The H-dibaryon at $m_\pi \simeq 200$ MeV from lattice QCD

Lattice 2026 @ University of Maryland

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UNIVERSITÄT BERN

Swiss National Science Foundation

LATTICE 2026

Collaborators

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Baryon Scattering Collaboration

H-dibaryon background

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- **Flavor-singlet doubly-strange dibaryon:** $J^P = 0^+\ ,\ I = 0\ ,\ S = -2.$

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- **Relevant in understanding:**
  - **Neutron stars strangeness content.** [Bombaci 2017, JPS Conf. Proc. 17, 101002] [Leong, Thomas & Guichon 2025, arXiv:2503.21171v1]
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- **Experimentally:**
  - $B_H < 7.9\,\mathrm{MeV}$ **from "Nagara" events.** [BELLE 2013, PRL 110.222002]
  - **a resonance in** $\Lambda\Lambda$**, or a bound/virtual state in** $N\Xi$ **?** [J-PARC E42, 2022-2026]

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- **The $\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ states form the singlet via:**

$$\begin{pmatrix} \langle \Lambda\Lambda | \\ \langle \Sigma\Sigma | \\ \langle N\Xi | \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{27}{40}} & -\sqrt{\frac{8}{40}} & -\sqrt{\frac{5}{40}} \\ -\sqrt{\frac{1}{40}} & -\sqrt{\frac{24}{40}} & \sqrt{\frac{15}{40}} \\ \sqrt{\frac{12}{40}} & \sqrt{\frac{8}{40}} & \sqrt{\frac{20}{40}} \end{pmatrix} \begin{pmatrix} \langle 27 | \\ \langle 8_s | \\ \langle 1 | \end{pmatrix}$$

[Inoue et al. (HAL QCD) 2010, 10.1143/PTP.124.591]
[Padmanath et al. 2021, arXiv:2111.11541v1]
[Laudicina et al. (BaSC) 2026, arXiv:2603.00698v1]

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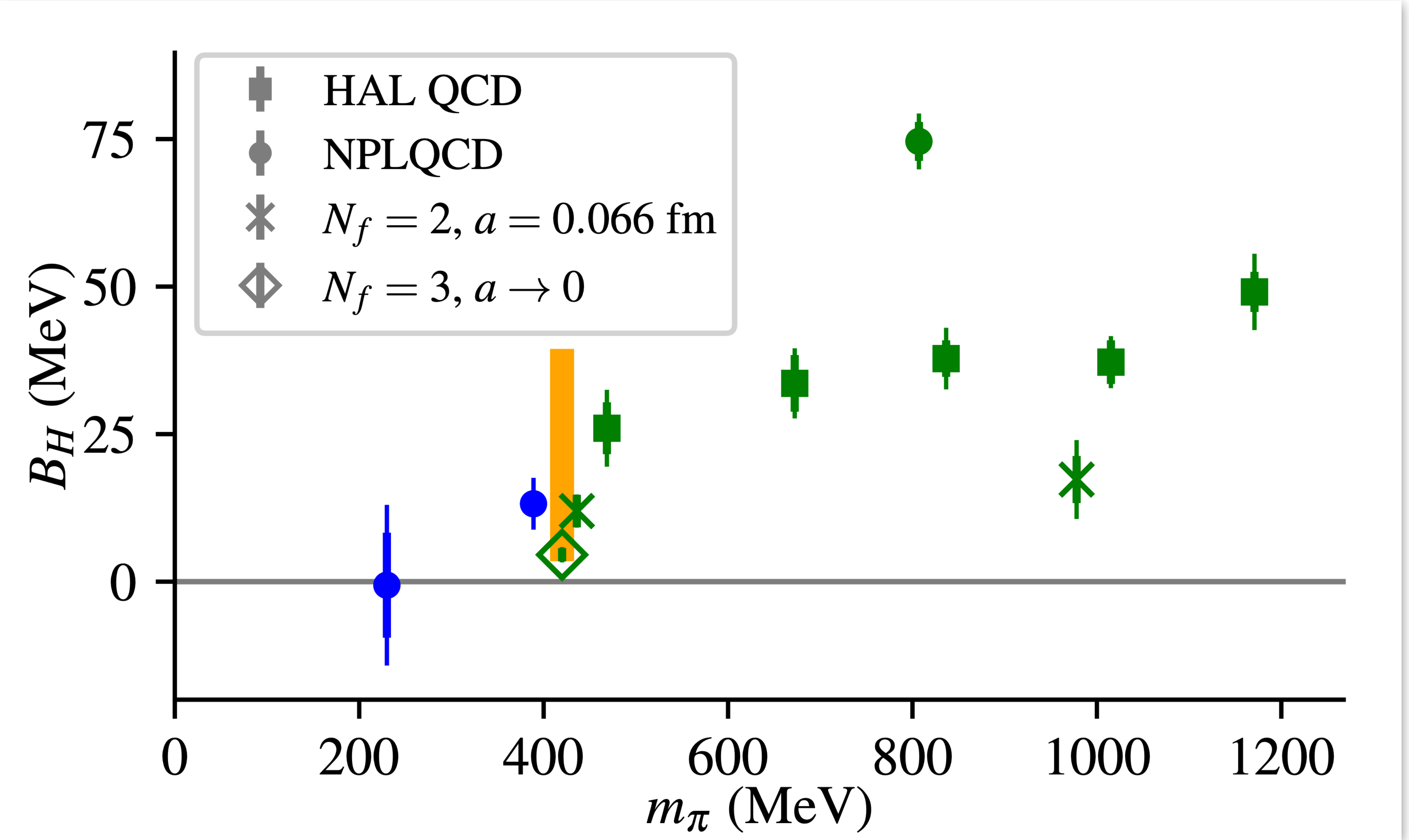
- **H-dibaryon is a shallow bound state in the continuum at the** $m_\pi = m_K \simeq 420\,\mathrm{MeV}$.
- $B_H^{SU(3)_f} = 4.56 \pm 1.13_{\mathrm{stat}} \pm 0.63_{\mathrm{syst}}\,\mathrm{MeV}$.
- **Finite-volume analysis on 8 ensembles then extrapolated to continuum limit.**



[Green et al. 2021, PRL 127.242003]

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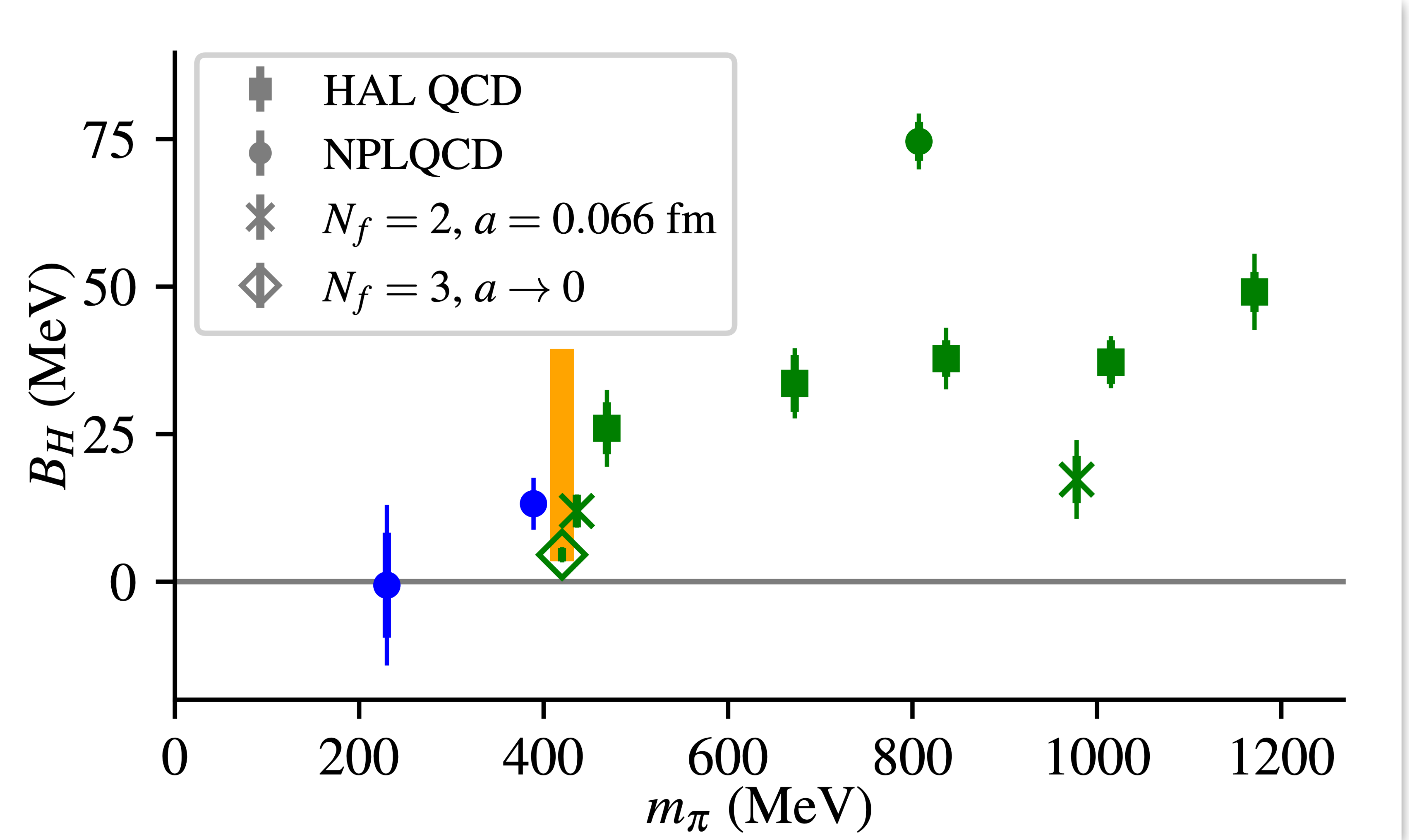

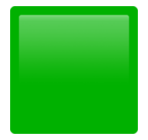 $\Longrightarrow$ $SU(3)_f$ **-symmetric point**

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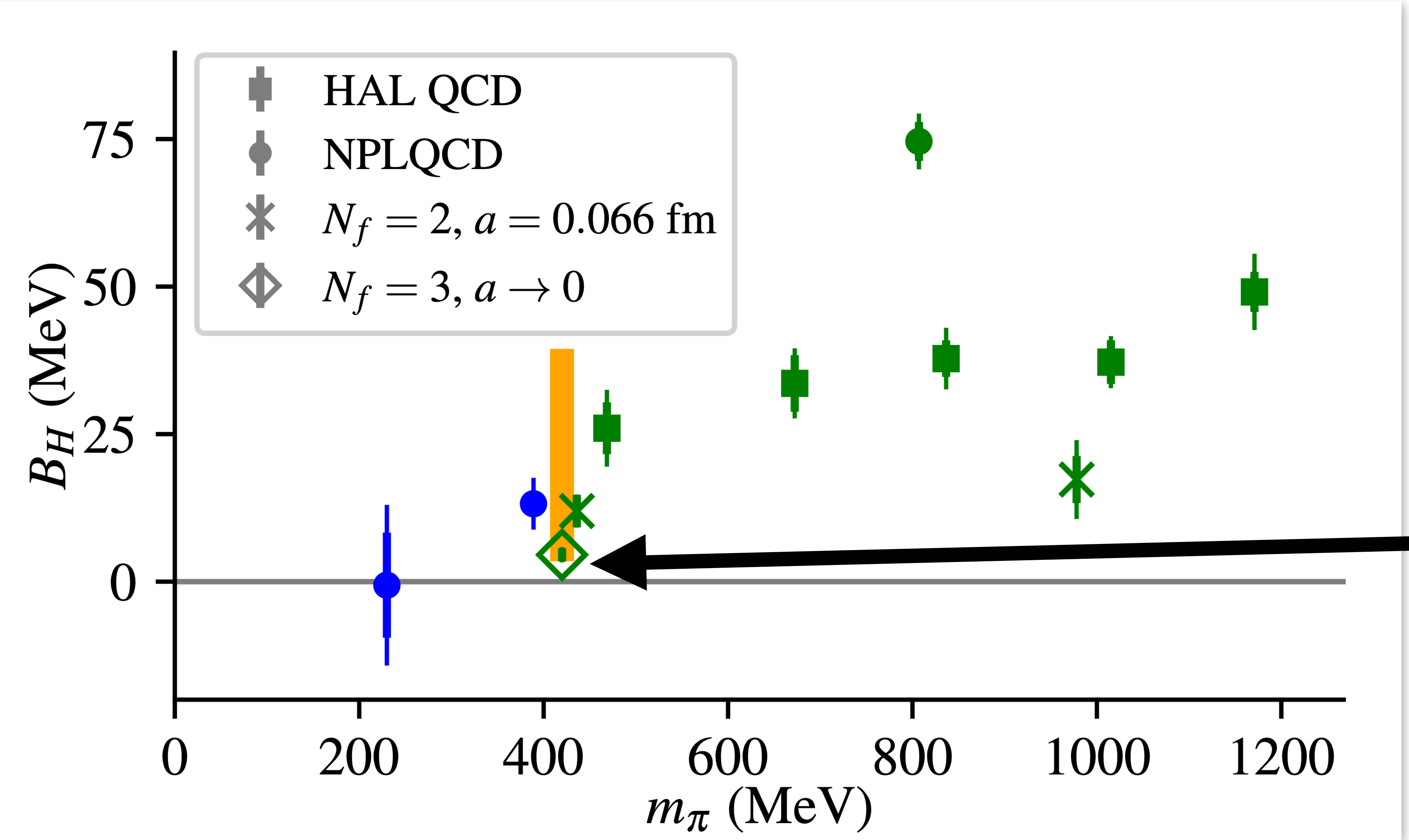

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Continuum limit

[Green et al. 2021, PRL 127.242003]

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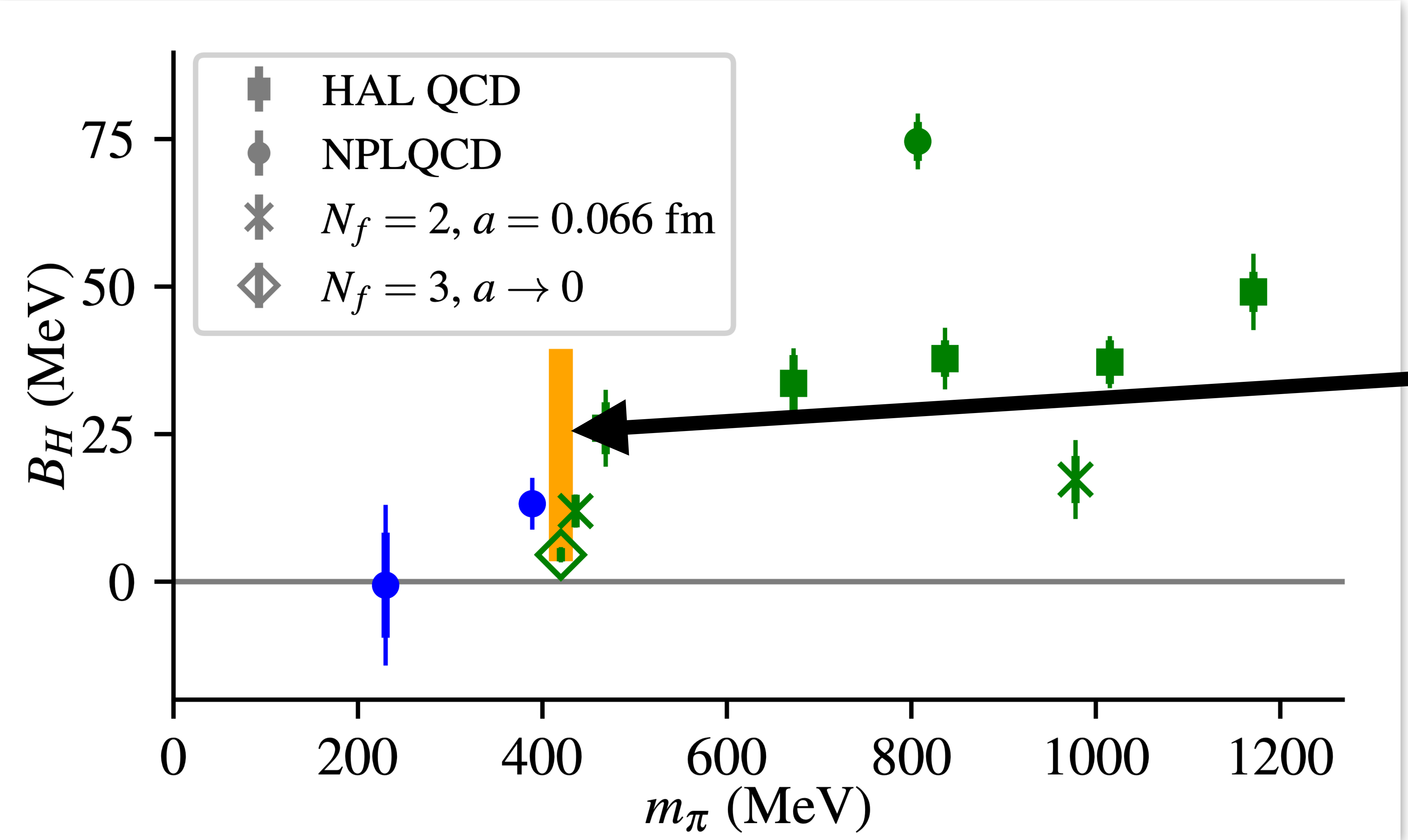

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Values at $a > 0$

[Green et al. 2021, PRL 127.242003]

D200 specs and operator basis

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a [fm]	β	$L^3 \times T$	m_π [MeV]	m_K [MeV]	N_{cfgs}	$N_{\mathrm{stoch-LapH}}$	N_{tsrc}
0.065	3.55	$64^3 \times 128$	200	480	2001	448	1

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e.g.,

$O_i(t) =$

```
1. PSQ=0 A1g L[0]L[0]
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5. PSQ=0 A1g S[0]S[0]
6. PSQ=0 A1g N[2]X[2]
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8. PSQ=0 A1g S[1]S[1]
9. PSQ=0 A1g N[3]X[3]
10.PSQ=0 A1g L[3]L[3]
```

[Dudek et al. (HadSpec) 2010, PRD 82.034508]
[Morningstar et al. (HadSpec) 2011, PRD 83.114505]
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GEVP for extracting N lowest levels

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- **Define correlation matrix for a given operators' basis.**

$$C_{ij}(t) \equiv \langle O_i(t)\, O_j^\dagger(0)\rangle = \sum_{k=0}^{\infty} \langle \Omega \,|\, O_i \,|\, k\rangle\langle k \,|\, O_j^\dagger \,|\, \Omega\rangle\, e^{-E_k t} \equiv \sum_{k=0}^{\infty} Z_i^{(k)} Z_j^{*(k)}\, e^{-E_k t}$$

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where $\lambda_n(t,t_0) \approx \alpha_n^{(0)}\, e^{-E_n t} + \mathrm{O}\left(e^{-(E_N - E_n)\,t}\right)$ **.**

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Energy spectrum in the finite volume !

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Next contribution is exponentially supressed by $(E_N - E_n)\,t$.

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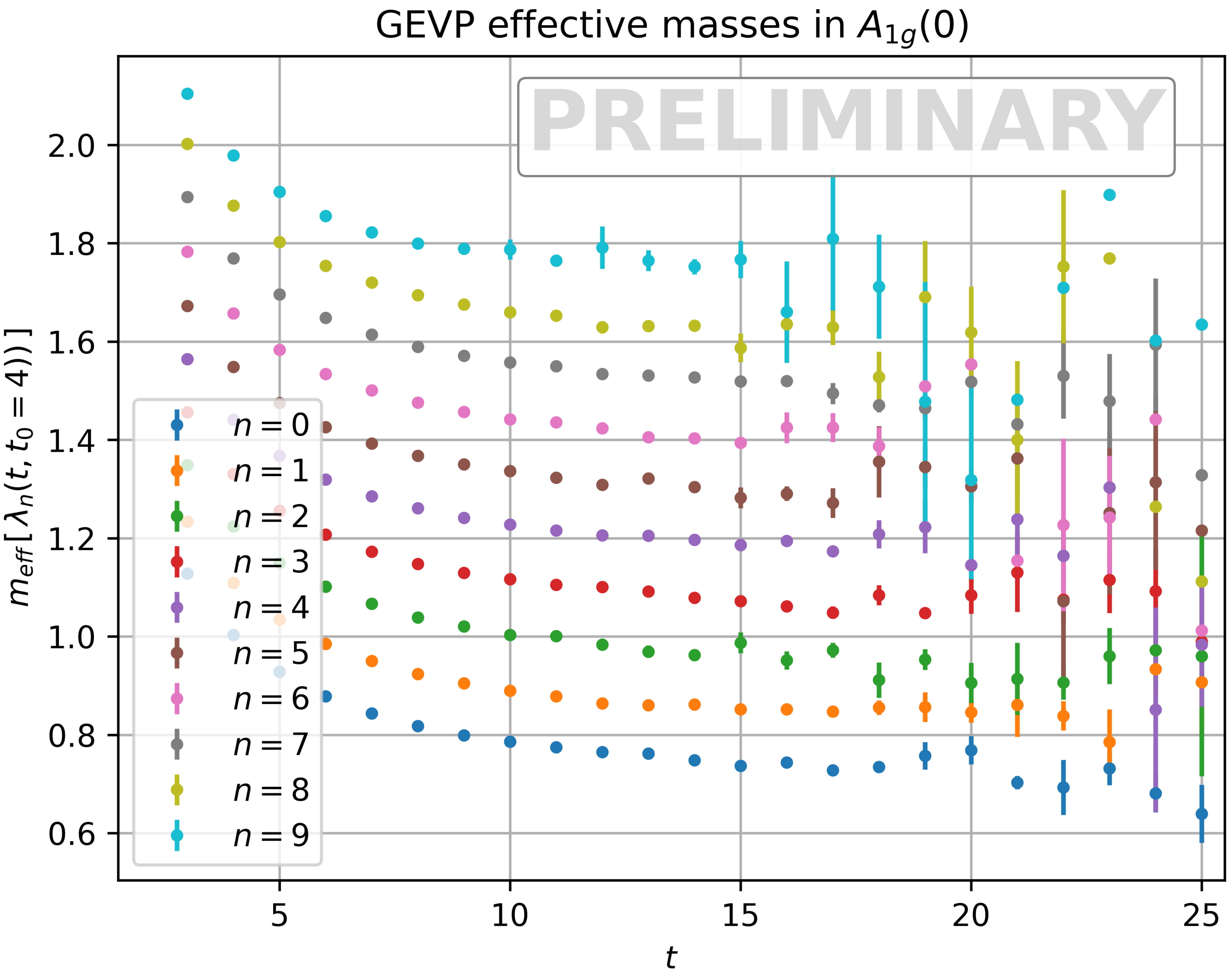

***y-shifted timeseries**

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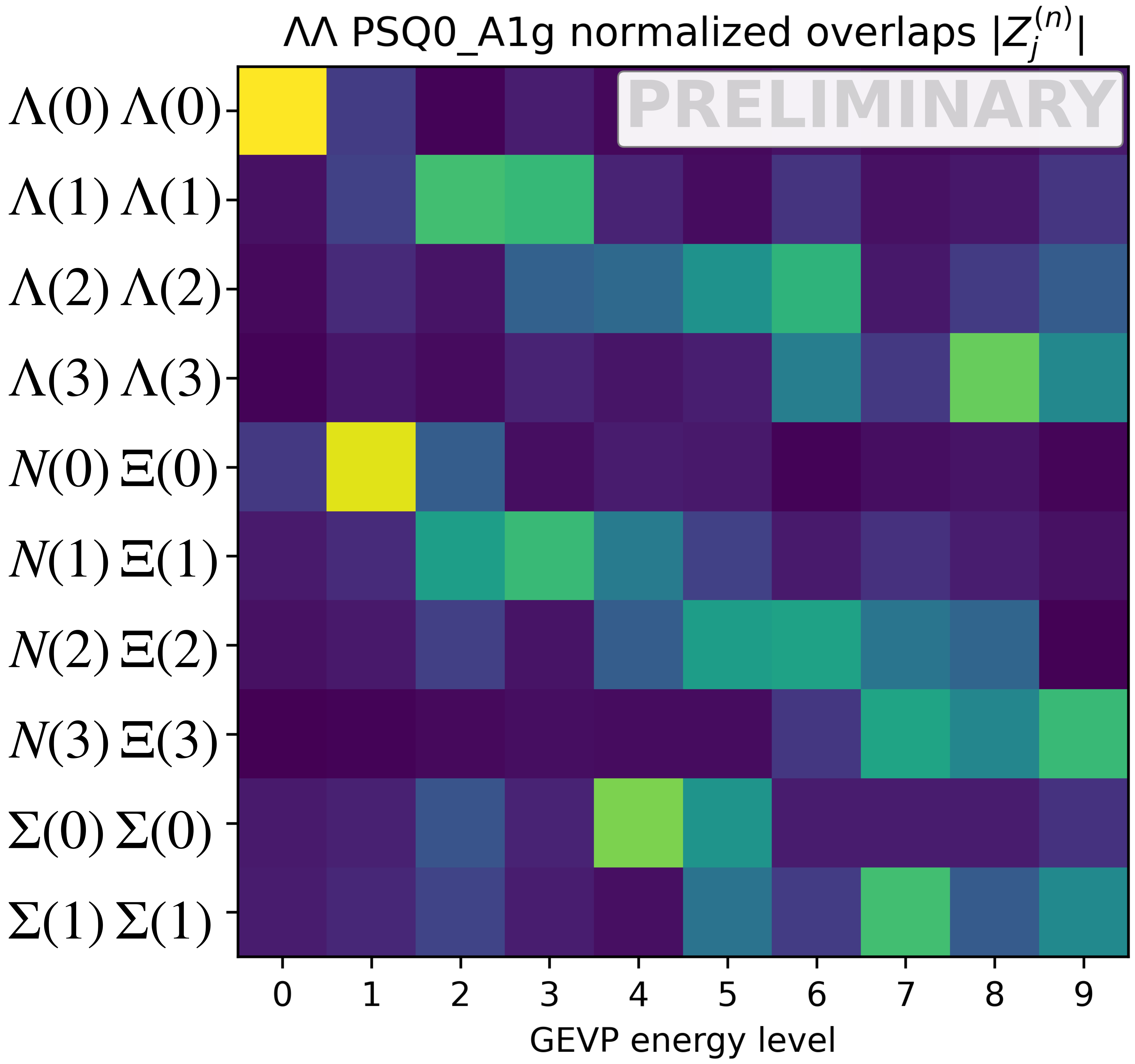

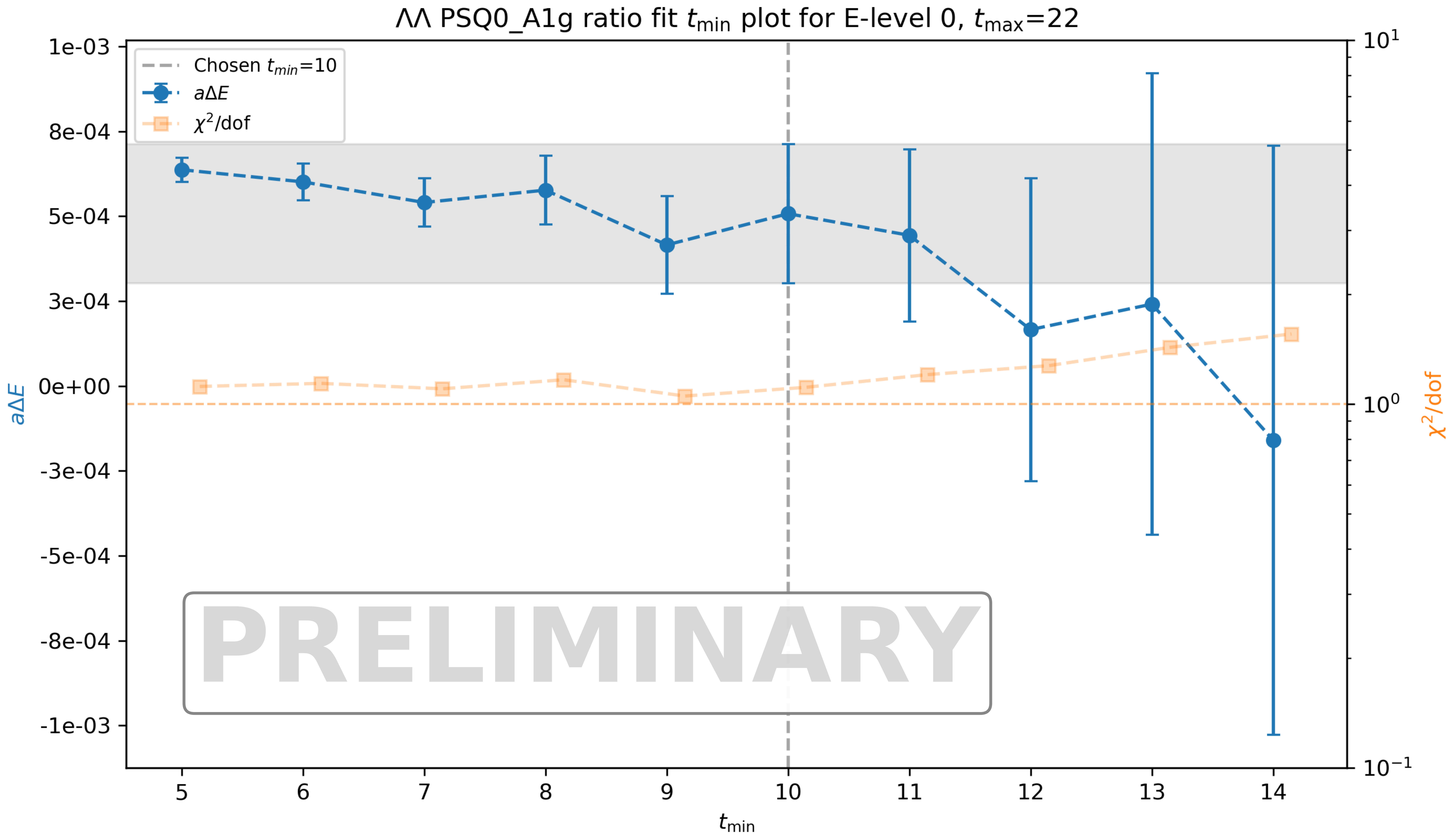
ΛΛ PSQ0_A1g ratio fit t_{min} plot for E-level 0, t_{max}=22
Chosen t_{min}=10
$a\Delta E$
χ^2/dof
1e-03
8e-04
5e-04
3e-04
0e+00
-3e-04
-5e-04
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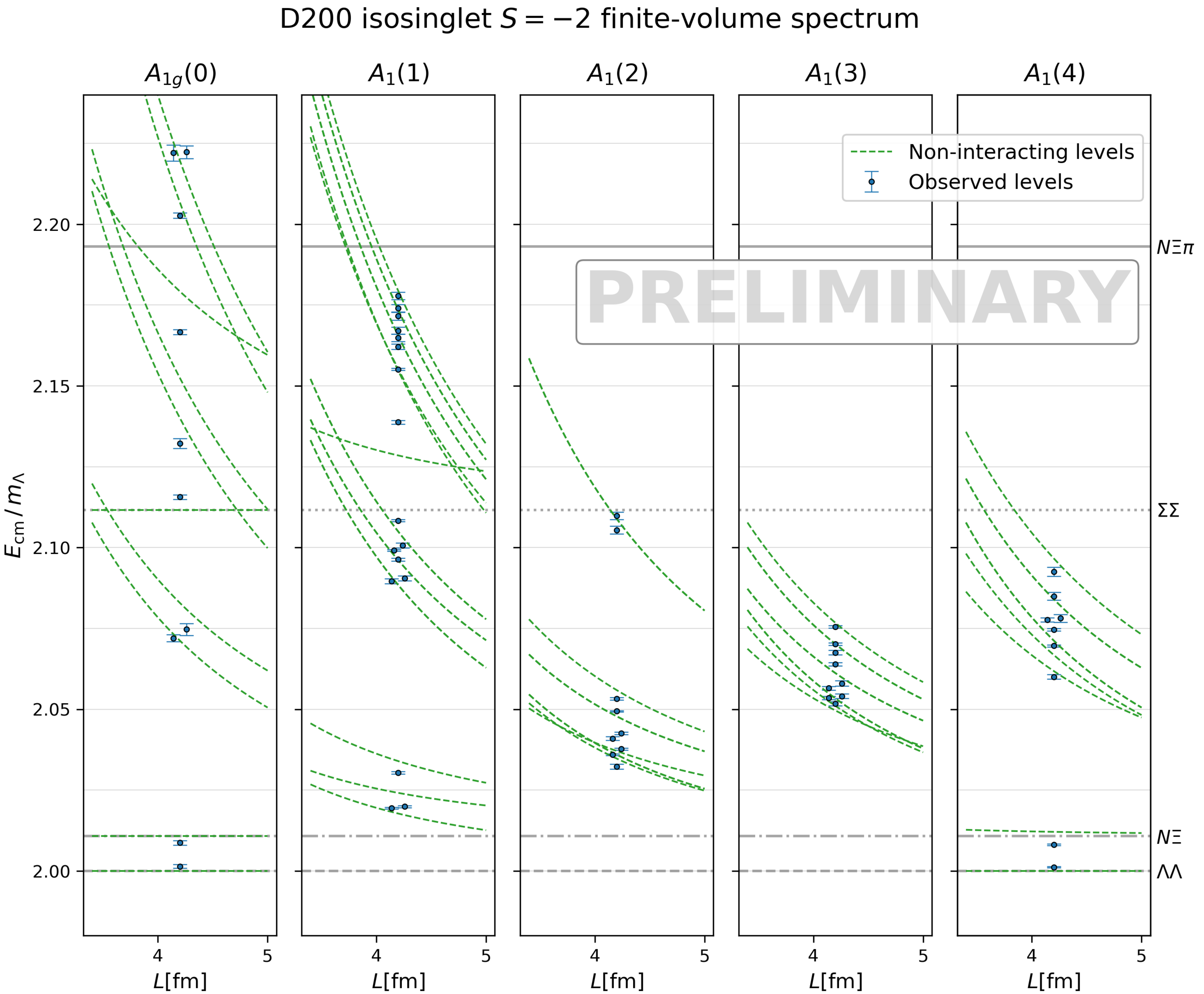
10^1
10^0
10^{-1}
χ^2/dof
5
6
7
8
9
10
11
12
13
14
t_{min}
PRELIMINARY

Finite-volume spectrum extraction

- $I = 0,\ S = -2$
- $m_\pi \simeq 200\,\text{MeV},\ m_K \simeq 480\,\text{MeV}$
- $(L^3, T) = (64^3, 128),\ a \simeq 0.064\,\text{fm}$



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$$\det_{\varphi,l,m,s}\left(K^{-1}(E) \;-\; B(\vec{P},L)\right)\Bigg|_{E=E_n} = 0$$

Scattering properties

Finite-volume properties

[Lüscher 1990, Nucl. Phys. B 354 531–578 + many others]

[Briceño 2014, Phys. Rev. D 89, 074507]

[Morningstar et al. 2017, Nucl. Phys. B 924 477–507]

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- **The determinant runs over scattering channels $\varphi = \{\Lambda\Lambda, N\Xi, \Sigma\Sigma\}$, l, m, s.**
- **We truncate to $l = 0$, total spin $s = 0$.**
- **A parametrization for $K^{-1}(E;\vec{\theta})$ is then proposed and the parameters $\vec{\theta}$ are optimized to match with E_n levels as observed on the lattice.**

Parameterization for $K^{-1}(E)$

$$\frac{K^{-1}(E)}{m_\Lambda} = \begin{pmatrix} 1/a_0 + b_0(E - m_{\Lambda\Lambda})/m_{\Lambda\Lambda} & f_{\Lambda\Lambda-N\Xi}(E) & f_{\Lambda\Lambda-\Sigma\Sigma}(E) \\ f_{\Lambda\Lambda-N\Xi}(E) & 1/a_1 + b_1(E - m_{N\Xi})/m_{N\Xi} & f_{N\Xi-\Sigma\Sigma}(E) \\ f_{\Lambda\Lambda-\Sigma\Sigma}(E) & f_{N\Xi-\Sigma\Sigma}(E) & 1/a_2 + b_2(E - m_{\Sigma\Sigma})/m_{\Sigma\Sigma} \end{pmatrix}$$

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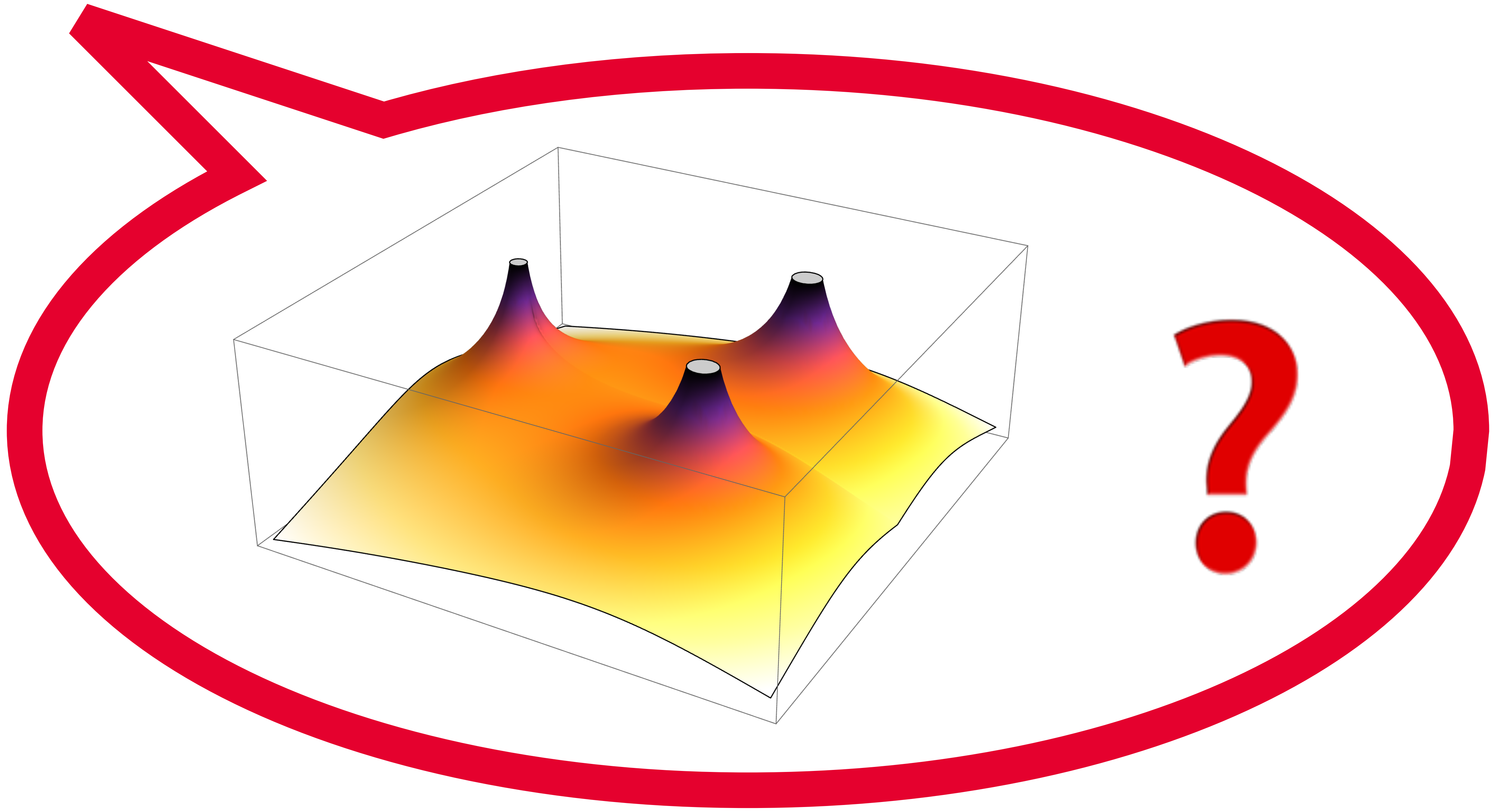
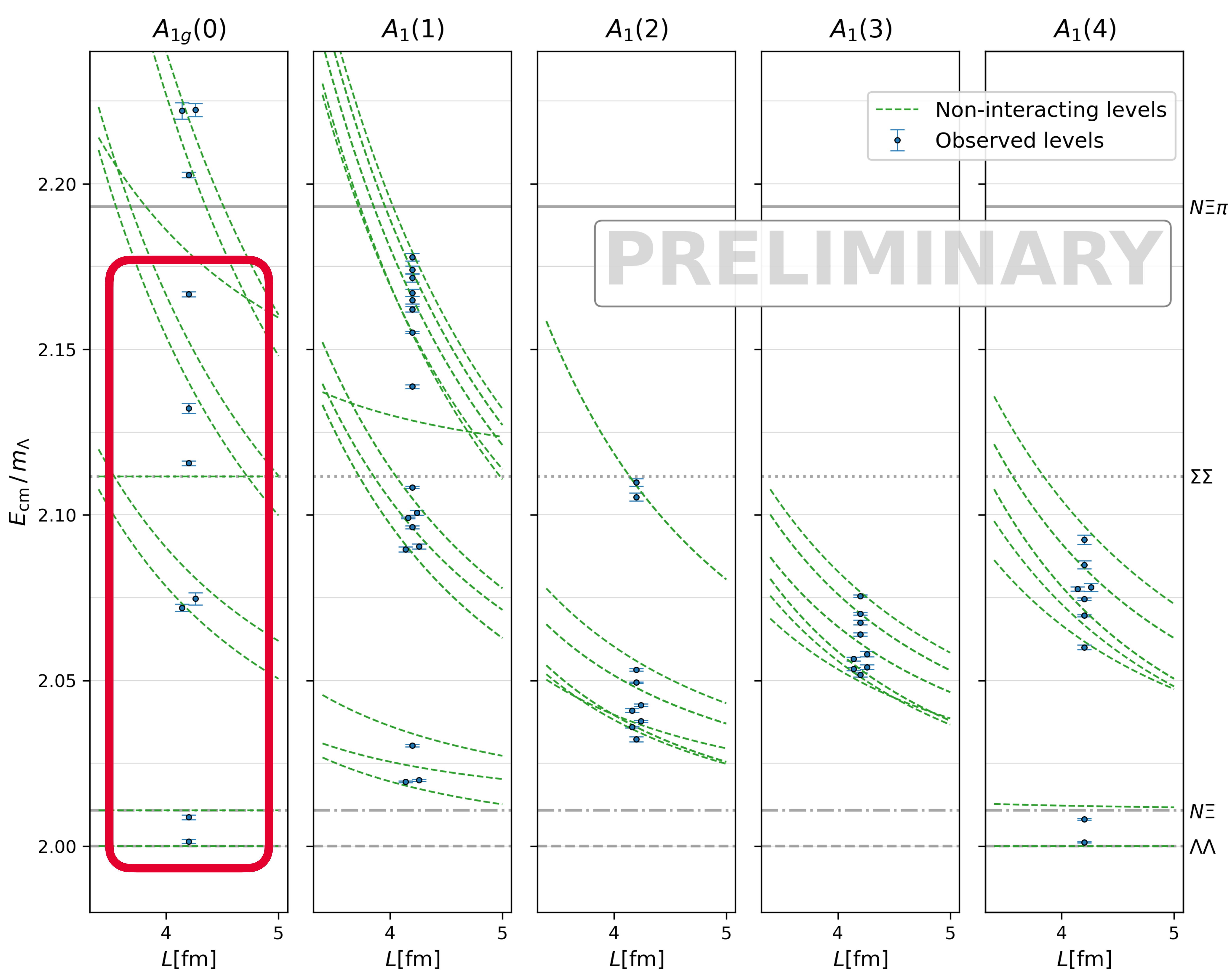
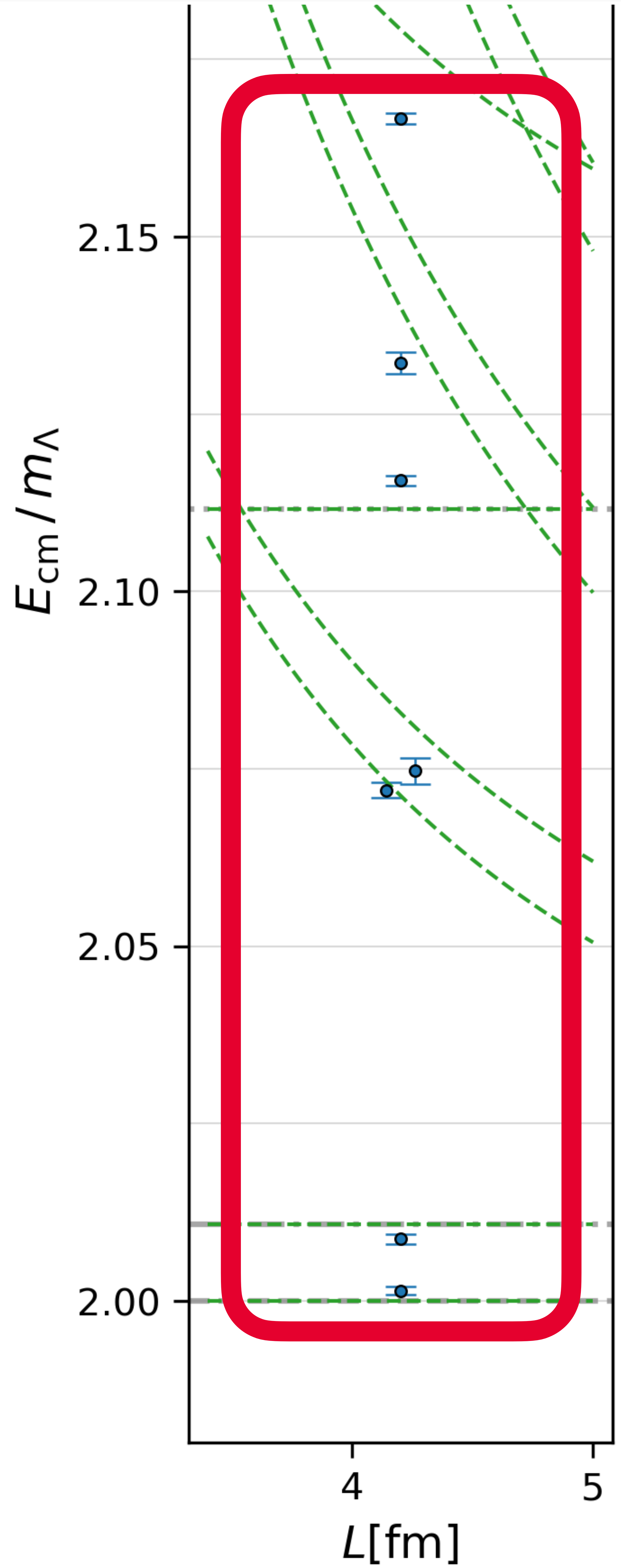


D200 isosinglet $S = -2$ finite-volume spectrum





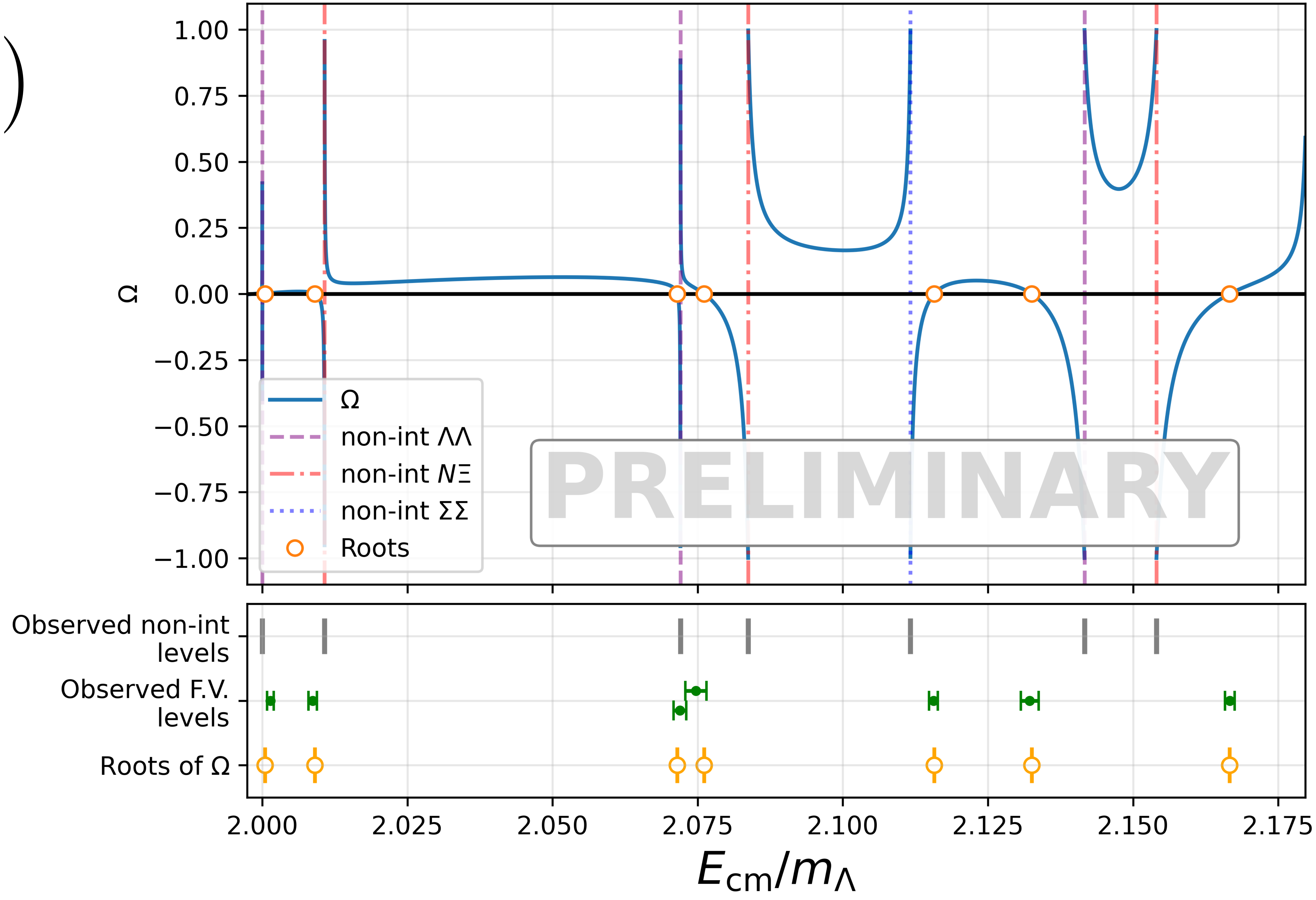
$K^{-1}(E)$ with 3 decoupled channels

$$QC = \det_{l=0}\left(K^{-1}(E) \;-\; B(\vec{0},L)\right)$$

$$\Omega = \frac{QC}{1+|QC|}$$

Parameter	Value
a_0	-0.43 ± 0.19
a_1	2.07 ± 0.38
a_2	-2.93 ± 1.92
b_0	23.2 ± 12.5
b_1	-2.14 ± 0.37

$$\chi^2/\mathrm{dof} = 3.75/2$$



Conclusions and Outlook

- ☑ Finite-volume spectrum: ratio fits to determine energy shifts.
- ⚠️☑ First finite-volume analysis: simple parametrizations.
- ⏳☐ Extraction of pole position via analytic continuation of fitted K^{-1}.
- ☐ Treatment of left-hand cuts.
- ☐ Bayesian / conspiracy fits finite spectrum extraction.

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Thanks !

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Finite-volume spectrum (left-hand cuts)

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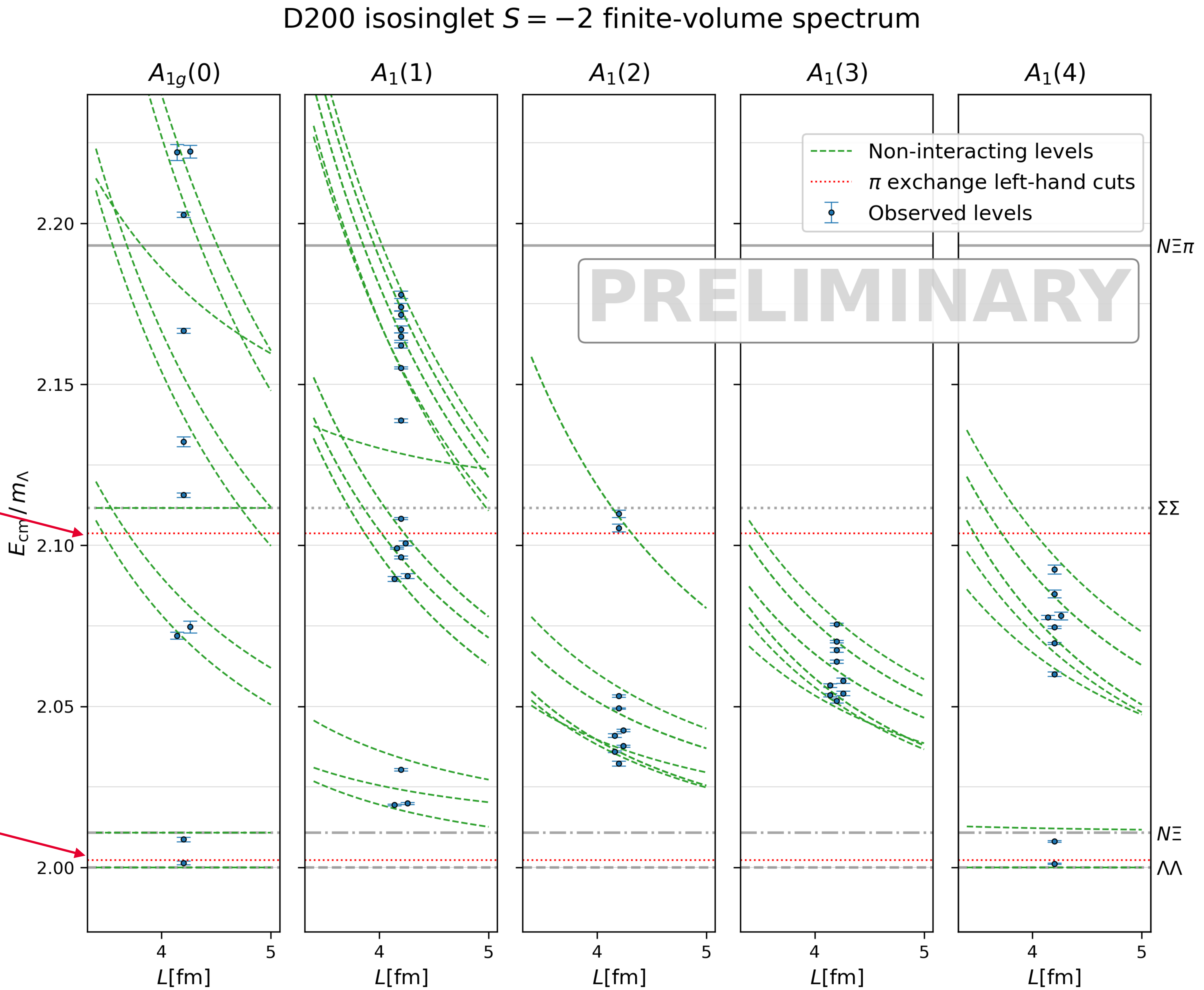
π exchange in ΣΣ

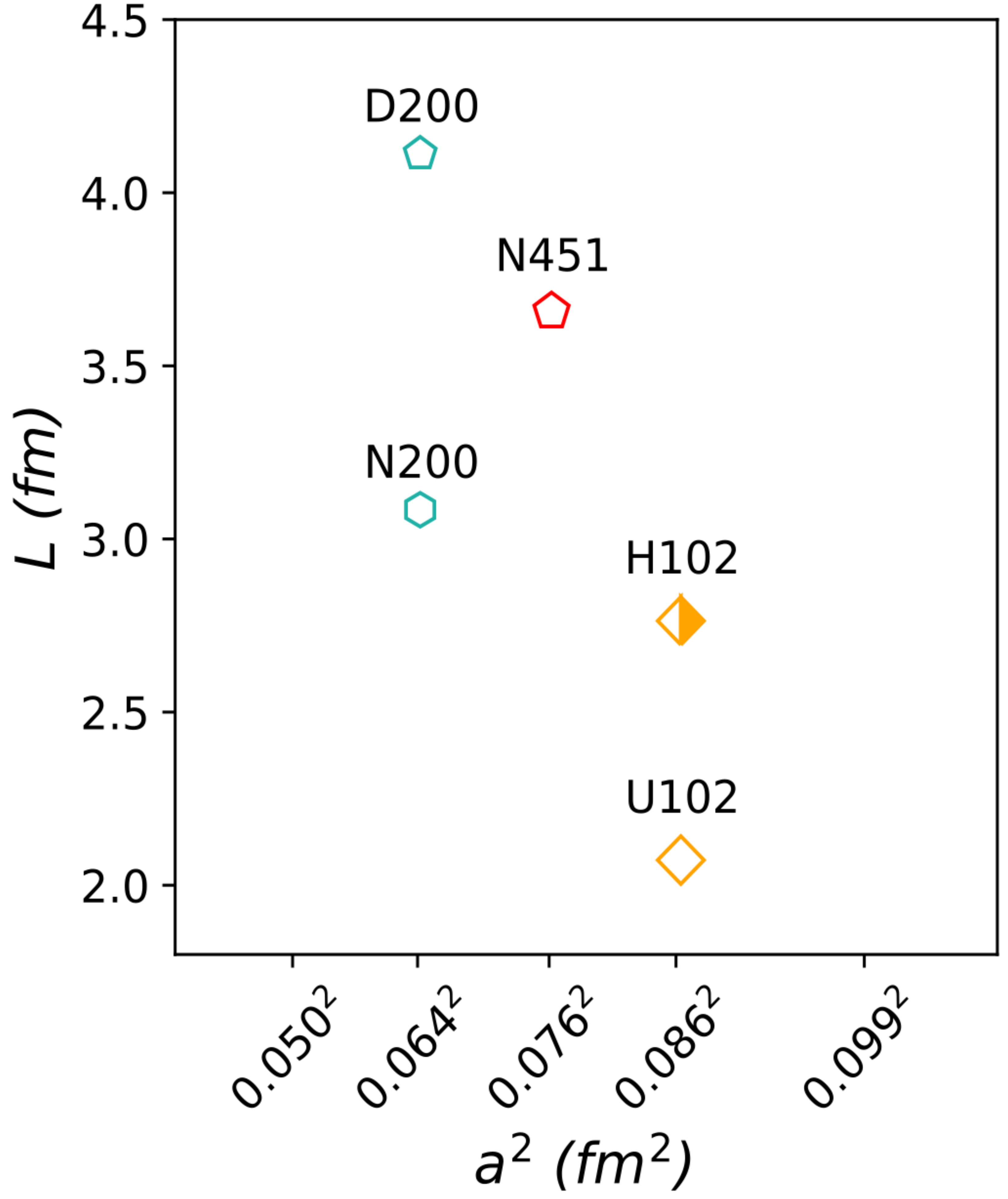
$$s_{\Sigma\Sigma-\pi} = (2m_\Sigma)^2 - m_\pi^2$$

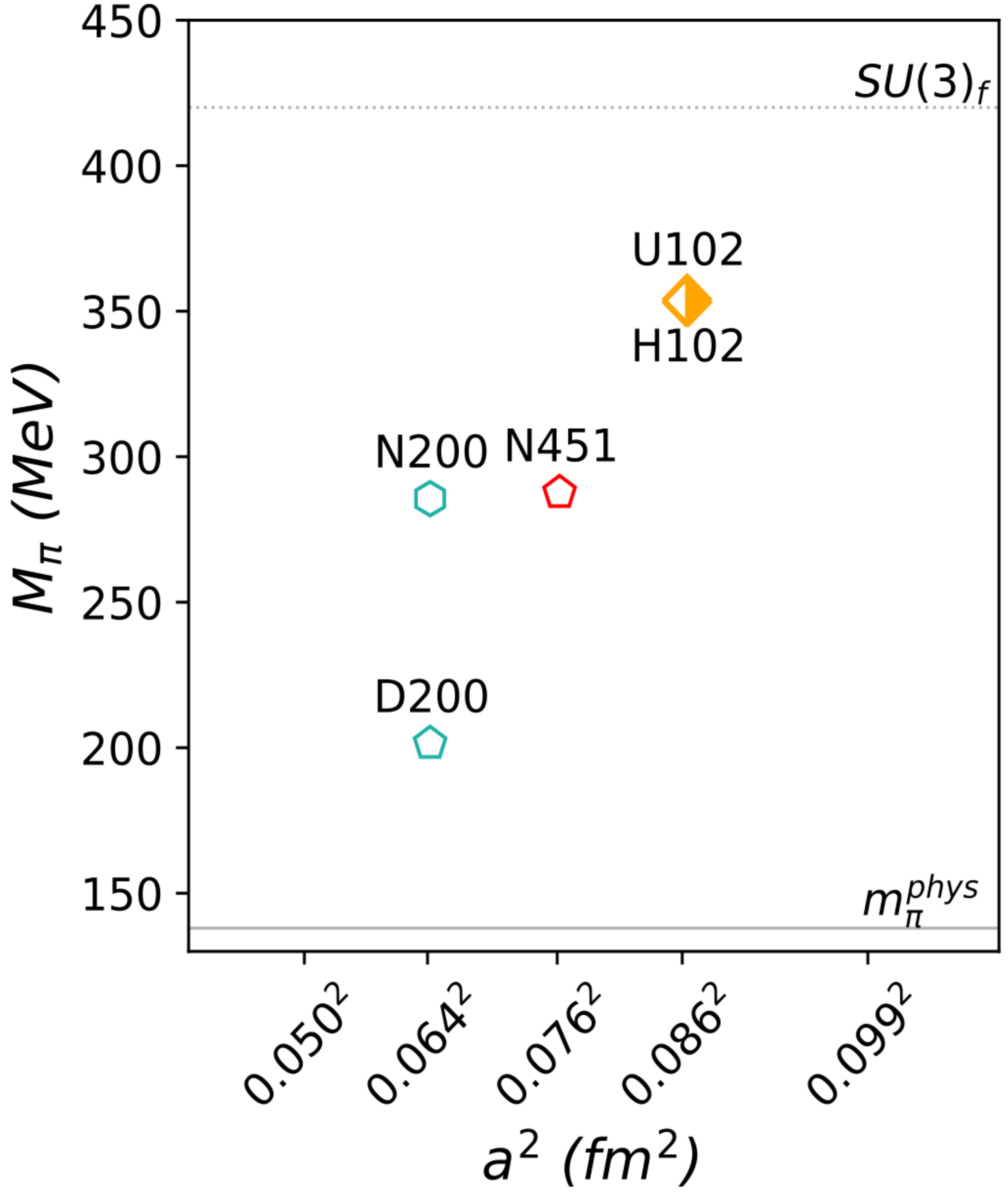
π exchange in NΞ

$$s_{N\Xi-\pi} = (m_N + m_\Xi)^2 - m_\pi^2$$



Going away from $SU(3)_f$ -symmetric point...





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