Parton Distribution Functions of the Nucleon from Twisted Mass Fermions with Physical Pion Mass

Constantia Alexandrou[1,2], Simone Bacchio[1], Minhuan Chu[3], Krzystof Cichy[3], Martha Constantinou[4], Giannis Koutsou[1], Joshua Miller[4], Gabriele Pierini[1,5]

[1]The Cyprus Institute, Cyprus; [2]University of Cyprus, Cyprus; [3]Adam Mickiewicz University, Poland; [4]Temple University, USA; [5]Technische Universitaet Berlin, Germany

Funded by the European Union
Grant Agreement No: 101072344

What will I be showing?

- PDFs on lattice;
- Lattice setup;
- Inverse problem: from z space to x space;
- Light-cone isovector unpolarized PDF;
- Moments of the PDF;

PDFs

- PDFs are key quantities for hadron dynamics and for 3D imaging of hadrons.

  They are defined on the light-cone: cannot be computed directly on the lattice;

PDFs

- PDFs are key quantities for hadron dynamics and for 3D imaging of hadrons. They are defined on the light-cone: cannot be computed directly on the lattice;
- LaMET [X. Ji et al., arXiv:2004.03543] approach to study them on a Euclidean space-time lattice;

PDFs

- PDFs are key quantities for hadron dynamics and for 3D imaging of hadrons. They are defined on the light-cone: cannot be computed directly on the lattice;
- LaMET [X. Ji et al., arXiv:2004.03543] approach to study them on a Euclidean space-time lattice;
- $F^{+}(x) = 2P^{+} \int \frac{dz^{-}}{4\pi} e^{-ixP^{+}z^{-}} \cdot \langle N(P)|\bar{\psi}(z^{-})\gamma^{+}\mathcal{W}(z^{-},0)\psi(0)|N(P)\rangle$

PDFs

- PDFs are key quantities for hadron dynamics and for 3D imaging of hadrons. They are defined on the light-cone: cannot be computed directly on the lattice;
- LaMET [X. Ji et al., arXiv:2004.03543] approach to study them on a Euclidean space-time lattice;
- $F^+(x) = 2P^+ \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \cdot \langle N(P)|\bar{\psi}(z^-)\gamma^+\mathcal{W}(z^-,0)\psi(0)|N(P)\rangle$
- $F^+(z,P) = \frac{1}{z\cdot P}\langle N(P)|\bar{\psi}(z^-)\gamma^+\mathcal{W}(z^-,0)\psi(0)|N(P)\rangle$

PDFs

- PDFs are key quantities for hadron dynamics and for 3D imaging of hadrons. They are defined on the light-cone: cannot be computed directly on the lattice;
- LaMET [X. Ji et al., arXiv:2004.03543] approach to study them on a Euclidean space-time lattice;
- $F^+(x) = 2P^+ \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \cdot \langle N(P)|\bar{\psi}(z^-)\gamma^+ \mathcal{W}(z^-,0)\psi(0)|N(P)\rangle$
- $F^+(z,P) = \frac{1}{z \cdot P} \langle N(P)|\bar{\psi}(z^-)\gamma^+ \mathcal{W}(z^-,0)\psi(0)|N(P)\rangle$

PDFs

- PDFs are key quantities for hadron dynamics and for 3D imaging of hadrons. They are defined on the light-cone: cannot be computed directly on the lattice;
- LaMET [X. Ji et al., arXiv:2004.03543] approach to study them on a Euclidean space-time lattice;
- $F^+(x) = 2P^+ \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \cdot \langle N(P)|\bar{\psi}(z^-)\gamma^+\mathcal{W}(z^-,0)\psi(0)|N(P)\rangle$
- $F^+(z,P) = \frac{1}{z\cdot P}\langle N(P)|\bar{\psi}(z^-)\gamma^+\mathcal{W}(z^-,0)\psi(0)|N(P)\rangle$

LaMET approach:

$$F^0(z,P) = \langle N(P)\left|\bar{\psi}(z)\gamma^0\mathcal{W}(z,0)\psi(0)\right| N(P)\rangle = \bar{u}(p,\lambda')\left[\frac{P^0}{m}\mathcal{H}(z)\right]u(p,\lambda)$$

From $\mathcal{H}(z)$ we can reconstruct the light-cone $F^+(x)$ $(H(x))$ PDF.

PDFs can be generalized to GPDs [Bhattacharya et al., arXiv:2209.05373v2]:

Martha Constantinou's talk on Tuesday at 14.20.

Lattice setup

Ensemble	a [fm]	$(L/a)^3 \times T/a$	m_π [GeV]	L [fm]
cB211.072.64	0.0801(4)	$64^3 \times 128$	0.1393(7)	5.12(3)

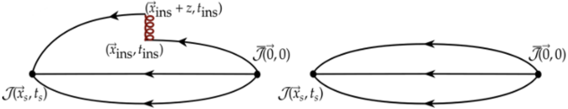
$N_f = 2 + 1 + 1$ with clover improved, twisted-mass fermions, $t_s = 0.8$ fm.

Lattice setup

Ensemble	a [fm]	$(L/a)^3 \times T/a$	m_π [GeV]	L [fm]
cB211.072.64	0.0801(4)	$64^3 \times 128$	0.1393(7)	5.12(3)

$N_f = 2+1+1$ with clover improved, twisted-mass fermions, $t_s = 0.8$ fm.

N_{conf}	652	709					
Boosts P_z [GeV]	0	0.24	0.48	0.72	1.21	1.45	1.69
N_{src}	16	1	2	4	12	24	48
Optimized momentum smearing $\vec{\Delta}$ at P_i	$(0, 0, P_z)$				$(2, 0, P_z)$		



Excited states

For $P_z = 1.21$ GeV we computed the three-point function at $t_s = \ 0.64, \ 0.8, \ 0.96$ fm.

$C^{2pt}(\Gamma_0, p; t_s) = \sum_n c_n(\vec{p}) e^{-E_n(\vec{p}) t_s}$

Two-state analysis

$C^{3pt}(\Gamma_0, p; t_s, t_{ins}) = \sum_{i,j} A_0^{ij} e^{-E_i(\vec{p})(t_s - t_{ins}) - E_j(\vec{p})(t_{ins})}$

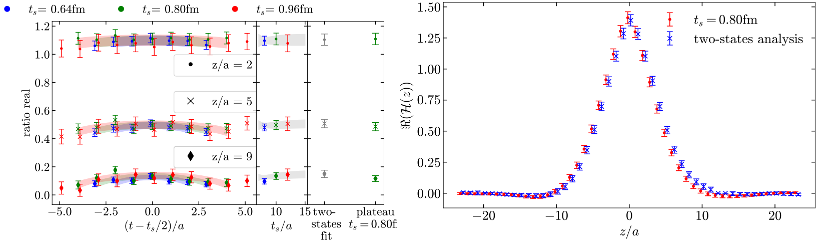
Excited states

For $P_z = 1.21$ GeV we computed the three-point function at $t_s =$ 0.64, 0.8, 0.96 fm.

$$C^{2pt}(\Gamma_0, p; t_s) = \sum_n c_n(\vec{p}) e^{-E_n(\vec{p}) t_s}$$

$$C^{3pt}(\Gamma_0, p; t_s, t_{ins}) = \sum_{i,j} A_0^{ij} e^{-E_i(\vec{p})(t_s - t_{ins}) - E_j(\vec{p})(t_{ins})}$$

Two-state analysis



Matrix element on lattice QCD

Appropriate ratio of three-point over two-point functions.

For $P_z = 0, 0.24, 0.48, 0.72$ GeV:

$$R(\Gamma_0, p; t_s, t_{ins}) = \frac{C^{3pt}(\Gamma_0, p; t_s, t_{ins})}{C^{2pt}(\Gamma_0, p, \Delta_0; t_s)}$$

Matrix element on lattice QCD

Appropriate ratio of three-point over two-point functions.

For $P_z = 0, 0.24, 0.48, 0.72$ GeV:

$$R(\Gamma_0, p; t_s, t_{ins}) = \frac{C^{3pt}(\Gamma_0, p; t_s, t_{ins})}{C^{2pt}(\Gamma_0, p, \Delta_0; t_s)}$$

For $P_z = 1.21, 1.45, 1.69$ GeV:

$$R(\Gamma_0, p; t_s, t_{ins}) = \frac{C^{3pt}(\Gamma_0, p; t_s, t_{ins})}{\sqrt{C^{2pt}(\Gamma_0, p, \Delta_0; t_s) C^{2pt}(\Gamma_0, p, \Delta; t_s)}}$$

Matrix element on lattice QCD

Appropriate ratio of three-point over two-point functions.

For $P_z = 0, 0.24, 0.48, 0.72$ GeV:

$$R(\Gamma_0, p; t_s, t_{ins}) = \frac{C^{3pt}(\Gamma_0, p; t_s, t_{ins})}{C^{2pt}(\Gamma_0, p, \Delta_0; t_s)}$$

For $P_z = 1.21, 1.45, 1.69$ GeV:

$$R(\Gamma_0, p; t_s, t_{ins}) = \frac{C^{3pt}(\Gamma_0, p; t_s, t_{ins})}{\sqrt{C^{2pt}(\Gamma_0, p, \Delta_0; t_s) C^{2pt}(\Gamma_0, p, \Delta; t_s)}}$$

Plateau fit to extract the Matrix element.

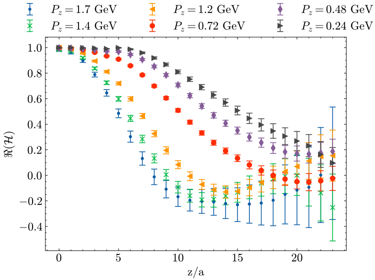
Renormalization

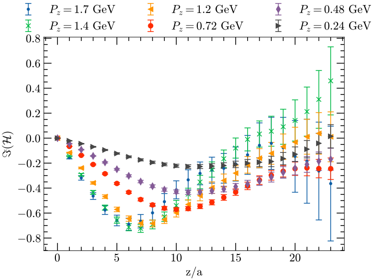
Latest developments in renormalization: Hybrid renormalization [Ding et al., arXiv:2407.03516]

$$\mathcal{H}_{ren}(z, P_z) = \frac{\mathcal{H}_{bare}(z, P_z)}{\mathcal{H}_{bare}(z, 0)} \qquad z \le z_s$$

$$\mathcal{H}_{ren}(z, P_z) = e^{\delta m(z - z_s)} \frac{\mathcal{H}_{bare}(z, P_z)}{\mathcal{H}_{bare}(z_s, 0)} \qquad z > z_s,$$

where $z_s = 4a$ and $\delta m \cdot a = 0.2660$.





x-reconstruction: Bayes-Gauss-Fourier technique

$$\mathcal{H}(x, P_z, \mu) = 2P_z \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{ixP_z z} \; \mathcal{H}(z, P_z, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{P_z^2}, \frac{M_N^2}{P_z^2}\right)$$

We studied both Backus-Gilbert (BG) and a newer approach: Bayes-Gauss-Fourier transformation technique (a gaussian process, GP) [Alexandrou et al., arXiv:2007.13800]. How does it work?

x-reconstruction: Bayes-Gauss-Fourier technique

$$\mathcal{H}(x, P_z, \mu) = 2P_z \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{ixP_z z} \, \mathcal{H}(z, P_z, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{P_z^2}, \frac{M_N^2}{P_z^2}\right)$$

We studied both Backus-Gilbert (BG) and a newer approach: Bayes-Gauss-Fourier transformation technique (a gaussian process, GP) [Alexandrou et al., arXiv:2007.13800]. How does it work?

- Find the continuous function $\mu(z)$ that better approximates $\rho(z)$ defined as

$$\mathcal{H}(z) = \rho(z) e^{i\phi(z)}$$

x-reconstruction: Bayes-Gauss-Fourier technique

$$\mathcal{H}(x, P_z, \mu) = 2P_z \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{ixP_z z}\, \mathcal{H}(z, P_z, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{P_z^2}, \frac{M_N^2}{P_z^2}\right)$$

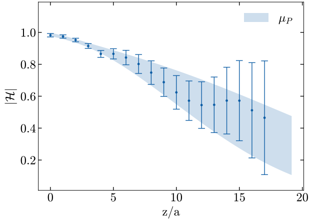
We studied both Backus-Gilbert (BG) and a newer approach: Bayes-Gauss-Fourier transformation technique (a gaussian process, GP) [Alexandrou et al., arXiv:2007.13800]. How does it work?

- Find the continuous function $\mu(z)$ that better approximates $\rho(z)$ defined as

$$\mathcal{H}(z) = \rho(z) e^{i\phi(z)}$$

- Fit ρ with a proper prior function

$$\mu_P(z; l', \sigma', \bar{z}) = l'^2 e^{-\frac{(|z|-\bar{z})^2}{2\sigma'^2}}$$



x-reconstruction: Bayes-Gauss-Fourier technique

- Given a covariance function

$$k_P(z, z') = l^2 e^{-\frac{(z-z')^2}{2\sigma^2}}, \; \tilde{K}_{ij} = k_P(z_i, z_j) + \Delta h_i^2 \delta_{ij}$$

whose parameters are chosen such that they minimize

$$p(\rho) = det(2\pi k_P)^{-1/2} e^{-\frac{1}{2}\sum_{ij}(\rho(z_i)-\mu_P(z_i))k_{Pij}^{-1}(\rho(z_j)-\mu_P(z_j))}$$

x-reconstruction: Bayes-Gauss-Fourier technique

- Given a covariance function

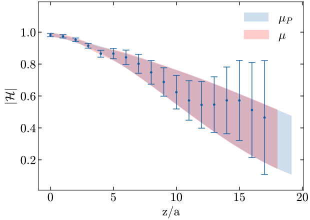
$$k_P(z,z') = l^2 e^{-\frac{(z-z')^2}{2\sigma^2}}, \ \tilde{K}_{ij} = k_P(z_i,z_j) + \Delta h_i^2 \delta_{ij}$$

whose parameters are chosen such that they minimize

$$p(\rho) = det(2\pi k_P)^{-1/2} e^{-\frac{1}{2}\sum_{ij}(\rho(z_i)-\mu_P(z_i))k_{Pij}^{-1}(\rho(z_j)-\mu_P(z_j))}$$

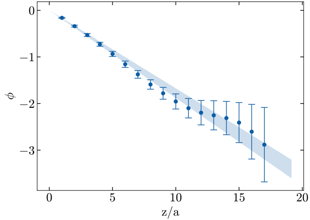
- we can get

$$\mu(z) = \mu_P(z) + \sum_{i,j} k_P(|z|,z_i)\tilde{K}_{ij}^{-1}(\rho(z_j)-\mu_P(z_j))$$



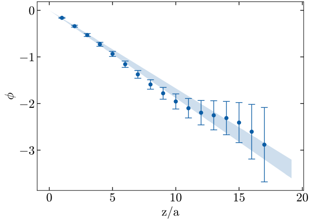
x-reconstruction: Bayes-Gauss-Fourier technique

- The phase $\phi(z)$ is parametrized with a linear function



x-reconstruction: Bayes-Gauss-Fourier technique

- The phase $\phi(z)$ is parametrized with a linear function

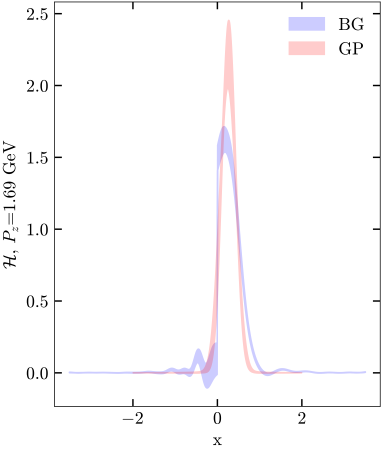


- The reconstructed $\mathcal{H}_{GP}(z)$ is thus

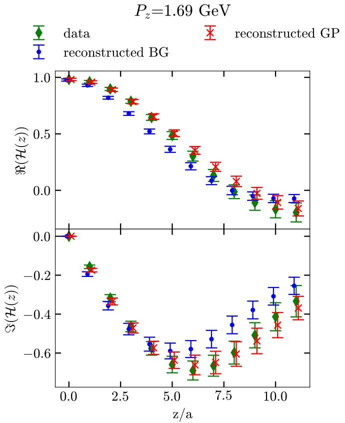
$$\mathcal{H}_{GP}(z) = \mu(z)e^{i\theta z}$$

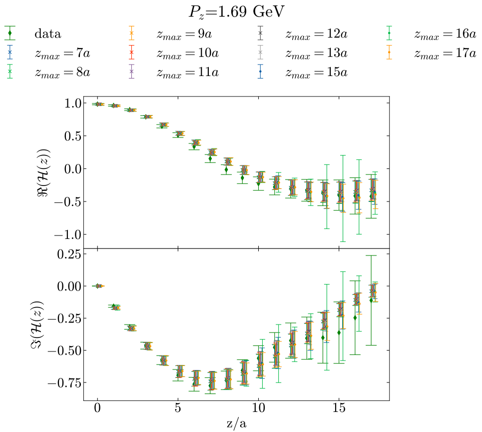
$\mathcal{H}(x)$ can be now computed in a closed form by doing the Fourier-Transform of $\mathcal{H}_{GP}(z)$.

x-reconstruction: Bayes-Gauss-Fourier technique



x-reconstruction: Bayes-Gauss-Fourier technique

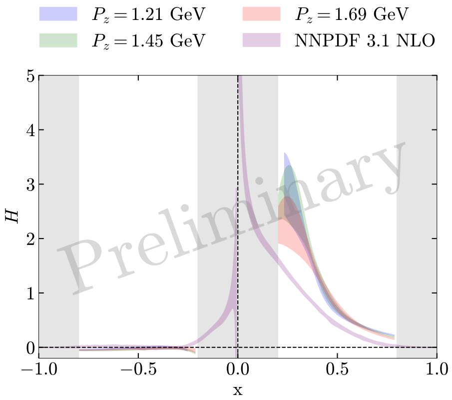




Light-cone matching

We computed the matching function $C\left(\frac{x}{y}, \frac{\mu}{P_z}\right)$ [Ding et al., arXiv:2407.03516] and get the light-cone PDF

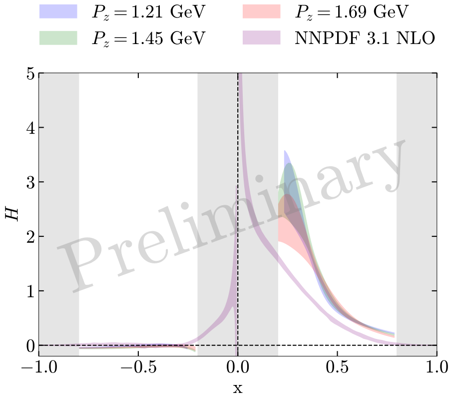
$$H(x, P_z, \mu^2) = \int \frac{dy}{y}\ C\left(\frac{x}{y}, \frac{\mu}{P_z}\right)\mathcal{H}(y, P_z, \mu^2) + \mathcal{O}(\Lambda^2_{QCD}/P_z^2, M_N^2/P_z^2)$$



Light-cone matching

We computed the matching function $C\left(\frac{x}{y}, \frac{\mu}{P_z}\right)$ [Ding et al., arXiv:2407.03516] and get the light-cone PDF

$$H(x, P_z, \mu^2) = \int \frac{dy}{y}\ C\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \mathcal{H}(y, P_z, \mu^2) + \mathcal{O}(\Lambda^2_{QCD}/P_z^2, M_N^2/P_z^2)$$



Studies on systematics of the hybrid renormalization ongoing.

Short-distance factorization

$$\mathcal{M}(z,P) = \frac{\mathcal{H}_{bare}\left(\vec{z},\vec{P}\right)}{\mathcal{H}_{bare}(\vec{z},0)} \frac{\mathcal{H}_{bare}(0,0)}{\mathcal{H}_{bare}\left(0,\vec{P}\right)} = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\bar{M}S}(\mu^2 z^2)}{C_0^{\bar{M}S}(\mu^2 z^2)} \langle x^n \rangle + \mathcal{O}\left(\Lambda_{QCD}^2 z^2\right),$$

with $C_n^{\bar{M}S}\left(\mu^2 z^2\right)$ being the Wilson coefficients at NLO [Gao et al., arXiv:2212.12569].

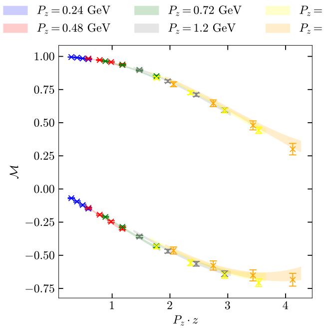
We use all the P_z and we set $n_{max} = 4$.

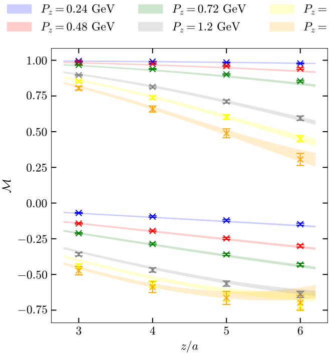
Short-distance factorization

$$\mathcal{M}(z,P) = \frac{\mathcal{H}_{bare}\left(\vec{z},\vec{P}\right)}{\mathcal{H}_{bare}\left(\vec{z},0\right)}\frac{\mathcal{H}_{bare}(0,0)}{\mathcal{H}_{bare}\left(0,\vec{P}\right)} = \sum_{n=0}\frac{(-izP)^n}{n!}\frac{C_n^{\bar{MS}}\left(\mu^2 z^2\right)}{C_0^{\bar{MS}}\left(\mu^2 z^2\right)}\langle x^n\rangle + \mathcal{O}\left(\Lambda_{QCD}^2 z^2\right),$$

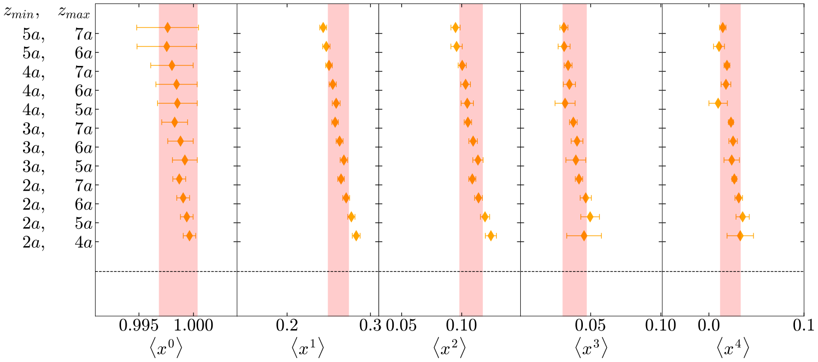
with $C_n^{\bar{MS}}\left(\mu^2 z^2\right)$ being the Wilson coefficients at NLO [Gao et al., arXiv:2212.12569].

We use all the P_z and we set $n_{max} = 4$.

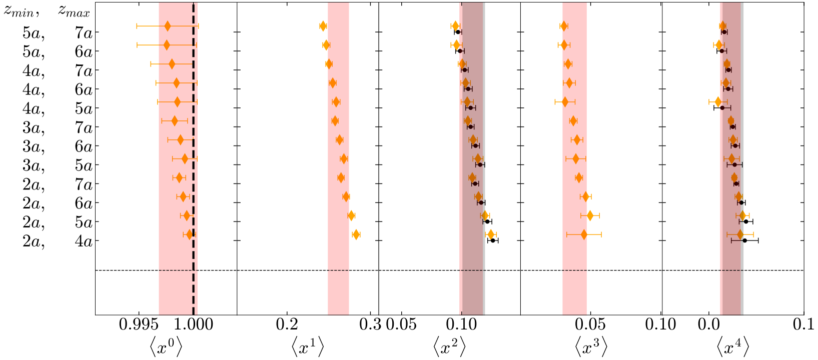




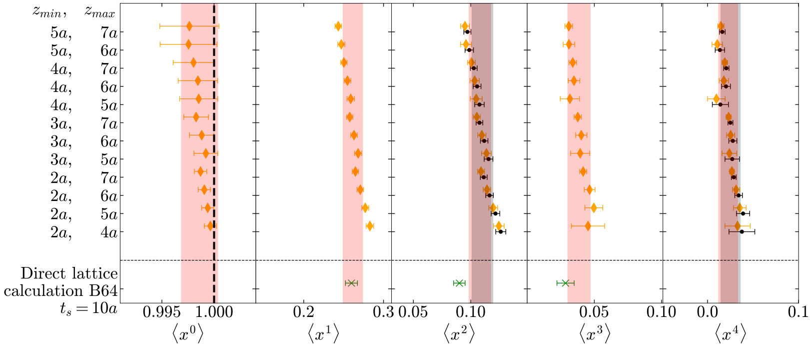
Short-distance factorization



Short-distance factorization



Short-distance factorization



Direct lattice calculation on B64 from [Alexandrou et al., arXiv:2607.20337] and [Alexandrou et al., arXiv:2607.03458]. Christian Kummer's talk on Tuesday at 15.20 and Constantia Alexandrou's talk on Thursday at 14.00.

Conclusion and outlook

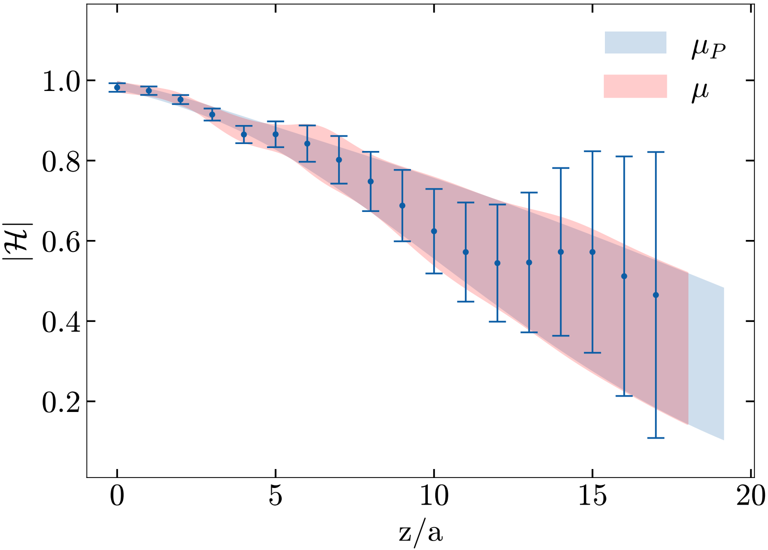
- Implementation of Bayes-Gauss-Fourier transform;
- Light-cone PDFs at physical mass for large boosts;
- Extraction of moments up to $\langle x^4 \rangle$, with agreement with lattice results from direct calculation.

- Wilson coefficients $C_n^{\bar{M}S}$ at NNLO;
- Hybrid renormalization for $\tilde{H}$;
- Systematics from hybrid renormalization;
- Two-state analysis for all boosts.

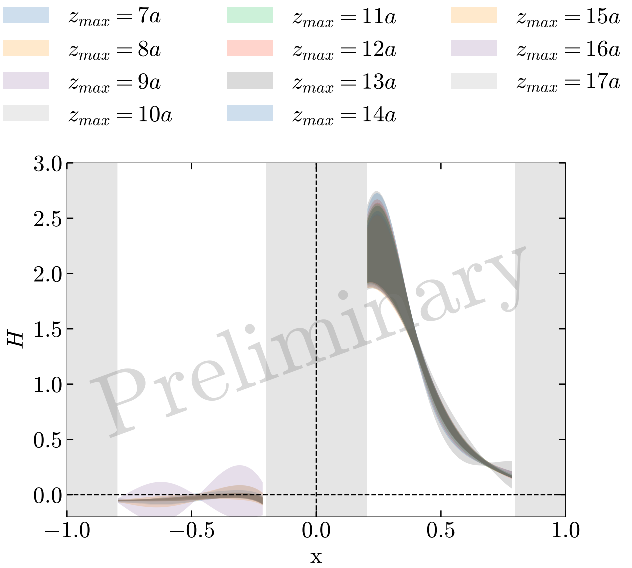

べんりだよ♡
まどーしょ

Thank You!

Bayes-Gauss-Fourier for different z



Bayes-Gauss-Fourier stability



Helicity-PDF moments

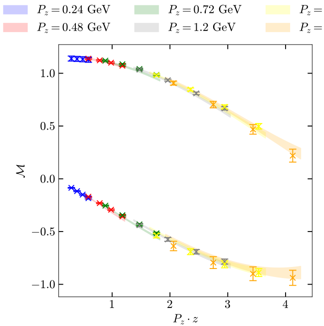
$$\mathcal{M}(z,P) = \frac{\tilde{\mathcal{H}}_{bare}\left(\vec{z},\vec{P},\right)}{\tilde{\mathcal{H}}_{bare}(\vec{z},0)} \frac{\tilde{\mathcal{H}}_{bare}(0,0)}{\tilde{\mathcal{H}}_{bare}\left(0,\vec{P}\right)} g_A = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\bar{MS}}\left(\mu^2 z^2\right)}{C_0^{\bar{MS}}\left(\mu^2 z^2\right)} \langle x^n \rangle + \mathcal{O}\left(\Lambda_{QCD}^2 z^2\right),$$

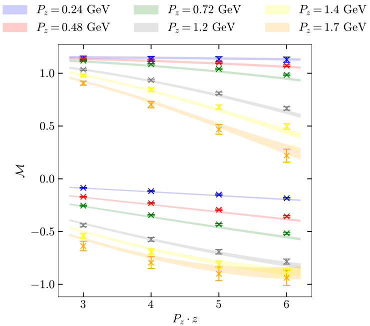
with $C_n^{\bar{MS}}\left(\mu^2 z^2\right)$ being the Wilson coefficients at NLO for the helicity case.

Helicity-PDF moments

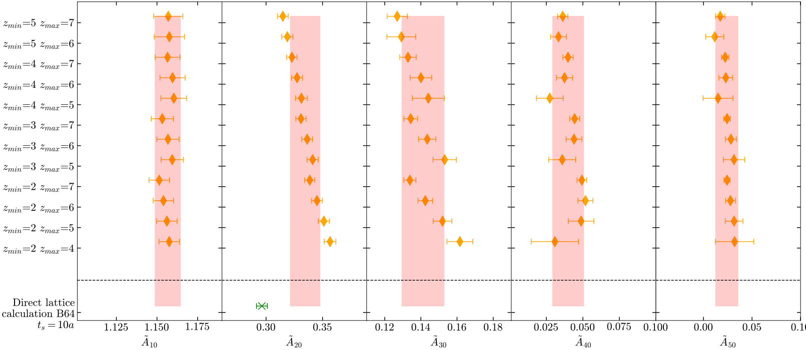
$$\mathcal{M}(z,P) = \frac{\tilde{\mathcal{H}}_{bare}\left(\vec{z},\vec{P},\right)}{\tilde{\mathcal{H}}_{bare}(\vec{z},0)} \frac{\tilde{\mathcal{H}}_{bare}(0,0)}{\tilde{\mathcal{H}}_{bare}\left(0,\vec{P}\right)} g_A = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\bar{MS}}(\mu^2 z^2)}{C_0^{\bar{MS}}(\mu^2 z^2)} \langle x^n \rangle + \mathcal{O}\left(\Lambda_{QCD}^2 z^2\right),$$

with $C_n^{\bar{MS}}(\mu^2 z^2)$ being the Wilson coefficients at NLO for the helicity case.





Helicity-PDF moments

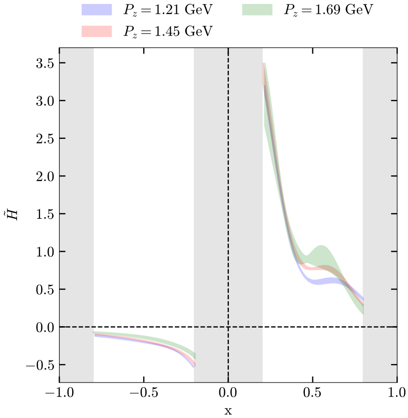


Direct lattice calculation on B64 from [Alexandrou et al., arXiv:2607.20337].

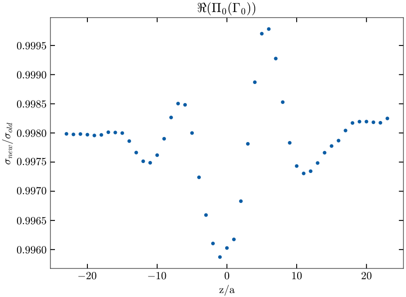
Helicity-PDF light cone

$$\tilde{F}^{\mu}(z,P) = \langle N(P)\left|\bar{\psi}(z)\gamma^{\mu}\gamma^{5}\mathcal{W}(z,0)\psi(0)\right|N(P)\rangle$$

[arXiv:2310.13114v2] We employ RI/MOM renormalization at $\mu = 2$ GeV. We use Bayes-Gauss-Fourier Transform and then we match them [Arxiv:1902.00307] to light-cone helicity PDF.



Different ratios



Ratio of the errors of

$$R(\Gamma_0, p; t_s, t_{ins}) = \frac{C^{3pt}(\Gamma_0, p; t_s, t_{ins})}{\sqrt{C^{2pt}(\Gamma_0, p, \Delta_0; t_s) C^{2pt}(\Gamma_0, p, \Delta; t_s)}}$$

and

$$R(\Gamma_0, p; t_s, t_{ins}) = \frac{C^{3pt}(\Gamma_0, p; t_s, t_{ins})}{C^{2pt}(\Gamma_0, p, \Delta_0; t_s)}$$

for $P_z = 1.21$ GeV.