ *k* ≤ 0.1/a Current is conserved
  - Finite with GIM' subtraction + conserved current
  - Must correct $m_{c'}^2 \rightarrow m_c^2$
  - Need 3 previous counter terms: Q_1^A, Q_2^A, Q_3^A
  - Choose coefficients to make 3-flavor agree with 4-flavor
  - Coef.'s ~log($m_c/m_{c'}$)



Class-B ambiguity

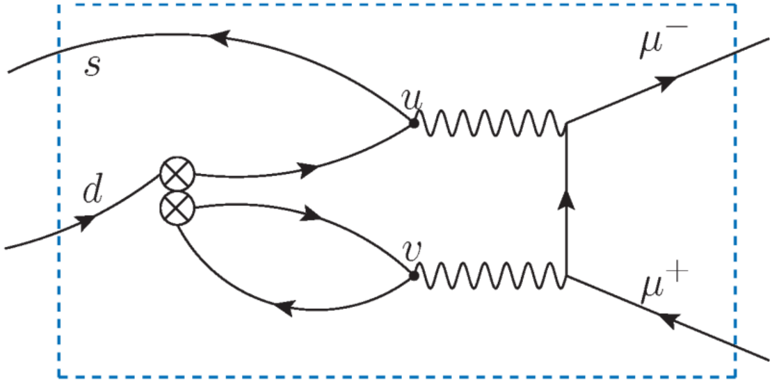
- Diagram has degree of divergence 1
- Linear "divergence" $\int d^4k\, k_\mu$ is finite but ambiguous to symmetrize
- *Adler ambiguity* corresponds to the local counter term:



$$\zeta\,(\overline{s}\gamma^\sigma\gamma^5 d)\epsilon^{\mu\nu\sigma\rho}\left\{(\partial^\rho A^\mu)A^\nu - A^\mu(\partial^\rho A^\nu)\right\}$$

- This violates gauge invariance but will appear if we use a non-conserved current
- We must add this counter term and adjust ζ to achieve current conservation

Class-C counter terms



- The whole diagram has degree of divergence 0 (also a four-fermi vertex)
- Not present in a four-flavor calculation.
- Requires a simple counter term,

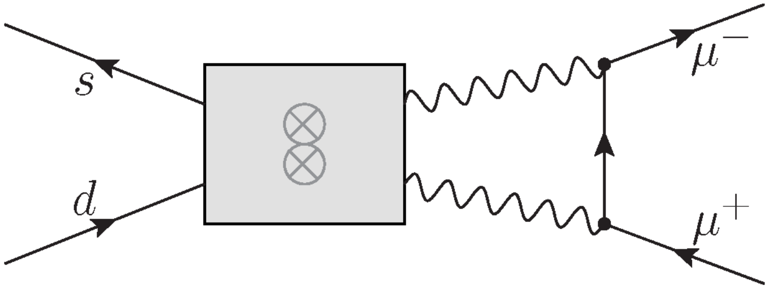
$$\mathrm{C}_1^{\mathrm{C}}(\bar{\mathrm{s}}\gamma^\mu\gamma^5\mathrm{d})(\overline{\mu}\gamma^\mu\gamma^5\mu)$$

- Determine C_1^{C} coefficient from 4-flavor, 3-flavor equality:

$$\begin{aligned}\langle\mu^+\mu^-|\mathrm{H}_{\mathrm{W}}^{\mathrm{N_f}=4}|\mathrm{K}\rangle^{\mathcal{K}}_{\mathrm{QCD+QED}} &= \langle\mu^+\mu^-|\mathrm{H}_{\mathrm{W}}^{\mathrm{N_f}=3}|\mathrm{K}\rangle^{\mathcal{K}}_{\mathrm{QCD+QED}} \\ &\quad +\mathrm{C}_1^{\mathrm{C}}\langle\mu^+\mu^-|(\bar{\mathrm{s}}\gamma^\mu\gamma^5\mathrm{d})(\overline{\mu}\gamma^\mu\gamma^5\mu)|\mathrm{K}\rangle^{\mathcal{K}}_{\mathrm{QCD}}\end{aligned}$$

  - Where $\mathcal{K}$ denotes heavy-quark, small-volume kinematics

Conclusion



- It is practical to perform a 3-flavor calculation of the two-photon contribution to $K_L \rightarrow \mu^+\mu^-$ decay
- Must determine 5 or 6 LECs

  from a 4-flavor calculation with an active charm quark
- This 4-flavor calculation can be simplified:
  - Use heavier than physical quark masses
  - Use small volume.
- Number of LECs and contractions can be reduced by expanding in $\alpha_s(m_c)$: ($N_{LEC} \geq 3$, $N_{cntrc} \geq 2$)
- In the next talk Ceran Hu will
  - Describe explicitly how these LEC's can be determined
  - Show first results