Leveraging Existing HISQ Eigenvectors for Improved Statistical Precision in the HVP Contribution to Muon $g-2$

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HVP with staggered fermions
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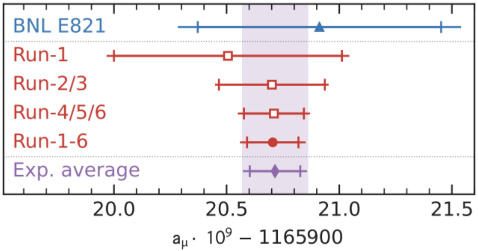
Outline

Muon $g-2$: Motivation and Experimental Status

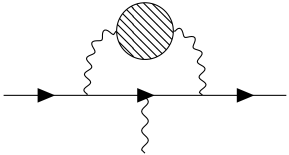
Why muon $g-2$?

- Anomalous magnetic moment: $a_\mu = (g-2)/2$
- Precision test of the Standard Model
- Theory error dominated by **hadronic vacuum polarization** (HVP)
- Lattice QCD provides a first-principles determination of HVP



- World average at 124 ppb [Donati et al. 2025]

HVP on the Lattice: Correlator Decomposition



In the time-momentum representation [Bernecker and Meyer 2011]:

$$a_\mu^{\text{HVP}} = 2\sum_{t=0}^{T/2} w(t)\, C(t)$$

where $w(t)$ is a known QED kernel and $C(t) = \frac{1}{3}\sum_{\vec{x},i}\langle J^i(\vec{x},t) J^i(0)\rangle$ is the 2-point current-current function.

Spectral decomposition of the propagator into low and high modes:

$$S(x,y) = \underbrace{S_L}_{\lambda \leq \lambda_{\text{cut}}} + \underbrace{S_H}_{\lambda > \lambda_{\text{cut}}}$$

$$\boxed{C(t) = C_{LL} + C_{LH} + C_{HL} + C_{HH}}$$

gives four correlator contributions [Giusti, Hernandez, Laine, Weisz, and Wittig 2004; Moningi, Aubin, Blum, Golterman, Jin, and Peris 2025]

- On the lattice the single-site current is not conserved, so we use a single link point-split **conserved** current.

On the finest ensemble $144^3 \times 288$, a=0.04 fm, we have

$$a_\mu^{\mathrm{HVP,\,lqc}} = (658.4 \pm 15.3) \times 10^{-10}$$

[Moningi, Aubin, Blum, Golterman, Jin, and Peris 2025; Moningi, Aubin, Blum, Golterman, Jin, and Peris 2026].

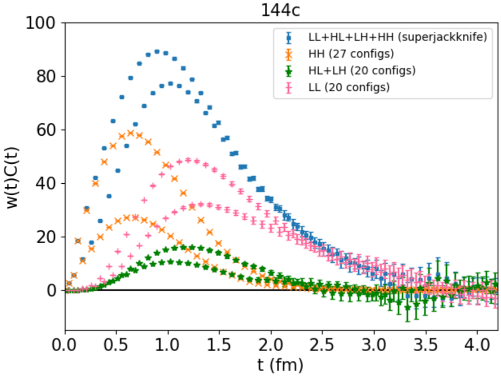
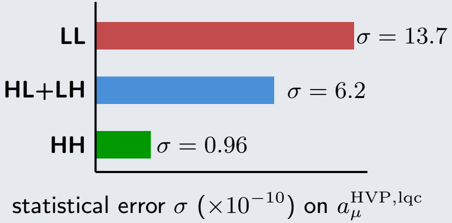


Figure 1: Summand in $w(t)C(t)$ (in units of 10^{-10}). Errors are statistical only.

Error Budget Breakdown on $a_\mu^{\mathrm{HVP,lqc}}$



Challenge

Statistical precision of HVP is the current bottleneck

Low-Low Contribution: The Computational Bottleneck

The low-low correlator:

$$C_{LL} = \sum_{m,n} \sum_{\vec{x}} \frac{\langle m|x+\mu\rangle\, U^\dagger_\mu(x)\, \langle x|n\rangle}{\lambda_m} \sum_{\vec{y}} \frac{\langle n|y\rangle\, U_\nu(y)\, \langle y+\nu|m\rangle}{\lambda_n} + \cdots$$

$$= \sum_{m,n} \frac{1}{\lambda_m \lambda_n} \Lambda^\dagger_\mu(x)_{mn}\, \Lambda_\nu(y)_{nm} + \cdots$$

with the meson field:

$$\left(\Lambda_\mu(t)\right)_{n,m} = \sum_{\vec{x}} \langle n|x\rangle\, U_\mu(x)\, \langle x+\mu|m\rangle$$

Two expensive steps

1. **Eigenvector generation**
   IRL/Lanczos on $D^\dagger D$
   $\mathcal{O}(N_{\text{low}} \times N_S^3 \times N_T)$
2. **Meson field**
   $\Lambda_\mu \sim \mathcal{O}(N_{\text{low}}^2 \times N_S^3 \times N_T)$
   Sparsening reduces $N_S^3 \to (N_S/\text{inc})^3$
   [Moningi, Aubin, Blum, Golterman, Jin, and Peris 2026]

Our strategy: eliminate Step 1 entirely

HISQ eigenvectors from MILC/Fermilab are **already available** $\to$ skip eigenvector generation entirely, proceed directly to meson field contraction with sparsening. This eliminates the single most expensive step in the C_{LL} pipeline.

The HISQ Dirac Operator

$$D_{\text{HISQ}} = c_1 D_{\text{Fat}} + c_2 D_{\text{Naik}}$$

Fat-Link Term D_{Fat}

- 1-link (nearest-neighbor) term with $c_1 = 1, c_2 = 0$
- ASQTAD fat7 smearing $\rightarrow$ reunitarization $\rightarrow$ another round of fat7 smearing
- Suppress taste-symmetry breaking
- Used in **our existing** calculation

Naik Term D_{Naik}

- 3-link (next-to-nearest-neighbor) term with $c_1 = 9/8, c_2 = -1/24$
- Removes free-field $\mathcal{O}(a^2)$ errors in the quark dispersion relation.
- Included in **MILC/Fermilab eigenvectors**

AMA-Style Bias Correction Strategy

The standard AMA estimator [Blum, Izubuchi, and Shintani 2013] applied to the action mismatch:

Our AMA mapping

$$\underbrace{C^F}_{\text{exact}} = \underbrace{C^H}_{\text{approx}} + \underbrace{\langle C^F - C^H \rangle}_{\Delta\text{: bias correction}}$$

Exact
(Fat-Link Action)

- Correlation function computed with fat-link operator D_F

→

Bias
(Both Actions)

- Small subset of exact measurements
- Estimate $\Delta = \langle C^F - C^H \rangle$

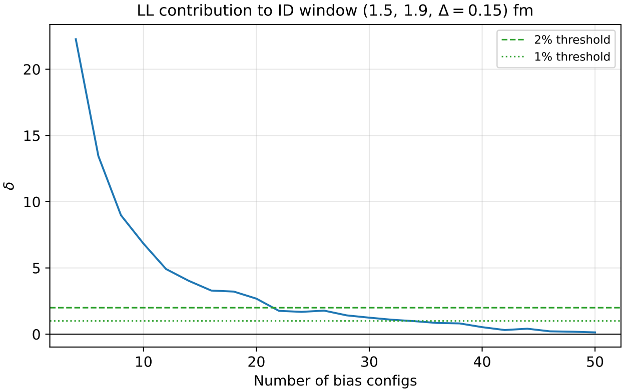
→

Approx
(HISQ Action)

- Correlation function from D_{HISQ}
- Cheap: evecs available

Key: the HISQ correlator (C^H) is the *approximation* (cheap) because eigenvectors are already available. The fat-link correlator (C^F) is the *exact* target.

Bias Correction Overhead vs. Number of Bias Configs



What is plotted

We define the **relative error overhead**:

$$\delta = \frac{\sigma_{\text{hisq corrected}} - \sigma_{\text{fat}}}{\sigma_{\text{fat}}} \times 100\%$$

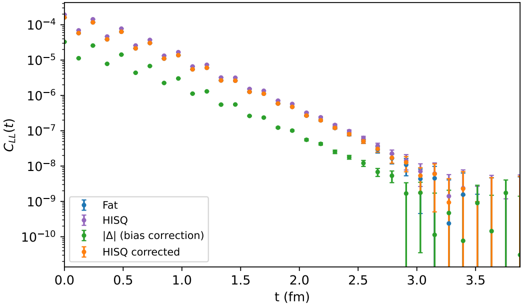
$\delta = 0$ means the correction adds **no extra noise**.

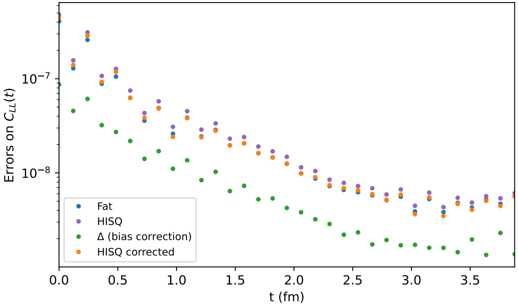
Key thresholds

- $N_{\text{bias}} \sim 5$: $\sim 22\%$ increase
- $N_{\text{bias}} \sim 10$: $\sim 7\%$ increase
- $N_{\text{bias}} \sim 20$: $\sim 2\%$ increase
- $N_{\text{bias}} \sim 30$: $\sim 1\%$ increase

Validation on $48^3 \times 64$: Low-Low Correlator

- $48^3 \times 64, a = 0.1212$ fm • $N_{\text{total}} = 51$ • $N_{\text{hisq}} = 51$ • $N_{\text{bias}} = 20$ configurations





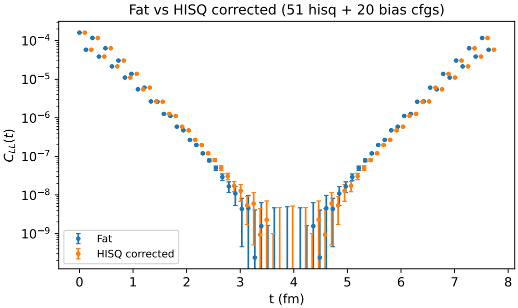
Correlator comparison

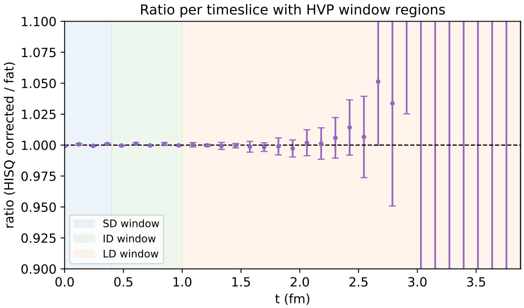
- $|\Delta|$ is 1–2 orders of magnitude below fat-link correlator
- Fat and HISQ corrected overlap
- Bias correction is a small perturbation at all timeslices

Error comparison

- $\sigma_\Delta < \sigma_{\text{HISQ}}$ across all timeslices
- Ratio $\sigma_\Delta / \sigma_{\text{HISQ}} \approx 0.27$
- Bias adds only $\sim 7\%$ to variance

- $48^3 \times 64, a = 0.1212$ fm • $N_{\text{total}} = 51$ • $N_{\text{hisq}} = 51$ • $N_{\text{bias}} = 20$ configurations



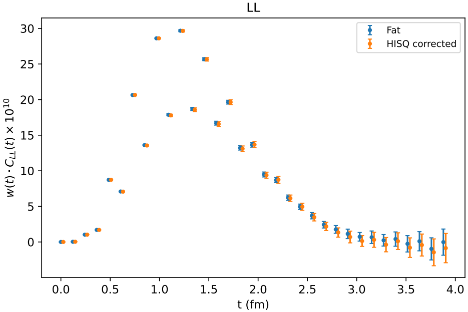


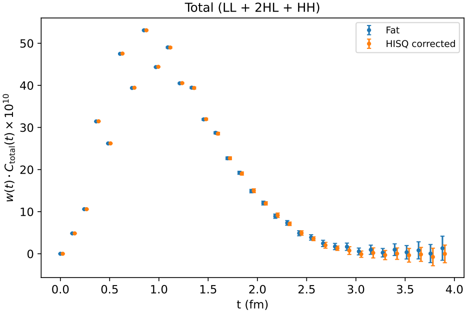
Region analysis

- $t = 0$–2 fm: Ratio $= 1.0002 \pm 0.0008$, Consistent with unity.
- $t = 2$–2.5 fm: Growing but controlled and still consistent with unity
- $t > 2.5$ fm: Signal lost in noise

Validation on $48^3 \times 64$: $w(t)C(t)$

- $48^3 \times 64, a = 0.1212$ fm
- $N_{\text{total}} = 51$
- $N_{\text{hisq}} = 51$
- $N_{\text{bias}} = 20$ configurations





- Fat-link and bias-corrected HISQ $w(t)C(t)$ overlap across all timeslices
- Errors grow at large t as correlator signal decays exponentially

Validation on $48^3 \times 64$: On Windows

- $48^3 \times 64, a = 0.1212$ fm • $N_{\text{total}} = 51$ • $N_{\text{hisq}} = 51$ • $N_{\text{bias}} = 20$ configurations
- Ratio $= a_\mu^{\text{HVP,lqc}}$ (hisq corrected)/$a_\mu^{\text{HVP,lqc}}$ (fat)

	Intermediate $(0.4,\ 1.0, \Delta = 0.15)$ fm	**Intermediate** $(1.5,\ 1.9, \Delta = 0.15)$ fm	**Long distance** $(t_1 = 1, \Delta = 0.15)$ fm
LL only			
ratio	1.0001 ± 0.0005	0.9988 ± 0.0041	1.0041 ± 0.0121
consistency	0.22σ ✓	0.31σ ✓	0.34σ ✓
Total			
ratio	1.00004 ± 0.00017	0.9991 ± 0.0030	1.0027 ± 0.0081
consistency	0.22σ ✓	0.31σ ✓	0.34σ ✓

Reduced bias sample

Even with only 10 bias configurations, the corrected result agrees with the fat-link result within 1σ.

Application to $144^3 \times 288$, $a = 0.042$ fm

Current status

- LL+HL on 20 configurations and HH on 27 configurations
- Fat-link eigenvectors only
- $a_\mu^{\mathrm{HVP,lqc}} = 658(15) \times 10^{-10}$ on this ensemble.

Proposed strategy

- Leverage 38 configurations with HISQ eigenvectors ($N_{\mathrm{low}} = 4000$) already available from MILC/Fermilab
- Apply AMA strategy bias correction to account for the fat-link $\leftrightarrow$ HISQ action difference

- Continue generating eigenvectors and correlators on additional 62 configurations, targeting **120 configurations total**

Projected error reduction: $\sqrt{(100+20)/20} \approx 2.4\times$

Bringing the statistical uncertainty on $a_\mu^{\mathrm{HVP,lqc}}$ from **2.3%** to **0.9%**

Summary

What we showed:

- ✓ Validated strategy to boost C_{LL} statistics using HISQ eigenvectors.
- ✓ AMA bias correction eliminates systematic difference between actions.
- ✓ Sub-percent bias, $\leq 0.34\sigma$ agreement on $48^3 \times 64$ (51 configs) across all windows.
- ✓ $< 2\%$ error overhead with 20 bias configurations.
- ✓ No systematic bias detected.
- ✓ Long distance divergence is signal loss, not method failure.

What we plan to:

- Use the validated AMA bias correction ($< 2.5\%$ overhead at $N_{\text{bias}} = 20$), this enables scaling from 20 to 100 configurations at a **fraction of the conventional cost**, projecting a $2.4\times$ improvement in a_μ^{HVP}.
- Also apply this AMA strategy on the sub-dominant, high-low part of correlation function to further our precision goals on $a_\mu^{\text{HVP,lqc}}$.

Thank you for your attention!

Any questions?

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