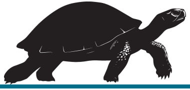
Jefferson Lab
Thomas Jefferson National Accelerator Facility

Direct Determination of the Baryon Octet Sigma Terms from $N_\mathrm{f} = 2 + 1$ Lattice QCD with Wilson fermions

with Gunnar Bali, Sara Collins, Jochen Heitger, Simon Weishäupl

[arXiv 2112.00586, arXiv 2301.03871, ...]

Pia Leonie Jones



Universität Münster

Graduiertenkolleg 2149 Research Training Group

RQCD

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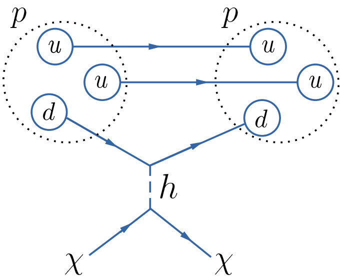

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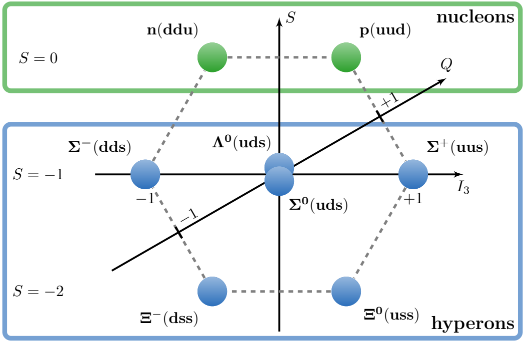

[RQCD 2305.04717]

Baryon Sigma Terms from Lattice QCD

Quark Mass Contributions to a Baryon

Want to determine

$$\boxed{\sigma_q^B = m_q \langle B|\bar{q}\,\mathbb{1}\,q|B\rangle}$$

$\Leftarrow$ B refers to the ground state of the baryon B at rest

INDIRECT

Feynman-Hellman theorem

$$\boxed{\sigma_q^B = m_q \frac{\partial m_B}{\partial m_q}}$$

DIRECT

"matrix element straight from the lattice"

RECAP: Spectral Decompositions

$$C_{\mathrm{2pt}}(t_{\mathrm{f}}) = \sum_{\vec{x}} \left\langle \mathcal{O}_{\mathrm{snk}}(\vec{x}, t_{\mathrm{f}}) \bar{\mathcal{O}}_{\mathrm{src}}(\vec{0}, 0) \right\rangle = \sum_{n} |Z_n|^2 e^{-E_n t_{\mathrm{f}}}$$

$Z_n = \langle \Omega | \mathcal{O}_{\mathrm{snk}} | n \rangle$ (vacuum state Ω) is the overlap of the interpolator $\mathcal{O}_{snk}$ onto the state n.

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t_f is the source-sink separation & t is the insertion time of the current

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How to access the ground-state matrix element

spectral decomposition

ratio method

$$R(t_\mathrm{f}, t) = \frac{C_{\mathrm{3pt}}(t_\mathrm{f}, t)}{C_{\mathrm{2pt}}(t_\mathrm{f})} \rightarrow \langle B|\bar{q}\,\mathbb{1}\,q|B\rangle + \mathcal{O}(e^{-\Delta E_1 t}) + \mathcal{O}(e^{-\Delta E_1 (t_f - t)}) + \mathcal{O}(e^{-\Delta E_1 t_f})$$

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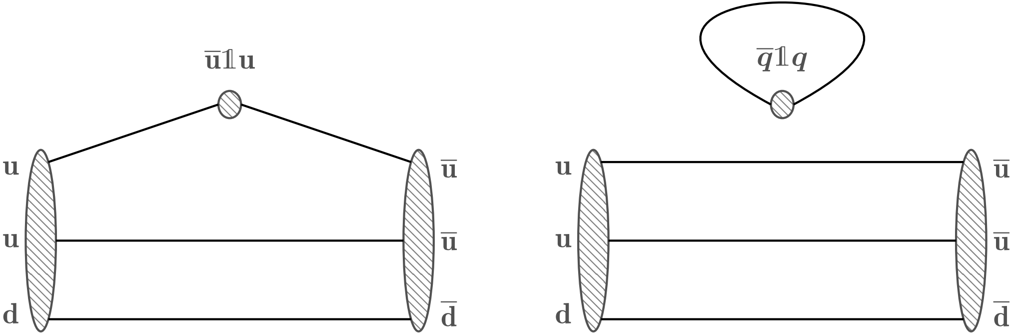

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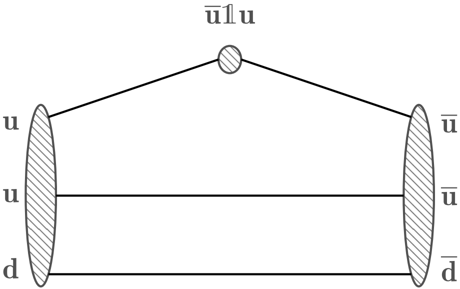

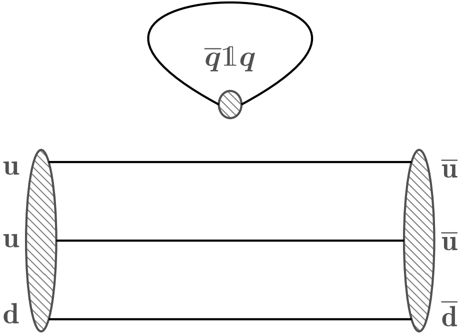

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EXCITED STATE CONTAMINATION

$$R(t_\mathrm{f}, t) = \frac{C_{3\mathrm{pt}}(t_\mathrm{f}, t)}{C_{2\mathrm{pt}}(t_\mathrm{f})} \to \langle B|\bar{q}\,\mathbb{1}\,q|B\rangle + \boxed{\mathcal{O}(e^{-\Delta E_1 t}) + \mathcal{O}(e^{-\Delta E_1 (t_f - t)}) + \mathcal{O}(e^{-\Delta E_1 t_f})}$$

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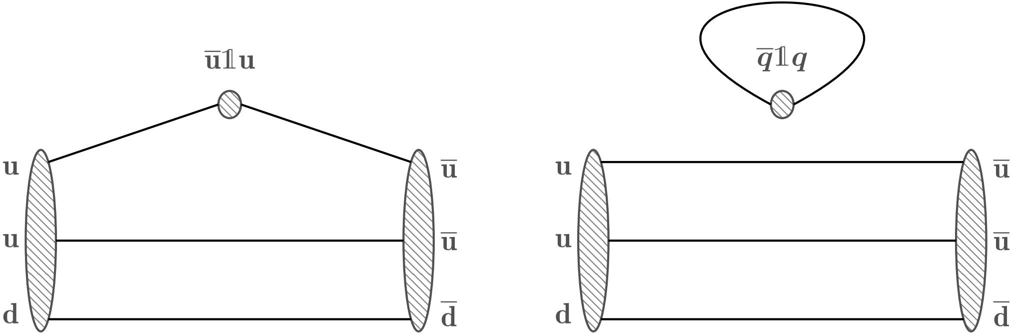
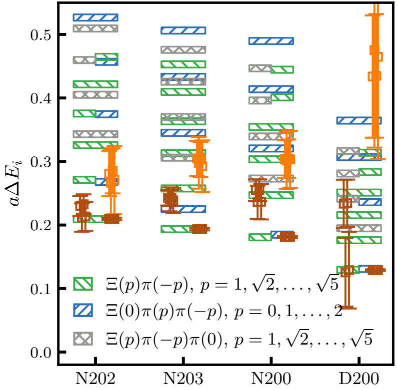


One-state fits -- ratio method with one excited state



'Octet baryon isovector charges from $N_\mathrm{f} = 2 + 1$ LQCD' [RQCD 2305.04717]

$$R(t_\mathrm{f}, t) = g^B_{q,S} + c_{0\leftrightarrow 1}\left(\mathrm{e}^{-\Delta E_1 \cdot t} + \mathrm{e}^{-\Delta E_1 \cdot (t_\mathrm{f} - t)}\right)$$

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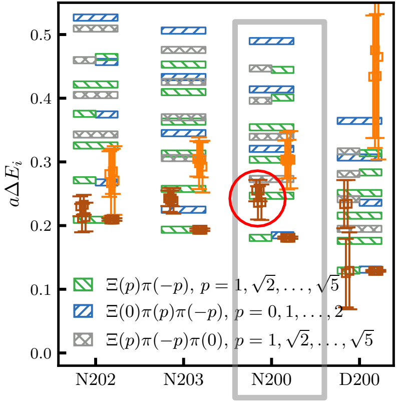

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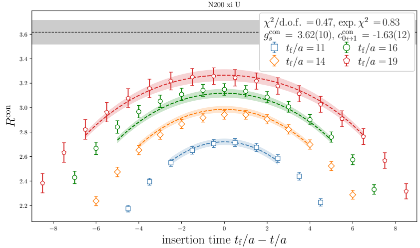
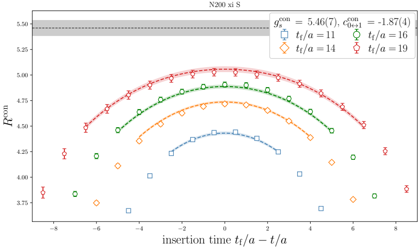
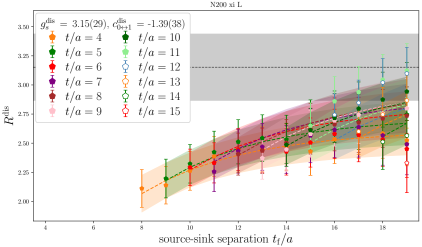
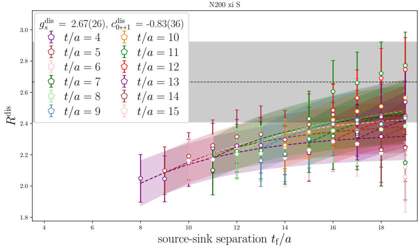
Simultaneous fits to connected & disconnected ratios









- **N200**
- **Ξ baryon**
- $a = 0.064\,\mathrm{fm}$
- $m_\pi = 286\,\mathrm{MeV}$
- **one-state fit:**
- $\chi^2/\chi^2_{\rm exp} \approx 0.83$
- $a\Delta E_1 = 0.234(12)$
- **top ($R^{\rm con}$):**

$\rightarrow$ $\bar{u}u$ current (lhs)

$\rightarrow$ $\bar{s}s$ current (rhs)
- **bottom ($R^{\rm dis}$):**

$\rightarrow$ $\bar{l}l$ current (lhs)

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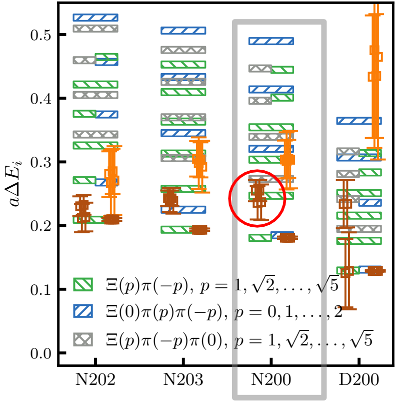

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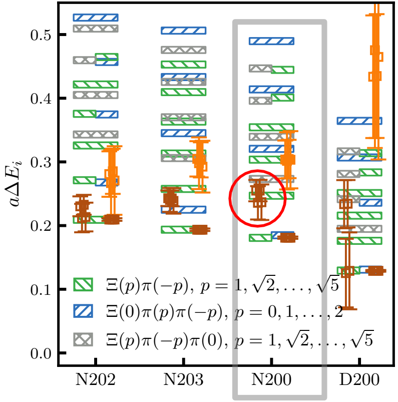

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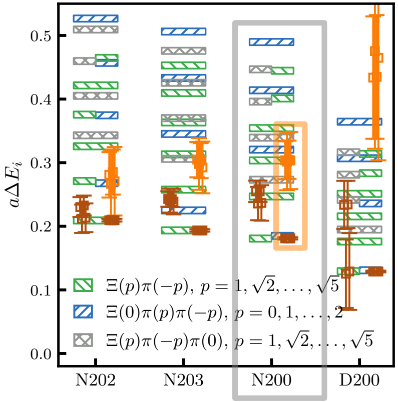
$$+ c_{0\leftrightarrow 2}\left(\mathrm{e}^{-\Delta E_2 \cdot t} + \mathrm{e}^{-\Delta E_2 \cdot (t_\mathrm{f} - t)}\right)$$

Two-state fits -- ratio method with two excited states



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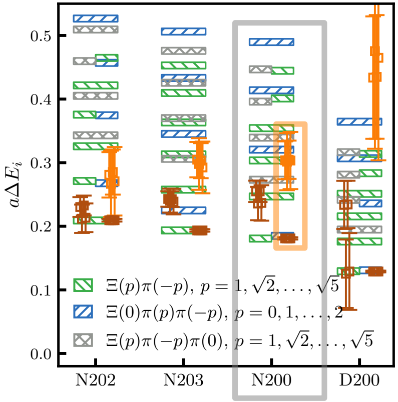

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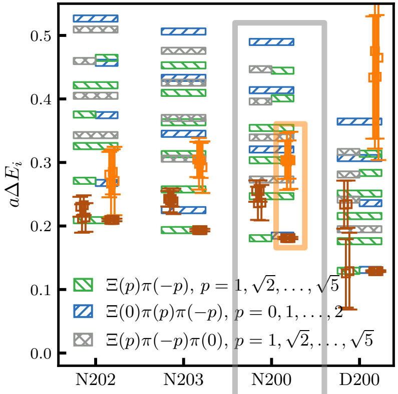

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- set prior for ΔE_1: $\boldsymbol{B(1)\pi(-1)}$ or $\boldsymbol{B(0)\pi(0)\pi(0)}$ (lowest bar, green/blue)
- set prior for ΔE_2 to **a mean of the results for $\boldsymbol{\Delta E_2}$** from simultaneous fits to the four channels $J \in \{A, S, T, (V)\}$ setting a prior for ΔE_1 to the lowest multi-particle state (orange points)

$\Rightarrow$ find compatibility across different methods

Extrapolate sigma terms

Construct sigma terms from scalar charges extracted at each a, L, m_q

$$\boxed{\sigma_q^B = m_q \langle B|\bar{q}\,\mathbb{1}\,q|B\rangle}$$

Baryon (B) pion sigma terms

$$\sigma_\pi^B = \underbrace{\sigma_u^B + \sigma_d^B}$$

$$\Uparrow$$

$$\sigma_u^B = m_u \langle B|\bar{u}\,\mathbb{1}\,u|B\rangle$$

$$\sigma_d^B = m_d \langle B|\bar{d}\,\mathbb{1}\,d|B\rangle$$

Baryon (B) strange sigma terms

$$\sigma_s^B = m_s \langle B|\bar{s}\,\mathbb{1}\,s|B\rangle$$

Numerical Setup

Constraining the physical point

- tree-level Symanzik-improved gauge action with $N_\mathrm{f} = 2 + 1$ O(a) improved Wilson fermions **(*CLS* ensembles)**

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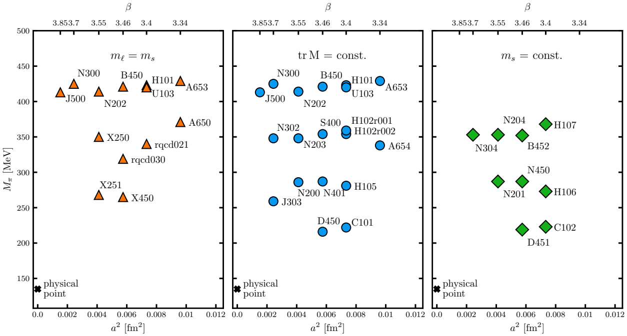

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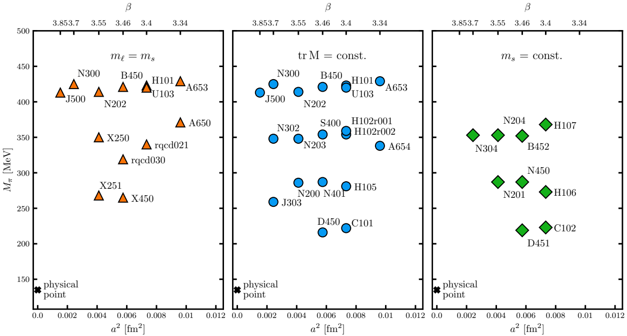

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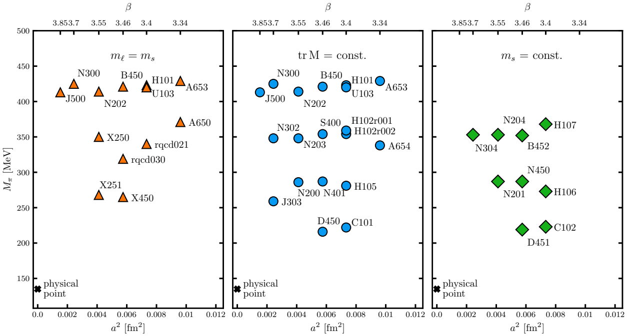

1. **three quark mass trajectories**
2. **high statistics**: 34 ensembles
   - $\sim$ 1500 configurations (from 400 up to 5000)

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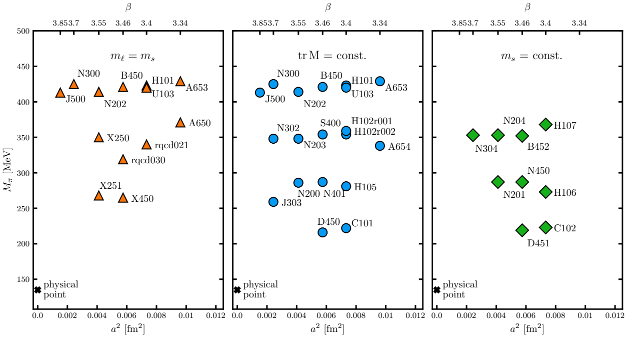

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3. **large spread of a, L, m_π**

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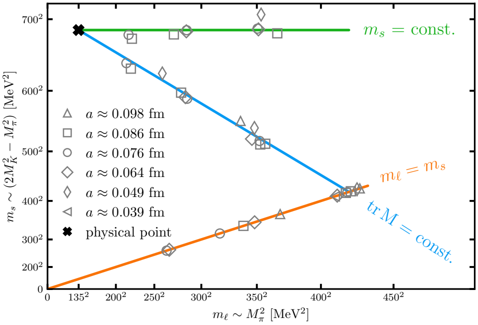
Constraining the physical point

- tree-level Symanzik-improved gauge action with $N_\mathrm{f} = 2 + 1$ O(a) improved Wilson fermions (***CLS*** **ensembles)**
- **src-snk separation $t_\mathrm{f} \approx 0.7\,\mathrm{fm} - 1.2\,\mathrm{fm}$**



1. **three quark mass trajectories**
2. **high statistics**: 34 ensembles
   - $\sim$ 1500 configurations (from 400 up to 5000)
3. **large spread of a, L, m_π**

Quark Mass Dependence, Cut-off & Finite-Volume Effects

Simultaneous Correlated Fit to all Pion and Strange Octet Baryon Sigma Terms

e.g.

$$\sigma_\pi^B \approx (1 + \mathbb{a}^2(c_B + \bar{c}_B\overline{\mathbb{M}}^2 + \delta c_B \delta\mathbb{M}^2)) \cdot \left[\mathbb{M}_\pi^2 \left\{\frac{2}{3}\overline{\mathbb{b}} - \delta\mathbb{b}_B + \frac{\mathbb{m}_0^2}{2(4\pi\mathbb{F}_0)^2}\left[\frac{g_{B,\pi}}{\mathbb{M}_\pi} f'\left(\frac{\mathbb{M}_\pi}{\mathbb{m}_0}\right) + \frac{g_{B,K}}{2\mathbb{M}_K} f'\left(\frac{\mathbb{M}_K}{\mathbb{m}_0}\right) + \frac{g_{B,\eta}}{3\mathbb{M}_\eta} f'\left(\frac{\mathbb{M}_\eta}{m_0}\right)\right]\right\} + \mathrm{FV}\right]$$

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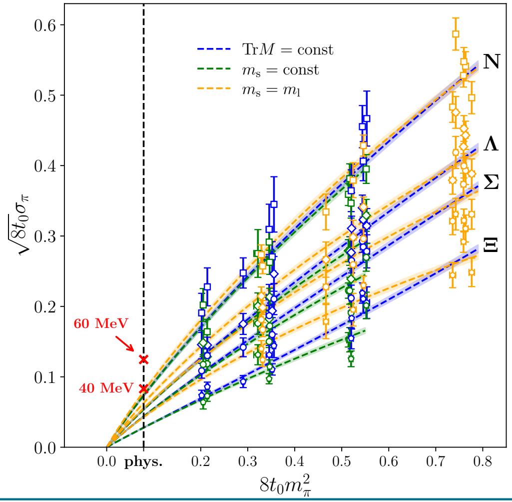
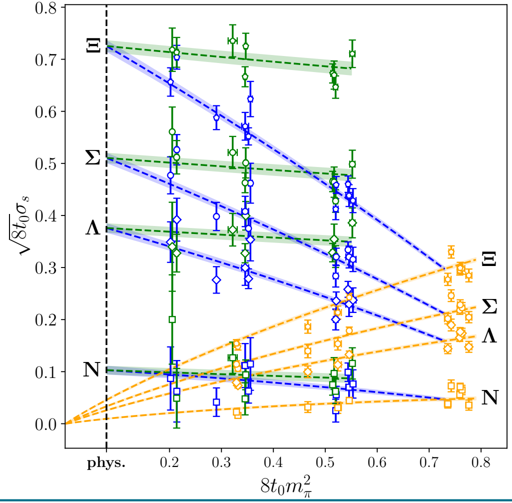
Physical Point Extrapolation - Quark Mass Dependence (BChPT)

e.g. data points based on One-State Fits & coarsest a excl. $\chi^2/\mathrm{d.o.f.} = 1.0$

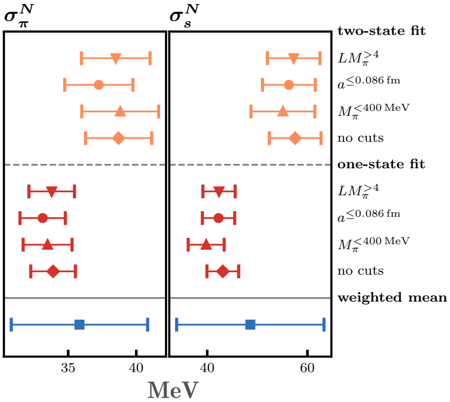




Preliminary Results - Baryon Octet Sigma Terms

Accounting for (1.) Varied Treatment of the Excited States & (2.) Varied Data Sets

NUCLEON



Ξ BARYON

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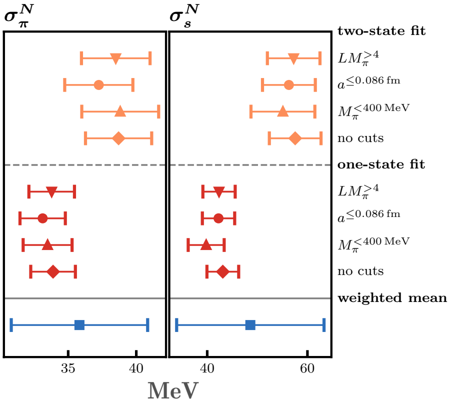

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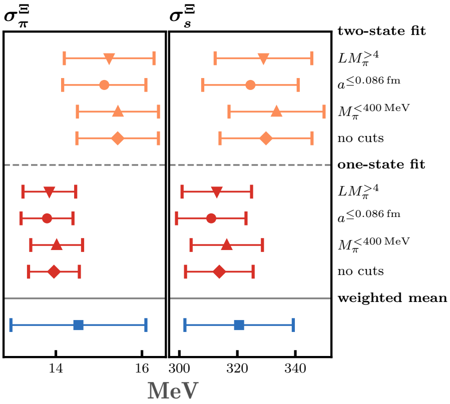
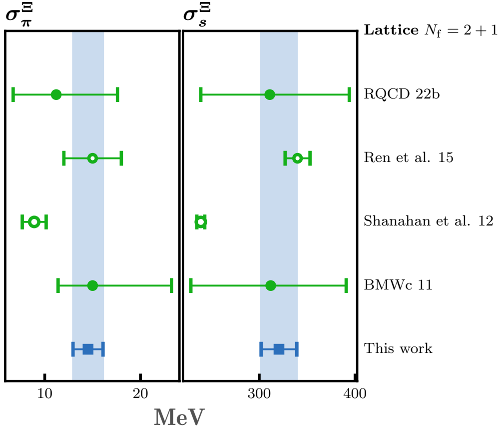


Preliminary Results Compared to Other Studies

Ξ BARYON

- Direct vs. Indirect determinations and vs. Combinations
  (Open symbols - no continuum extrapolation)



$\Rightarrow$ **improved precision** for the Ξ sigma terms, similarly for the Λ and Σ (not for the nucleon N)

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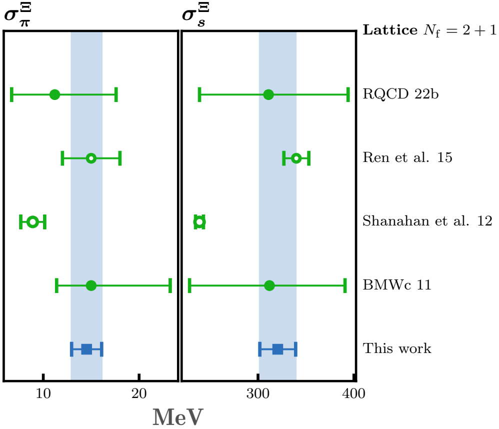

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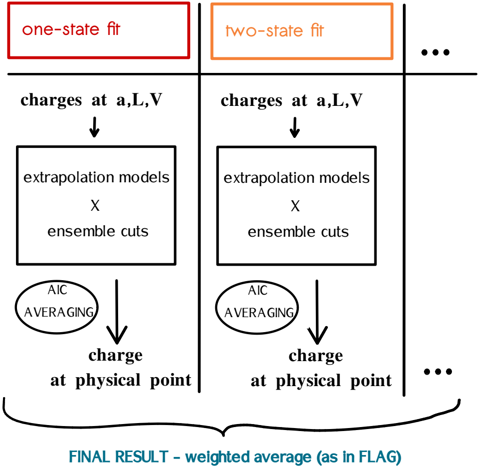
- **first direct determination of all four baryon octet sigma terms** (not only N)

Working out the error budget in detail

- model averaging procedure to account for systematics from
  - Varied Treatment of Excited States (✓)
  - Varied Data Sets (✓)
  - **Extrapolation Models - Variations**
- take errors of κ_{crit} and renormalisation r_{m} [arXiv:2101.10969] consistently into account
  - fit to the bare charges rather than the renormalised sigma terms (✓)
- statistical error estimation via the Γ-method (✓)
  - store information on correlation for a careful error propagation using the `pyerrors` package [arXiv:2209.14371]



Conclusion

on our determination of the quark mass contributions - the so-called sigma terms

Challenges

- signal-to-noise ratio
  - excited state contamination
- physical point extrapolation
  1. quark mass dependence
  2. cut-off effects
  3. finite-volume effects

How we deal with them

- varied treatment of excited states
  - one-state vs. two-state ratio fits
- large set of CLS ensembles
  - **high statistics**
  - large range of a, L, m_π
- varied data sets (various cuts)
- vary extrapolation models

- **first direct determination of all four baryon octet sigma terms** (not only N)
  $\Rightarrow$ **improved precision** (for $B \neq N$) despite very conservative error for now
  $\Rightarrow$ **to do - model averaging procedure to account for systematics**

BACKUP

Quark Mass Dependence, Cut-off & Finite-Volume Effects

Simultaneous Correlated Fit to all Pion and Strange Octet Baryon Sigma Terms

e.g.

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- all dimensionful quantities rescaled e.g. $\mathbb{a} = \frac{a}{\sqrt{8t_0^*}}$, $\mathbb{M}_\pi = \sqrt{8t_0} M_\pi$
- cut-off effects modelled as $\mathrm{O}(a^2)$: $c's$ depend on B but constraints from symmetric point

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  - loop function f (covariant BChPT in the extended on-mass-shell scheme)

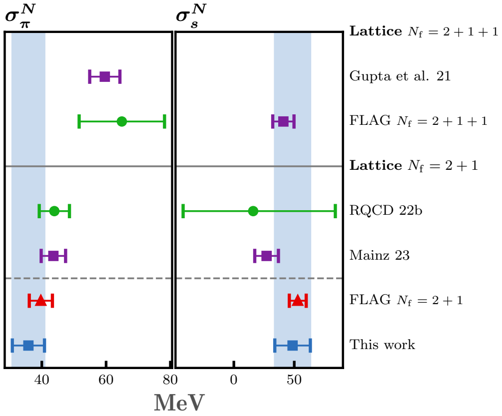
[Bernard et al.: Nucl. Phys. B 388 (1992) 315; Gasser et al.: Nucl. Phys. B 307 (1988) 779]

Preliminary Results Compared to Other Studies

NUCLEON

- Direct vs. Indirect determinations and vs. Combinations (Open symbols - no continuum extrapolation)

*FLAG2021



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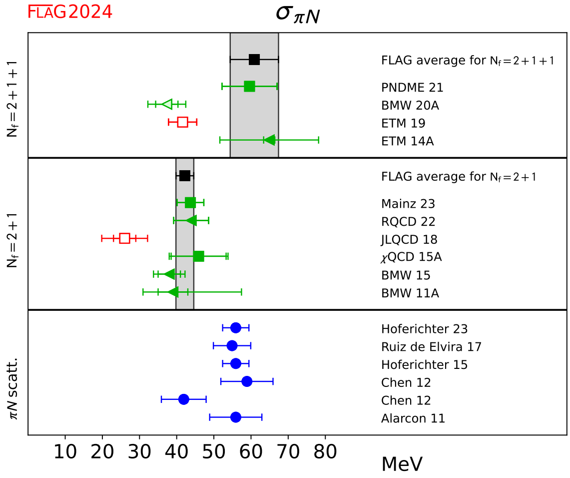

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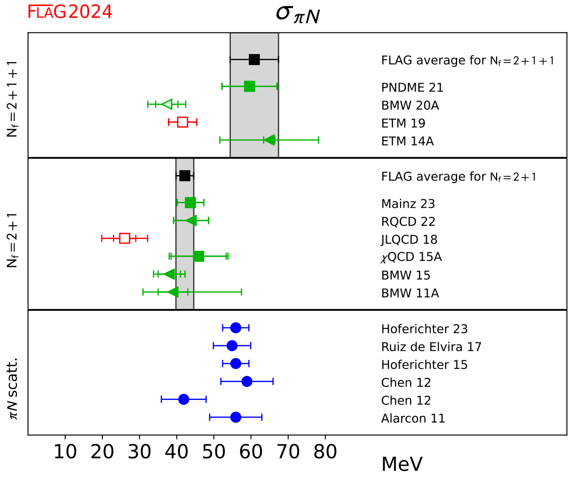

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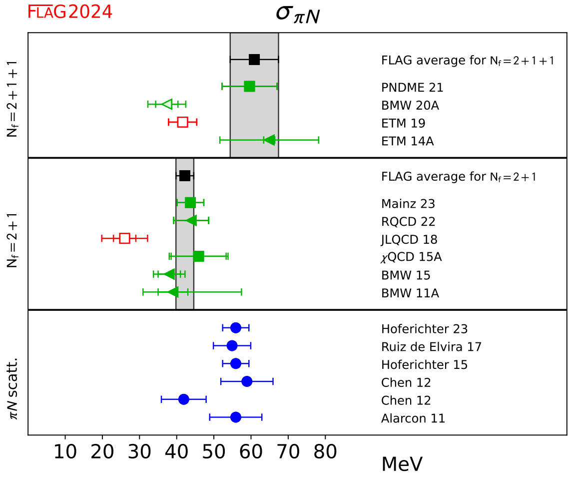

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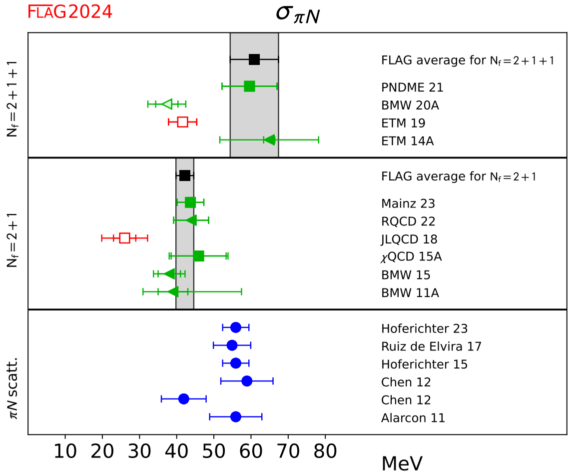

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$\Leftarrow$ $N_f = 2+1$ results mostly $40\,\mathrm{MeV}$ **(This work too)**

$\Leftarrow$ phenomenology mostly $60\,\mathrm{MeV}$

$\Rightarrow$ tension with phenomenology for the nucleon-pion sigma term persists

Systematics

Systematics

1. **Varied treatment of the excited states**

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   1.1 **One-state fits**

Systematics

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2. **Varied data sets**

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2. **Varied data sets**
   2.1 **No cuts: including all 34 ensembles**
   2.2 $M_\pi^{<400\,\mathrm{MeV}}$: **excluding all ensembles with a pion mass** $M_\pi > 400\,\mathrm{MeV}$
   2.3 $a \leq 0.086\,\mathrm{fm}$: **excluding all ensembles at the coarsest lattice spacing**

Systematics

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   1.1 **One-state fits**

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   2.1 **No cuts: including all 34 ensembles**

   2.2 $M_\pi^{<400\,\mathrm{MeV}}$**: excluding all ensembles with a pion mass** $M_\pi > 400\,\mathrm{MeV}$

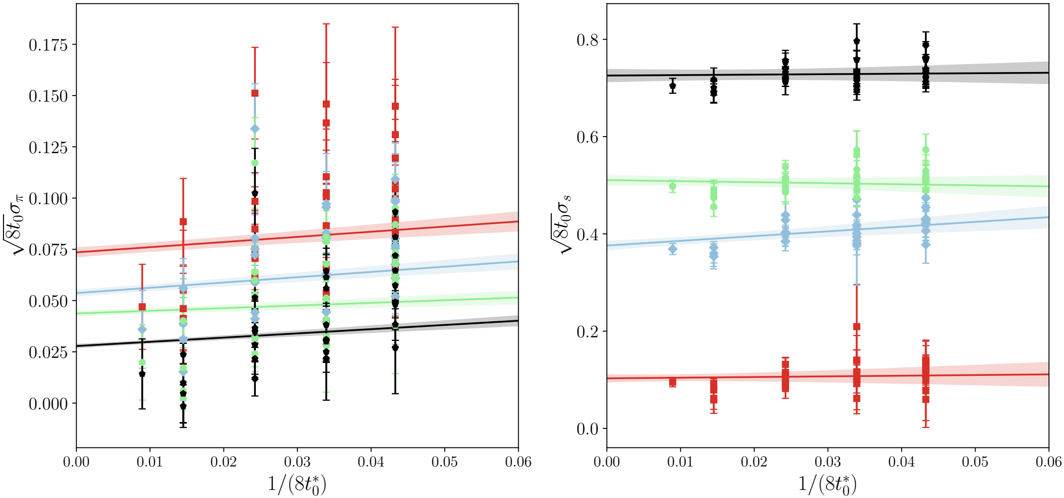
   2.3 $a \leq 0.086\,\mathrm{fm}$**: excluding all ensembles at the coarsest lattice spacing**

   2.4 $LM_\pi^{>4}$**: excluding all ensembles with** $LM_\pi < 4$

Physical Point Extrapolation -- Cut-Off Effects

Data points based on One-state fits

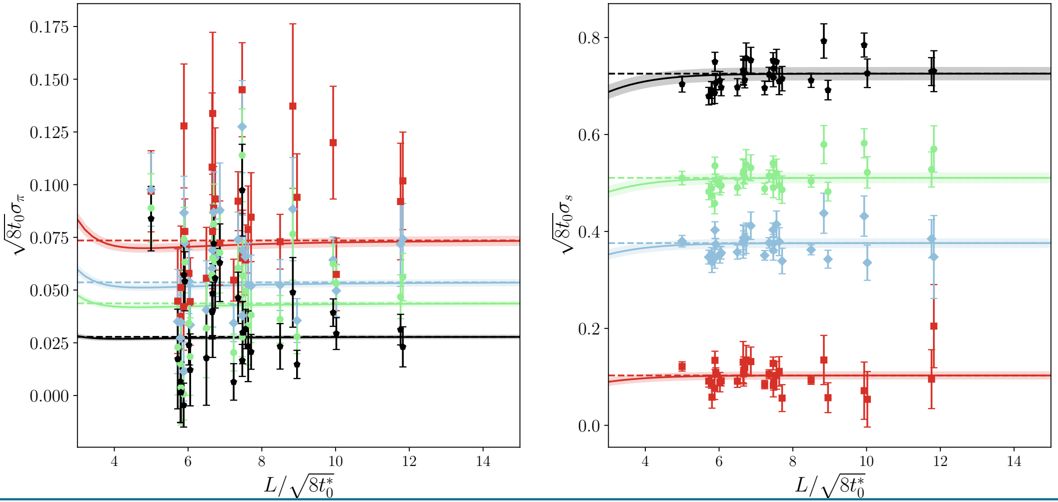
$\chi^2/\chi_{\mathrm{d.o.f.}} = 1.0$



Physical Point Extrapolation -- Finite-Volume Effects

Data points based on One-state fits

$\chi^2/\chi_{\mathrm{d.o.f.}} = 1.0$



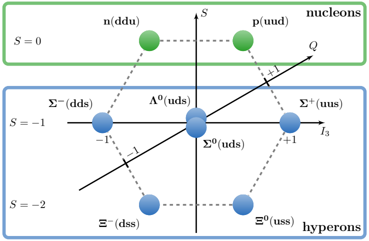
How do we get at the ground states of the octet baryons?

Operators used to **generate the quantum number of the octet baryons** at rest:

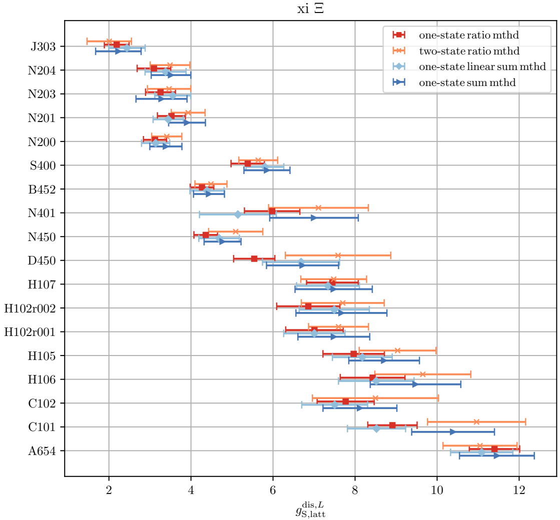
$$\mathcal{O}^{\alpha,\mathrm{N}}_{\mathrm{snk}} = \epsilon^{abc} u^{\alpha}_{a} \left(u^{\beta}_{b} (C\gamma_5)^{\beta\gamma} d^{\gamma}_{c} \right) \quad \text{and} \quad \mathcal{O}^{\alpha,\Lambda}_{\mathrm{snk}} = \epsilon^{abc} s^{\alpha}_{a} \left(u^{\beta}_{b} (C\gamma_5)^{\beta\gamma} d^{\gamma}_{c} \right),$$

$$\mathcal{O}^{\alpha,\Sigma}_{\mathrm{snk}} = \epsilon^{abc} u^{\alpha}_{a} \left(u^{\beta}_{b} (C\gamma_5)^{\beta\gamma} s^{\gamma}_{c} \right) \quad \text{and} \quad \mathcal{O}^{\alpha,\Xi}_{\mathrm{snk}} = \epsilon^{abc} s^{\alpha}_{a} \left(s^{\beta}_{b} (C\gamma_5)^{\beta\gamma} u^{\gamma}_{c} \right).$$

- a, b, c are colour indices
- α, β, γ are spin indices
- $\mathcal{O}^{\alpha}_{\mathrm{src}} = \mathcal{O}^{\alpha}_{\mathrm{snk}}$ and $\bar{\mathcal{O}}_{\mathrm{src}} = \mathcal{O}^{\dagger}_{\mathrm{src}} \gamma_4$
- C stands for the charge conjugation operator



Different fitting methods



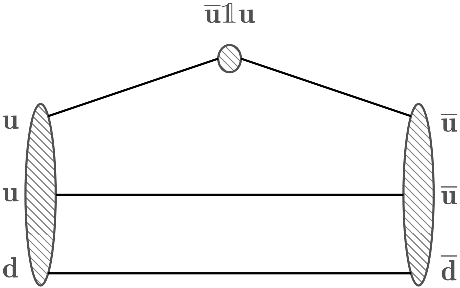
How to access the ground-state matrix element

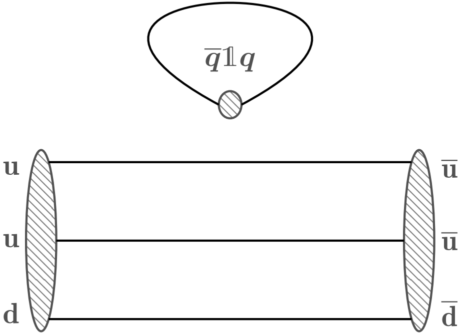
spectral decomposition

summation method

$$\frac{\mathrm{d}}{\mathrm{d}(t_\mathrm{f}/a)} \sum_{t/a} R(t_\mathrm{f}, t) = g^{B}_{q,S} + \mathrm{O}(t_\mathrm{f}/a \cdot \mathrm{e}^{-\Delta E_1 \cdot t_\mathrm{f}})$$

($\Delta E_1 = E_1 - E_0$, energy gap between ground & first excited state)





How to access the ground-state matrix element

spectral decomposition

summation method **ESC**

$$\frac{\mathrm{d}}{\mathrm{d}(t_\mathrm{f}/a)} \sum_{t/a} R(t_\mathrm{f}, t) = g^B_{q,S} + \boxed{\mathrm{O}(t_\mathrm{f}/a \cdot \mathrm{e}^{-\Delta E_1 \cdot t_\mathrm{f}})}$$

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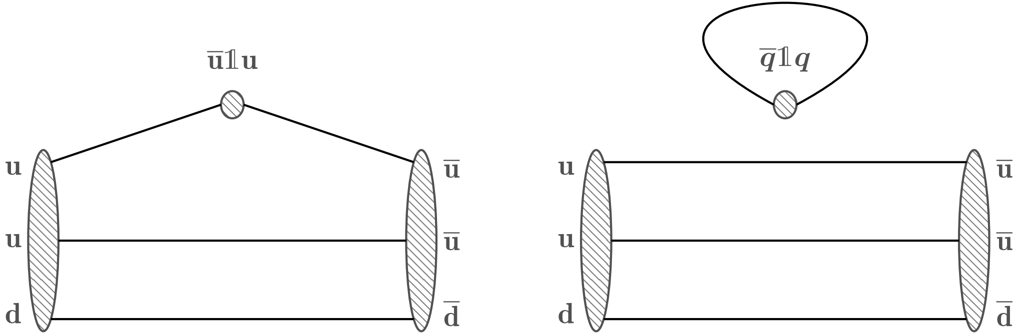


Error estimation: Γ-method

Markov chain Monte Carlo time series a^i_α ● ● ● ● ● ● ● ● ●

Let's look at any function of the primary observables A_α (exact value) e.g. $f(A_\alpha) = \frac{A_1}{A_2}$
The autocorrelation function of the observable f,

$$\boldsymbol{\Gamma_f(t) = \frac{1}{N-t}\sum_{i=1}^{N-t} \delta_f^i \delta_f^{i+t}}$$

based on a Taylor series

$$\delta_f^i = \sum_\alpha \overline{f}_\alpha \delta_\alpha^i + \dots, \quad \overline{f}_\alpha = \left.\frac{\partial f}{\partial A_\alpha}\right|_{\overline{a}_\alpha}, \quad \delta_\alpha^i = \overline{a}_\alpha^i - a_\alpha^i$$

[N. Madras and A.D. Sokal, J. Statist. Phys. 50 (1988) 109.; Wolff: arXiv:hep-lat/0306017]

Error estimation: Γ-method

Markov chain Monte Carlo time series

a^i_α 

Error estimation: Γ-method

Markov chain Monte Carlo time series

a_α^i 

$$\Gamma_f(t) = \frac{1}{N-t}\sum_{i=1}^{N-t} \delta_f^i \delta_f^{i+t} \quad \xRightarrow[\text{standard variance}]{\text{NO autocorrelation}} \quad \Gamma_f(0) = \mathrm{var}(\delta_f^i) = v_f$$

Error estimation: Γ-method

Markov chain Monte Carlo time series a^i_α ● ● ● ● ● ● ● ● ●

$$\Gamma_f(t) = \frac{1}{N-t}\sum_{i=1}^{N-t}\delta_f^i\delta_f^{i+t} \xrightarrow[\text{standard variance}]{\text{NO autocorrelation}} \Gamma_f(0) = \mathrm{var}(\delta_f^i) = v_f$$

Autocorrelation expected to decay exponentially for large distances t, $\frac{\Gamma_f(t)}{\Gamma_f(0)} \sim \exp\left(-\frac{t}{\tau_{\exp,f}}\right)$

Integrated autocorrelation time for infinitely long chain

$$\boldsymbol{\tau_{\mathrm{int},f} = \frac{1}{2} + \sum_{t=1}^{\infty}\frac{\Gamma_f(t)}{\Gamma_f(0)} \approx \int_0^\infty \mathrm{d}t\, \mathrm{e}^{-t/\tau_{\exp,f}} = \tau_{\exp,f}}$$

Error estimation: Γ-method

- the estimator $\Gamma_f(t)$ only reliable for $0 \leq t \ll N$
- so sum not only naturally finite but has to be cut deliberately at some earlier point W in time

The standard error of the observable f

$$\sigma_f^2 = 2\tau_{\text{int},f}\frac{v_f}{N} = \frac{1}{N}\left(\Gamma_f(0) + 2\sum_{t=1}^{W}\Gamma_f(t)\right)$$

and a **direct estimate of the autocorrelation time**

- **automatic windowing procedure** to determine W:
  - find minimum of $g(W) \sim \frac{\delta_{\text{tot}}\sigma_f}{\sigma_f}$ÿ

$\rightarrow$ often large N and complicated non-linear functions

$\Rightarrow$ Γ-method cheaper than e.g. bootstrap/jackknife

Error estimation: Γ-method - fit parameters

$$\chi^2(p_i, y_a) = \sum_{a=1}^{N_{\rm data}} \left(\frac{f(x_a; p_i) - y_a}{\sigma(y_a)} \right)^2$$

where $x_a, y_a (a = 1..N_{\rm data})$ are the data points to be fitted and $p_i (i = 1..N_{\rm data})$ the fit parameters. **Condition: $\boldsymbol{\chi^2}$ function remains at its global minimum (at $\bar{p}_i$, $\bar{y}_a$) for small fluctuations in the fit parameters and the data.**

- expand $\boldsymbol{\chi^2}$ around its minimum w.r.t to shifts in the data, $\boldsymbol{\delta d_a}$ and in the fit parameters $\boldsymbol{\delta p_i}$ at leading order:

$$\chi^2(p_i, \bar{d}_a + \delta d_a) = \chi^2|_{(p_i, \bar{d}_a)} + \partial_{d_a} \chi^2|_{(p_i, \bar{d}_a)} \delta d_a$$

$$\chi^2(\bar{p}_i + \delta p_i, \bar{d}_a + \delta d_a) = \chi^2|_{(\bar{p}_i, \bar{d}_a)} + \partial_{d_a} \chi^2|_{(\bar{p}_i, \bar{d}_a)} \delta d_a + \chi^2|_{(\bar{p}_i, \bar{d}_a)} + \partial_{p_i} \chi^2|_{(\bar{p}_i, \bar{d}_a)} \delta p_i$$

Error estimation: Γ-method - fit parameters

So the χ^2 function stays at its minimum shifts in the data points must be compensated for by the fit parameters:

$$\partial_{p_j}\chi^2(\bar{p}_i+\delta p_i,\bar{d}_a+\delta d_a)=0$$
$$\leftrightarrow \partial_{p_j}(\partial_{d_a}\chi^2|_{(\bar{p}_i,\bar{d}_a)}\delta d_a+\chi^2|_{(\bar{p}_i,\bar{d}_a)}+\partial_{p_i}\chi^2|_{(\bar{p}_i,\bar{d}_a)}\delta p_i)=0$$

$$\rightarrow \frac{\delta p_i}{\delta \bar{d}_a}=-\frac{\partial_{p_j}\partial d_a\chi^2}{\partial_{p_j}\partial_{p_i}\chi^2}\Big|_{(\bar{p}_i,\bar{d}_a)}=\sum_{j=1}^{N_{\text{data}}}(H)^{-1}_{ij}\partial_{p_j}\partial_{d_a}\chi^2|_{(\bar{p}_i,\bar{d}_a)}$$

$\Rightarrow$ extract δp_i by multiplying by $\delta\bar{d}_a$ (formerly called δ_f!)

Error estimation: Γ-method

Γ-method
application of non-linear functions **once**
$+$ derivatives of the result w.r.t
every input observable **once**

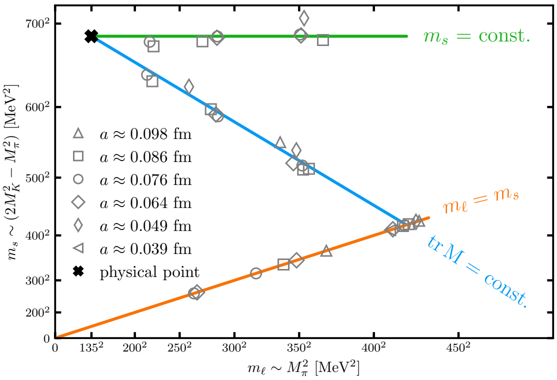
bootstrap/jackknife
evaluation of function on each sample
(e.g. **500 times**)

ADVANTAGES

- often large N and complicated non-linear functions $\Rightarrow$ **Γ-method cheaper**
- **direct estimate of the autocorrelation time**

Numerical Setup

Constraining the physical point via **three trajectories** in the quark mass plane



- **High statistics**: 34 ensembles
  - $\sim$ 1500 configurations (from 400 up to 5000)
- **src-snk separation $t_\mathrm{f} \approx 0.7\,\mathrm{fm} - 1.2\,\mathrm{fm}$**
- Statistical error estimation in the analysis via the **Γ-method**
  - direct estimate of **autocorrelation** + benefits in computing time

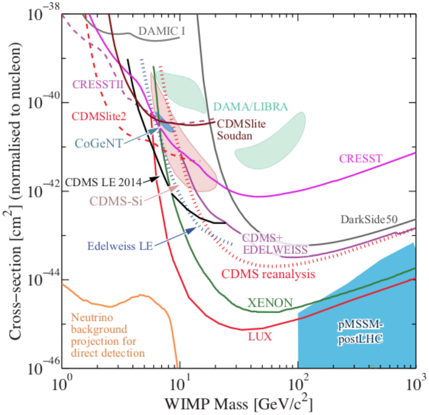
    [Wolff: arXiv:hep-lat/0306017]

    [Ramos: arXiv:1809.01289]
  - pyerrors python package
    - **combined fits**

[Joswig et al.: arXiv:2209.14371]

Comparing model predictions to the exclusion bounds

obtained from direct dark matter experiments



[The 2015 PDG review]

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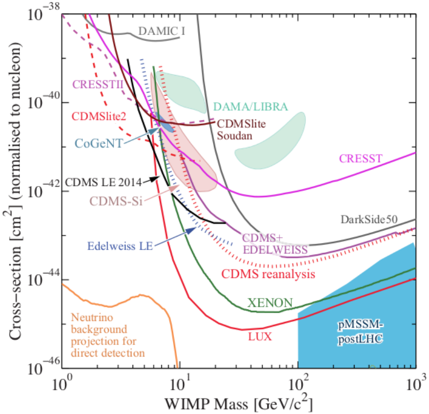
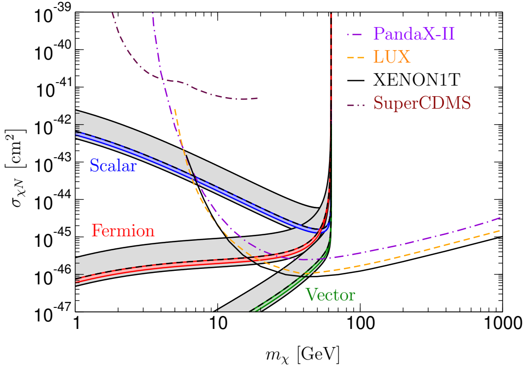


[The 2015 PDG review]

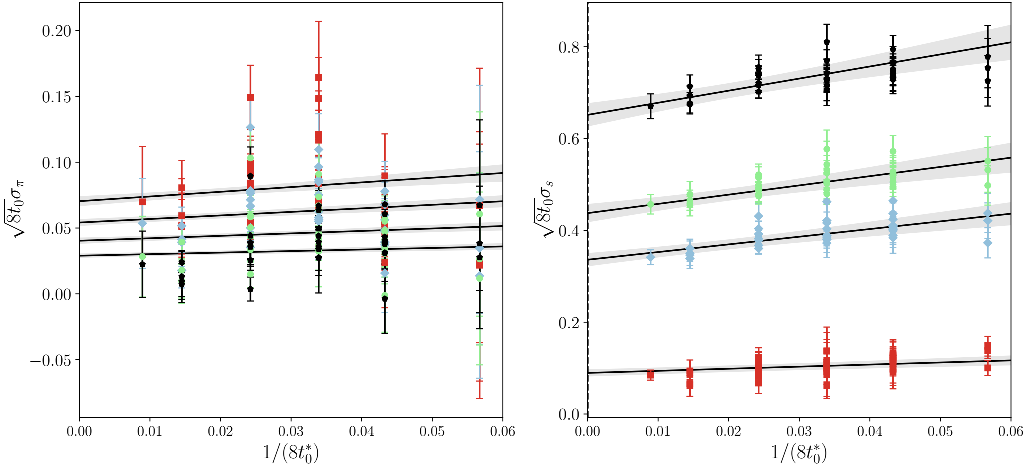


[Hoferichter, Klos, Menéndez, Schwenk: arXiv:1708.02245]

Physical Point Extrapolation -- Cut-Off Effects

Data points based on One-state fits

$\chi^2/\chi^2_{\mathrm{exp}} = 1.3$



Physical Point Extrapolation -- Finite-Volume Effects

Data points based on One-state fits

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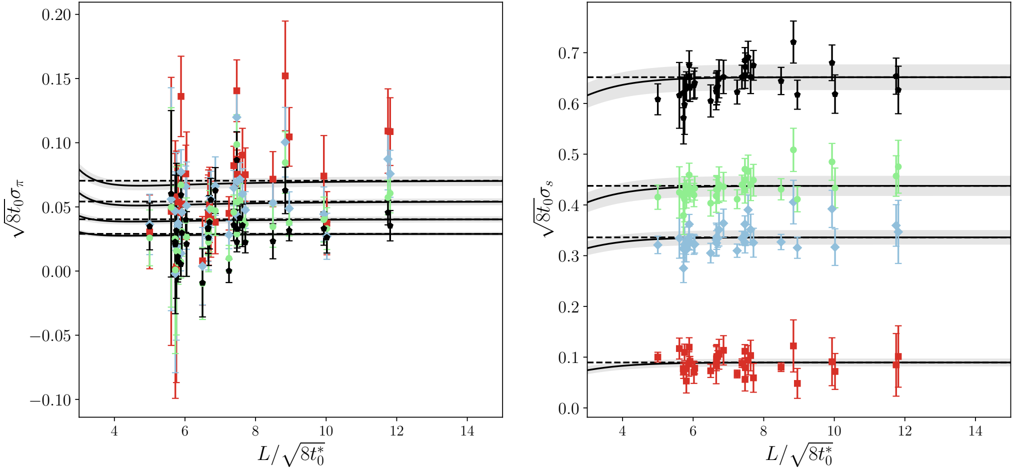


Why determine the sigma terms?

Decomposition of the hadron mass

$$
\begin{aligned}
M_B &= \langle T_{44}\rangle_B \\
&= \underbrace{\sum_q \boldsymbol{\sigma_{qB}}}_{\langle H_\mathrm{m}\rangle_B} + \underbrace{\frac{1}{2}\langle B^2 - E^2\rangle_B + \sum_q \langle \bar{q} D\cdot\gamma q\rangle_B}_{\langle H_\mathrm{kin}\rangle_B = 3\langle H_\mathrm{a}\rangle_B} \\
&\quad - \underbrace{\frac{1}{4}\left\{\gamma_\mathrm{m}(\alpha_s)\sum_q \boldsymbol{\sigma_{qB}} + \frac{\beta(\alpha_s)}{4\alpha_s}\langle E^2 + B^2\rangle_B\right\}}_{\langle H_\mathrm{a}\rangle_B = \frac{1}{4}(\boldsymbol{M_B} - \langle H_\mathrm{m}\rangle_B)}
\end{aligned}
$$

1. **quark mass contributions**
   = sigma terms
2. **quarks' and gluons' kinetic energies**
3. **the trace anomaly**

$\rightarrow$ Knowledge of the sigma terms $\boldsymbol{\sigma_{qB}}$ and the baryon masses $\boldsymbol{M_B}$ - sufficient to determine all three components of the mass decomposition!

Why determine the sigma terms?

Decomposition of the hadron mass

- have non-perturbative determination of the sigma terms for the light and strange quarks
- not for the charm, bottom and top quarks
  $\Rightarrow$ **heavy quark limit**

at leading order in $1/m_h$ and α_s:

$$\sigma_h = \frac{2}{27}\left(m_H - \sum_{q=u,d,s} \sigma_{qH}\right)$$

$\rightarrow$ all the sigma terms available to determine full decomposition

- investigated most so far: **nucleon sigma terms $\sigma_{q\mathrm{N}}$**
- appear in the expressions for **WIMP-nucleon scattering cross-sections** $\Rightarrow$ relevant for comparing model predictions to the exclusion bounds obtained from **direct dark matter experiments** (e.g. XENON1T)
- **discrepancies** between results for the **nucleon pion sigma term from LQCD and phenomenology** still to be resolved $40\mathrm{MeV}$ vs. $60\mathrm{MeV}$

e.g. scalar singlet dark matter model

$\mathcal{L} = \frac{1}{2}\mu_s^2 S^2 + \frac{1}{2}\lambda_{hs} S^2 |H|^2 + \frac{1}{4}\lambda_s S^4 + \frac{1}{2}\partial_\mu S \partial^\mu S$

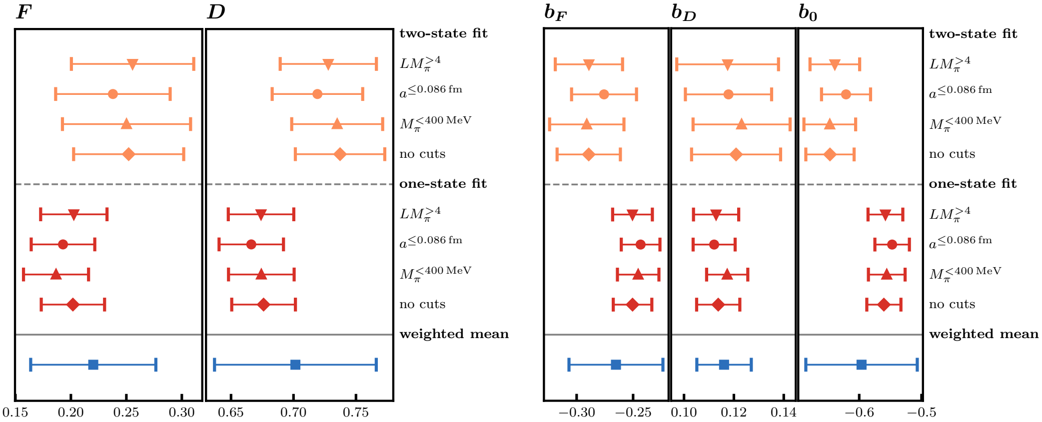
spin-independent cross-section

$$\boldsymbol{\sigma_{\mathrm{SI}} \sim \frac{\lambda_{hs}^2 M_{\mathrm{N}}^4}{4\pi m_{\mathrm{H}}^4 (M_{\mathrm{N}} + m_s)^2} \cdot f_{\mathrm{N}}^2}$$

- effective Higgs-N coupling f_{N}
- quark fractions f_{T_q}

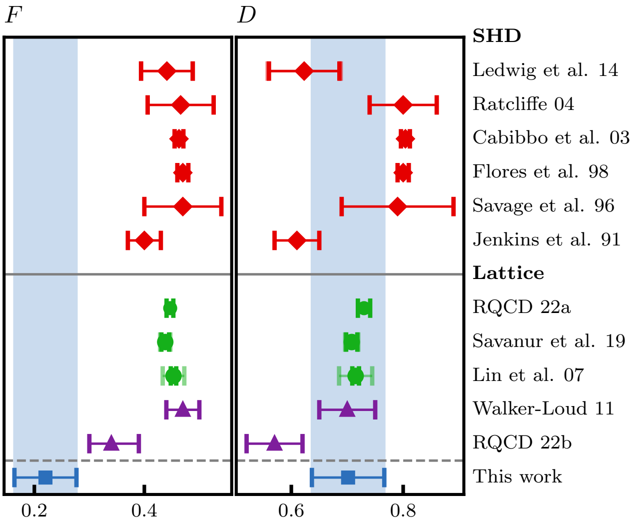
$$\boldsymbol{f_{\mathrm{N}} = \sum_q f_{\mathrm{T}_q}, \quad f_{\mathrm{T}_q} = \frac{\sigma_q}{M_{\mathrm{N}}}}$$

Results - Low Energy Constants



SU(3) LECs F & D

- measurements of **s**emileptonic **h**yperon **d**ecays (selected results)
- **Lattice**
  - calculations of the hyperon axial charges (green points) - albeit for physical quark masses, rather than in the chiral limit
  - baryon masses (purple points)
  - **This work**: Preliminary results from extrapolation of the sigma terms based on two-state fits



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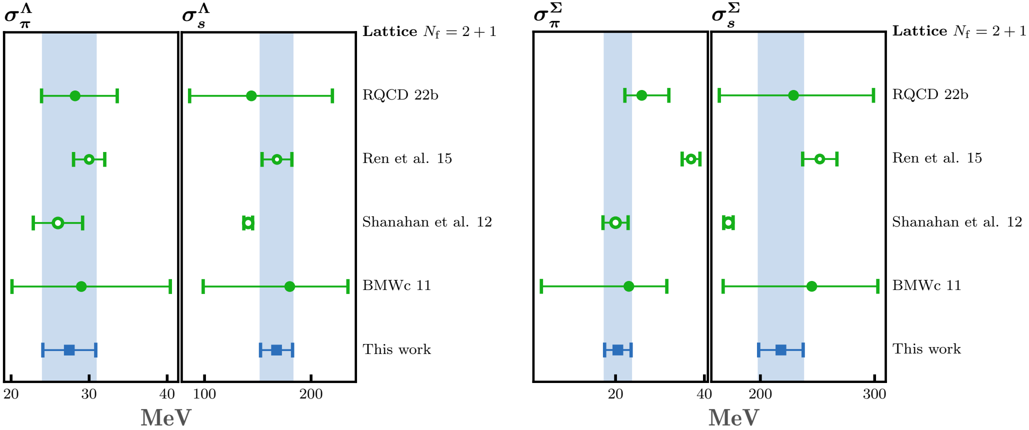
Preliminary Results Compared to Other Studies

- Direct vs. Indirect determinations and vs. Combinations
  (Open symbols - no continuum extrapolation)



The spectrum - the nucleon

$$C_{\mathrm{3pt}}(t_{\mathrm{f}}, t) = C_{\mathrm{3pt}}^{\mathrm{con}}(t_{\mathrm{f}}, t) + C_{\mathrm{3pt}}^{\mathrm{dis}}(t_{\mathrm{f}}, t) = \sum_{n,n'} Z_{n'} Z_n^* \langle \mathrm{n}' | \mathrm{J} | \mathrm{n} \rangle e^{-E_n t} e^{-E_{n'}(t_{\mathrm{f}}-t)}$$

with the **overlap factors** $Z_n^* = \langle n | \bar{\mathcal{O}}_{\mathrm{src}} | \Omega \rangle$ (vacuum state Ω)
$\Rightarrow$ All energy levels E_n belong to states $|n\rangle$ that couple to $\bar{\mathcal{O}}_{\mathrm{src}}$.

What does the spectrum look like for $C_{\mathrm{3pt}}^{\mathrm{con}}(t_{\mathrm{f}}, t)$ and $C_{\mathrm{3pt}}^{\mathrm{dis}}(t_{\mathrm{f}}, t)$?

- IDEA: Construct $C_{\mathrm{3pt}}(t_{\mathrm{f}}, t)$ that only has con. contributions/dis. contributions:
  $\rightarrow$ introduce three different currents $J^{u\pm d} = \bar{u}\Gamma u \pm \bar{d}\Gamma d$ and $J^{v} = \bar{v}\Gamma v$
  where v is a partially quenched, auxiliary quark flavour, mass degenerate w.r.t the u quark
  - **connected** $\bar{u}\Gamma u$ contribution from $\langle N | \frac{1}{2}\left[J^{u+d} + J^{u-d} - 2J^{v}\right] | N \rangle$
  - **connected** $\bar{d}\Gamma d$ contribution from $\langle N | \frac{1}{2}\left[J^{u+d} - J^{u-d} - 2J^{v}\right] | N \rangle$
  - **disconnected** $\bar{u}\Gamma u$ or $\bar{d}\Gamma d$ contribution from $\langle N | J^{v} | N \rangle$

$\Rightarrow$ Connected and disconnected contributions can be made up of linear combinations of matrix elements of these currents.

Challenges

- **signal-to-noise ratio** - large enough src-snk separations for ground state dominance ✗
  - correlation functions: smearing to increase overlap with ground state

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Approach:

Varied treatment of excited states to **estimate systematics**

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+ ...

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- **Physical Point Extrapolation**

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Approach:

+ ...

Varied treatment of excited states to **estimate systematics**

- **Physical Point Extrapolation**

Approach:

High statistics | **Lots of spacings, volumes, masses** | **Lots of models**