ETMC calculation of a_μ^{HVP} in isospin-symmetric QCD

Marco Garofalo on behalf of ETMC

University of Bonn

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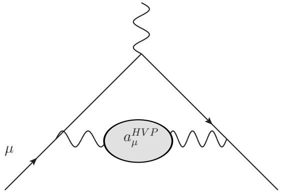
Hadron Vacuum Polarization from Lattice QCD

- Time-Momentum representation [D. Bernecker, H. B. Meyer (2011)]

$$a_\mu^{\mathrm{HVP}} = 3\alpha_{em}^2 \int_0^\infty dt\, K(t) V(t) \qquad V(t) = -\frac{1}{3}\sum_{i=1}^{3}\int d\boldsymbol{x}\, \langle J_i(\boldsymbol{x},t), J_i(0)\rangle$$

$$J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c$$



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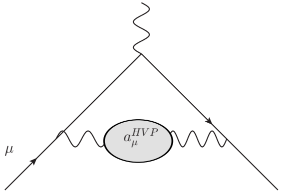
- Isospin decomposition [Mainz/CLS (2019)]

$$a_\mu^{\rm HVP}(\text{iso}) = a_\mu^{\rm HVP}(I=1) + a_\mu^{\rm HVP}(I=0)$$

- $a_\mu^{\rm HVP}(I=1) = \frac{9}{10} a_\mu^{\rm HVP}(\ell)$
- $a_\mu^{\rm HVP}(I=0) = a_\mu^{\rm HVP}(\text{oct}) + a_\mu^{\rm HVP}(c) + \delta a_\mu^{c,\text{disc}}$



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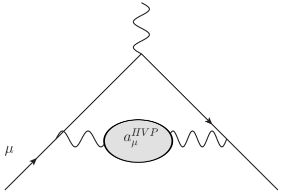
- Mixed action setup: two regularizations of $a_\mu^{\mathrm{HVP}}(I=1)$ with common continuum limit



ETMC $N_f = 2+1+1$ Wilson-clover twisted-mass Gauge ensembles

- 4 lattice spacings: $a \simeq 0.049 - 0.080$ fm
- Spatial volumes: $L \in [5.0, 7.7]$ fm
- $M_\pi \sim 135$ MeV
- FLAG and WP25 iso-QCD definitions

ensemble	V/a^4	a^{iso} [fm]	L [fm]
B64	$64^3 \times 128$	~ 0.080	~ 5.10
B96	$96^3 \times 192$	~ 0.080	~ 7.65
C80	$80^3 \times 160$	~ 0.068	~ 5.44
D96	$96^3 \times 192$	~ 0.057	~ 5.46
E112	$112^3 \times 224$	~ 0.049	~ 5.47

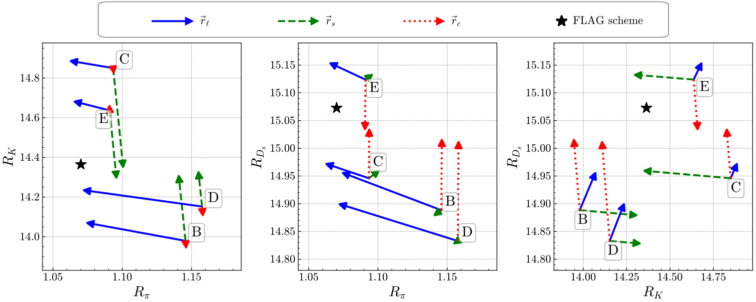
Scale Setting: FLAG

- FLAG iso QCD definition: $R_\pi^{\rm FLAG} = \left[\frac{m_\pi^2}{f_\pi^2}\right]^{\rm FLAG}$, $R_K^{\rm FLAG} = \left[\frac{m_K^2}{f_\pi^2}\right]^{\rm FLAG}$, $R_{D_s}^{\rm FLAG} = \left[\frac{m_{D_s}}{f_\pi}\right]^{\rm FLAG}$
- Solve for (m_ℓ, m_s, m_c, m_0)

$$\vec{R}^{\,\rm sim} + (m_0 - m_0^{\rm sim}) \left.\frac{d\vec{R}}{dm_0}\right|_{\rm sim} + \sum_{f=\ell,s,c} (m_f - m_f^{\rm sim}) \left.\frac{d\vec{R}}{dm_f}\right|_{\rm sim} = \vec{R}^{\,\rm FLAG} ,$$

$$m_{\rm PCAC}^{\rm sim} + (m_0 - m_0^{\rm sim}) \left.\frac{dm_{\rm PCAC}}{dm_0}\right|_{\rm sim} = 0 ,$$

- $a^{\rm FLAG} = a f_\pi / f_\pi^{\rm FLAG}$
- Results in the WP25 scheme are also provided



Blinding Procedure

- Additive blinding of the $I = 1$ data

$$C^{\mathrm{reg}}(an) \to C^{\mathrm{reg}}(an) + \frac{a^3}{(Z_V^{\mathrm{reg}})^2} \sum_{k=1}^{K} A_k e^{-m_k \frac{T}{2}} \cosh\left(m_k \left(\frac{T}{2} - an\right)\right)$$

  - large enough K (we chose $K > 12$) does not allow for unblinding data
  - does not alter the cut-off effects $\longrightarrow$ removal of free-theory lattice artifacts possible;
  - Finite Volume Effects (FVEs) not altered;

Low Mode Averaging (LMA)

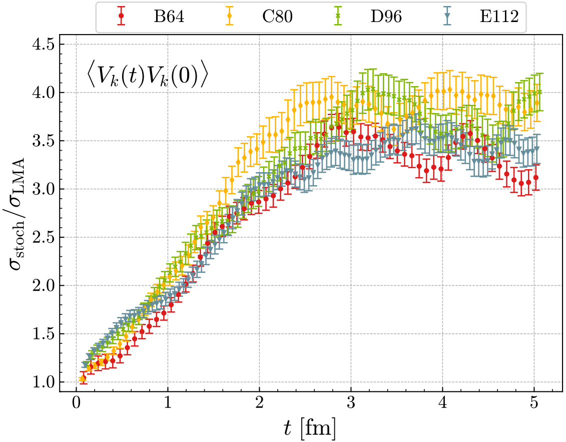
[H. Neff, et al. (2001)] [L. Giusti, et al. (2004)] [T. A. DeGrand, S Schaefer, (2004)]

- Exact treatment of lowest eigenvectors for $L \simeq 5.1 - 5.5$ fm

Ensemble	N_U	$N_{\rm ev}^{\rm IR}$	N_η
B64	1300	400	1024
C80	870	530	1600
D96	670	530	960
E112	680	530	1344

- Multigrid LMA for $L \simeq 7.6$ fm

Ensemble	N_U	$N_\eta^{\rm MG}$	N_η
B96	490	11200	768



Bounding Method

[C. Lehner (2016)] [S. Borsanyi et al (2017)]

- For $t > t_c$ and $X = \{I = 1, 8\}$

$$V^X(t_c)\, e^{-m_{\rm eff}^X(t^*)(t-t_c)} \leq V^X(t) \leq V^X(t_c)\, e^{-E_0^X(t-t_c)},$$

- $m_{\rm eff}^X(t^*)$ effective mass of the correlator $V^X(t)$ at time $t^* < t_c$

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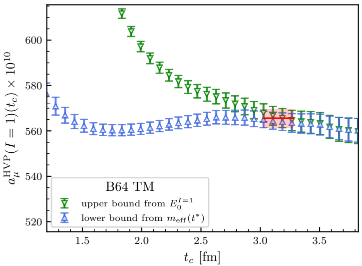
- **For** $I = 1$**:** we used $E_0^{I=1} = 0.98\, E_{0,\rm free}^{I=1}$
  - OS regularization

$$E_{0,\rm free}^{I=1} = 2\sqrt{m_\pi^2 + \left(\frac{2\pi}{L}\right)^2}\,.$$

  - tm regularization (twisted mass neutral pion π^0)

$$E_{0,\rm free}^{I=1} = \sqrt{m_\pi^2 + \left(\frac{2\pi}{L}\right)^2} + \sqrt{m_{\pi^0}^2 + \left(\frac{2\pi}{L}\right)^2}\,.$$



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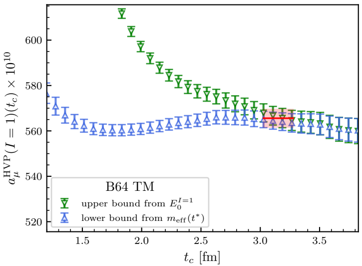
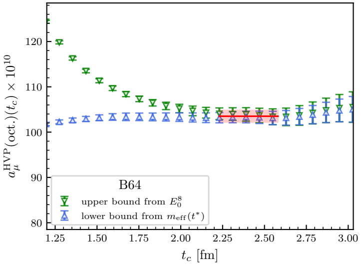
- **For $I = 0$ octet:**

$$E_0^8 = 2\sqrt{m_\pi^2 + \left(\frac{2\pi}{L}\right)^2} + m_{\pi^0}\ ,$$



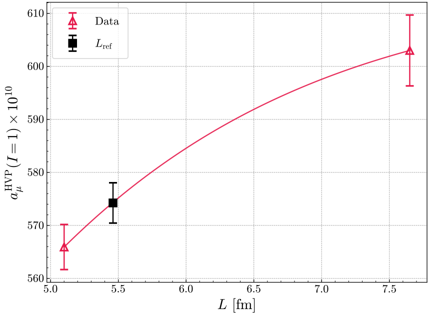


Finite-Volume Effects

- Continuum extrapolation done at $L_{\mathrm{ref}} = 5.46$fm
- Fit the B64 $L = 5.1$ and B96 $L = 7.65$

$$a_\mu^{\mathrm{HVP}}(I=1; a_B, L) = c_0 + c_1 L\, e^{-m_\pi L}\,.$$

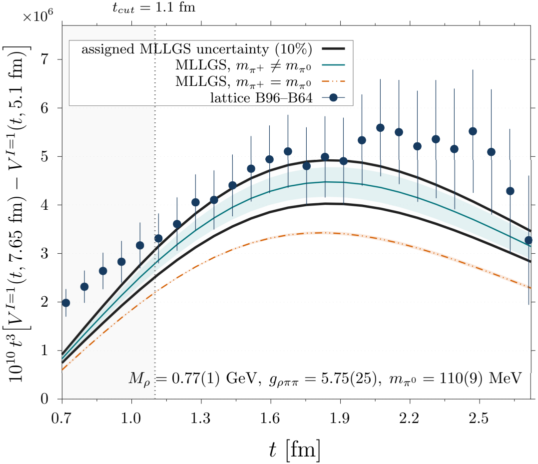
- The other ensembles have $L = 5.44, 5.46, 5.47$



In the continuum we extrapolate to infinite volume with the MLLGS model:

Validation of MLLGS

- Validated by other collaboration, for instance [Mainz/CLS (2025), BMW (2020)]
- Validation with our data at $L = 5.1, 7.65$ fm
- TM correlator has $\pi^+\pi^0$ states
- Agreement with our data for $t > 1.1$ fm
- We assign 10% uncertainties



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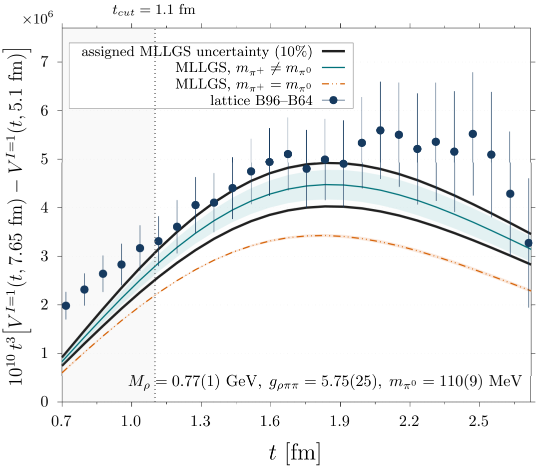
- Data driven correction in the region $t < 1.1$ fm based on B96-B64
    - For $t < 1.1$ we expect small FVE
    - In the continuum we recover $\pi^0 = \pi^\pm$
    - Difference data driven - MLLGS model taken as uncertainty



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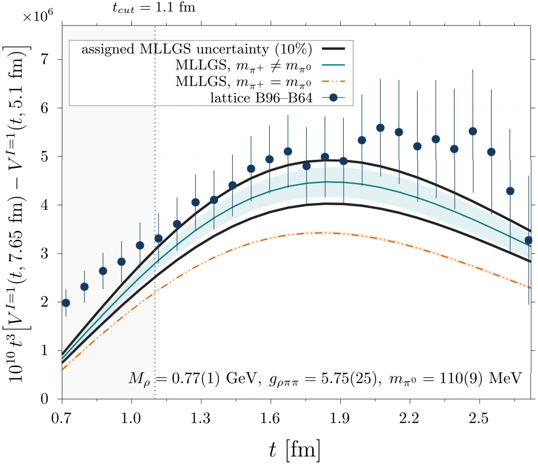
- Total:

$$\Delta_{\mathrm{FV}} a_\mu^{\mathrm{HVP}}(I=1; L_{\mathrm{ref}}) = 27.1(2.5)(1.0)[2.7] \times 10^{-10}$$

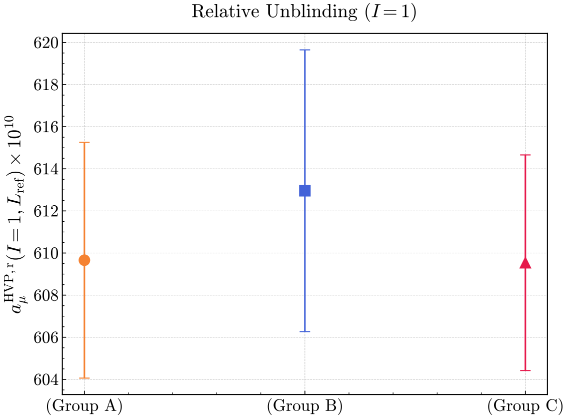


$I = 1$ Contribution: Results

- Reference volume: $L_{\mathrm{ref}} = 5.46\,\mathrm{fm}$
- fit function

$$a_\mu^{\mathrm{HVP,r}}(I=1; a, L_{\mathrm{ref}}) = a_\mu^{\mathrm{HVP}}(I=1; L_{\mathrm{ref}}) + c_1^r a^2 + c_{1L}^r \frac{a^2}{\left|\log\left(a^2\Lambda_0^2\right)\right|^\gamma} + c_2^r a^4\,, \qquad r = \mathrm{TM}, \mathrm{OS}\,.$$

- Several variants from removing terms from the fit function
- BAIC average



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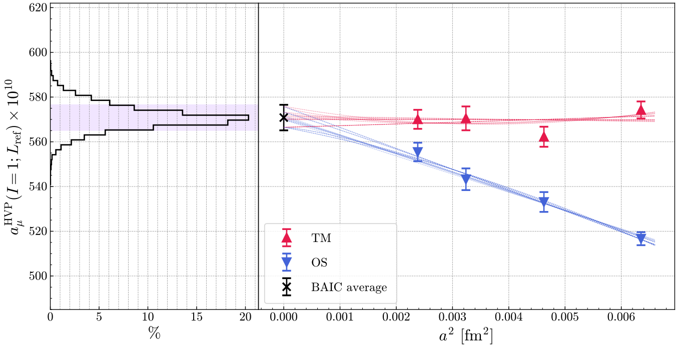

$$a_\mu^{\text{HVP,FLAG}}(I=1) = 597.9(4.9)_{\text{stat}}(2.9)_{\text{syst}}(2.7)_L[6.3] \times 10^{-10}$$

$I = 0$ Octet Contribution

- Decomposition:

$$a_\mu^{\mathrm{HVP}}(I=0) = a_\mu^{\mathrm{HVP}}(\mathrm{oct.}) + a_\mu^{\mathrm{HVP}}(c) + \delta a_\mu^{c,\mathrm{disc}}$$

$$j_\mu^{(8)}(x) = (1/6)[\, j_\mu^u + j_\mu^d - 2 j_\mu^s \,]$$

- Disconnected diagrams frequency splitting: [L. Giusti et al. (2019)]

$$\begin{aligned} \langle j_\mu^{(8)}(x)\rangle_\psi &= B_\mu(m_\ell) - B_\mu(m_s) \\ &= [B_\mu(m_\ell) - B_\mu(m_1)] + [B_\mu(m_1) - B_\mu(m_2)] \\ &+ \ldots + [B_\mu(m_N) - B_\mu(m_s)] \end{aligned}$$

- WTI

$$B_\mu(m_\ell) - B_\mu(m_s) = \frac{1}{3}(\mu_\ell + \mu_s) \int d^4x \langle P^1(x) A_\mu^2(z)\rangle_\psi$$

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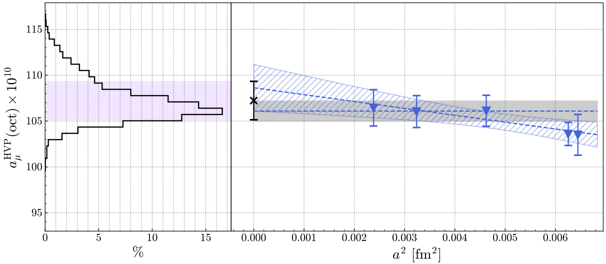
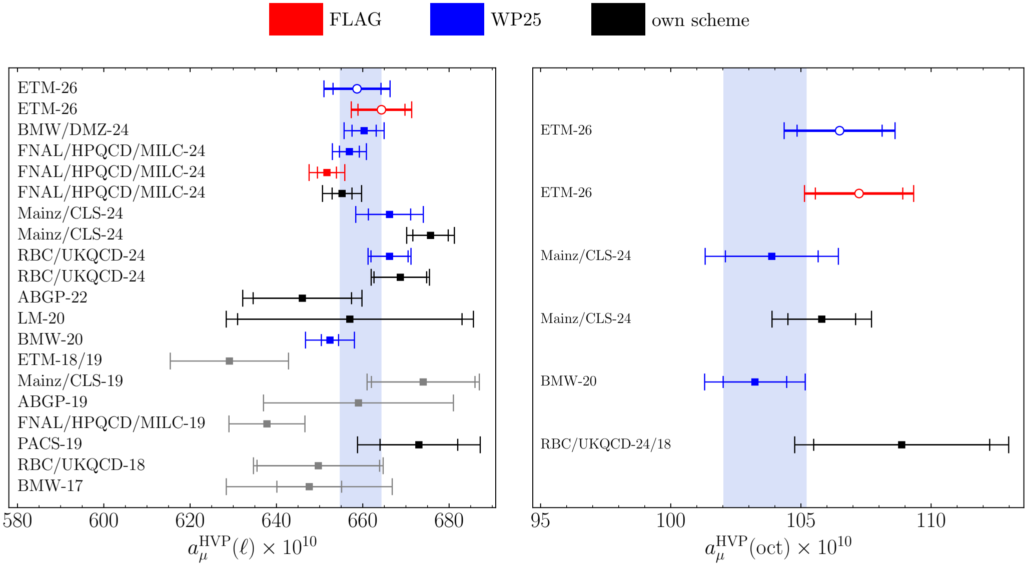


$$a_\mu^{\mathrm{HVP,FLAG}}(\mathrm{oct}) = 107.2\,(1.7)_{\mathrm{stat}}\,(1.3)_{\mathrm{syst}}\,[2.1] \times 10^{-10}\,.$$



FLAG
WP25
own scheme
ETM-26
ETM-26
BMW/DMZ-24
FNAL/HPQCD/MILC-24
FNAL/HPQCD/MILC-24
FNAL/HPQCD/MILC-24
Mainz/CLS-24
Mainz/CLS-24
RBC/UKQCD-24
RBC/UKQCD-24
ABGP-22
LM-20
BMW-20
ETM-18/19
Mainz/CLS-19
ABGP-19
FNAL/HPQCD/MILC-19
PACS-19
RBC/UKQCD-18
BMW-17
580
600
620
640
660
680
$a_\mu^{\mathrm{HVP}}(\ell) \times 10^{10}$
ETM-26
ETM-26
Mainz/CLS-24
Mainz/CLS-24
BMW-20
RBC/UKQCD-24/18
95
100
105
110
$a_\mu^{\mathrm{HVP}}(\mathrm{oct}) \times 10^{10}$

Conclusion

- $\sim 1\%$ precision for $I = 1$
- $\sim 2\%$ precision for $I = 0$
- Finalizing up the results
  - $a_\mu^{\mathrm{HVP}}(c)$ and $a_\mu^{\mathrm{HVP}}(s)$
  - Window observables
- Outlook
  - Strong IB and electromagnetic correction

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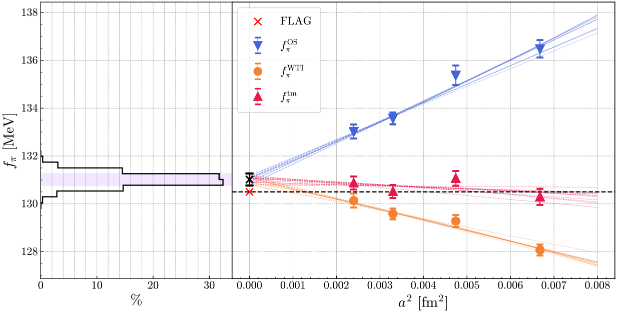
Thank you for your attention

WP25 scheme

- WP25 iso QCD definition $P_\pi^{\mathrm{WP25}} = \left[m_\pi^2 w_0^2\right]^{\mathrm{WP25}}, \quad P_K^{\mathrm{WP25}} = \left[m_K^2 w_0^2\right]^{\mathrm{WP25}}, \quad P_{D_s}^{\mathrm{WP25}} = \left[m_{D_s} w_0\right]^{\mathrm{WP25}}$
- $O(a^2)$ modification to reduce charm mistuning $P_{D_s}^{\mathrm{WP25}'} = \left[m_{D_s} w_0\right]^{\mathrm{WP25}} + Ca^2$
- Solve

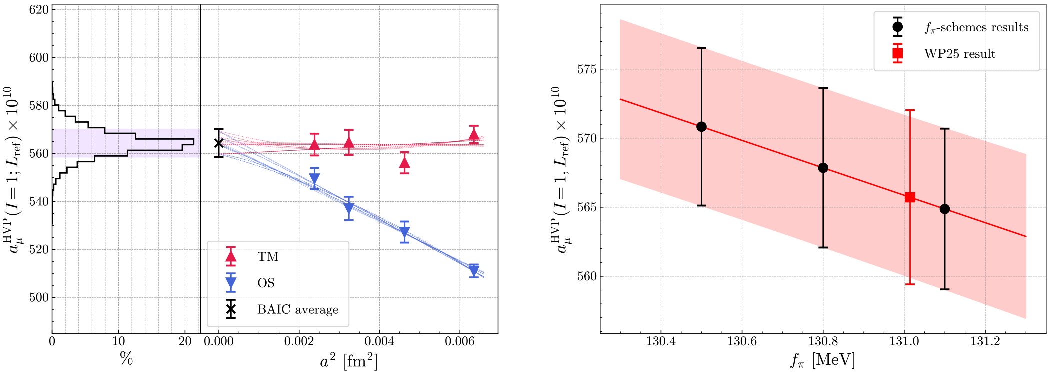
$$\vec{P}^{\mathrm{sim}} + (m_0 - m_0^{\mathrm{sim}}) \left.\frac{d\vec{P}}{dm_0}\right|_{\mathrm{sim}} + \sum_{f=\ell,s,c} (m_f - m_f^{\mathrm{sim}}) \left.\frac{d\vec{P}}{dm_f}\right|_{\mathrm{sim}} = \vec{P}^{\mathrm{WP25}'} ,$$

- Compute f_π from conserved current and local current tm and OS

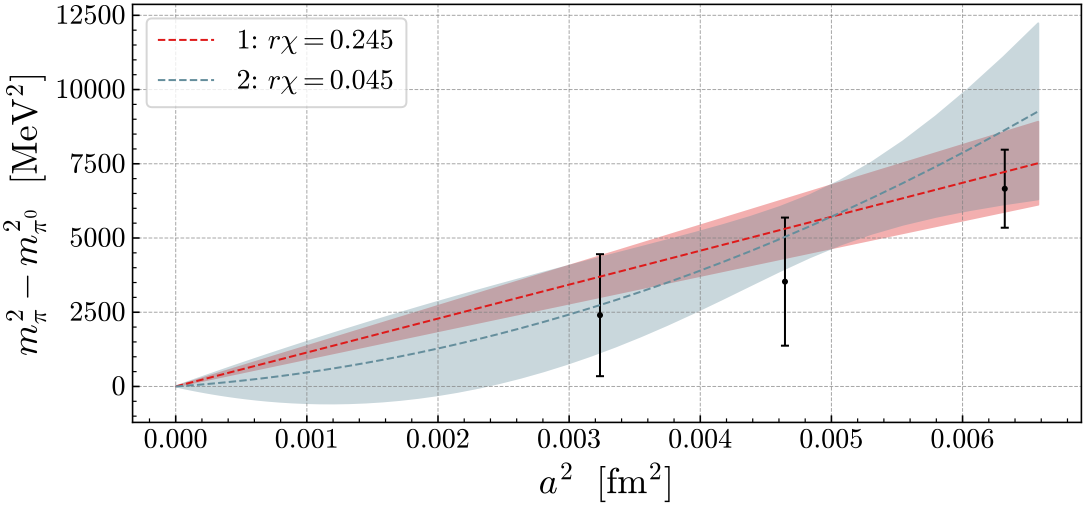


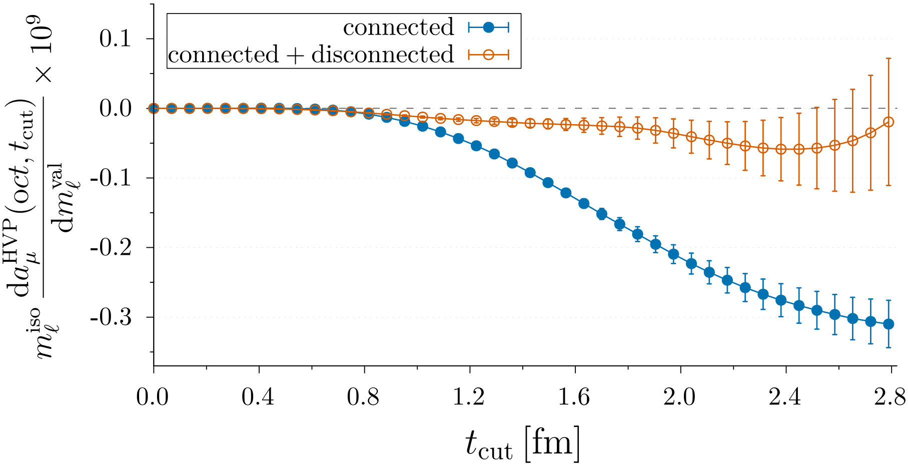
WP25 scheme

- Define few iso QCD schemes for some f_π
- Interpolate to f_π^{WP25}



Pion mass splitting





connected
connected + disconnected
$m_\ell^{\mathrm{iso}} \frac{\mathrm{d}a_\mu^{\mathrm{HVP}}(\mathrm{oct}, t_{\mathrm{cut}})}{\mathrm{d}m_\ell^{\mathrm{val}}} \times 10^9$
0.1
0.0
-0.1
-0.2
-0.3
0.0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
t_{cut} [fm]