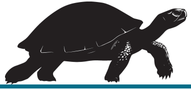
Jefferson Lab
Thomas Jefferson National Accelerator Facility

Direct Determination of the Baryon Octet Sigma Terms from $N_\mathrm{f} = 2 + 1$ Lattice QCD with Wilson fermions

with Gunnar Bali, Sara Collins, Jochen Heitger, Simon Weishäupl

[arXiv 2112.00586, arXiv 2301.03871, ...]

Pia Leonie Jones



Universität Münster
Graduiertenkolleg 2149 Research Training Group
RQCD

Baryon Charges

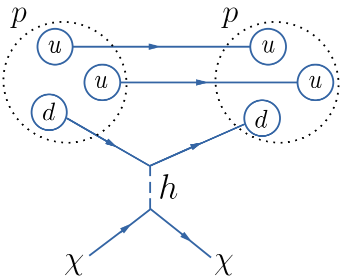
$$\boldsymbol{g_J^B = \langle B|J|B\rangle}$$

- $|B\rangle$ refers to the ground state of a baryon B at rest
- current J can be flavour diagonal or flavour changing
  e.g. $J = \bar{q}\Gamma q$ or $J = \bar{u}\Gamma d$
- Γ insertion - choice of γ-matrices
  - isosinglet **scalar** ($\mathbb{1}$)
    $\rightarrow \boldsymbol{m_q g_S^{q,B}}$ = **quark mass contribution**
    $\Rightarrow$ **the so-called sigma terms**

isosinglet (flavour diagonal) scalar current

$$\boldsymbol{J = \bar{q}\mathbb{1}q}$$

- e.g. dark matter mediated:



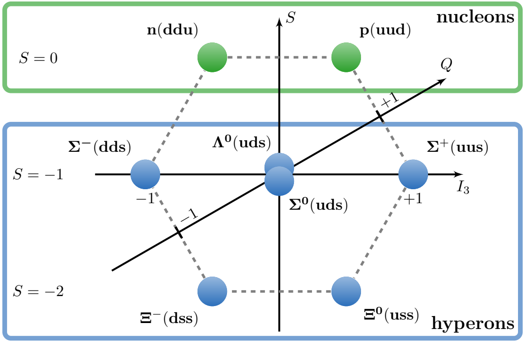
Why determine the sigma terms?

quark mass contributions

- **WIMP-nucleon scattering cross-sections** (e.g. XENON1T)
- **decomposition of the hadron mass**
- **discrepancy** between the **nucleon pion sigma term** from Lattice QCD & phenomenology
- access **L**ow **E**nergy **C**onstants from ChPT

$\Rightarrow$ study σ_s^B & $\sigma_\pi^B = \sigma_u^B + \sigma_d^B$ of the **baryon octet** (in $N_\mathrm{f} = 3$ & $m_\mathrm{u} = m_\mathrm{d}$)

$\Rightarrow$ nucleon N, lambda Λ, sigma Σ and xi Ξ



[RQCD 2305.04717]

Baryon Sigma Terms from Lattice QCD

Quark Mass Contributions to a Baryon

Want to determine

$$\boxed{\sigma_q^B = m_q \langle B | \bar{q}\, \mathbb{1}\, q | B \rangle}$$

$\Leftarrow$ B refers to the ground state of the baryon B at rest

INDIRECT

Feynman-Hellman theorem

$$\boxed{\sigma_q^B = m_q \frac{\partial m_B}{\partial m_q}}$$

DIRECT

“matrix element straight from the lattice”

RECAP: Spectral Decompositions

$$C_{2\mathrm{pt}}(t_\mathrm{f}) = \sum_{\vec{x}} \left\langle \mathcal{O}_\mathrm{snk}(\vec{x}, t_\mathrm{f}) \bar{\mathcal{O}}_\mathrm{src}(\vec{0}, 0) \right\rangle = \sum_n |Z_n|^2 e^{-E_n t_\mathrm{f}}$$

$Z_n = \langle \Omega | \mathcal{O}_\mathrm{snk} | n \rangle$ (vacuum state Ω) is the overlap of the interpolator $\mathcal{O}_{snk}$ onto the state n.

$$\begin{aligned} C_{3\mathrm{pt}}(t_\mathrm{f}, t) &= \sum_{\vec{x},\vec{y}} \left\langle \mathcal{O}_\mathrm{snk}(\vec{x}, t_\mathrm{f}) J(\vec{y}, t) \bar{\mathcal{O}}_\mathrm{src}(\vec{0}, 0) \right\rangle - \sum_{\vec{x},\vec{y}} \langle J(\vec{y}, t) \rangle \left\langle \mathcal{O}_\mathrm{snk}(\vec{x}, t_\mathrm{f}) \bar{\mathcal{O}}_\mathrm{src}(\vec{0}, 0) \right\rangle \\ &= \sum_{n,n'} Z_{n'} Z_n^* \langle \mathbf{n'} | \mathbf{J} | \mathbf{n} \rangle e^{-E_n t} e^{-E_{n'}(t_\mathrm{f} - t)} \end{aligned}$$

t_f is the source-sink separation & t is the insertion time of the current

RECAP: Spectral Decompositions

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t_f is the source-sink separation & t is the insertion time of the current

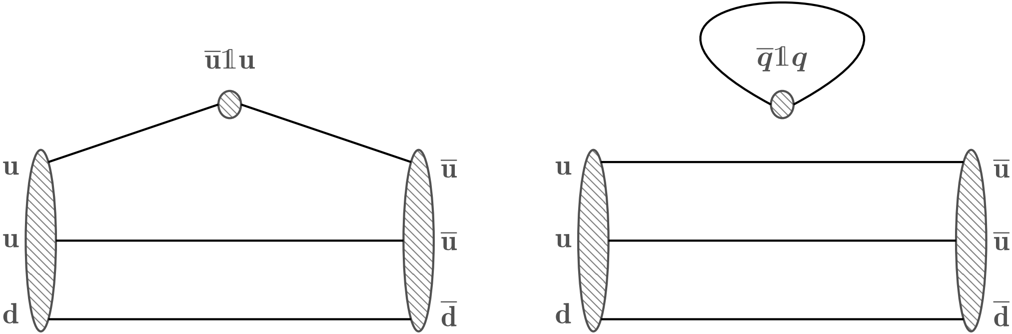
How to access the ground-state matrix element

spectral decomposition

ratio method

$$R(t_\mathrm{f}, t) = \frac{C_\mathrm{3pt}(t_\mathrm{f}, t)}{C_\mathrm{2pt}(t_\mathrm{f})} \to \langle B|\bar{q}\,\mathbb{1}\,q|B\rangle + \mathcal{O}(e^{-\Delta E_1 t}) + \mathcal{O}(e^{-\Delta E_1 (t_f - t)}) + \mathcal{O}(e^{-\Delta E_1 t_f})$$

($\Delta E_1 = E_1 - E_0$, energy gap between ground & first excited state)



How to access the ground-state matrix element

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SIGNAL-TO-NOISE PROBLEM

$$R(t_\mathrm{f}, t) = \frac{C_\mathrm{3pt}(t_\mathrm{f}, t)}{C_\mathrm{2pt}(t_\mathrm{f})} \to \langle B|\bar{q}\,\mathbb{1}\,q|B\rangle + \boxed{\mathcal{O}(e^{-\Delta E_1 t}) + \mathcal{O}(e^{-\Delta E_1 (t_f - t)}) + \mathcal{O}(e^{-\Delta E_1 t_f})}$$

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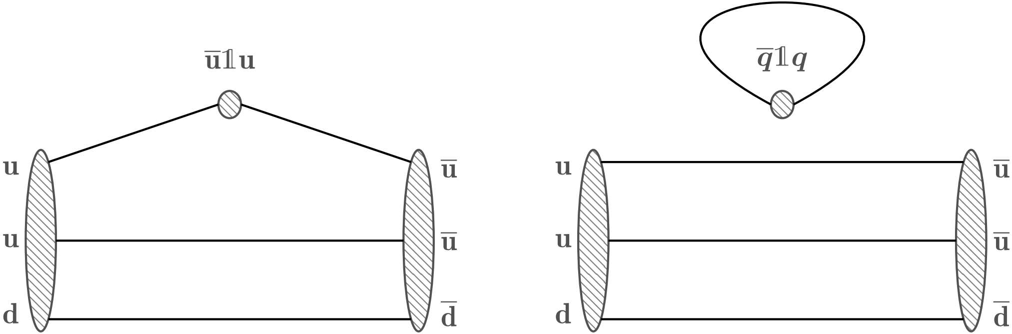

How to access the ground-state matrix element

spectral decomposition

ratio method

EXCITED STATE CONTAMINATION

$$R(t_f, t) = \frac{C_{3pt}(t_f, t)}{C_{2pt}(t_f)} \to \langle B|\bar{q}\,\mathbb{1}\,q|B\rangle + \boxed{\mathcal{O}(e^{-\Delta E_1 t}) + \mathcal{O}(e^{-\Delta E_1 (t_f - t)}) + \mathcal{O}(e^{-\Delta E_1 t_f})}$$

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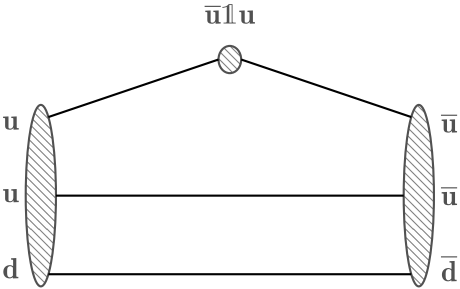
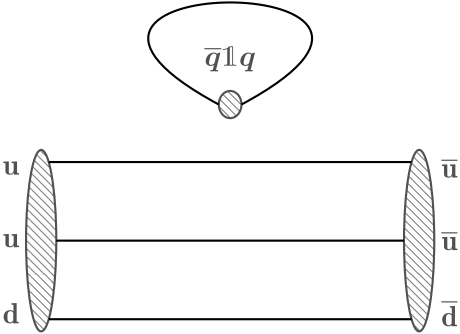
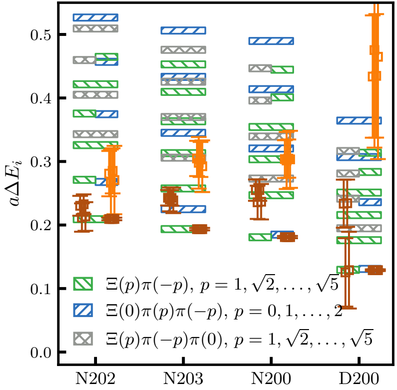




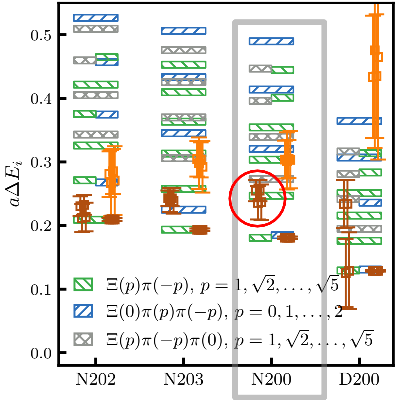
One-state fits -- ratio method with one excited state



$$R(t_{\mathrm{f}}, t) = g^B_{q,S} + c_{0\leftrightarrow 1}\left(\mathrm{e}^{-\Delta E_1 \cdot t} + \mathrm{e}^{-\Delta E_1 \cdot (t_{\mathrm{f}} - t)}\right)$$

'Octet baryon isovector charges from $N_{\mathrm{f}} = 2 + 1$ LQCD' [RQCD 2305.04717]

One-state fits -- ratio method with one excited state



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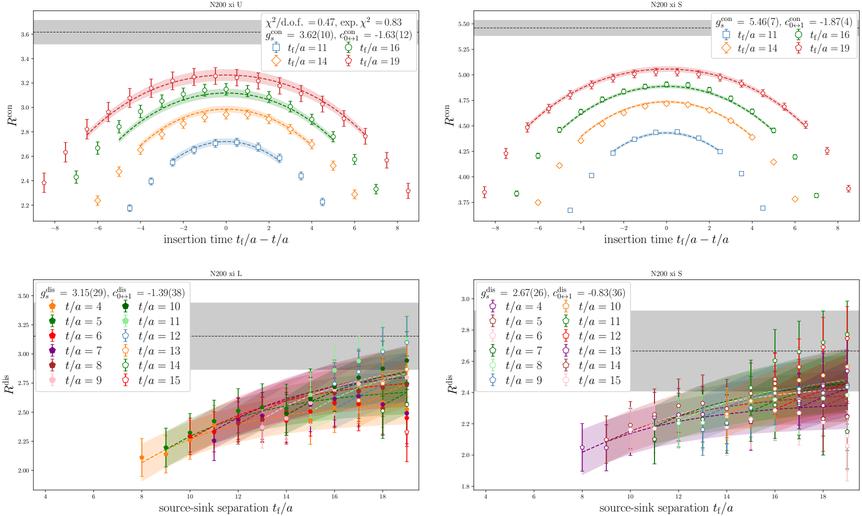
Simultaneous fits to connected & disconnected ratios



- **N200**
- **Ξ baryon**
- $a = 0.064\,\text{fm}$
- $m_\pi = 286\,\text{MeV}$
- **<u>one-state fit</u>:**
- $\chi^2/\chi^2_{\text{exp}} \approx 0.83$
- $a\Delta E_1 = 0.234(12)$
- **top (R^{con}):**

$\rightarrow$ $\bar{u}u$ current (lhs)

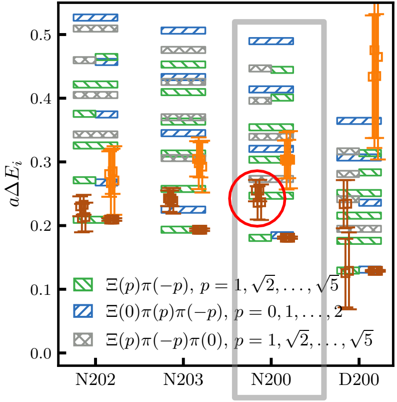
$\rightarrow$ $\bar{s}s$ current (rhs)

- **bottom (R^{dis}):**

$\rightarrow$ $\bar{l}l$ current (lhs)

$\rightarrow$ $\bar{s}s$ current (rhs)

One-state fits -- ratio method with one excited state

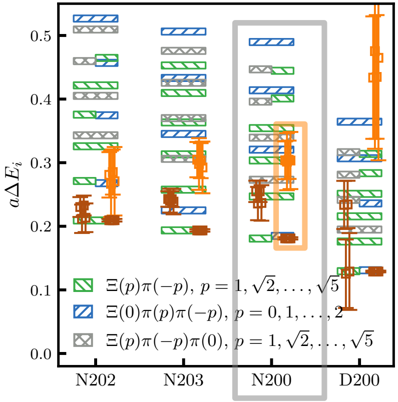


'Octet baryon isovector charges from $N_\mathrm{f} = 2 + 1$ LQCD' [RQCD 2305.04717]

$$R(t_\mathrm{f}, t) = g^B_{q,S} + c_{0\leftrightarrow 1}\left(\mathrm{e}^{-\Delta E_1 \cdot t} + \mathrm{e}^{-\Delta E_1 \cdot (t_\mathrm{f} - t)}\right)$$

$$+ c_{0\leftrightarrow 2}\left(\mathrm{e}^{-\Delta E_2 \cdot t} + \mathrm{e}^{-\Delta E_2 \cdot (t_\mathrm{f} - t)}\right)$$

Two-state fits -- ratio method with two excited states



'Octet baryon isovector charges from $N_\mathrm{f}=2+1$ LQCD' [RQCD 2305.04717]

$$R(t_\mathrm{f},t) = g_S^q + c_{0\leftrightarrow 1}\left(\mathrm{e}^{-\Delta E_1\cdot t}+\mathrm{e}^{-\Delta E_1\cdot(t_\mathrm{f}-t)}\right) + c_{0\leftrightarrow 2}\left(\mathrm{e}^{-\Delta E_2\cdot t}+\mathrm{e}^{-\Delta E_2\cdot(t_\mathrm{f}-t)}\right)$$

- set prior for ΔE_1: $\boldsymbol{B(1)\pi(-1)}$ or $\boldsymbol{B(0)\pi(0)\pi(0)}$ (lowest bar, green/blue)
- set prior for ΔE_2 to **a mean of the results for $\boldsymbol{\Delta E_2}$** from simultaneous fits to the four channels $J\in\{A,S,T,(V)\}$ setting a prior for ΔE_1 to the lowest multi-particle state (orange points)

$\Rightarrow$ find compatibility across different methods

Extrapolate sigma terms

Construct sigma terms from scalar charges extracted at each a, L, m_q

$$\boxed{\sigma_q^B = m_q \langle B | \bar{q} \, \mathbb{1} \, q | B \rangle}$$

Baryon (B) pion sigma terms

$$\sigma_\pi^B = \underbrace{\sigma_u^B + \sigma_d^B}$$

$$\Uparrow$$

$$\sigma_u^B = m_u \langle B | \bar{u} \, \mathbb{1} \, u | B \rangle$$
$$\sigma_d^B = m_d \langle B | \bar{d} \, \mathbb{1} \, d | B \rangle$$

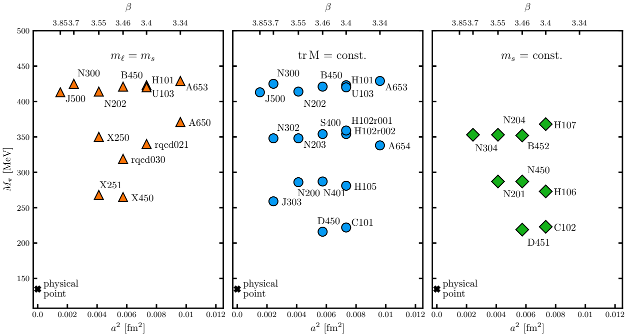
Baryon (B) strange sigma terms

$$\sigma_s^B = m_s \langle B | \bar{s} \, \mathbb{1} \, s | B \rangle$$

Numerical Setup

Constraining the physical point

- tree-level Symanzik-improved gauge action with $N_\mathrm{f} = 2 + 1$ O(a) improved Wilson fermions (***CLS*** **ensembles)**
- **src-snk separation $t_\mathrm{f} \approx 0.7\,\mathrm{fm} - 1.2\,\mathrm{fm}$**



1. **three quark mass trajectories**
2. **high statistics**: 34 ensembles
   - $\sim$ 1500 configurations (from 400 up to 5000)
3. **large spread of a, L, m_π**

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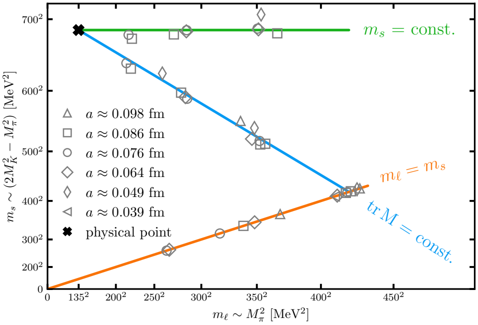


1. **three quark mass trajectories**
2. **high statistics**: 34 ensembles
   - $\sim$ 1500 configurations (from 400 up to 5000)
3. **large spread of a, L, m_π**

Quark Mass Dependence, Cut-off & Finite-Volume Effects

Simultaneous Correlated Fit to all Pion and Strange Octet Baryon Sigma Terms

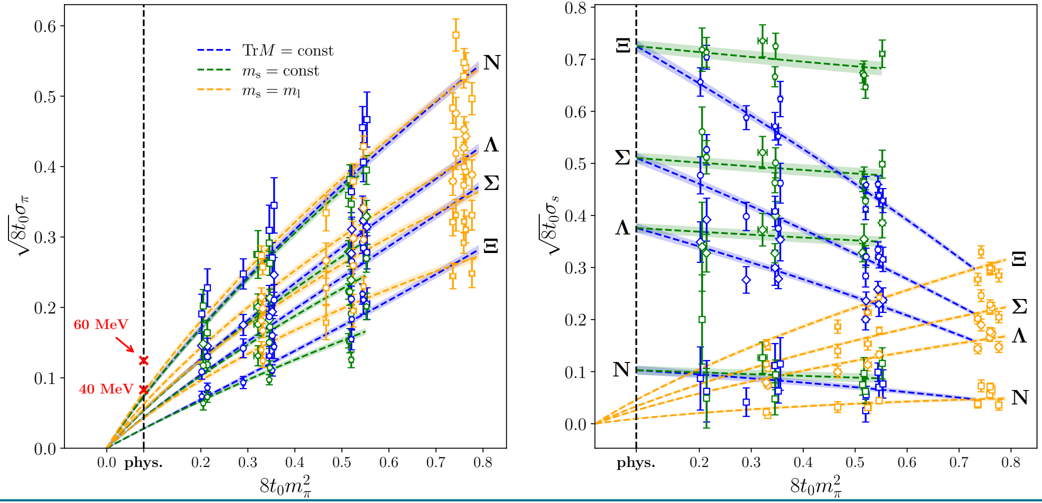
e.g.

$$\sigma_\pi^B \approx (1 + \mathbb{a}^2(c_B + \bar{c}_B\overline{\mathbb{M}}^2 + \delta c_B \delta\mathbb{M}^2))$$
$$\cdot \left[\mathbb{M}_\pi^2 \left\{\frac{2}{3}\overline{\mathbb{b}} - \delta\mathbb{b}_B + \frac{\mathbb{m}_0^2}{2(4\pi\mathbb{F}_0)^2}\left[\frac{g_{B,\pi}}{\mathbb{M}_\pi} f'\left(\frac{\mathbb{M}_\pi}{\mathbb{m}_0}\right) + \frac{g_{B,K}}{2\mathbb{M}_K} f'\left(\frac{\mathbb{M}_K}{\mathbb{m}_0}\right) + \frac{g_{B,\eta}}{3\mathbb{M}_\eta} f'\left(\frac{\mathbb{M}_\eta}{m_0}\right)\right]\right\} + \mathrm{FV}\right]$$

- cut-off effects modelled as $\mathrm{O}(a^2)$: $c's$ depend on B but constraints from symmetric point
- NNLO SU(3) **BaryonChPT** for quark mass dependence & finite-volume effects
  $\rightarrow$ pion and strange baryon sigma terms describable by **the same L**ow **E**nergy **C**onstants
  - **couplings** $\boldsymbol{g_B}$ = combinations of **LO LECs** $\boldsymbol{F}$ & $\boldsymbol{D}$ $(F + D = g_\mathrm{A})$
  - $\delta\mathbb{b}_B$ = combinations of **NLO LECs** $\boldsymbol{b_D, b_F}$ and $\boldsymbol{\bar{b} = -6b_0 - 4b_D}$
  - **no additional parameters** for the finite-volume effects (FV)

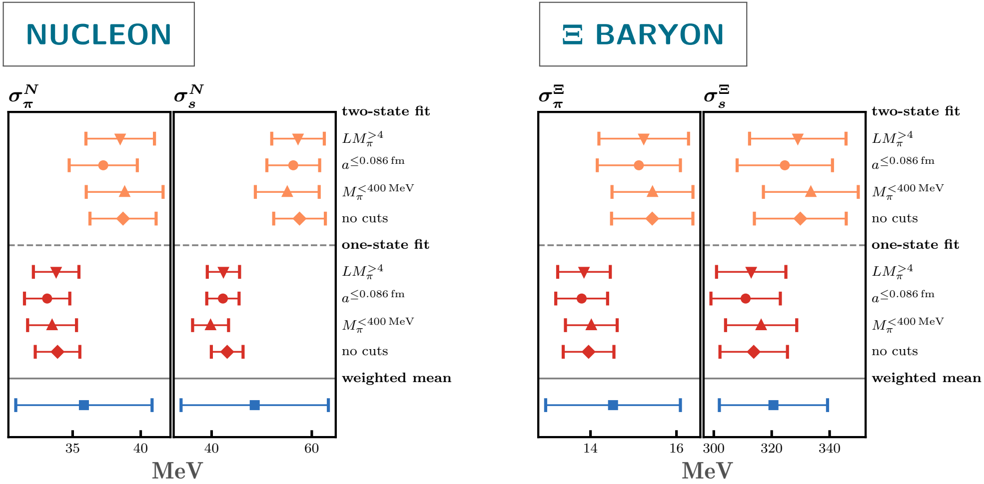
Physical Point Extrapolation - Quark Mass Dependence (BChPT)

e.g. data points based on One-State Fits & coarsest a excl. $\chi^2/\mathbf{d.o.f.} = \mathbf{1.0}$



Preliminary Results - Baryon Octet Sigma Terms

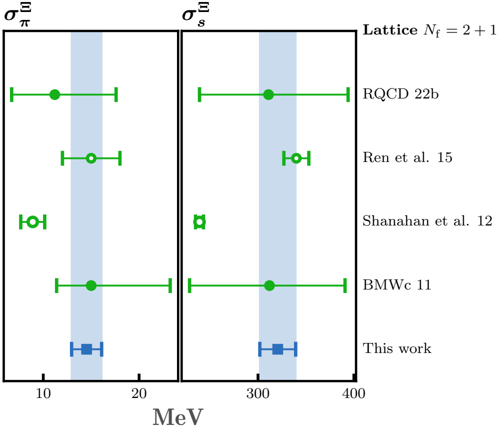
Accounting for (1.) Varied Treatment of the Excited States & (2.) Varied Data Sets



Preliminary Results Compared to Other Studies

Ξ BARYON

- Direct vs. Indirect determinations and vs. Combinations
  (Open symbols - no continuum extrapolation)

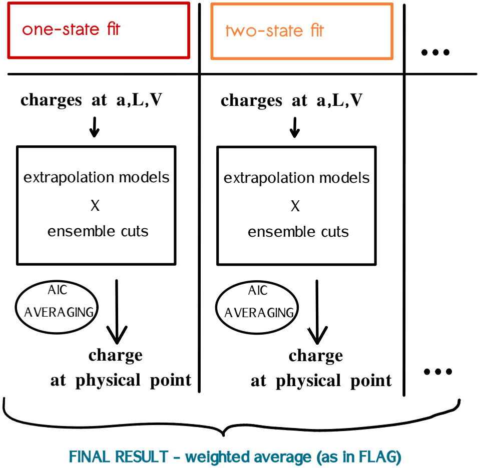


$\Rightarrow$ **improved precision** for the Ξ sigma terms, similarly for the Λ and Σ (not for the nucleon N)

- **first direct determination of all four baryon octet sigma terms** (not only N)

Working out the error budget in detail

- model averaging procedure to account for systematics from
  - Varied Treatment of Excited States (✓)
  - Varied Data Sets (✓)
  - **Extrapolation Models - Variations**
- take errors of κ_{crit} and renormalisation r_{m} [arXiv:2101.10969] consistently into account
  - fit to the bare charges rather than the renormalised sigma terms (✓)
- statistical error estimation via the Γ-method (✓)
  - store information on correlation for a careful error propagation using the `pyerrors` package [arXiv:2209.14371]



Conclusion

on our determination of the quark mass contributions - the so-called sigma terms

Challenges

- signal-to-noise ratio
  - excited state contamination
- physical point extrapolation
  1. quark mass dependence
  2. cut-off effects
  3. finite-volume effects

How we deal with them

- varied treatment of excited states
  - one-state vs. two-state ratio fits
- large set of CLS ensembles
  - **high statistics**
  - large range of a, L, m_π
- varied data sets (various cuts)
- vary extrapolation models

- **first direct determination of all four baryon octet sigma terms** (not only N)
  $\Rightarrow$ **improved precision** (for $B \neq N$) despite very conservative error for now
  $\Rightarrow$ **to do - model averaging procedure to account for systematics**