Inclusive $\bar{B}_s \to X_{\bar{s}c}\ell\bar{\nu}$ decay from lattice QCD

Based on arXiv:2607.01116

Alessandro De Santis

July 31, 2026

Bs. = **Backup Slides**

clickable link

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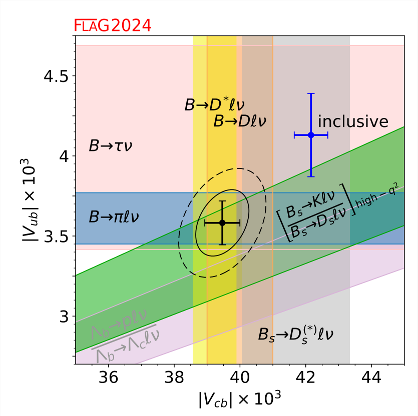
Motivations: $|V_{cb}|$ puzzle



"$|V_{cb}|$ *is the new HVP*" A. Juttner's talk

- **Inclusive so far**: sum of exclusive channels measured experimentally or directly measured using tailored techniques + OPE/perturbative methods
- A first-principles lattice QCD **inclusive calculation** may shed light on the puzzle
- Shown that it is possible by studying $D_s \mapsto X\ell\bar{\nu}$
  - PRL 135 (2025) 12,121901
  - PRD 112 (2025) 5, 054503
  - PRD 112 (2025) 1, 014501
  - R. Kellermann's talk
- In this work we move to $\bar{B}_s \mapsto X_{\bar{s}c}\ell\bar{\nu}$

Work outline

What we have in this work

- Continuum limit with three lattice spacing
- Improved calculation of correlators
- Simulation strategy for lighter-than-physical b-quark masses
- Interpolation at physical point using OPE perturbative information

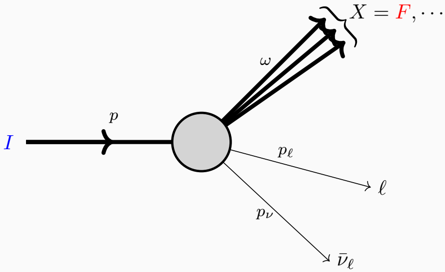
What we do not have yet

- Infinite-volume limit (B48 and B96 planned)
- Fourth lattice spacing (E112 ongoing)
- $\bar{B}_s \mapsto X_{\bar{s}u} \ell \bar{\nu}$ channel

ETMC configurations at physical point

ID	L/a	a [fm]	L [fm]	status
B64	64	0.080	5.09	completed
C80	80	0.068	5.46	completed
D96	96	0.057	5.46	completed
E112	112	0.049	5.48	ongoing
B48	48	0.080	3.82	planned
B96	96	0.080	7.63	planned

Simulation strategy



$$|\boldsymbol{\omega}| \in \left[0, \left(\frac{1 - {r_{IF}}^2}{2}\right)\right]$$

$$\omega_0 \in \left[\sqrt{{r_{IF}}^2 + \boldsymbol{\omega}^2},\ 1 - \sqrt{\boldsymbol{\omega}^2}\right]$$

$$e_\ell \in \left[\frac{1 - \omega_0 - \sqrt{\boldsymbol{\omega}^2}}{2}, \frac{1 - \omega_0 + \sqrt{\boldsymbol{\omega}^2}}{2}\right]$$

$$r_{IF} = \frac{M_F}{M_I} \qquad r_{IF}^{\text{phys}} = \frac{M_{D_s}}{M_{\bar{B}_s}} = 0.367$$

- This determines the triple-differential decay rate

$$\Gamma \propto \int \mathrm{d}\boldsymbol{\omega}^2 \int \mathrm{d}\omega_0 \int \mathrm{d}e_\ell \, \frac{\mathrm{d}\Gamma}{\mathrm{d}\boldsymbol{\omega}^2 \mathrm{d}\omega_0 \mathrm{d}e_\ell}$$

- $am_b > 1$ prevents direct simulation of $M_I = M_{B_s}$

Simulation strategy

- The phase space only depends on r_{IF}

$$\Phi = \int \mathrm{d}\boldsymbol{\omega}^2 \int \mathrm{d}\omega_0 \int \mathrm{d}e_\ell$$

$$= \frac{(1-r_{IF}^2)(1-5r_{IF}^2-2r_{IF}^4)-6r_{IF}^4\log\left(r_{IF}^2\right)}{48}$$

- **Solution**: compute the **dimensionless decay rate**

$$\hat{\Gamma}(M_I) = \frac{\Gamma(M_I)}{\bar{\Gamma}(M_I)} \qquad \bar{\Gamma}(M_I) = \frac{M_I^5 G_F^2 S_{\mathrm{EW}}}{48\pi^4}$$

- for several pairs of initial and final lighter-than-physical meson masses while keeping the **dimensionless phase-space constant**

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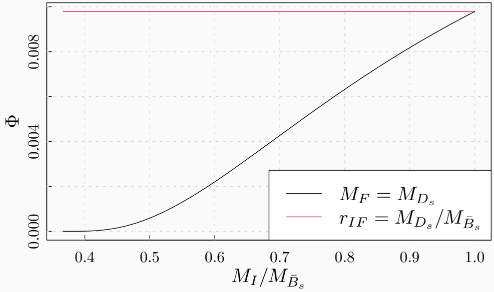
- We follow the red line



- Extrapolate the $\hat{\Gamma}$ at physical $M_I = M_{\bar{B}_s}$
- Restore physical units at the end via $\bar{\Gamma}(M_{B_s})$

Simulation strategy

Recipe:

- At given $m_i^{(n)}$ compute $M_I^{(n)}(m_i^{(n)})$
- Compute $M_F^{(n)}(m_f)$ for several m_f and determine $m_f^\star$ via interpolation (Bs.) such that

$$\frac{M_F^{(n)}(m_f^\star)}{M_I^{(n)}(m_i^{(n)})} = r_{IF}^{\mathrm{phys}}$$

- increase $m_i^{(n+1)} = 1.28\, m_i^{(n)}$ and repeat
- initial condition: $m_i^{(0)}$ such that $M_I^{(0)} = M_{D_s}$

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n	$M_I^{(n)}$ [GeV]	$M_F^{(n)}$ [GeV]	$M_{\bar{B}_s}/M_I^{(n)}$
0	1.97	0.72	2.72
1	2.28	0.84	2.35
2	2.66	0.97	2.01
3	3.12	1.14	1.72
4	3.67	1.35	1.46
5	4.33	1.59	1.24

Initial and Final simulated meson masses

How to extract in practice Γ?

Smeared decay rate

- Master equation

$$\hat{\Gamma} = \int_0^{(|\boldsymbol{\omega}|^{\max})^2} d\boldsymbol{\omega}^2 \sum_{p=0}^{2} \lim_{\sigma \mapsto 0} \left[|\boldsymbol{\omega}|^{3-p} \int_{\omega_0^{\min}}^{\infty} d\omega_0 \, \Theta_\sigma^{(p)}(\omega_0^{\max} - \omega_0) \, Z^{(p)}(\omega_0, \boldsymbol{\omega}^2) \right]$$

- Integral over $|\boldsymbol{\omega}|$ done numerically by simulating 10 momenta
- $\Theta_\sigma^{(p)} = x^p \Theta_\sigma(x)$ with $\Theta_\sigma(x)$ smoothed version of $\Theta(x)$ to impose upper cut in final-energy space

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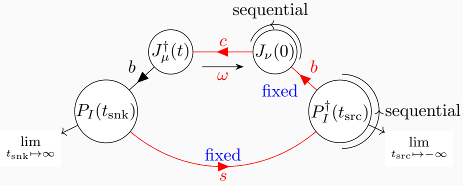
- $\Theta_\sigma^{(p)} = x^p \Theta_\sigma(x)$ with $\Theta_\sigma(x)$ smoothed version of $\Theta(x)$ to impose upper cut in final-energy space
- $Z^{(p)}$ are linear combination of **spectral functions** associated with a **four-point correlator**

$$C_{\mu\nu}(t_{\mathrm{snk}}, t, t_{\mathrm{src}}, \boldsymbol{\omega}^2) = a^9 \sum_{\boldsymbol{x}_{\mathrm{snk}}, \boldsymbol{x}_{\mathrm{src}}, \boldsymbol{x}} e^{iM_I \boldsymbol{\omega} \cdot \boldsymbol{x}} \cdot T \langle 0 | P_I(x_{\mathrm{snk}}) J_\mu^\dagger(x) J_\nu(0) P_I^\dagger(x_{\mathrm{src}}) | 0 \rangle$$

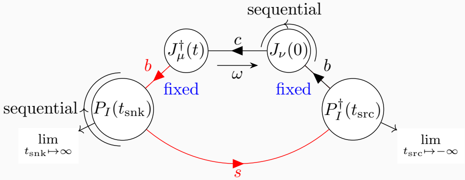
$$T/2 \gg t_{\mathrm{snk}} \gg t > 0 \gg t_{\mathrm{src}} \gg -T/2 \qquad (\text{Bs.})$$

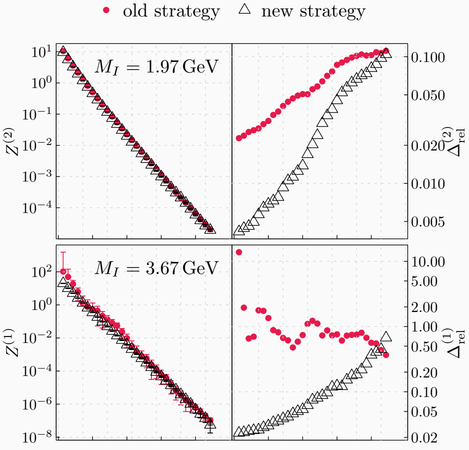
Improved determination of four-point functions

old strategy



new strategy





computationally more expensive but rewarding

Extraction of the smeared decay rate

$$\hat{\Gamma} = \int_0^{(|\boldsymbol{\omega}|^{\max})^2} \mathrm{d}\boldsymbol{\omega}^2 \sum_{p=0}^{2} \lim_{\sigma \mapsto 0} \underbrace{|\boldsymbol{\omega}|^{3-p} \int_{\omega_0^{\min}}^{\infty} \mathrm{d}\omega_0 \, \Theta_\sigma^{(p)}(\omega_0^{\max} - \omega_0) \, Z^{(p)}(\omega_0, \boldsymbol{\omega}^2)}_{\frac{\mathrm{d}\Gamma^{(p)}(\sigma)}{\mathrm{d}\boldsymbol{\omega}^2}}$$

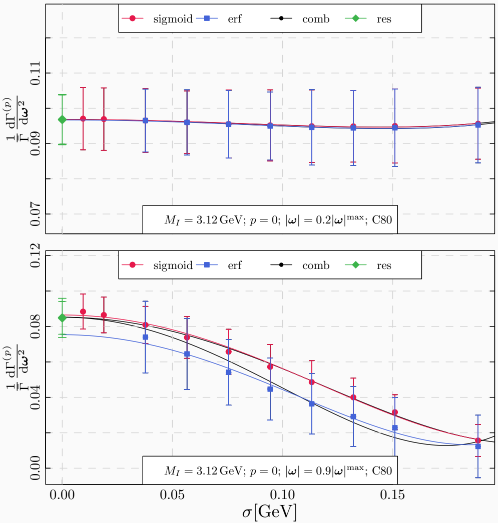
- **Inverse problem**: extraction of smeared spectral function from Euclidean correlator using the Hansen-Lupo-Tantalo method
- Bs. and F. Margari's talk before me

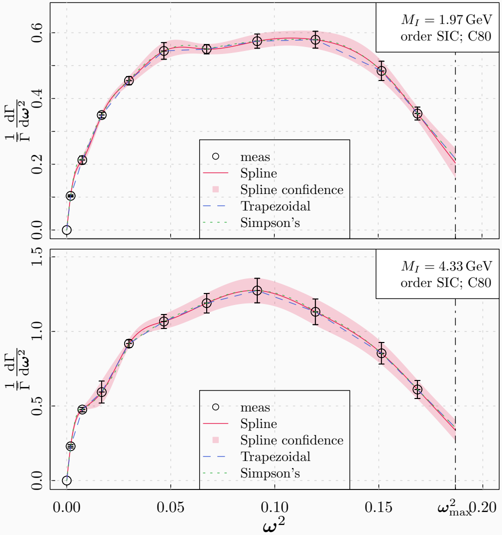
Operations order

$$\Gamma^{\text{phys}} = \bar{\Gamma}(M_{B_s}) \cdot \lim_{x\to 1} \left[\int_0^{(|\boldsymbol{\omega}|^{\max})^2} \mathrm{d}\boldsymbol{\omega}^2 \sum_{p=0}^{2} \lim_{\sigma\mapsto 0} \lim_{a\mapsto 0} \frac{\mathrm{d}\Gamma^{(p)}(\sigma, a, x)}{\mathrm{d}\boldsymbol{\omega}^2} \right] \qquad x = \frac{M_{\bar{B}_s}}{M_I}$$

- Three interchangeable steps
  - Integration (I) over momenta
  - Smearing (S) parameter to zero
  - Continuum (C) limit
- To assess systematics we consider three orders
  - CSI $\to$ maximum number of continuum limits
  - SCI
  - SIC $\to$ mininum number of continuum limits

SIC snapshots





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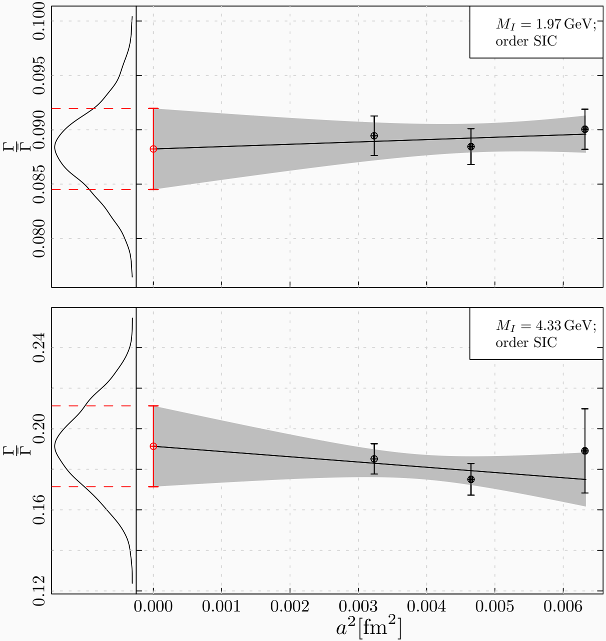
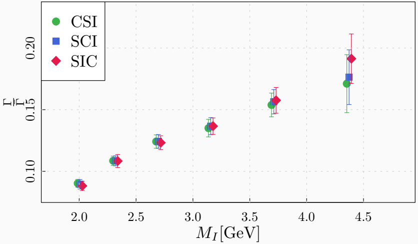


- Only one continuum limit for each M_I
- **weak a^2-dependence** $\rightarrow$ significant error reduction with fourth lattice spacing at a^2 ~0.0024 fm^2

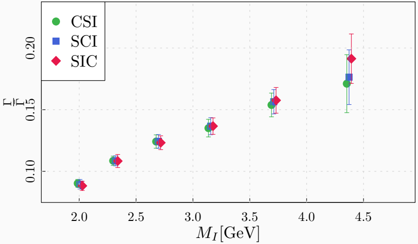


Agreement across procedures
$\rightarrow$ solidity of our results

two ways of extrapolate $M_I \rightarrow M_{\bar{B}_s}$:

- fitting-function approach
- functional-space approach based on Gaussian Processes (Bs.)

Extrapolation to physical point: fitting-function approach



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- **OPE result**: $I \rightarrow X_f \ell \bar{\nu}$ in the limit $M_I \rightarrow \infty$ is purely partonic $\rightarrow$ non-perturbative inputs are **not** needed

$$\hat{\Gamma}^{(n)}_{\text{part}}(x) = \frac{\pi}{4} \sum_{k=0}^{n} \frac{\alpha_s^k(\mu_s(x))}{\pi^k} X_k \qquad X_k = X_k(r_{IF})$$

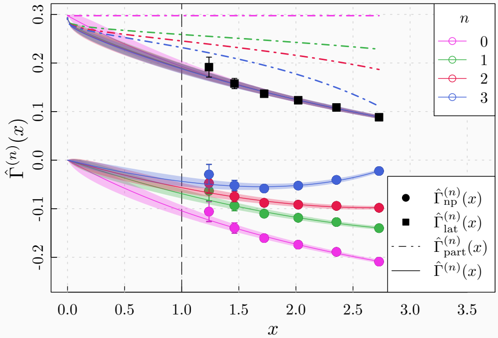
- Combine this information to **guide** the extrapolation

$$\hat{\Gamma}^{(n)}(x) = \hat{\Gamma}^{(n)}_{\text{part}}(x) + \hat{\Gamma}^{(n)}_{\text{np}}(x)$$

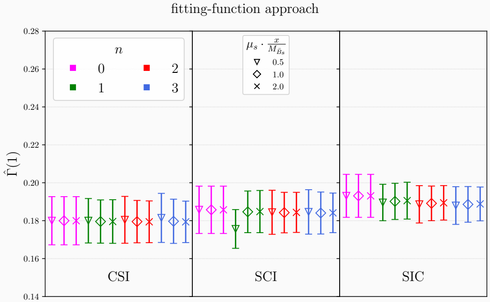
- Lattice non-perturbative inputs parametrized by

$$\hat{\Gamma}^{(n)}_{\text{np}}(x) = x \, \frac{a_0^{(n)} + a_1^{(n)} \, x}{1 + b_0^{(n)} \, x}$$

Extrapolation to physical point: fitting-function approach

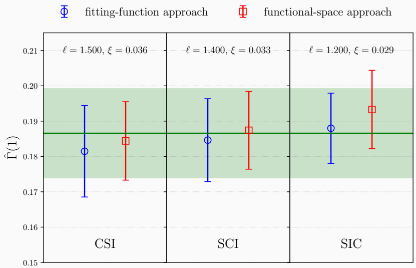


Examples in SIC order



Weak dependence on n and μ_s in the non-perturbative region

Decay rate and $|V_{cb}|^2$



$$\frac{\Gamma^{\mathrm{latt-incl}}[\bar{B}_s \to X_{\bar{s}c}\ell\bar{\nu}]}{|V_{cb}|^2}$$

$$= 2.47(13)(10)[17] \times 10^{-11}\ \mathrm{GeV}$$

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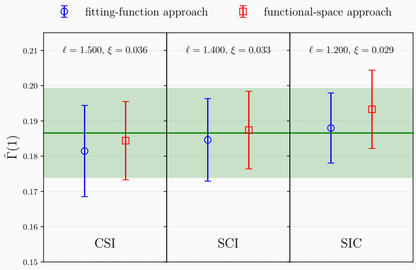

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- Comparison to experiments

$$\Gamma^{\mathrm{Belle}}[\bar{B}_s \to X\ell\bar{\nu}] = 4.14(35) \times 10^{-14}\ \mathrm{GeV}$$

$$= |V_{cb}|^2 \cdot \Gamma[\bar{B}_s \to X_{\bar{s}c}\ell\bar{\nu}] \cdot \left(1 + \frac{|V_{ub}|^2}{|V_{cb}|^2} \cdot \frac{\Gamma[\bar{B}_s \to X_{\bar{s}u}\ell\bar{\nu}]}{\Gamma[\bar{B}_s \to X_{\bar{s}c}\ell\bar{\nu}]}\right)$$

- $\frac{|V_{ub}|^2}{|V_{cb}|^2} \simeq 0.008$ and $\mathcal{O}(\Gamma/\Gamma) = 1$

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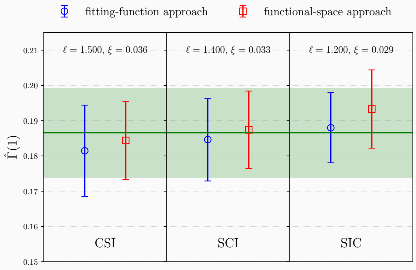

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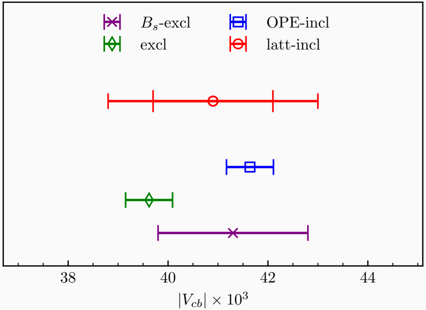
- $\frac{|V_{ub}|^2}{|V_{cb}|^2} \simeq 0.008$ and $\mathcal{O}(\Gamma/\Gamma) = 1$

- **First first-principles determination** of $|V_{cb}|^2$ from inclusive semi-leptonic decay of B_s

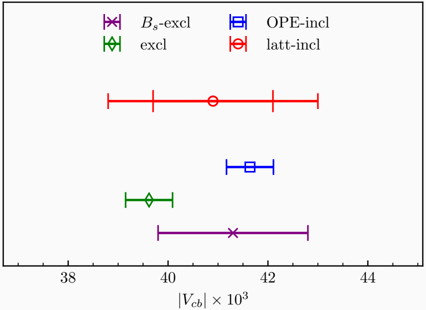
$$|V_{cb}|^{\mathrm{latt-incl}} = 40.9(1.2)^{\mathrm{latt}}(1.7)^{\mathrm{exp}}[2.1]^{\mathrm{tot}} \times 10^{-3}$$



7% precision dominated by experimental uncertainty

- **This is not our ultimate result**
  - A substantial reduction in the total error is expected once the fourth lattice spacing is included

Outlooks



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  - Complete generation of correlators
  - Compute three-point functions for exclusive analysis $B_s \mapsto D_s \ell \bar{\nu}$
  - Estimate $\bar{B}_s \mapsto X_{\bar{s}u} \ell \bar{\nu}$
  - ... move finally to $B \mapsto X \ell \bar{\nu}$

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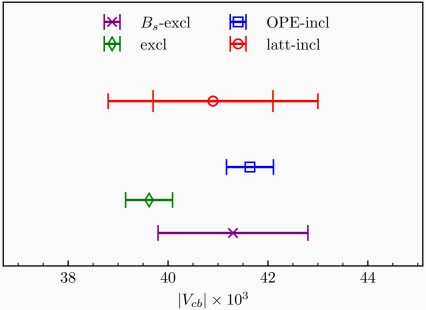

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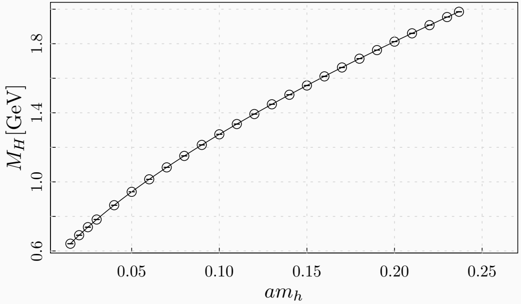
Thank you for the attention!

Backup slides (Bs.)

Mass interpolation

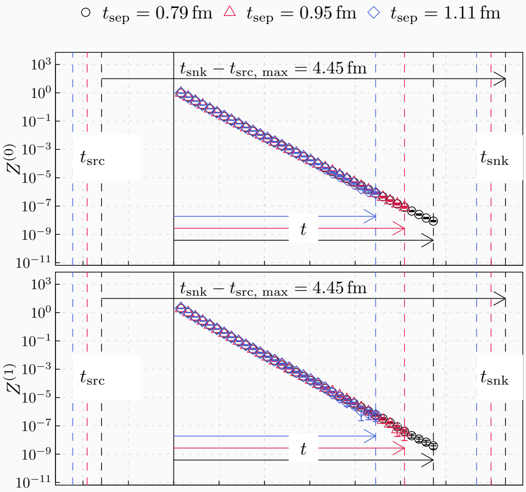
Padé-fit to interpolate $M_H(m_f)$

$$M_H(m_f) = \frac{n_0 + n_1 m_f + n_2 m_f^2}{1 + d_1 m_f + d_2 m_f^2}$$



Tuning of four-point functions

- We have to ensure that we choose $t_{\text{sep}} = 0 - t_{\text{src}} = t_{\text{snk}} - t$ such that the asymptotic limits $t_{\text{src}} \to -\infty$, $t_{\text{snk}} \to \infty$, $T \to \infty$ are fulfilled
- All signals agree within the statistical errors, and we choose the setup $t_{\text{sep}} = 0.95$ fm for our further simulations



HLT method

- We want to extract

$$\rho[K] = \int_0^\infty \mathrm{d}\omega\, K(\omega)\rho(\omega) \qquad \text{e.g.} \qquad K(\omega) = \omega^p \Theta_\sigma(\omega)$$

from

$$C(an) = \int_0^\infty \mathrm{d}\omega\, e^{-\omega a n}\rho(\omega) + \delta^{\mathrm{stat}}(t) \qquad n = 1, \cdots, N \leq T/2$$

- Expressing $K(\omega)$ over the basis given by exponentials

$$A[g] = \int_0^\infty \mathrm{d}\omega\, W(\omega)\Big[K(\omega) - \sum_{n=1}^{N} g_n(N) e^{-\omega a n}\Big]^2$$

- The systematic error associated with $N < \infty$ can be controlled since

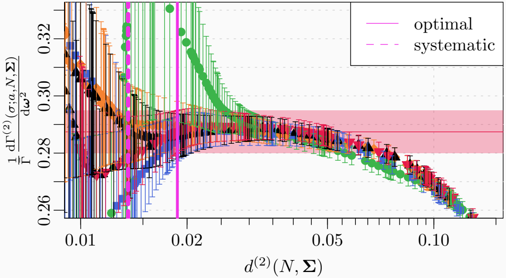
$$\rho[K] \equiv \lim_{N\to\infty} \sum_{n=1}^{N} g_n(N) \cdot C(an)$$

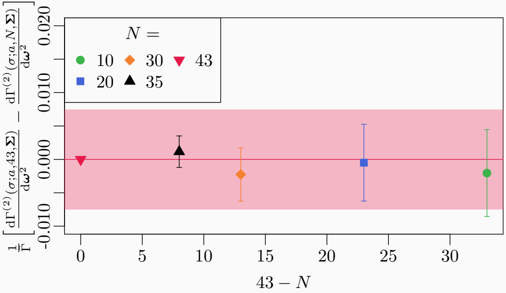
HLT method: stability analysis

- Due to the presence of statistical error we cannot minimize directly $A[g]$ but rather

$$W[g] = (1-\lambda)A[g] + \lambda \boldsymbol{g}^T \mathrm{C\hat{O}V}[C] \cdot \boldsymbol{g}$$

- $\lambda \to 0$ reduction of systematic error but large statistical uncertainty
- $\lambda \to 1$ suppression of statistical error but large systematics
- **plateau analysis**: find optimal balance between statistical and systematic error by progressively reducing the latter until it is negligible within statistical errors





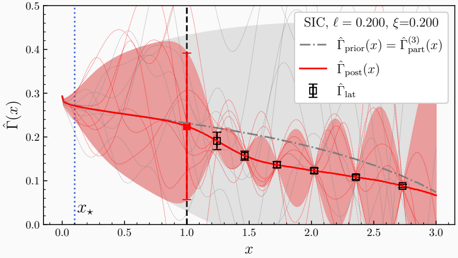
Extrapolation to physical point: functional-space approach

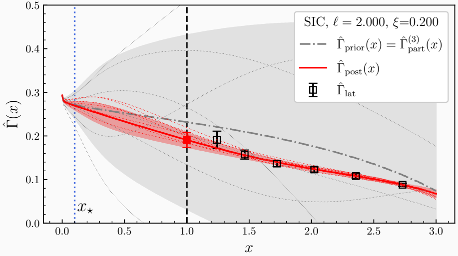
- **Gaussian Processes** (GP) generalize the concept of probability distribution to functions
- Given **prior** mean function and **covariance kernel**, a GP can be conditioned based on observed data
- The **posterior** distribution allows to infer $\hat{\Gamma}(x)$ outside the range of observed points

$$\hat{\Gamma}_{\text{prior}}(x) = \hat{\Gamma}^{(n)}_{\text{part}}(x)$$

$$\mathcal{K}_{\text{prior}}(x, x') = \xi(x)\xi(x') \exp\left(-\frac{(x-x')^2}{2\ell^2}\right)$$

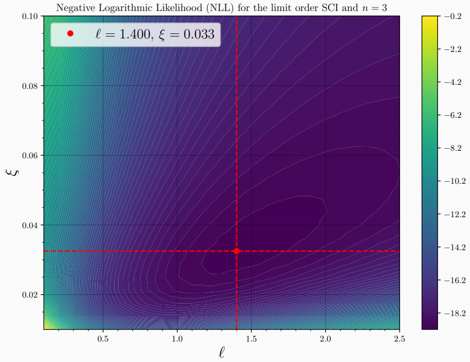
- $\xi(x) \to$ uncertainty of GP
- $\ell \to$ correlation (smoothness) of functions sampled from the GP

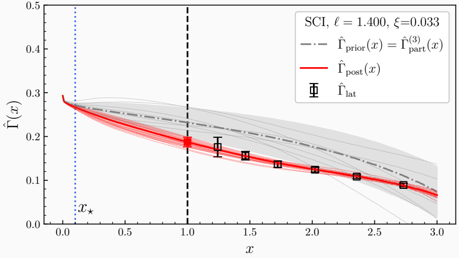




Extrapolation to physical point: functional-space approach

In pure Bayesian spirit, ℓ and ξ are suggested by minimizing a Negative Logarithmic Likelihood expressing the probability of observing the data given a choice of the priors





Similar result to fitting-function approach but **we didn't have to choose a fitting function!**

Gaussian Process Regression

- **Prior distribution** before any observation

$$\hat{\Gamma}(x) \sim \mathcal{GP}\Big(\hat{\Gamma}_{\text{prior}}(x)\ ,\ \mathcal{K}_{\text{prior}}(x,x)\Big)$$

- **Observed data** $(\hat{\Gamma}_{\text{obs}})_i$ in correspondence of $x_i^{\text{obs}},\ \ i = 1, 2, \cdots, N_{\text{obs}}$. Covariance matrix $\hat{\Sigma}_{\text{obs}}$

- Joint distribution to describe data at new value x

$$\begin{pmatrix} \hat{\Gamma}(x) \\ \hat{\mathbf{\Gamma}}_{\mathbf{obs}} \end{pmatrix} \sim \mathcal{GP}\left(\begin{pmatrix} \hat{\Gamma}_{\text{prior}}(x) \\ \hat{\mathbf{\Gamma}}_{\mathbf{prior}} \end{pmatrix}, \begin{pmatrix} \mathcal{K}_{\text{prior}}(x,x) & \mathbf{\Delta}^T(x) \\ \mathbf{\Delta}(x) & \hat{\Sigma}_{\text{prior}} + \hat{\Sigma}_{\text{obs}} \end{pmatrix}\right)$$

$$\Delta_i(x) = \mathcal{K}_{\text{prior}}(x_i^{\text{obs}}, x) \qquad (\hat{\Sigma}_{\text{prior}})_{ij} = \mathcal{K}_{\text{prior}}(x_i^{\text{obs}}, x_j^{\text{obs}}) \qquad (\hat{\Gamma}_{\text{prior}})_i = \hat{\Gamma}_{\text{prior}}(x_i^{\text{obs}})$$

Gaussian Process Regression

- Conditioning of the joint distribution still gives a Gaussian Process

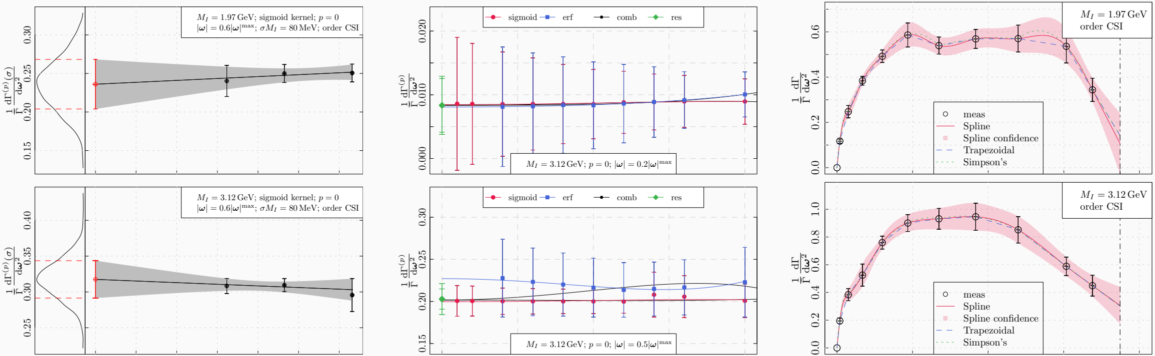
$$\hat{\Gamma}(x)\big|\hat{\mathbf{\Gamma}}_{\mathbf{obs}} \sim \mathcal{GP}\left(\hat{\Gamma}_{\mathrm{post}}(x), \Delta_{\mathrm{post}}(x)\right)$$

- Posterior distribution of $\hat{\Gamma}(x)$ given the observations $\hat{\mathbf{\Gamma}}_{\mathbf{obs}}$

$$\hat{\Gamma}_{\mathrm{post}}(x) = \hat{\Gamma}_{\mathrm{prior}}(x) + \mathbf{\Delta}^T \frac{1}{\hat{\Sigma}_{\mathrm{prior}} + \hat{\Sigma}_{\mathrm{obs}}} \left(\hat{\mathbf{\Gamma}}_{\mathbf{obs}} - \hat{\mathbf{\Gamma}}_{\mathbf{prior}}\right),$$

$$\Delta_{\mathrm{post}}(x) = \mathcal{K}_{\mathrm{prior}}(x, x) - \mathbf{\Delta}^T \frac{1}{\hat{\Sigma}_{\mathrm{prior}} + \hat{\Sigma}_{\mathrm{obs}}} \mathbf{\Delta}\,.$$

CSI snapshots



SCI snapshots