



Nucleon Unpolarized PDF at Physical Point with High Boost

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Why PDF matter & How they are obtained

LHC:

Precision Standard Model tests

Drell-Yan & SIDIS:

Nucleon sea flavor decomposition



Why

PDF: probability of finding a quark or gluon carrying x of the hadron



How

Global QCD Analyses:

Fit DIS, Drell-Yan and Jet data

Lattice QCD: first-principles

Independent cross-check of global fits

LaMET: Two Requirements in Balance

PRL 110 (2013) 262002
SCPMA 57 (2014) 1407
RMP 93 (2021) 035005

Boost the nucleon to a large & finite momentum: Quasi-PDF $\Rightarrow$ Light-cone PDF

Large Momentum P_z

📌 Power corrections scale as

$$\left(\frac{\Lambda_{\text{QCD}}}{xP_z}\right)^2 \text{ and } \left(\frac{m}{xP_z}\right)^2$$

Fine Lattice Spacing a

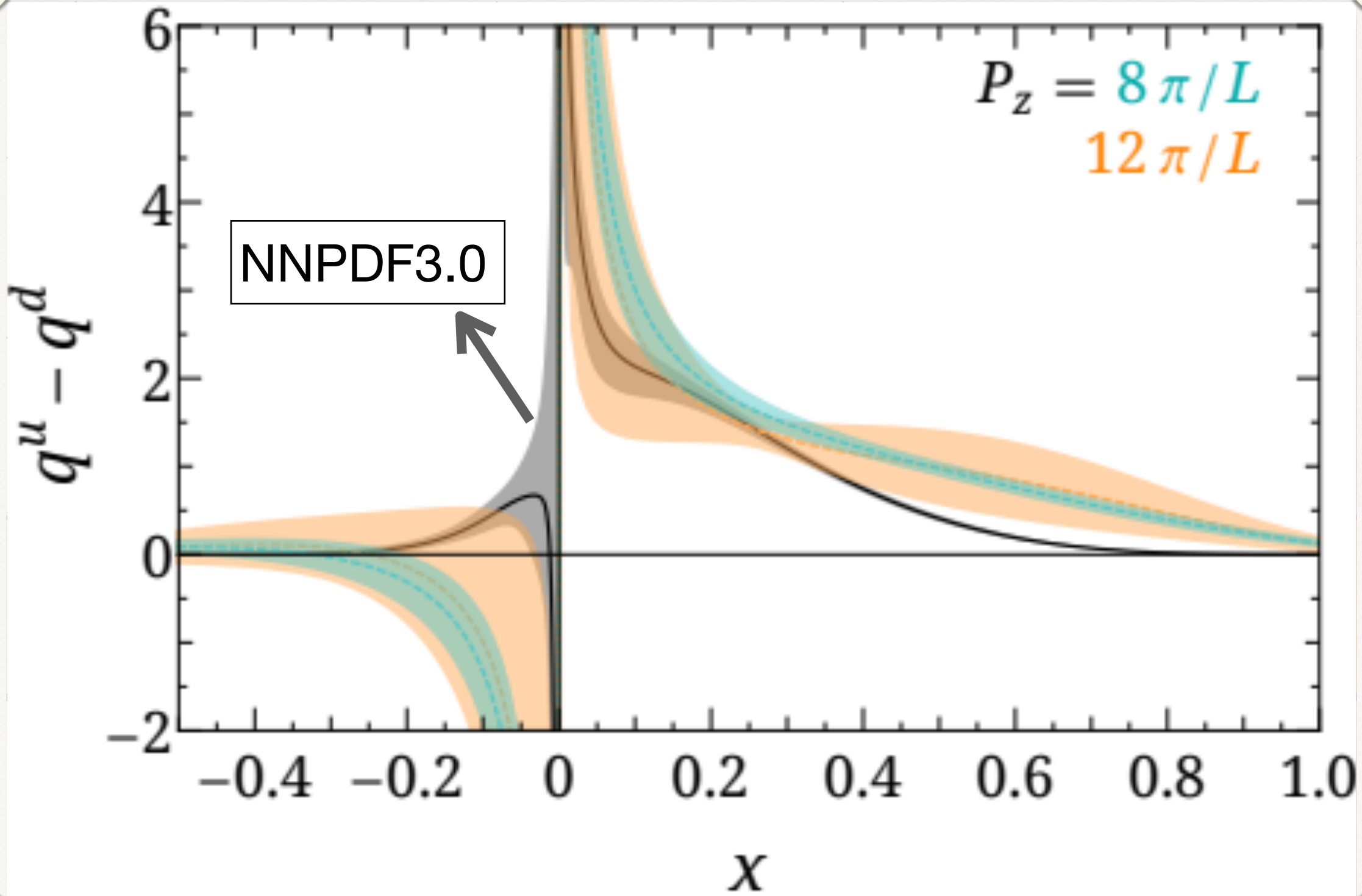
📌 Discretization artifacts grow with aP_z
and become uncontrolled as $aP_z \rightarrow 1$

$$aP_z < 1$$

Complementary approaches: pseudo-PDF, lattice cross-section method, HOPE

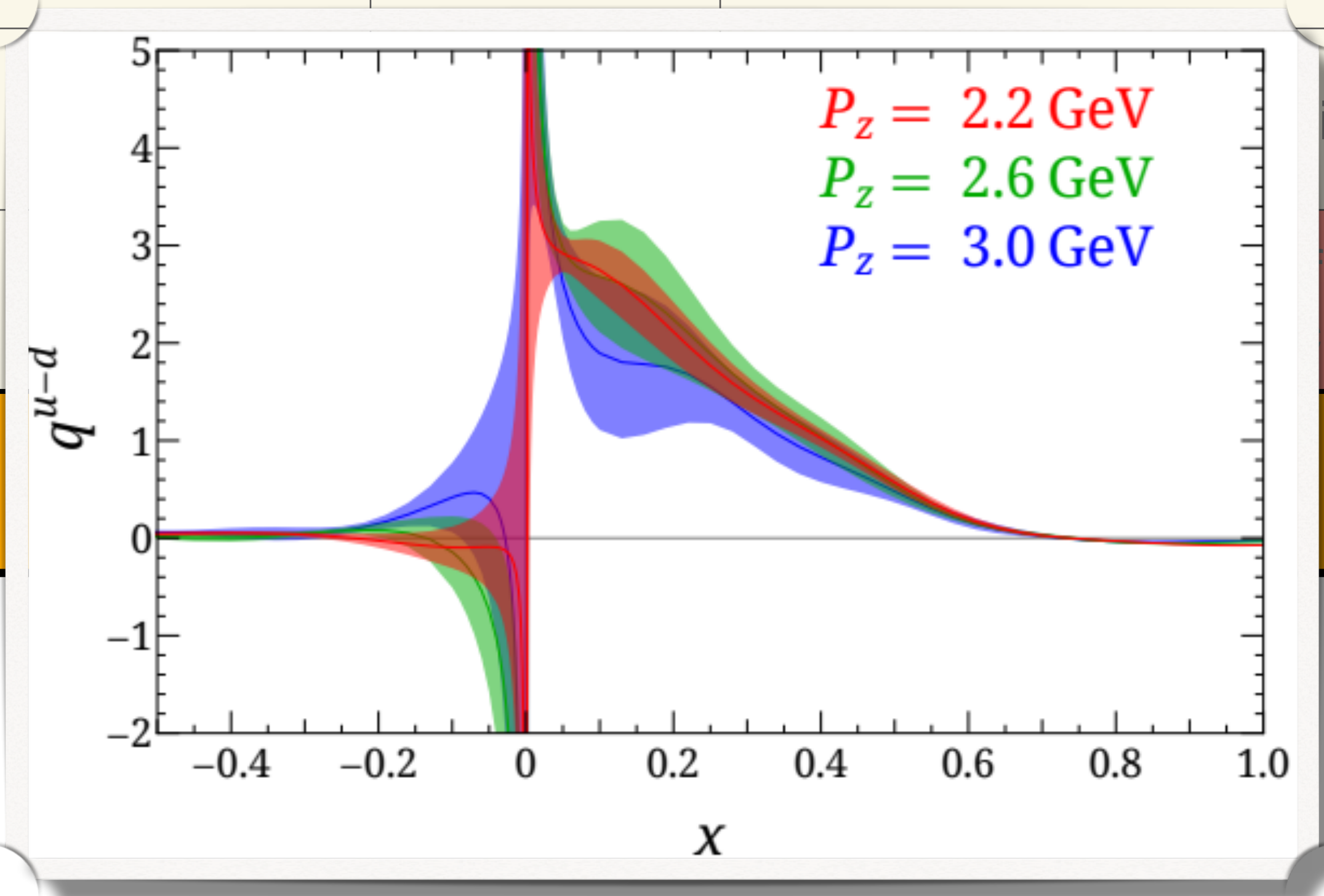
Lattice nucleon unpolarized PDF: P_z & m_π

Study	a [fm]	m_π [MeV]	P_z [GeV]	aP_z	Outcome
PRD 98 (2018) 5, 054054	0.09	Physical	1.29	< 0.6	Higher P_z needed
arXiv: 1803.04393	0.09	Physical			
PRD 107 (2023) 7, 074509	0.076	Physical			
arXiv: 2602.11283	0.06	300			
This work	0.076	Physical			



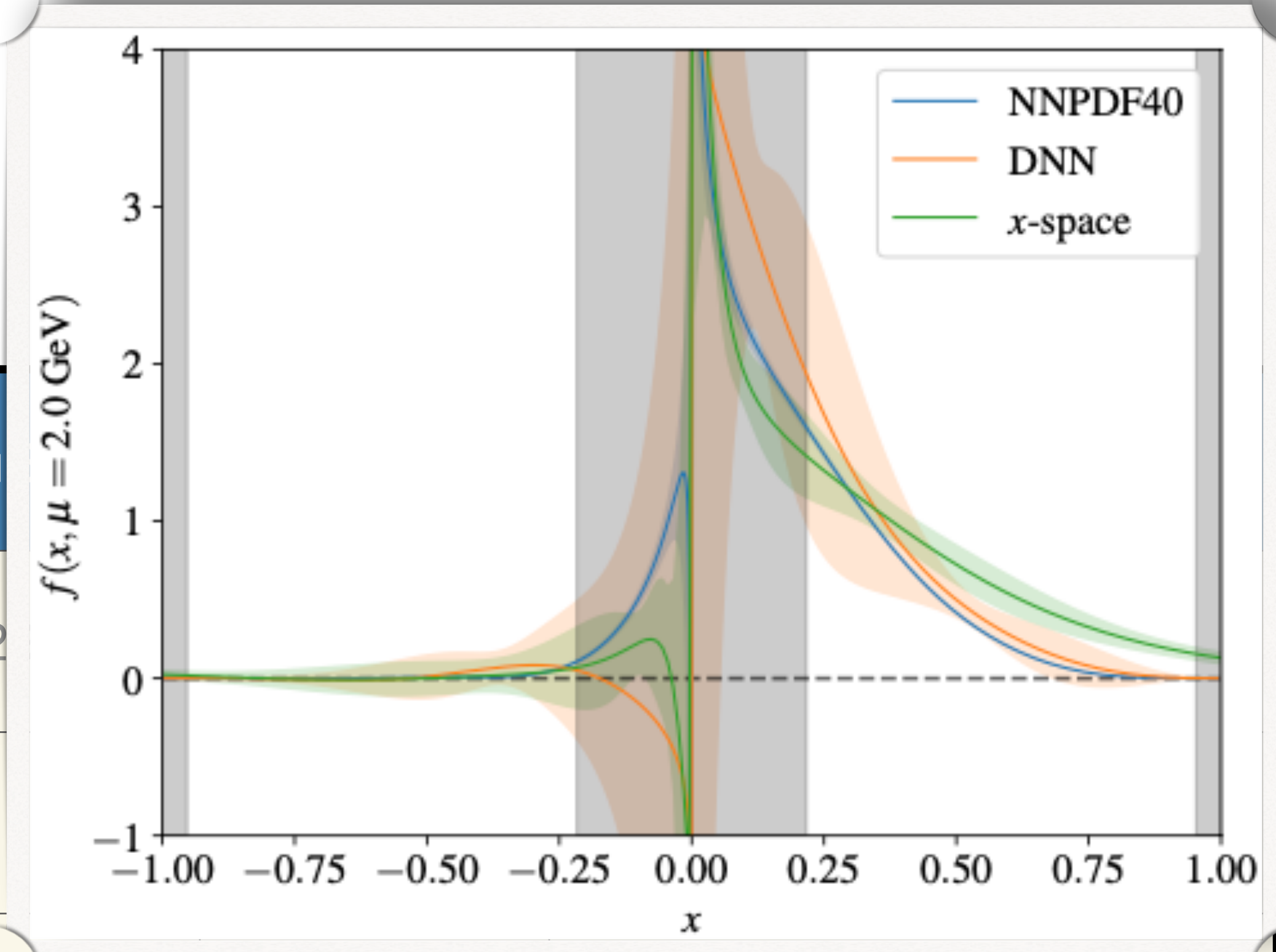
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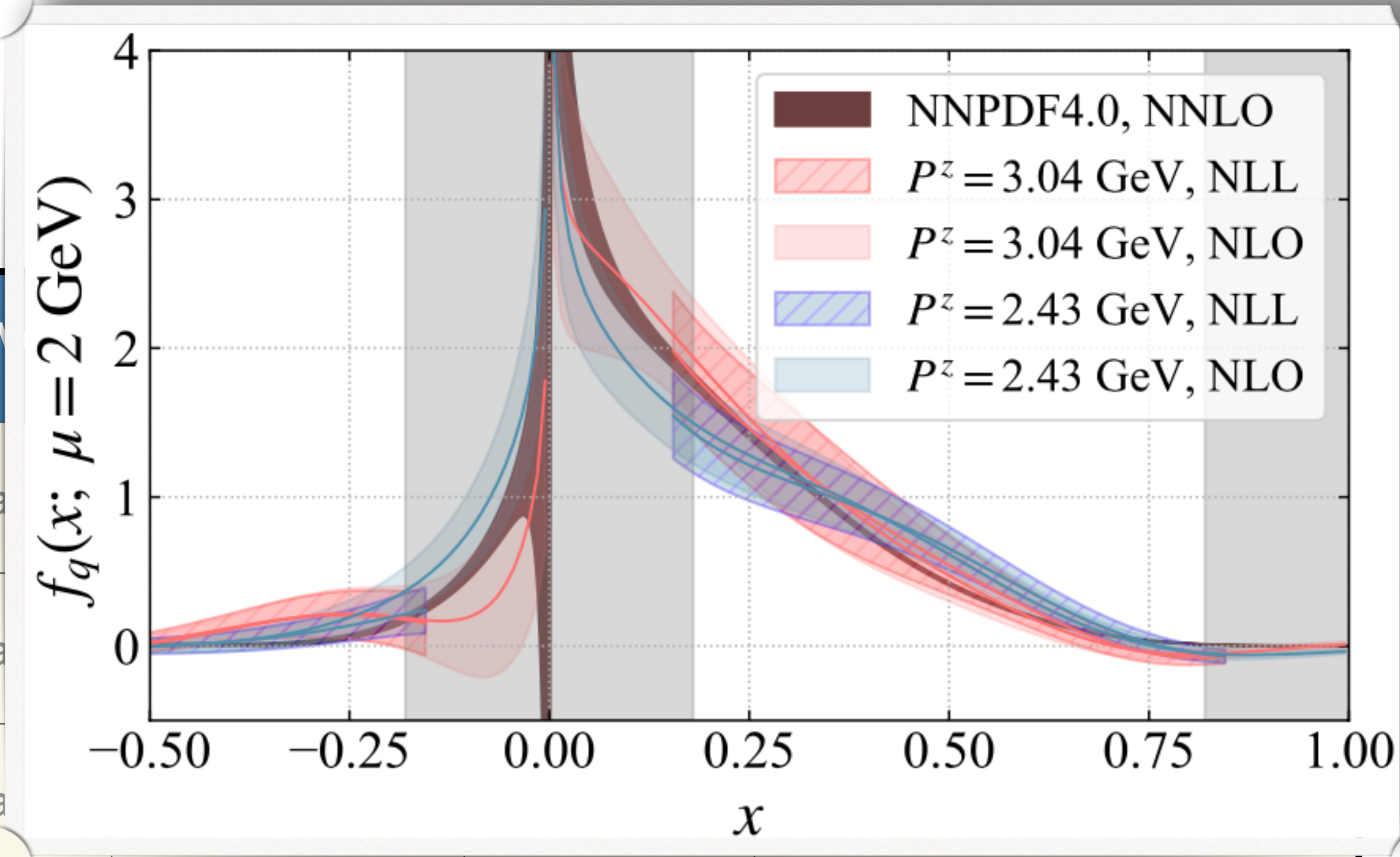
Lattice nucleon unpolarized PDF: P_z & m_π

Study	a [fm]	m_π [MeV]	P_z [GeV]		
PRD 98 (2018) 5, 054054	0.09	Physical	1.2		
arXiv: 1803.04393	0.09	Physical	3		
PRD 107 (2023) 7, 074509	0.076	Physical	1.53	< 0.6	Deviates from global fits
arXiv: 2602.11283	0.06	300	3	< 1	Compatible w. Global fits; Good P_z convergence
This work	0.076	Physical	2.29	< 0.9	?



Lattice nucleon unpolarized PDF: P_z & m_π

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Motivation

Study	a [fm]	m_π [MeV]	P_z [GeV]	aP_z	Outcome
PRD 98 (2018) 5, 054054	0.09	Physical	1.29	< 0.6	Higher P_z needed
arXiv: 1803.04393	0.09	Physical	3	> 1.3	Uncontrolled aP_z
PRD 107 (2023) 7, 074509	0.076	Physical	1.53	< 0.6	Deviates from global fits
arXiv: 2602.11283	0.06	300	3	< 1	Compatible w. Global fits; Good P_z convergence
Dominance of P_z or Accidental coincidence ?					
This work	0.076	Physical	2.29	< 0.9	?

A Universality Test:

The lattice determination reflects the underlying physics rather than a particular choice of ensemble or extraction method

Large P_z with Controlled Systematics

★ $a = 0.076$ fm, $L_s \times L_t = 64^3 \times 64$; Physical Point

★ Isovector ($u - d$) & connected isoscalar ($u + d$)

$$\Rightarrow P_z = \{1.78, 2.29\} \text{ GeV}, \quad aP_z < 0.9$$

Coulomb-Gauge Quark Bilinears

$$C_{3\text{pt}}^q(\mathbf{P}, \tau, t_s; z) = \sum_{\mathbf{x}, \mathbf{y}} e^{-i\mathbf{P} \cdot \mathbf{x}} \mathcal{P}_{\alpha\beta}^{\text{unpol}} \langle N_\beta(\mathbf{x}, t_s) \bar{q}(z) \gamma^t q(0) \bar{N}_\alpha(\mathbf{0}, 0) \rangle$$

- 🔧 Avoids the divergence in the Wilson line
- 🔧 Simpler renormalization, better SNR at **large z**
- 🔧 Belongs to universality class in LaMET

PRD 109 (2024) 094506

Enhanced Interpolating Operators

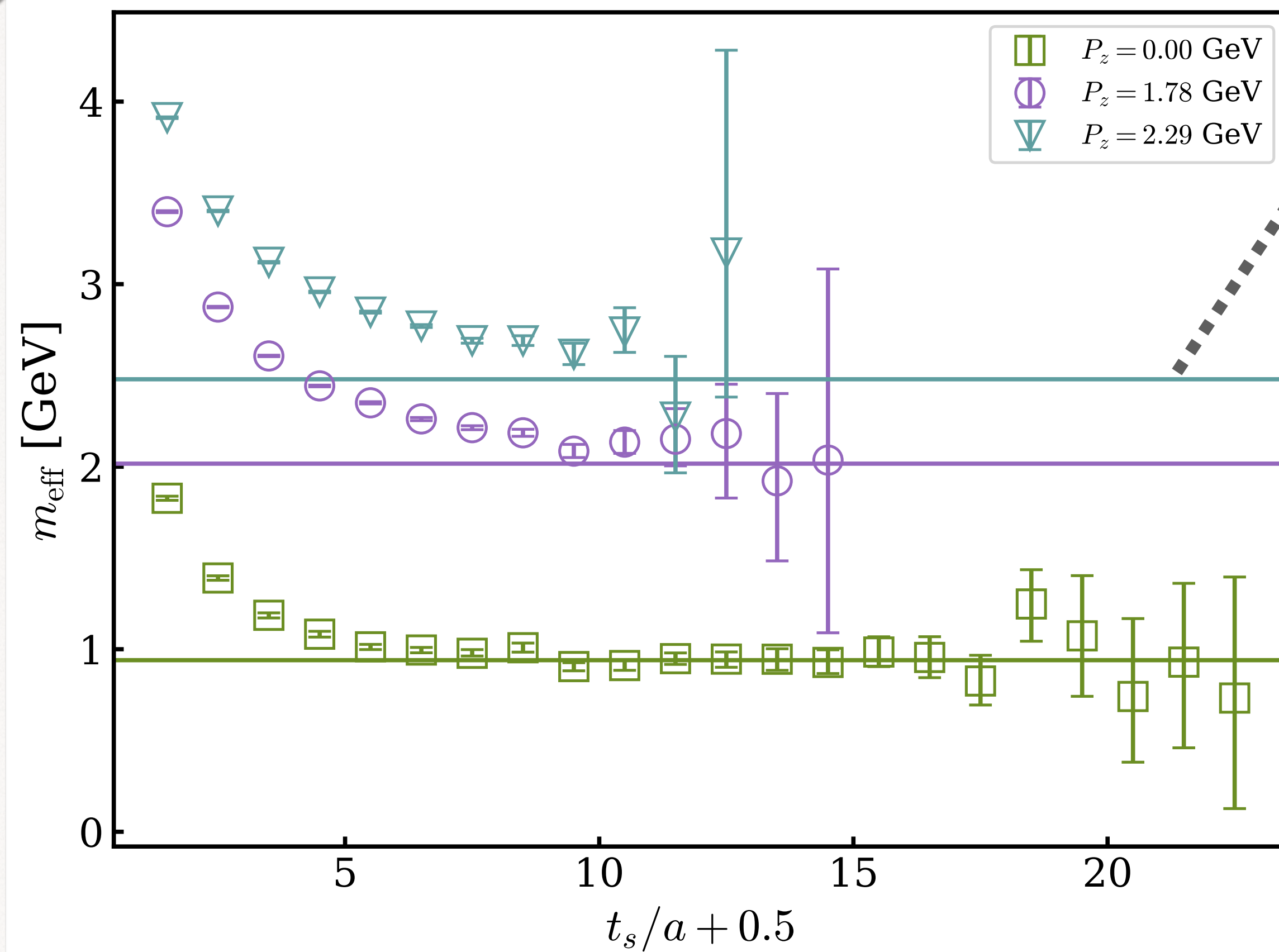
$$N_\alpha(\mathbf{x}, t) = \epsilon_{abc} u_{a,\alpha}(\mathbf{x}, t) \left[u_b^T(\mathbf{x}, t) C \gamma_t \gamma_5 d_c(\mathbf{x}, t) \right]$$

- 🔧 Ground-state overlap grows with boost
- 🔧 SNR improves at **large P_z**

PRD 112 (2025) L051502

Bare Matrix Element h^B

$$E(P_z) = \sqrt{m_N^2 + P_z^2}, \quad m_N = 0.94 \text{ GeV}$$



$$\begin{cases} C_{2\text{pt}}(\mathbf{P}, t_s) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t_s}, \\ C_{3\text{pt}}(\mathbf{P}, \tau, t_s; z) = \sum_{m,n=0}^{\infty} A_m h_{mn} A_n^* e^{-E_m(t_s-\tau)} e^{-E_n \tau} \end{cases}$$

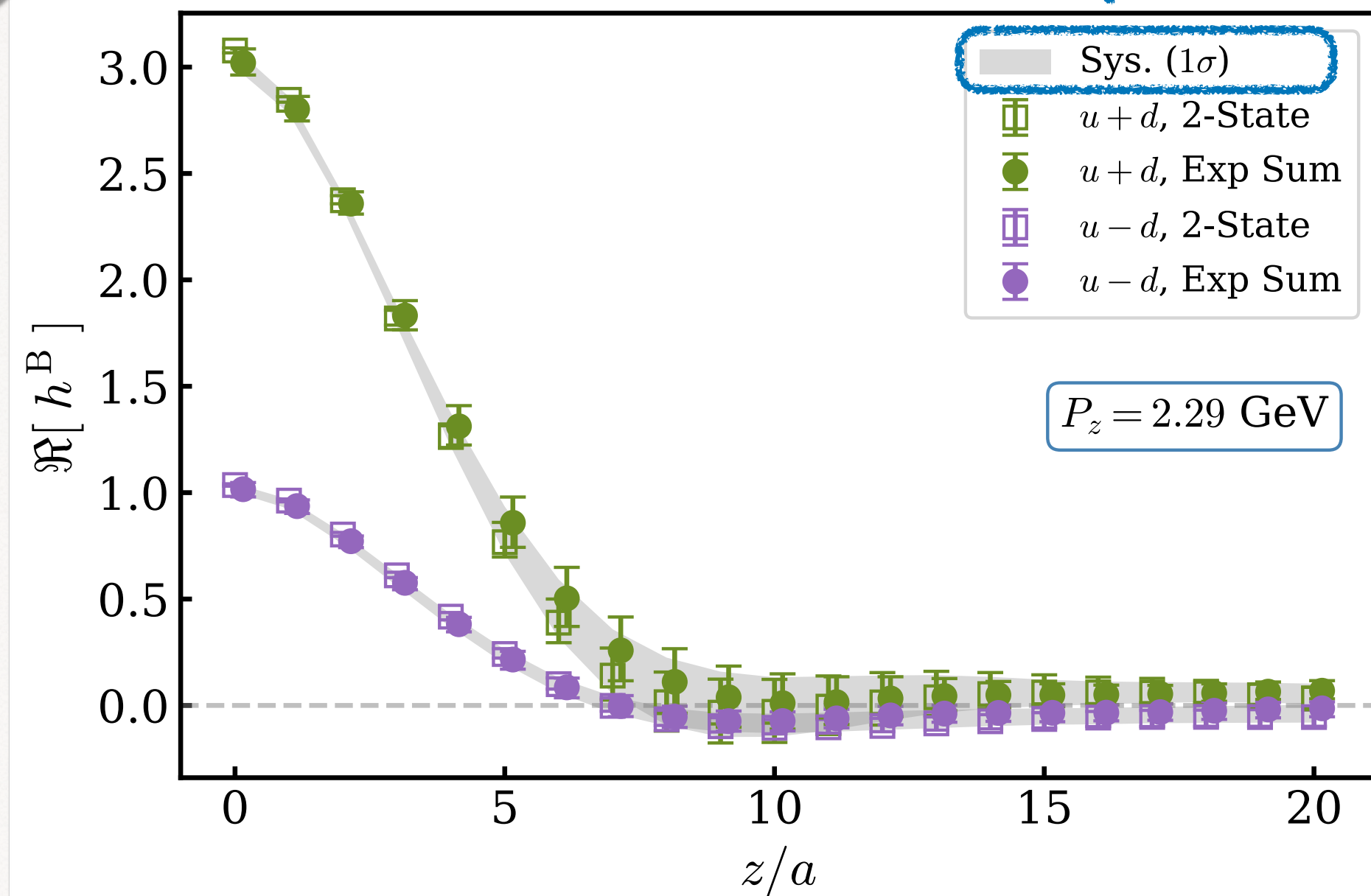
$$\Rightarrow R(\mathbf{P}; \tau, t_s; z) \equiv \frac{C_{3\text{pt}}(\mathbf{P}, \tau, t_s; z)}{C_{2\text{pt}}(\mathbf{P}, t_s)}$$

$\downarrow t_s \rightarrow \infty$

$$h^B = h_{00}$$

h^B & h^R

Statistical + Systematic



NPB 964 (2021) 115311

★ UV divergences in $h^B \Rightarrow$ Hybrid renormalization:

$$h^R(z, z_s, P_z) = N \frac{h^B(z, P_z, a)}{h^B(z, 0, a)} \theta(z_s - |z|) + N \frac{h^B(z, P_z, a)}{h^B(z_s, 0, a)} \theta(|z| - z_s),$$

$$N = h^B(0, 0, a) / h^B(0, P_z, a)$$



Unphysical oscillations in PDF $\Rightarrow$

arXiv: 2601.12189

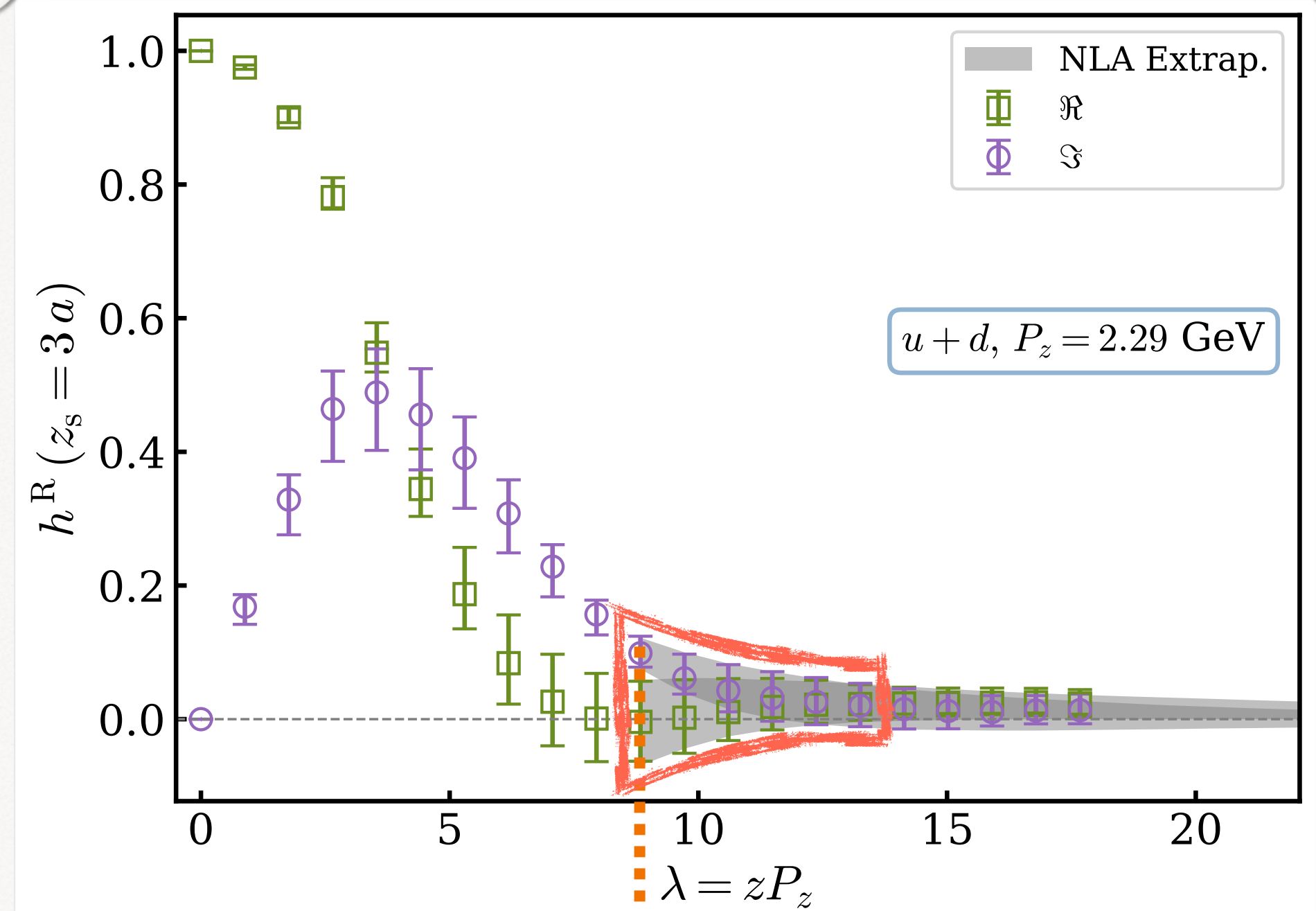
Extrapolation w. Next-to-leading-asymptotic (NLA) model:

$$h^{\text{NLA}}(z, P_z) = \left[A_2 e^{i \text{sgn}(\lambda) \phi_2} + \frac{A'_2}{|\lambda|} e^{i \text{sgn}(\lambda) \phi'_2} \right] \frac{e^{-m|z|}}{|\lambda|^n},$$

$\lambda = zP_z, \{A_2, A'_2, \phi_2, \phi'_2, m, n\}$ fit params



$$h^R(z, P_z) = \begin{cases} h^R(z, P_z), & |z| \leq z_{\text{extrap}}, \\ h^{\text{NLA}}(z, P_z), & |z| > z_{\text{extrap}}. \end{cases}$$



★ Quasi-PDF: $\tilde{q}(x, P_z, z_s) = P_z \int \frac{dz}{2\pi} e^{-ixP_z z} h^R(z, P_z, z_s)$

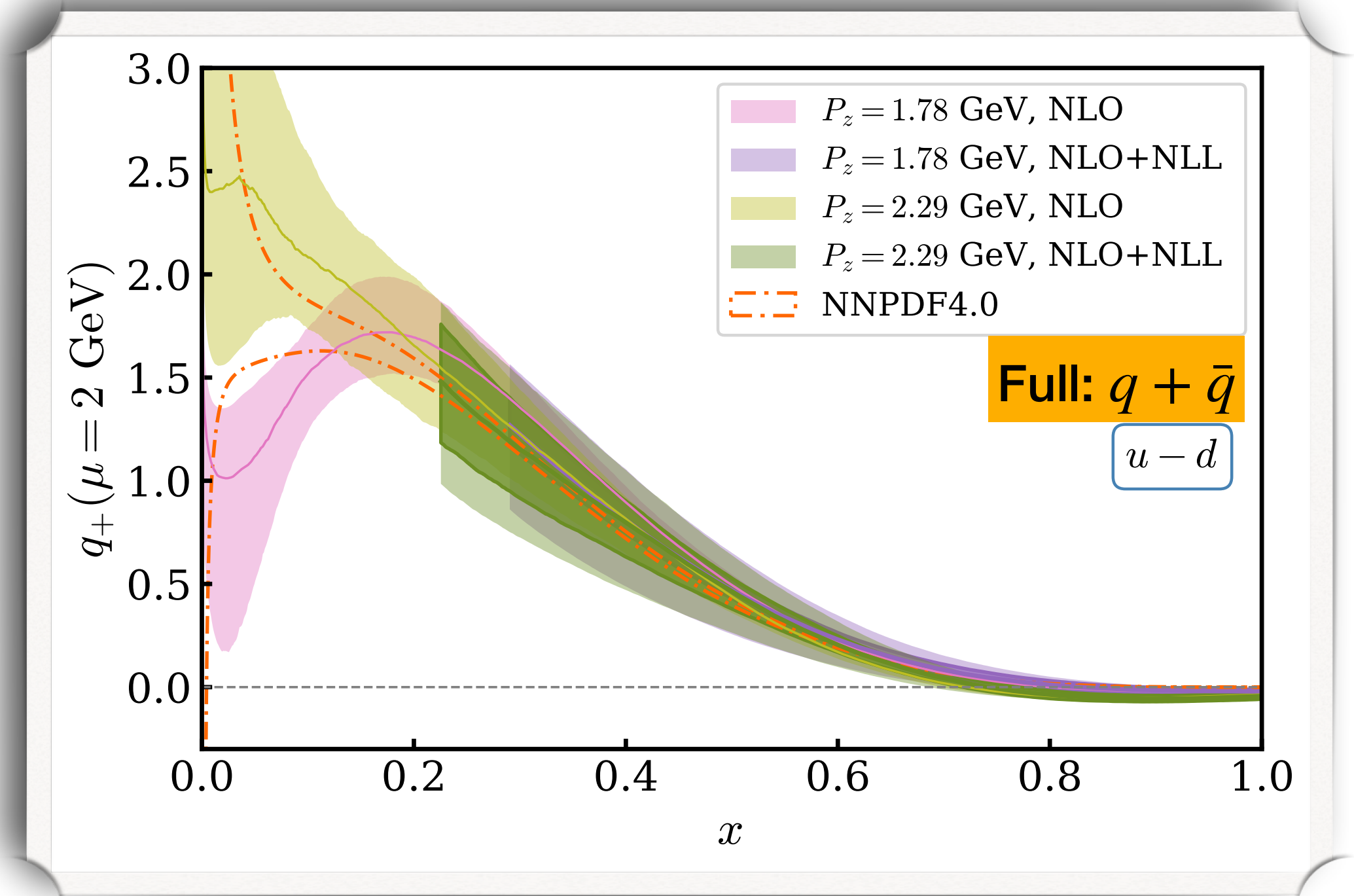
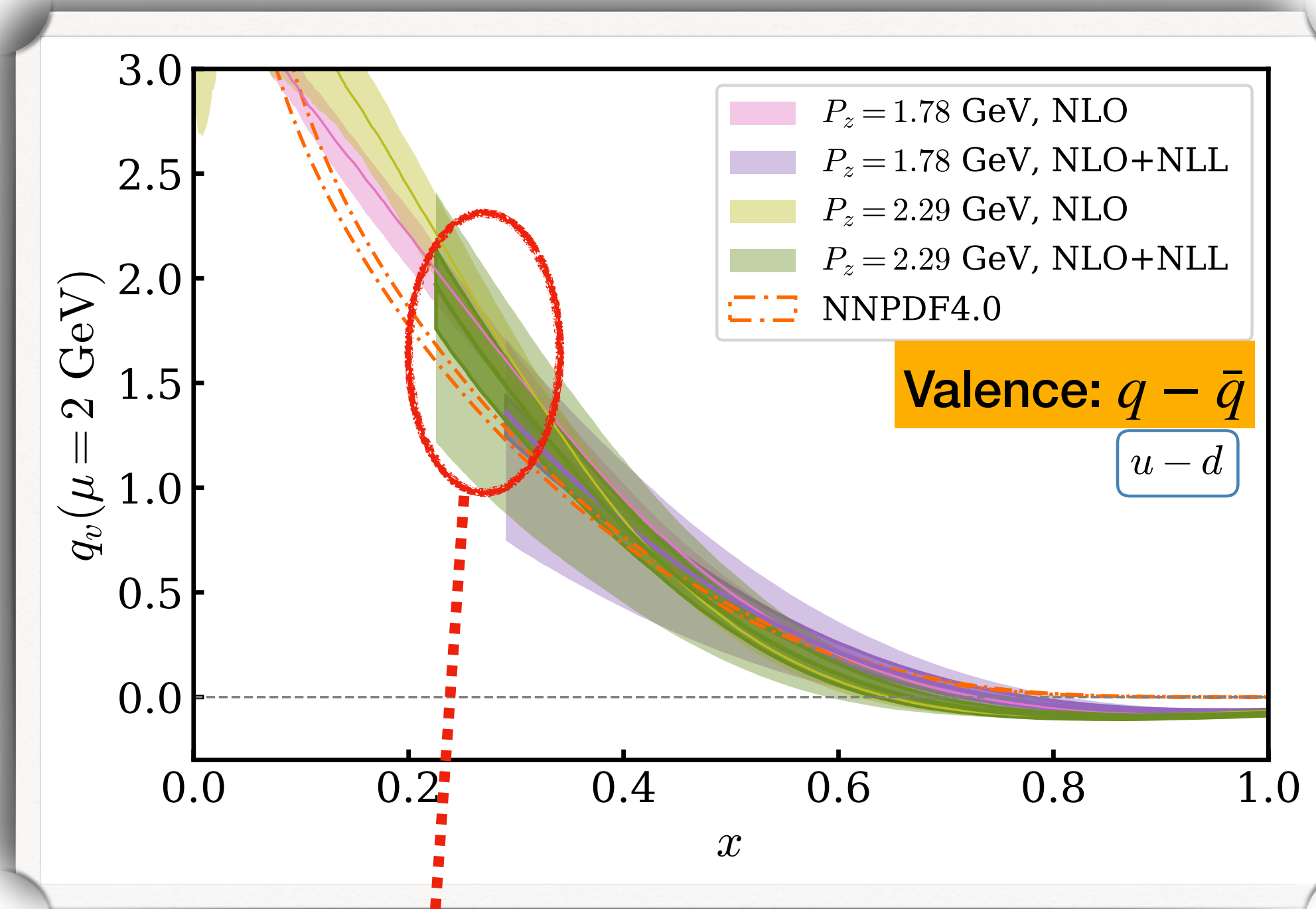
★ Fixed order at NLO: large logarithms $\propto \ln(\mu^2 / (2yP_z)^2)$

$$q(x, \mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \left[\delta\left(1 - \frac{x}{y}\right) - \frac{\alpha_s(\mu) C_F}{2\pi} C_{\text{hyb.}}^{(1)}\left(\frac{x}{y}, \frac{\mu}{p_z}, z_s p_z\right) \right] \tilde{q}(y, P_z, z_s), \quad p_z = xP_z$$

★ RGR improved NLO+NLL: $\mu_0 = 2 \kappa z P_z$, $\ln(\mu_0^2 / (2yP_z)^2) \Rightarrow \mu = 2 \text{ GeV}$ NPB 991 (2023) 116201

$$q(x, \mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \int_{-\infty}^{\infty} \frac{dz}{|z|} C_{\text{evo}}^{-1}\left(\alpha_s(\mu_0), \frac{x}{z}, \frac{\mu}{\mu_0}\right) \times \left[\delta\left(1 - \frac{z}{y}\right) - \frac{\alpha_s(\mu_0) C_F}{2\pi} C_{\text{hyb.}}^{(1)}\left(\frac{z}{y}, \frac{\mu_0}{p_z}, z_s p_z\right) \right] \tilde{q}(y, P_z, z_s)$$

Light-cone PDF: $u - d$



NLO+NLL:

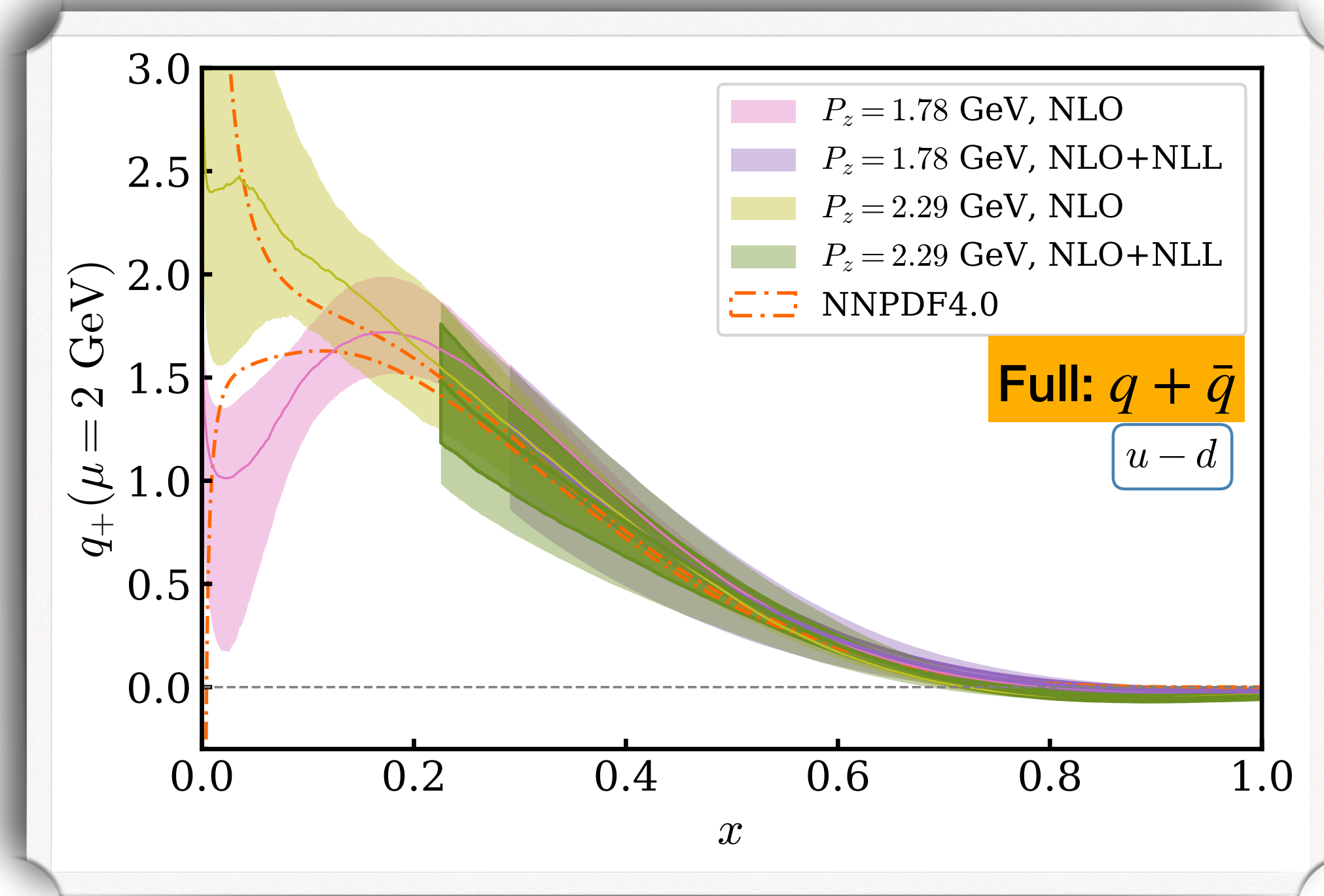
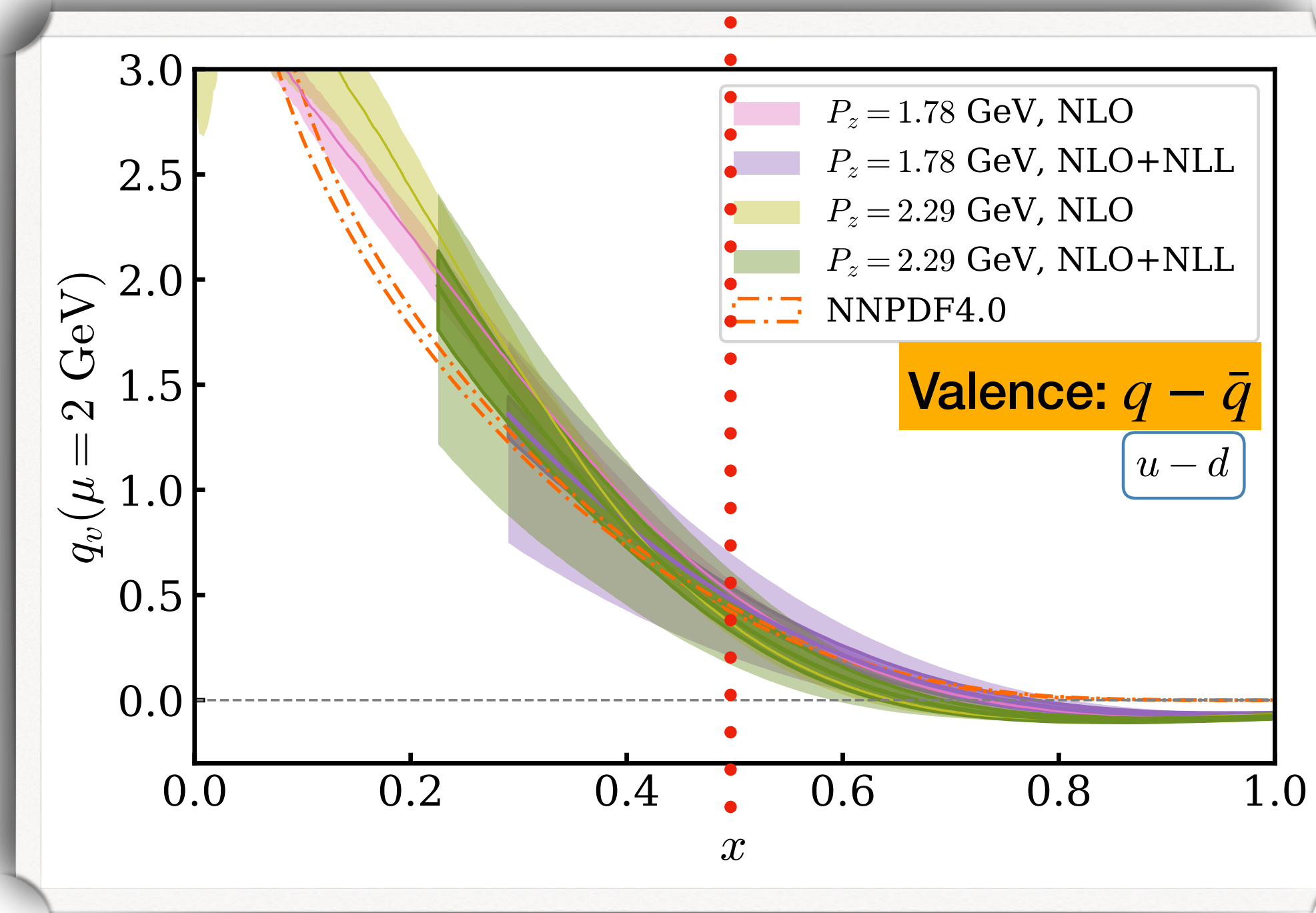
① Bands:

Central scale $\kappa = 1$

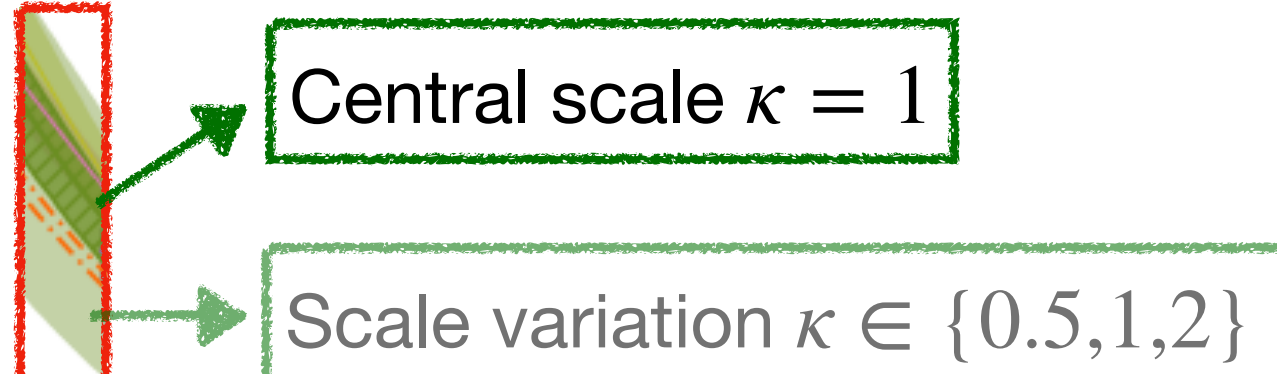
Scale variation $\kappa \in \{0.5, 1, 2\}$

② Cutoff: $\mu_0 = 2\kappa x P_z, \alpha_s(\mu_0) < 1 \rightarrow x_{\min}$

Light-cone PDF: $u - d$



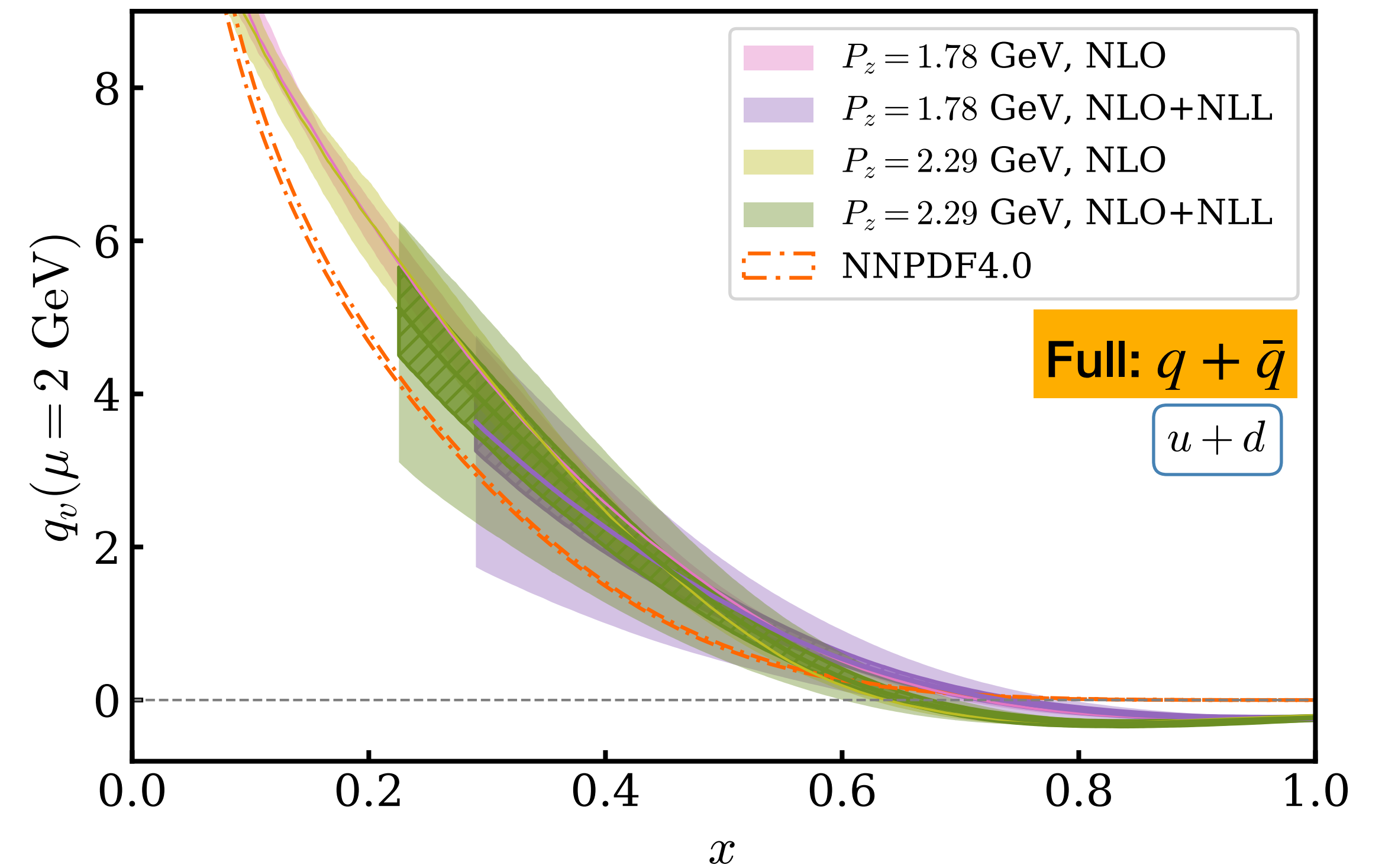
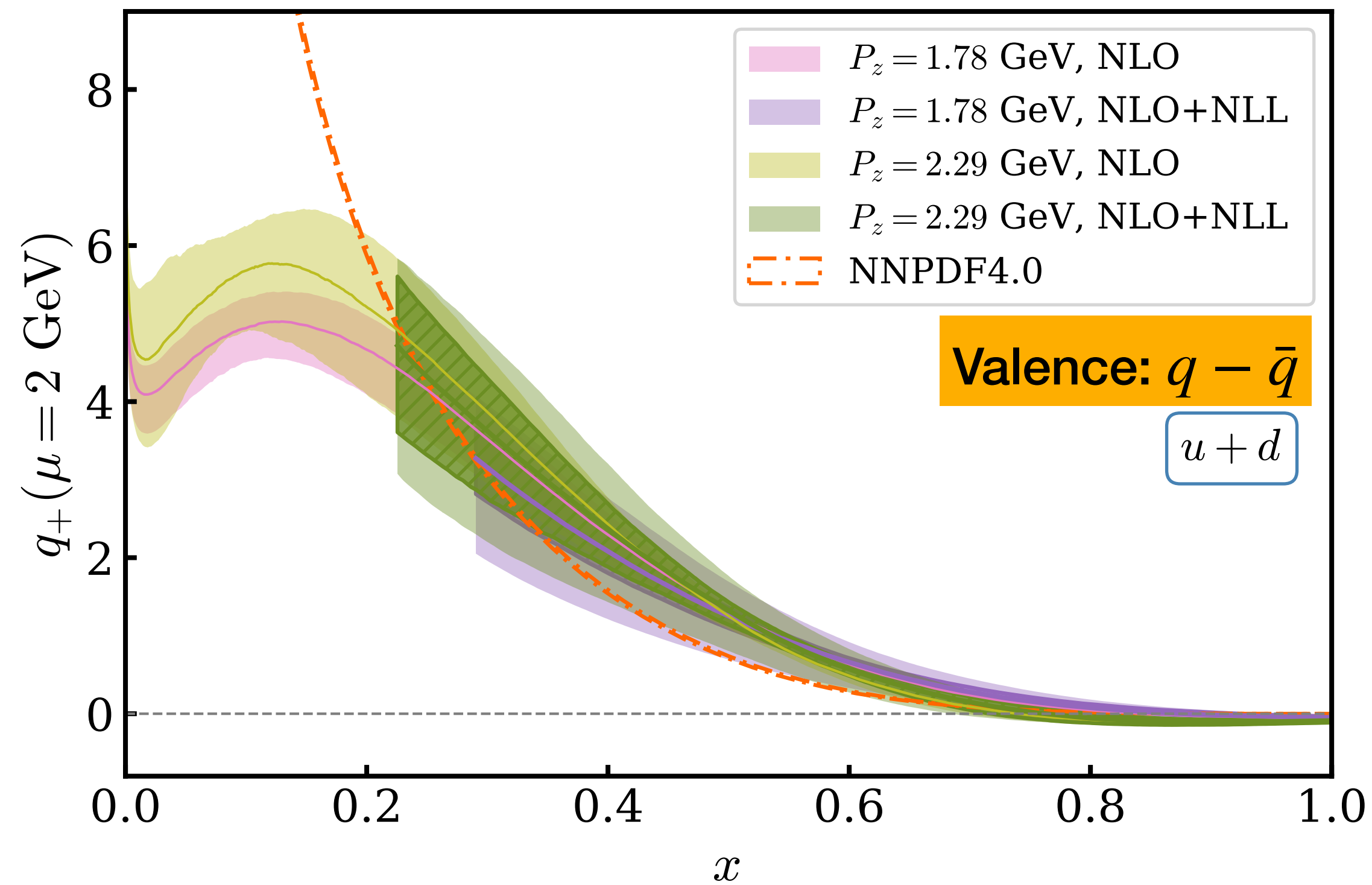
NLO+NLL:

- ① Bands: 
 - Central scale $\kappa = 1$
 - Scale variation $\kappa \in \{0.5, 1, 2\}$
- ② Cutoff: $\mu_0 = 2\kappa x P_z, \alpha_s(\mu_0) < 1 \rightarrow x_{\min}$

-  $x \downarrow$ — scale dependence $\uparrow$
-  $x \lesssim 0.5$: $\mu_0 / \mu < 1$, RGR correction
-  $0.25 \lesssim x \lesssim 0.75$: Consistent with global fits
-  $x \rightarrow 1$: threshold resummation in the future

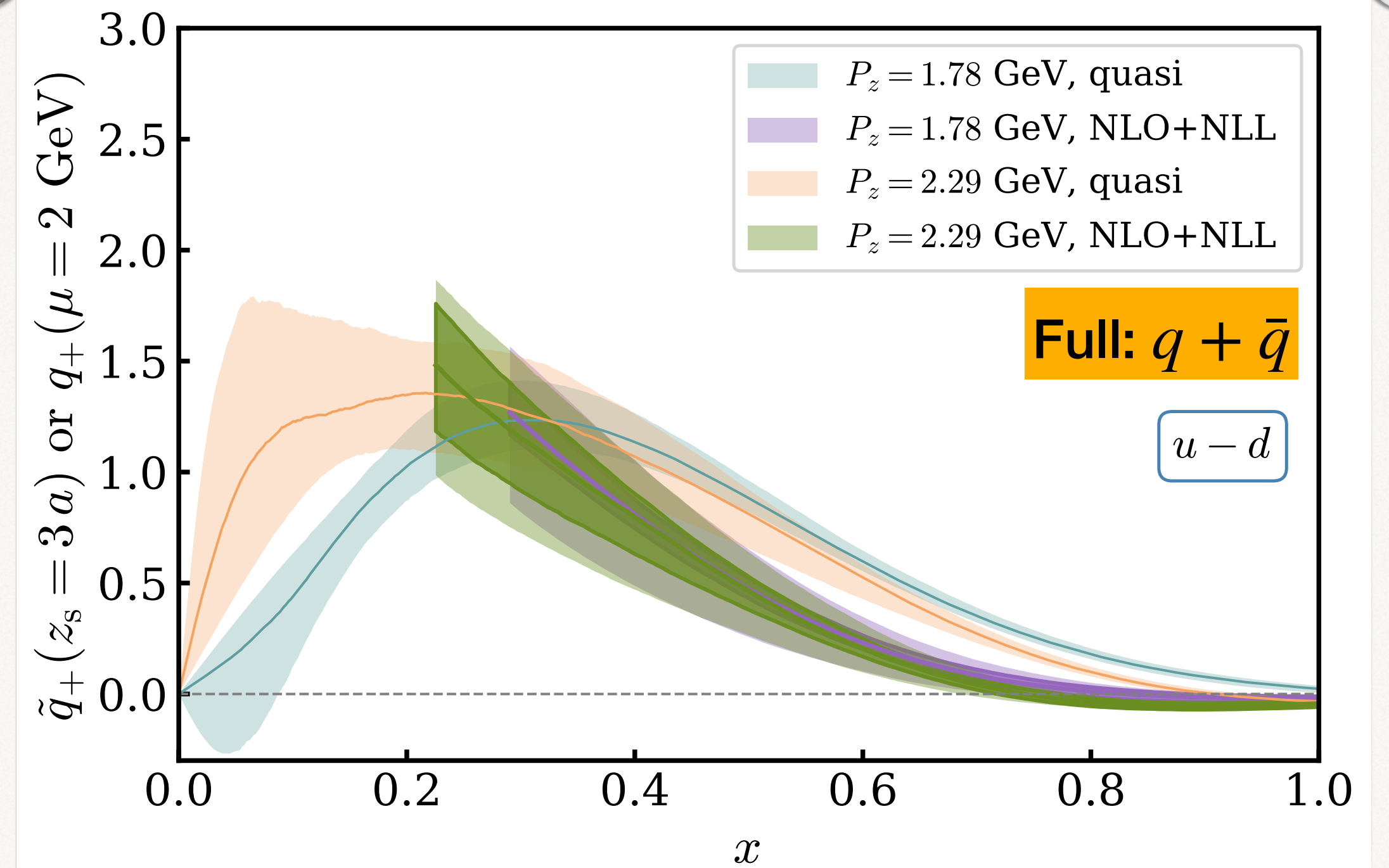
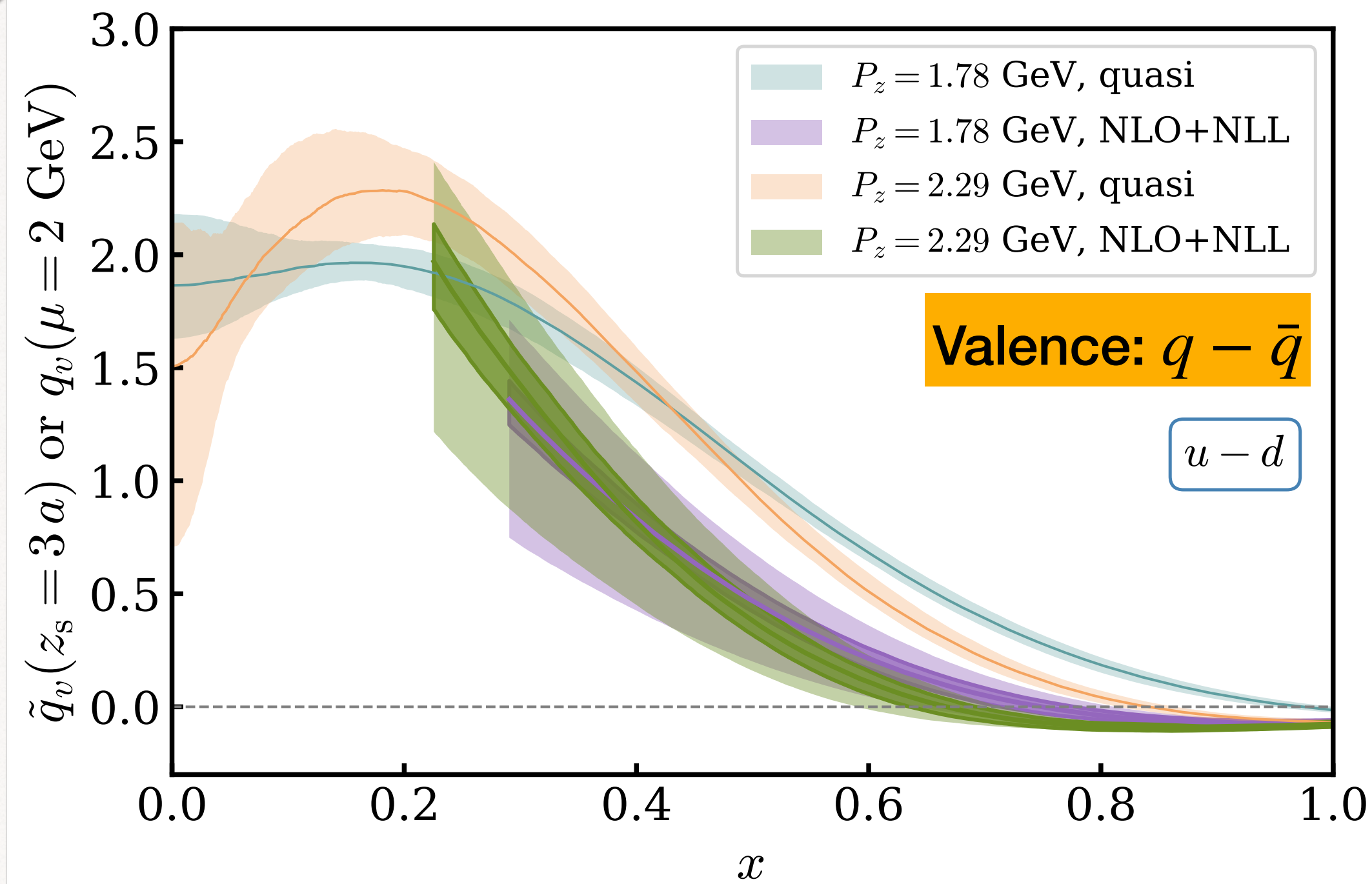
Light-cone PDF: connected $u + d$

NNPDF4.0: EPJC 82 (2022) 428



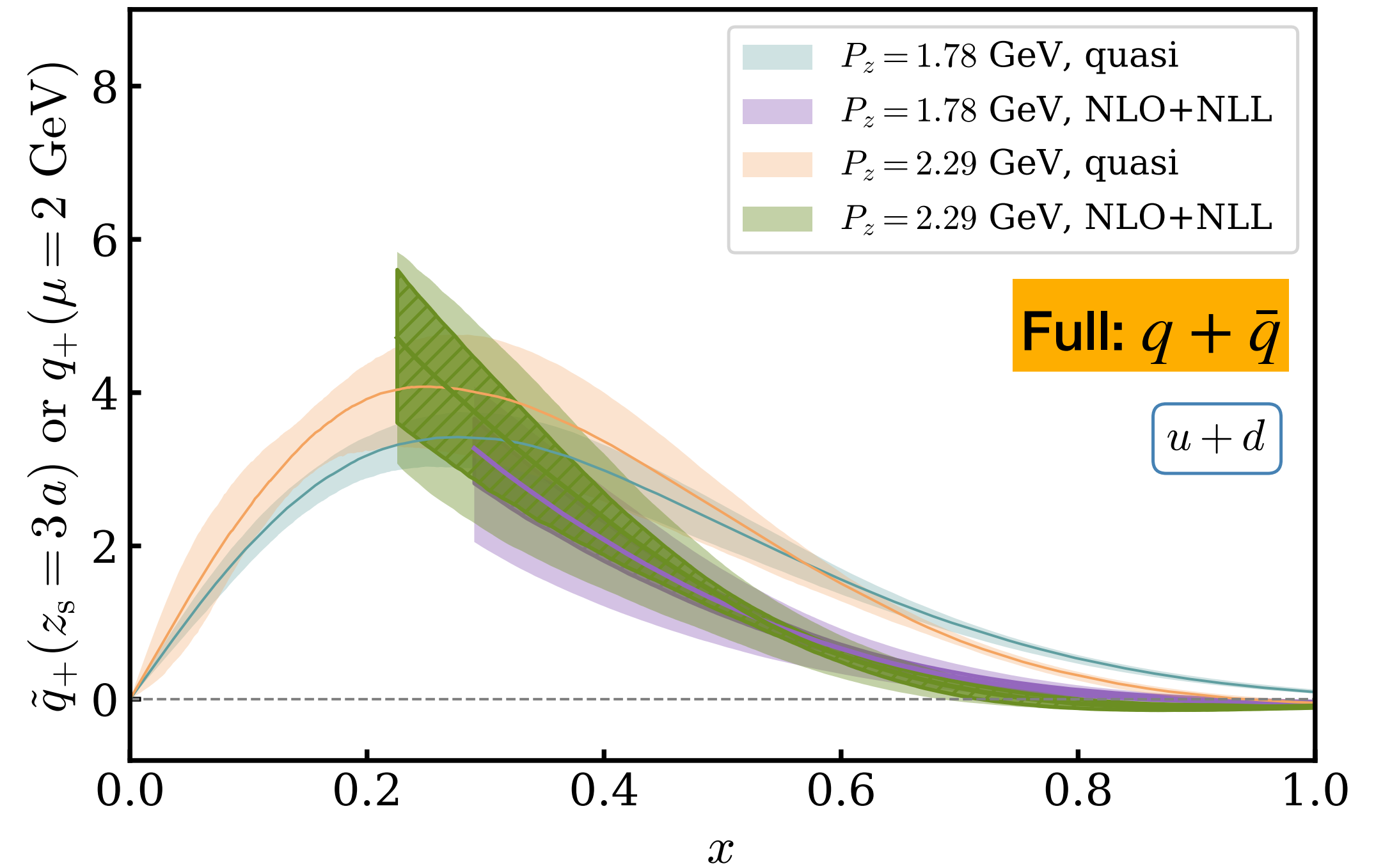
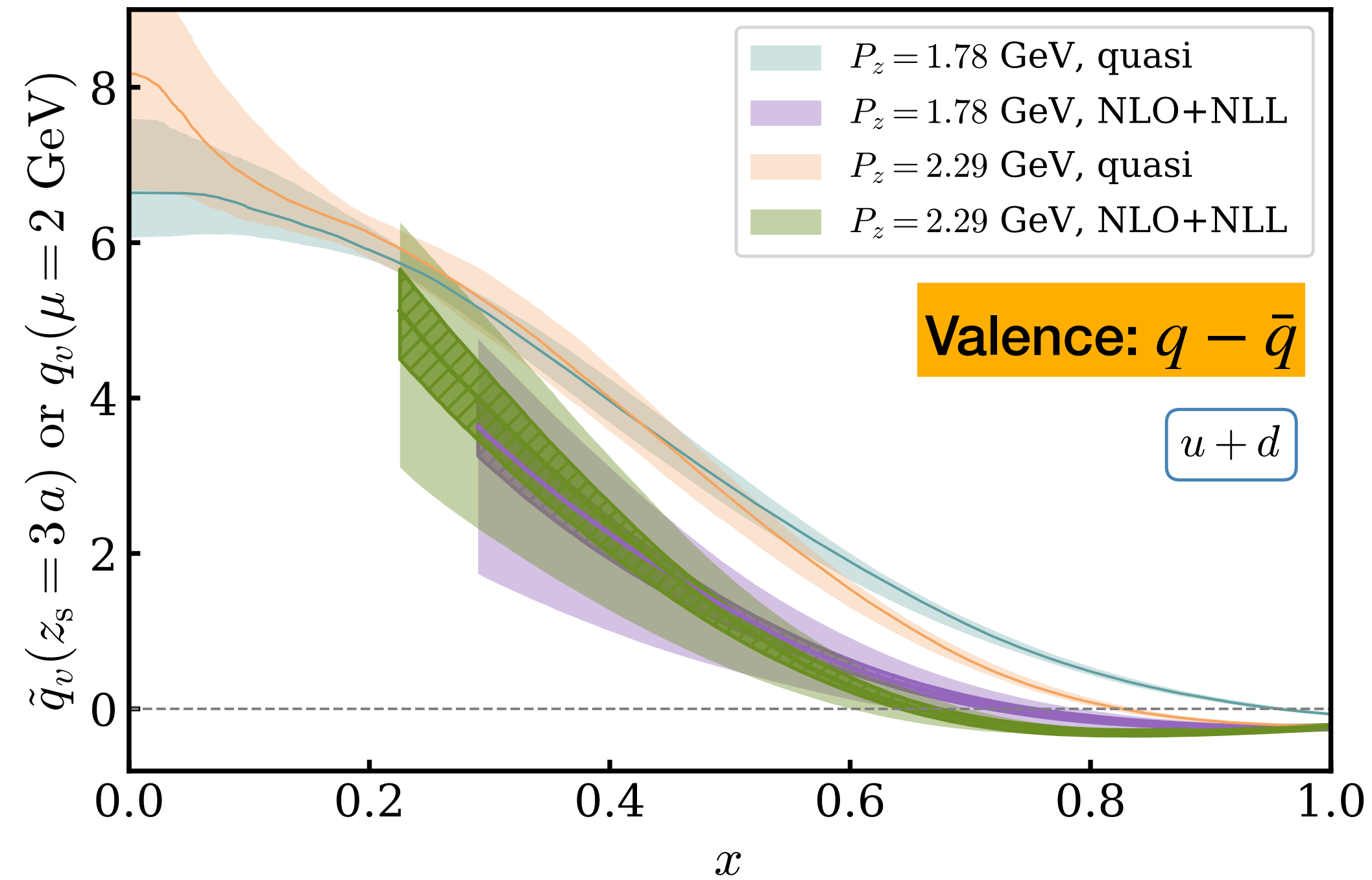
- Consistent within the scale variation
- Omit the disconnected diagrams $\rightarrow$ Offset in the central curves

P_z -dependence: $u - d$



- ⌚ Perturbative matching improves the consistency between diff. momenta
- ⌚ LaMER expectation: momentum-independent light-cone results

P_z -dependence: connected $u + d$



Valence: $q - \bar{q}$

Full: $q + \bar{q}$

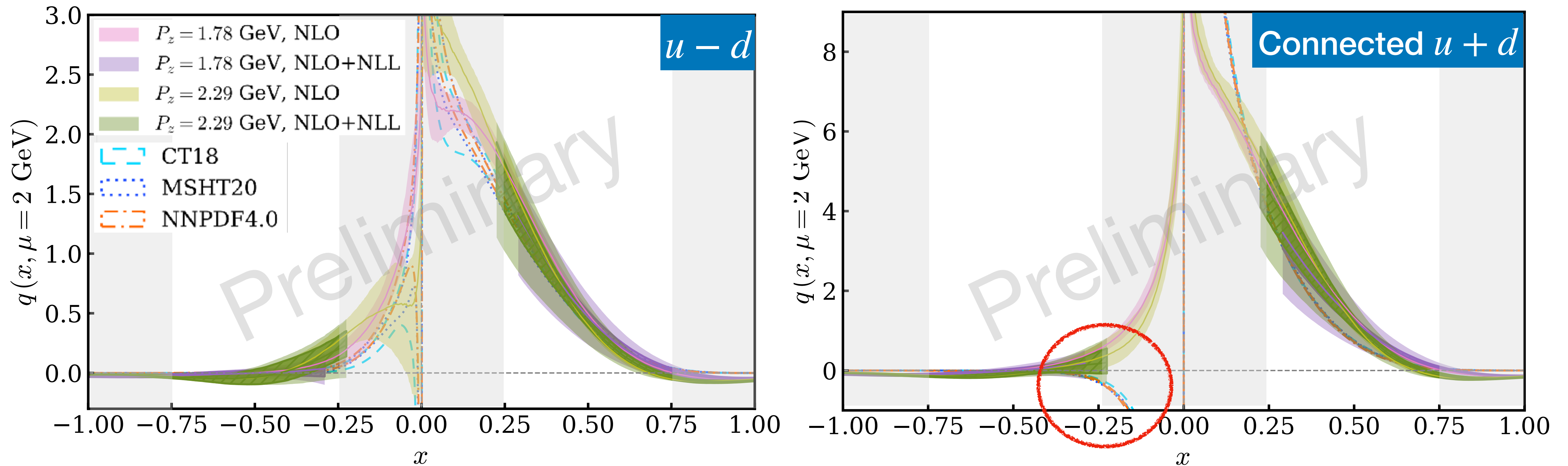


Positive x : q

$x \in [-1, 1]$

Negative x : $\bar{q}$

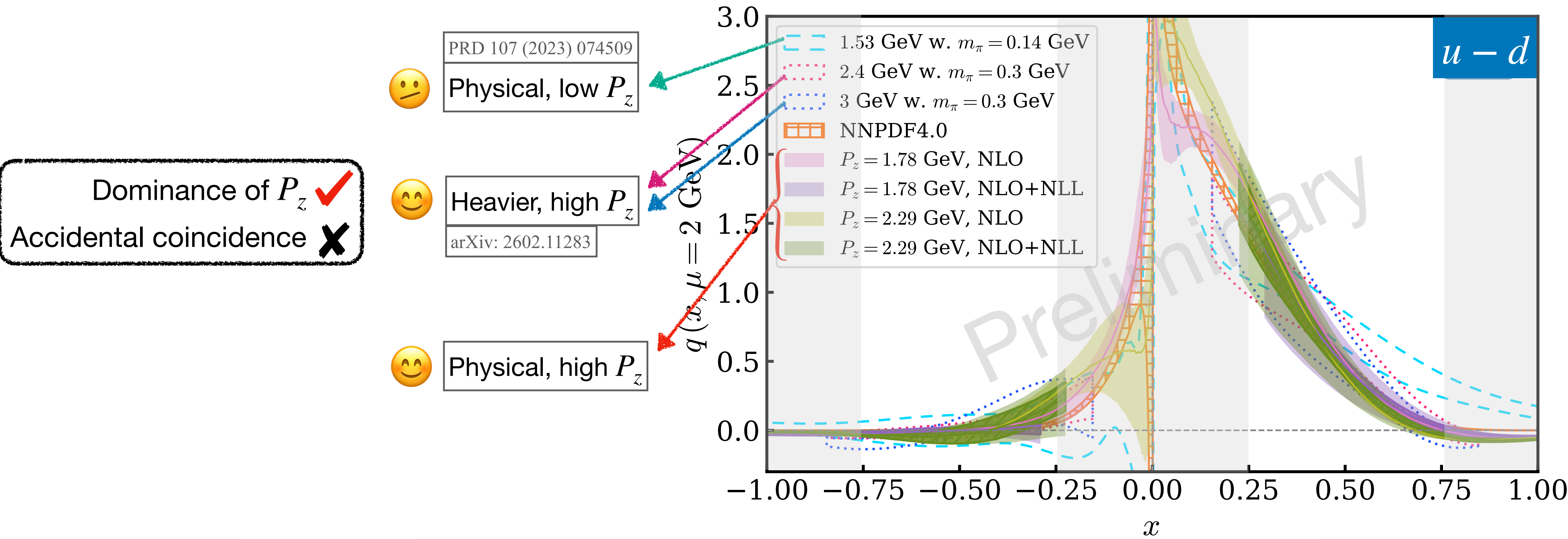
Entire- x PDF: Compare w. Global fits



CT18:
PRD 103 (2021) 014013
MSHT20:
EPJC 81 (2021) 341
NNPDF4.0:
EPJC 82 (2022) 428

- 📌 RGR-improved results are needed
- 📌 Isovector: good agreement with global analyses; antiquark: $\bar{d}(x) > \bar{u}(x)$ for $x > 0$
- 📌 Connected isoscalar:
quark: agree within error
antiquark: $-\left[\bar{u}(x) + \bar{d}(x)\right] \leq 0$ for $x > 0$, overlook the disconnected diagrams

Entire- x PDF: Compare w. Other lattice results



The universality of unpolarized PDF from first principle QCD

Summary & Outlook

- ☑ Nucleon unpolarized PDF
 - physical point
 - large, discretization-controlled P_z
 - isovector and connected isoscalar
- ☑ Consistent with global fits
 - NLO & NLL RGR
 - $0.25 \lesssim x \lesssim 0.75$
 - $\bar{u} + \bar{d}$ is underestimated
- ☑ Universality of Lattice QCD determination
 - combining previous works

- ☐ NNLO + NNLL RGR
 - improve the perturbative accuracy
 - study the perturbative convergence
- ☐ Threshold resummation
 - control the large logarithms near $x \rightarrow 1$
- ☐ Disconnected diagrams in $u + d$
- ☐ Extend to multiple a & continuum limit
 - remove the residual discretization artifacts
 - more definitive test of the universality

Thanks for your attention!

Backup

Statistics

$$N_{\text{meas}} = N_{\text{cfg}} \times N_{\text{src}} \text{ with } N_{\text{cfg}} = 700$$

P_z [GeV]	n_z	ζ	t_{sep}/a	N_{src}	N_{meas}
0	0	0	6, 8	16	11,200
			10, 12	32	22,400
1.78	7	3/7	6, 8	32	22,400
			10	128	89,600
			12	512	358,400
2.29	9	1/3	6, 8	32	22,400
			10	512	358,400
			12	1024	716,800

Two Fit Strategies

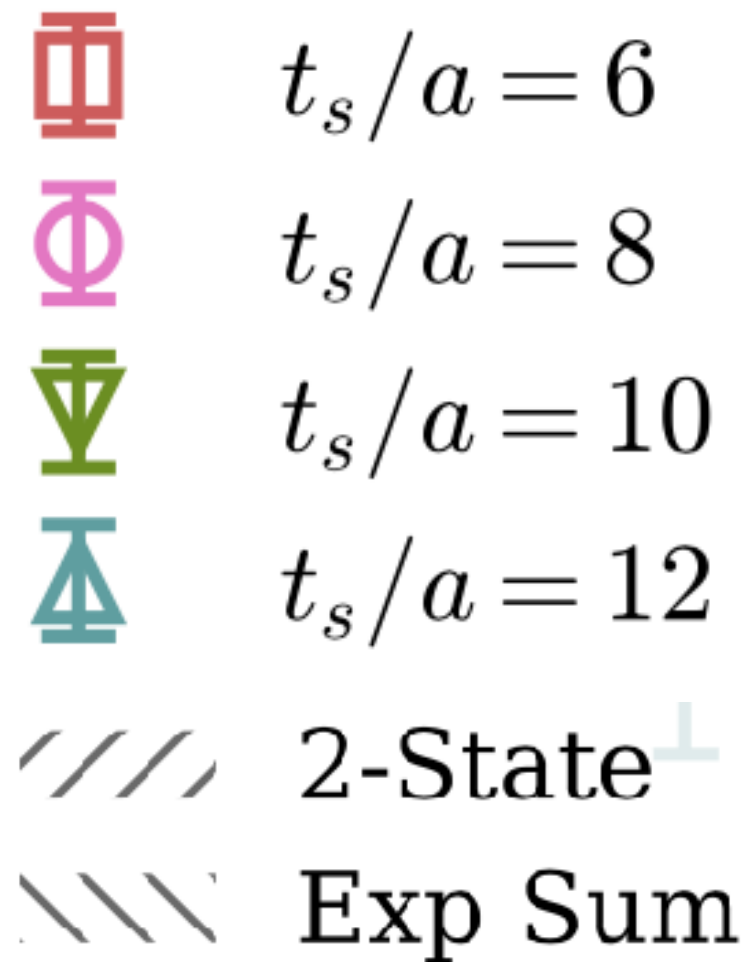
★ Two-state Fit:

$$R(\tau, t_s) \approx \frac{h_{00} + h_{11} \frac{|A_1|^2}{|A_0|^2} e^{-\Delta E t_s} + h_{01} \frac{|A_1|}{|A_0|} e^{-\Delta E \tau} + h_{10} \frac{|A_1|}{|A_0|} e^{-\Delta E (t_s - \tau)}}{1 + \frac{|A_1|^2}{|A_0|^2} e^{-\Delta E t_s}}, \quad \Delta E = E_1 - E_0$$

★ Exponential Summation Fit:

$$R_{\text{sum}}(t_s) = \sum_{\tau=\tau_{\text{skip}}}^{t_s - \tau_{\text{skip}}} R(\tau, t_s) \approx n h_{00} + n B_1 e^{-\Delta E t_s} + B_0, \quad n = (t_s - 2\tau_{\text{skip}})/a + 1$$

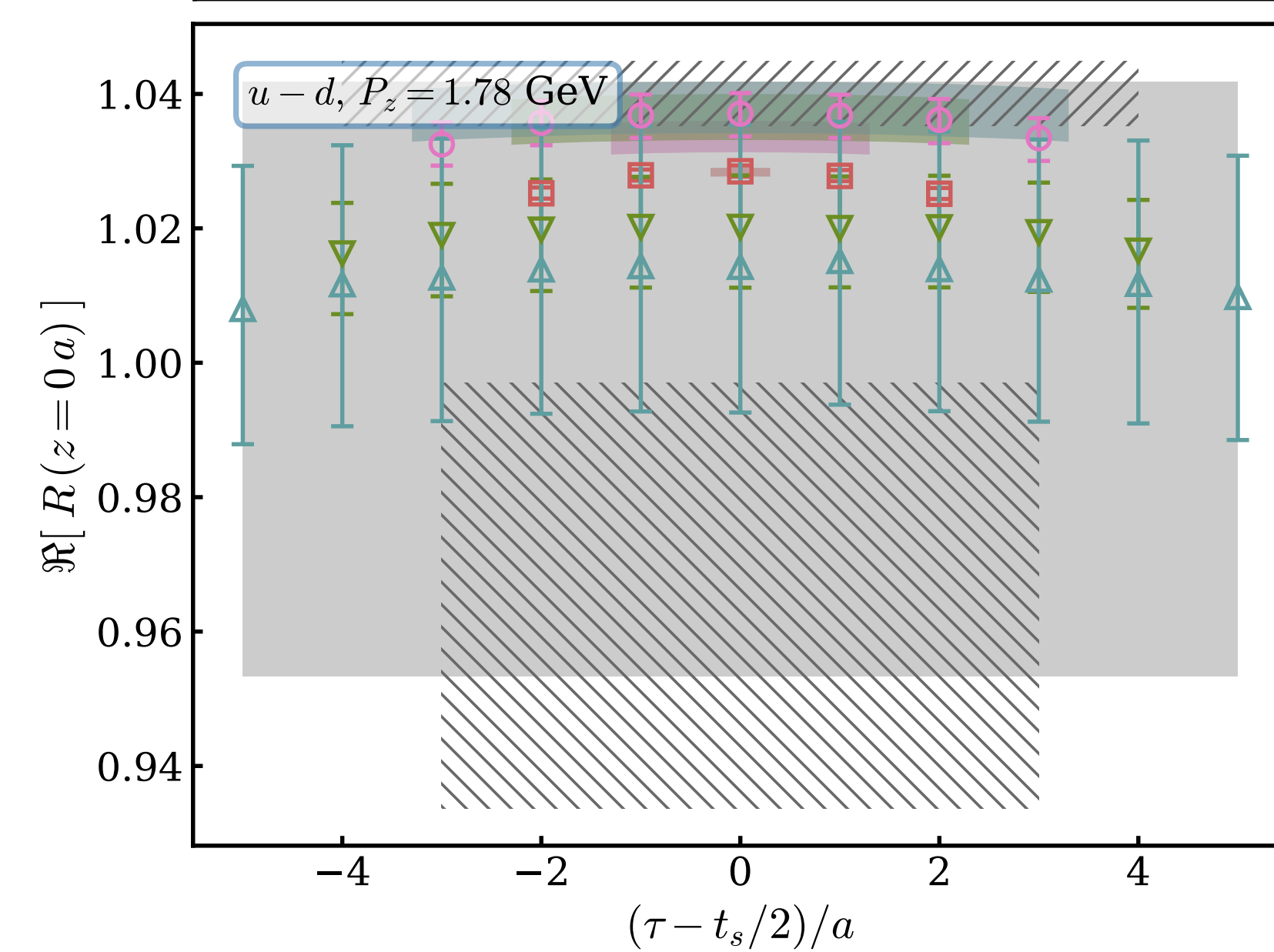
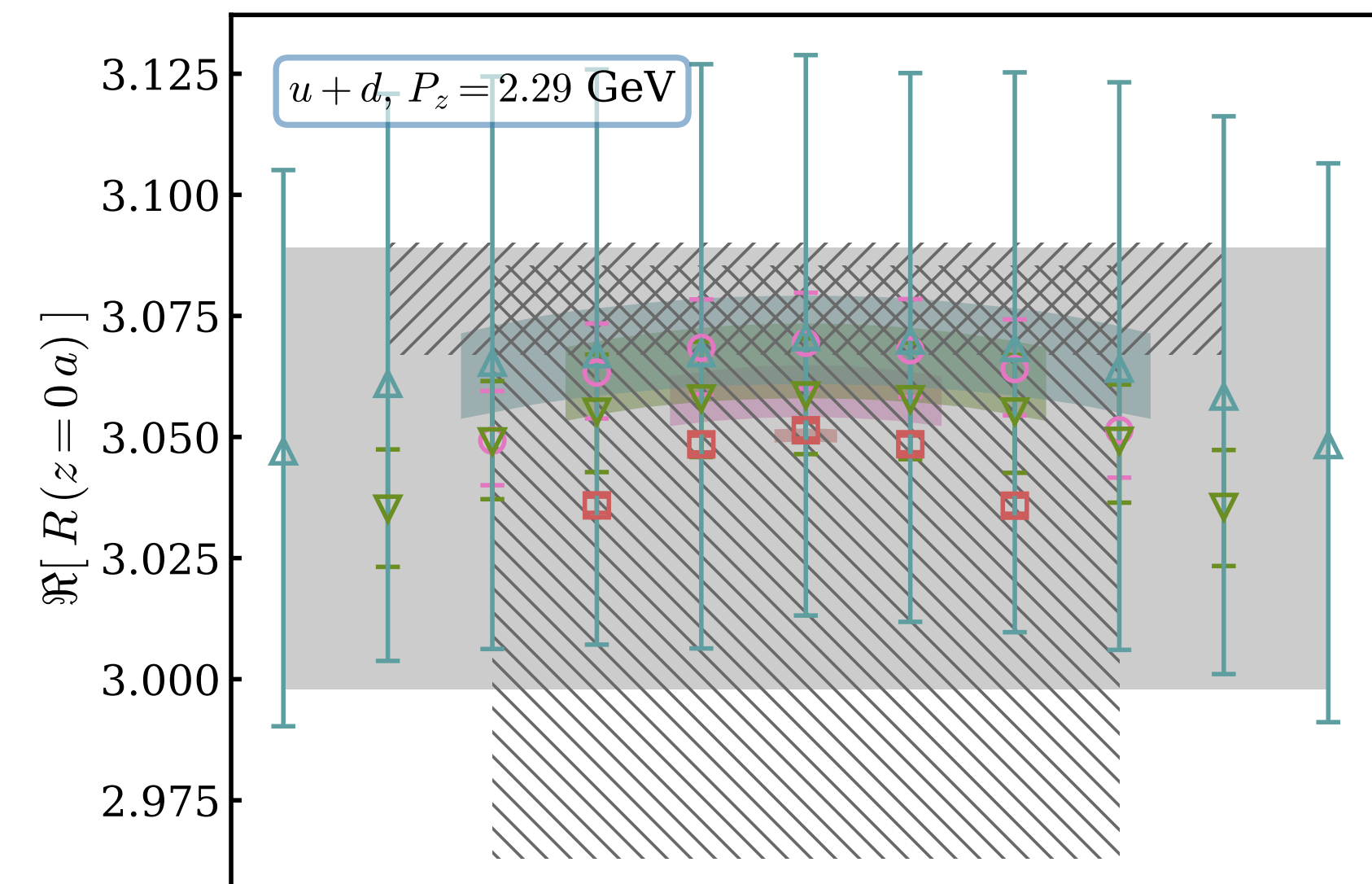
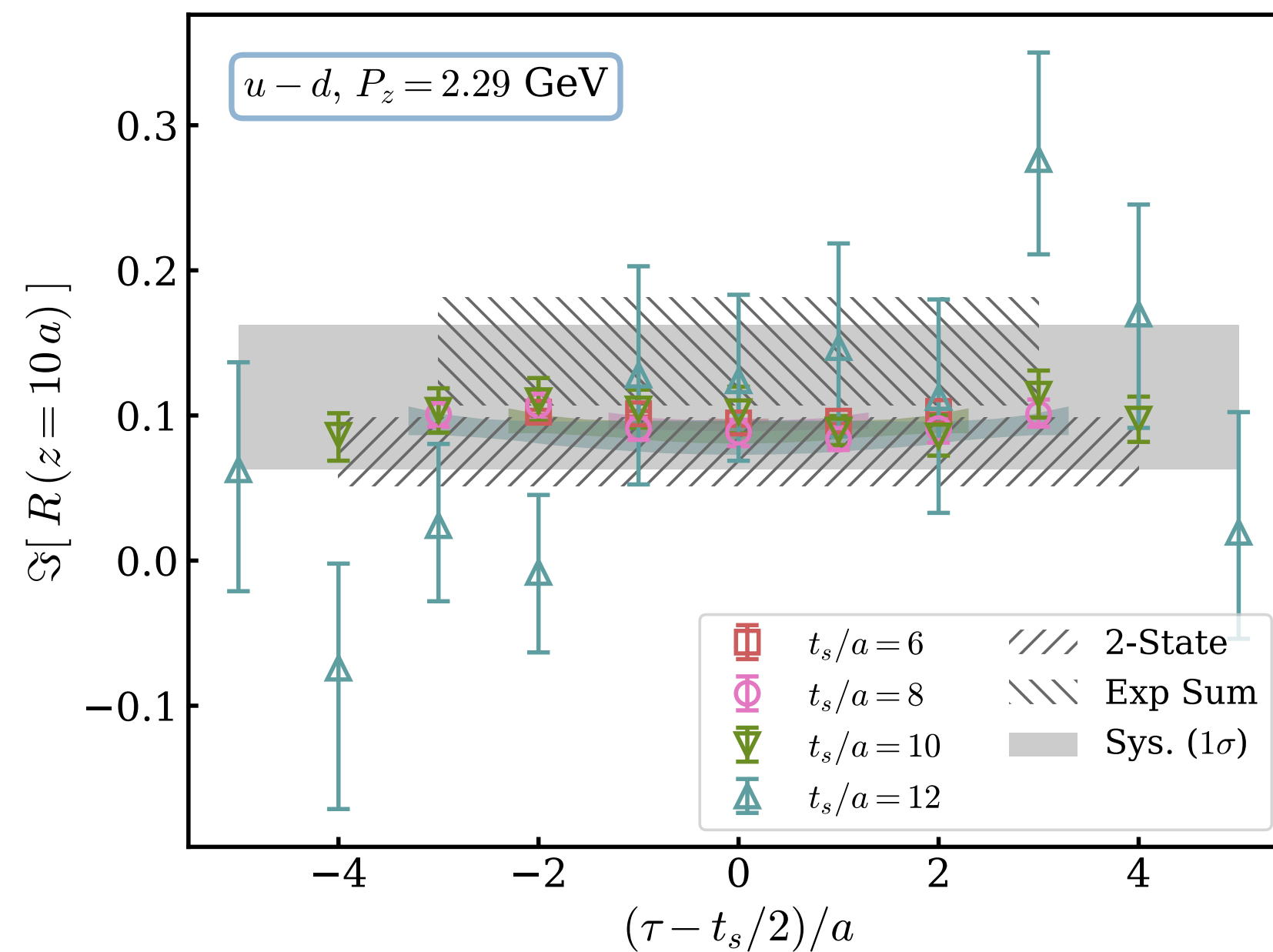
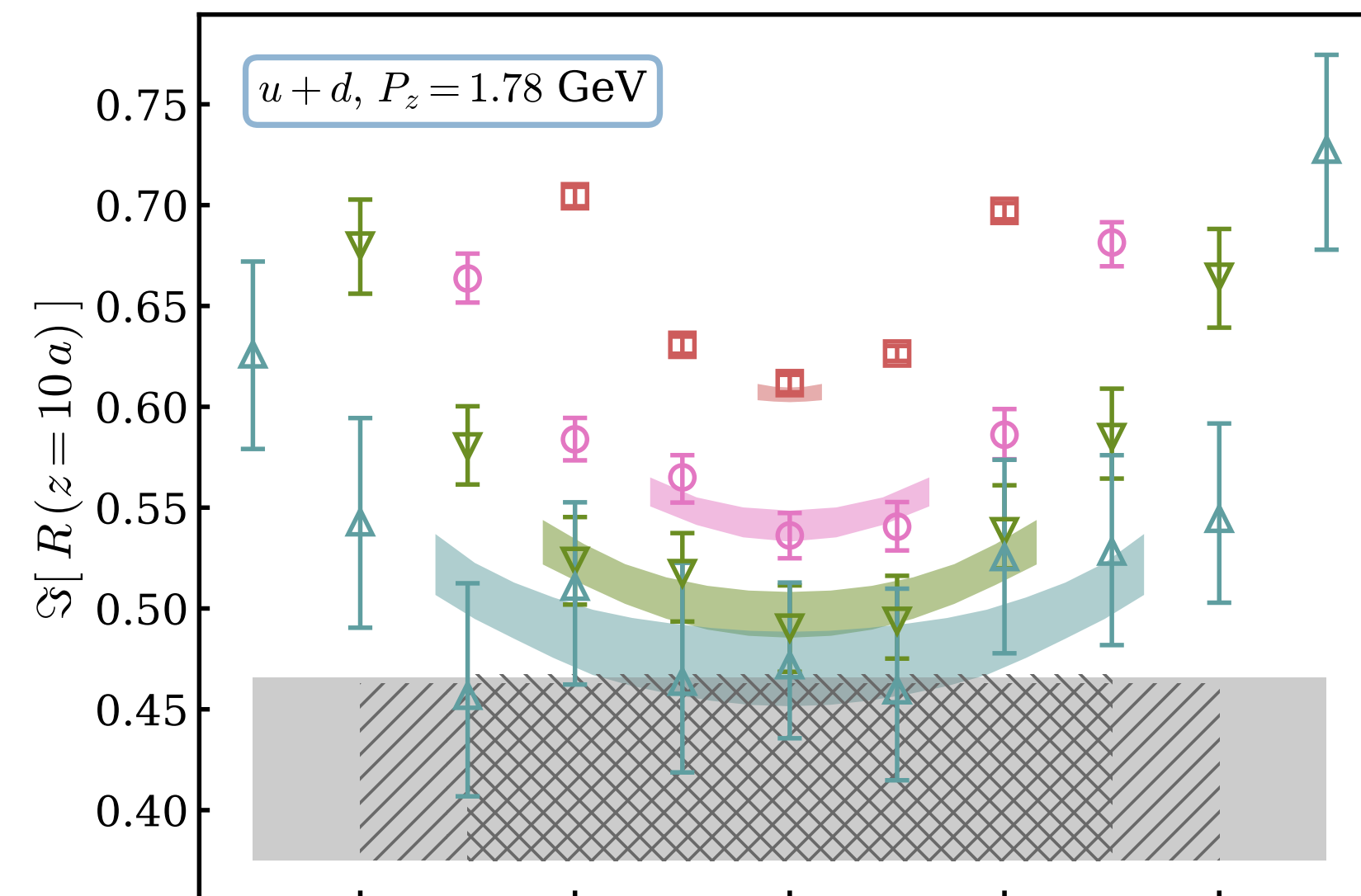
Representative Examples



Statistical
+
Systematic



Sys. (1σ)

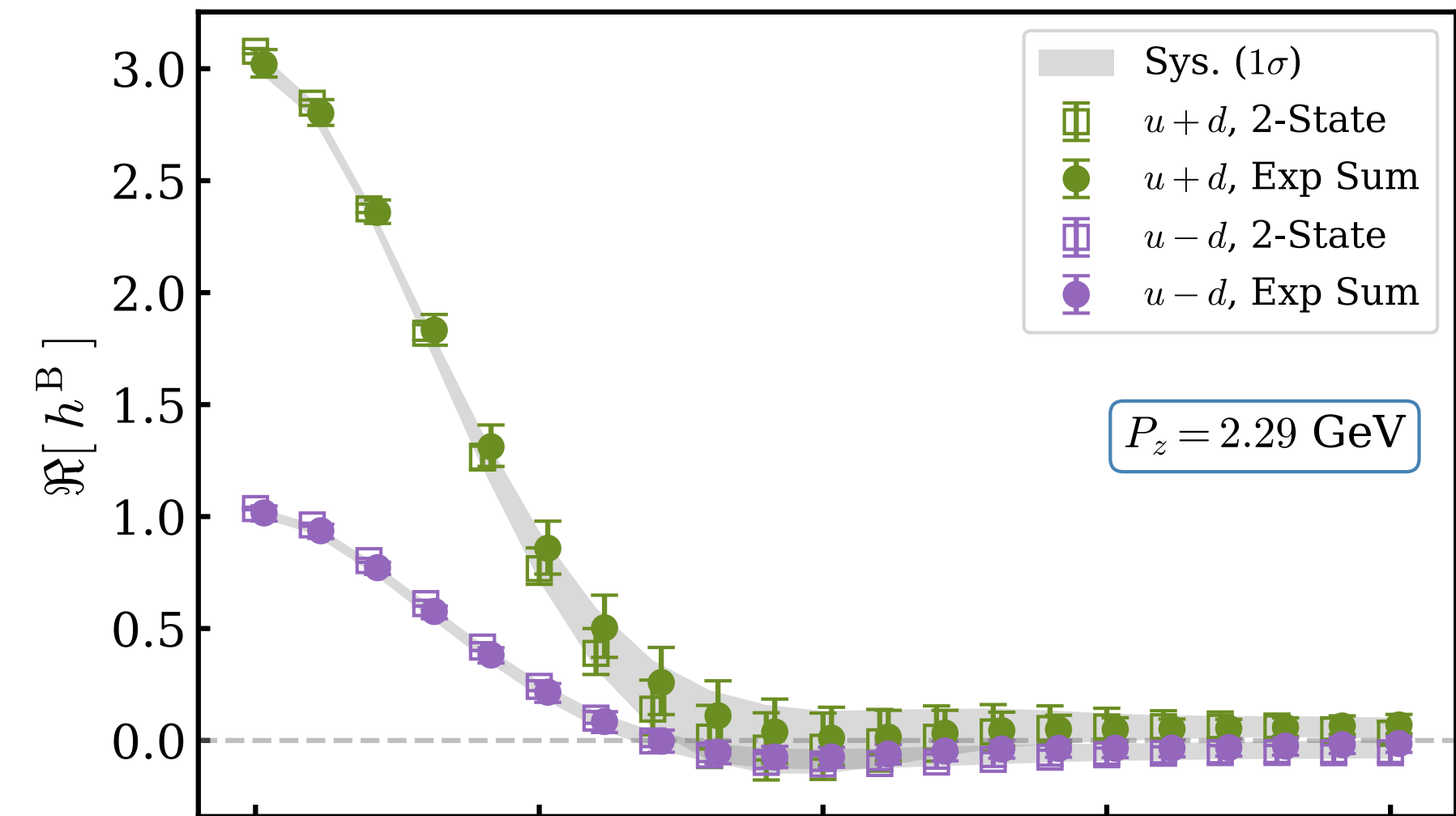
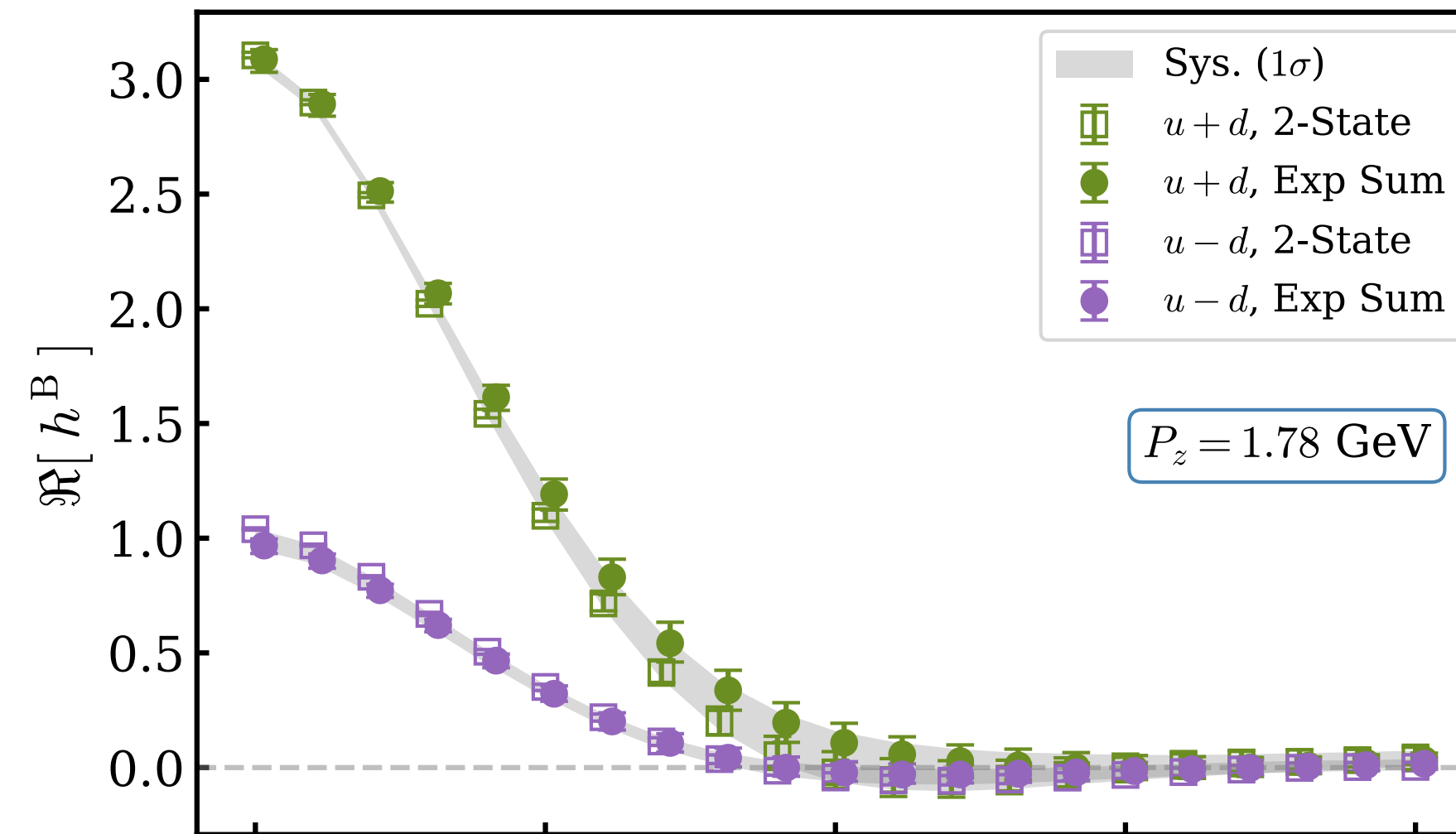


h^B v.s. z/a

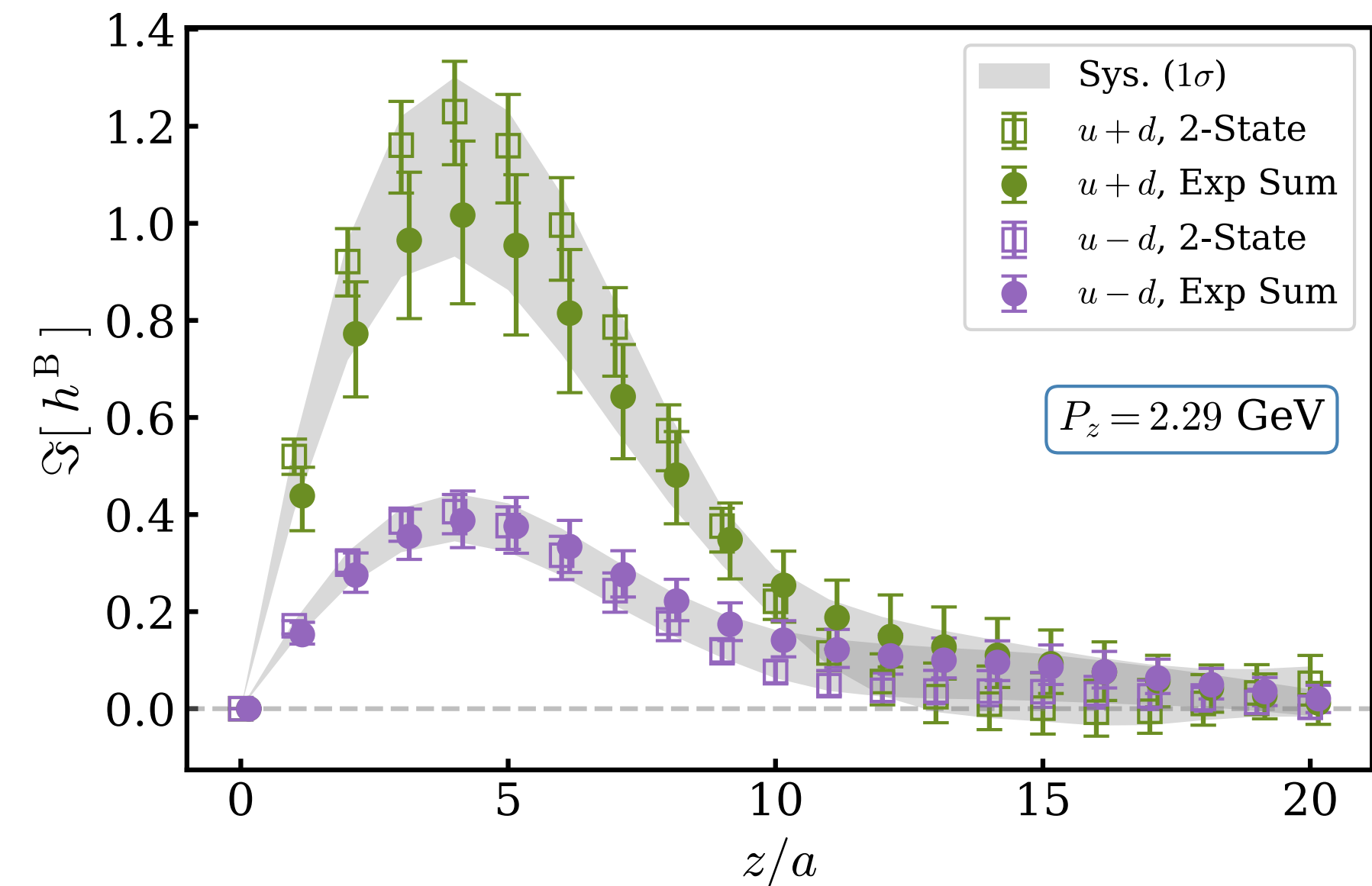
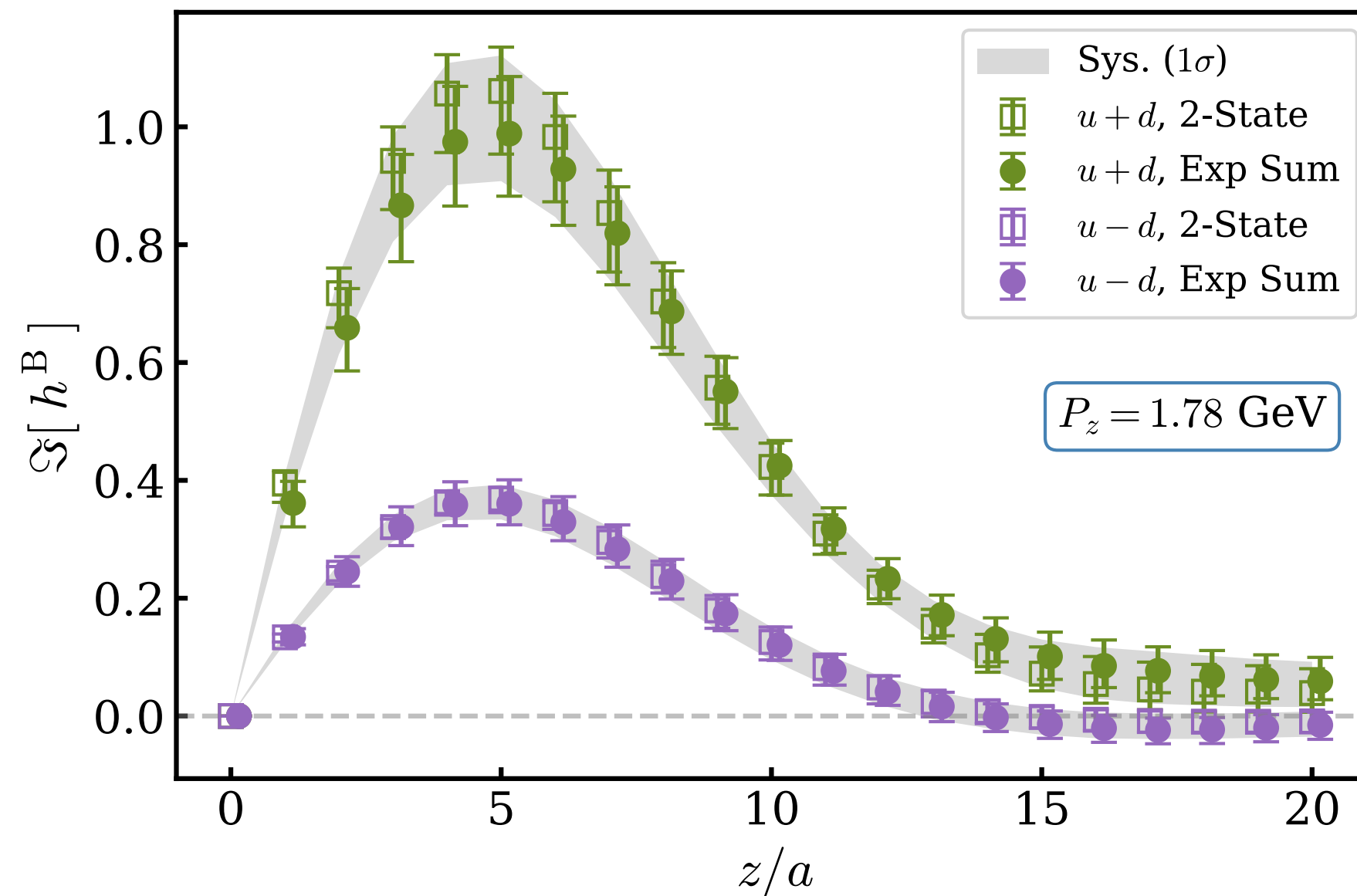
$P_z = 1.78$ GeV

$P_z = 2.29$ GeV

Real



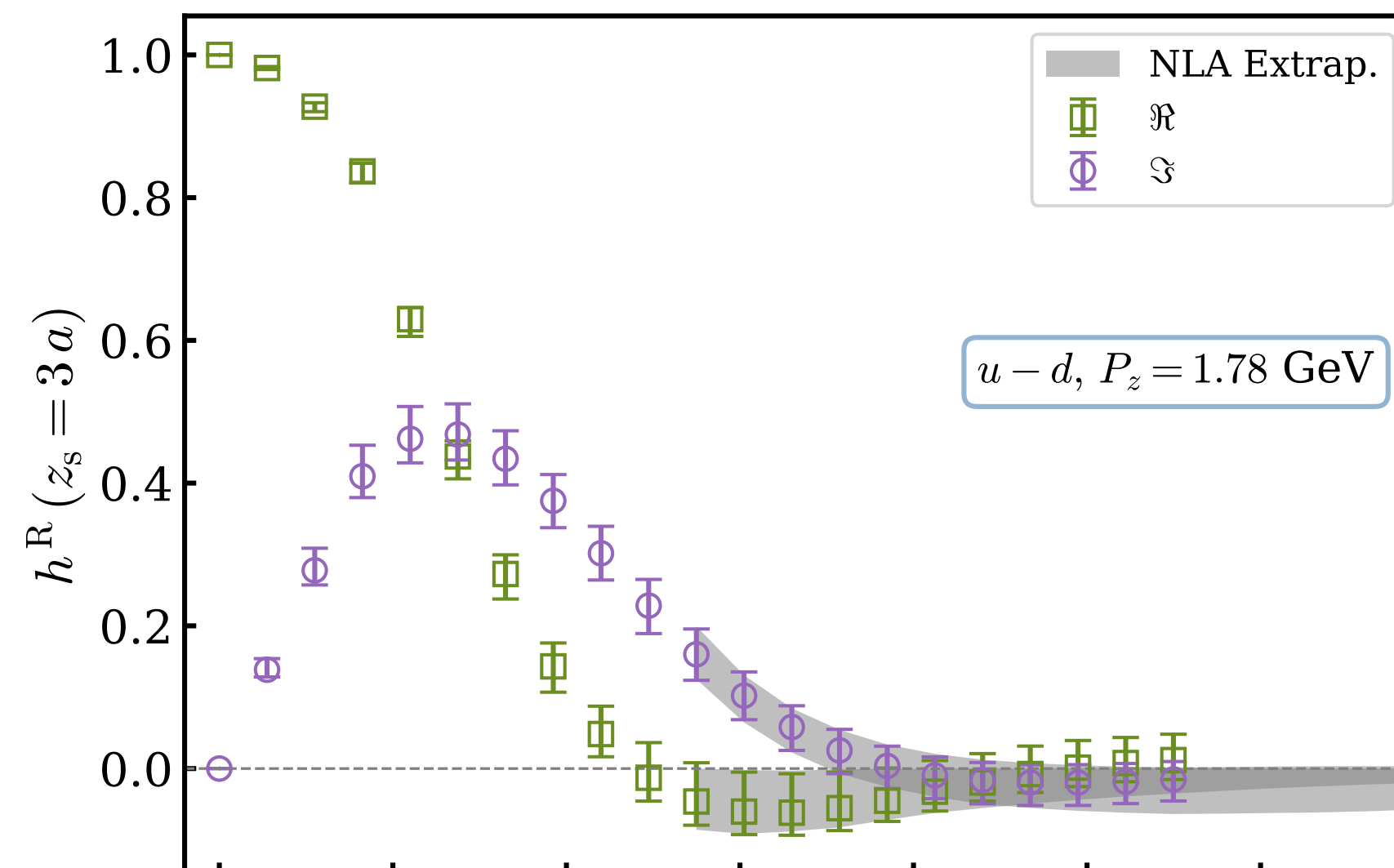
Imaginary



h^R v.s. λ

Isovector

$$P_z = 1.78 \text{ GeV}$$



Isoscalar

$$P_z = 2.29 \text{ GeV}$$

