Kaon semileptonic form factor using $N_f = 2+1+1$ PACS10$_c$ configurations

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for PACS Collaboration

Refs: PRD101,9,094504(2020), PRD106,9,094501(2022), PoS(LATTICE2024)227(2025)

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Introduction

Urgent task in particle physics
Search for signal beyond standard model (BSM)

one of CKM matrix elements

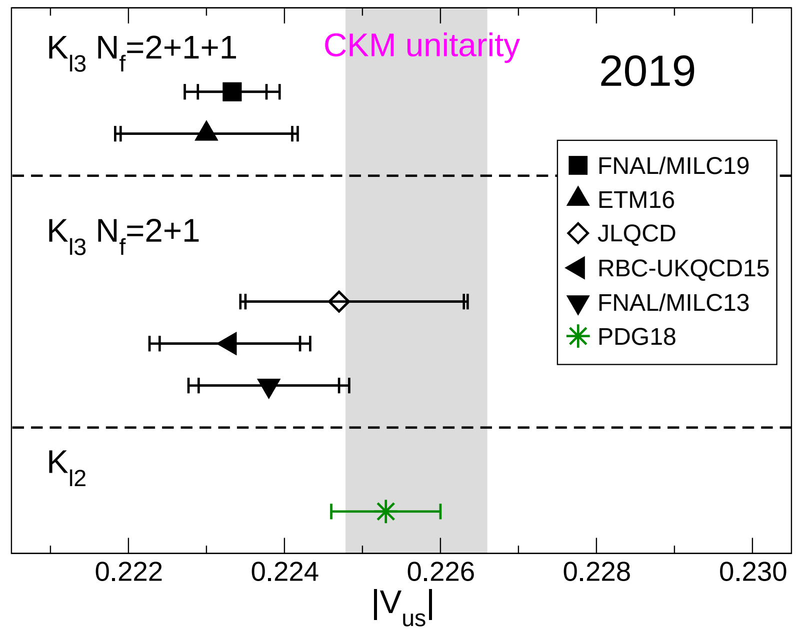
$|V_{us}|$: a candidate of BSM signal

Three determinations of $|V_{us}|$

1. CKM unitarity $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
   constraint in SM (grey band)
   $|V_{us}| \approx \sqrt{1 - |V_{ud}|^2}$

2. $K_{\ell 2}$ $K \to \ell\nu$ decay

3. $K_{\ell 3}$ $K \to \pi\ell\nu$ decay
   Most accurate value ['19 FNAL/MILC]



Around 2019, $|V_{us}|^{1.} = |V_{us}|^{2.} \neq |V_{us}|^{3.}$ $\sim 2\sigma$ difference

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1. CKM unitarity $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

   constraint in SM (grey band)

   $|V_{us}| \approx \sqrt{1 - |V_{ud}|^2}$ w/ new $|V_{ud}|$

   Radiative correction in $|V_{ud}|$ reestimate

   ['18 Seng *et al.*, '20 Hardy, Towner]

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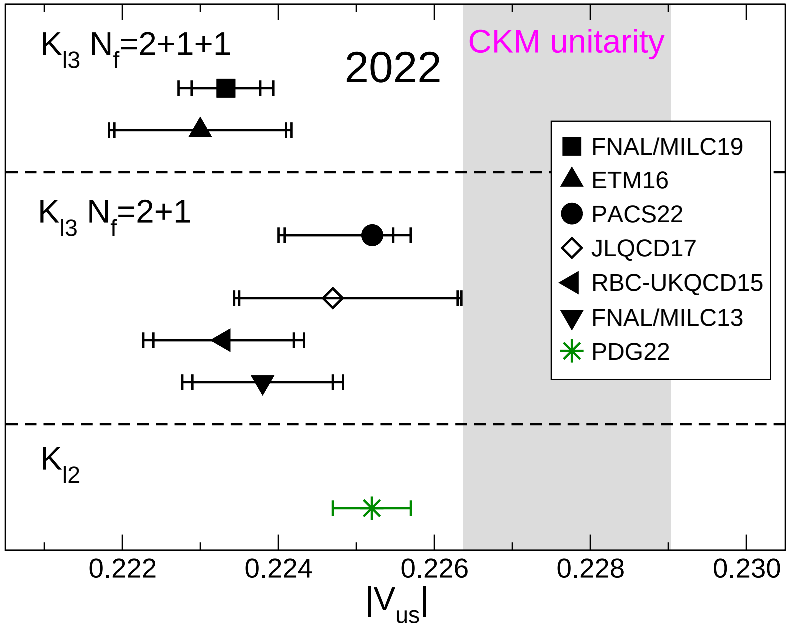
   Most accurate value ['19 FNAL/MILC]



After 2020, $|V_{us}|^{1.} \neq |V_{us}|^{2.} \neq |V_{us}|^{3.}$ $\sim 3\sigma$ differences

Important to confirm by several independent calculations

Simulation parameters

PACS10$_c$ configurations: $L \gtrsim 10$[fm] at very close to physical point

$N_f = 2+1+1$ six-stout-smeared non-perturbative $\mathcal{O}(a)$-improved Wilson + Iwasaki gauge actions

$K_{\ell 3}$ form factor calculation systematic errors controlled: chiral extrap., finite V, finite a

PACS10/L128[PRD101,9,094504(2020)]; PACS10/L160[PRD106,9,094501(2022)]

N_f	2+1 (PACS10)			2+1+1 (PACS10$_c$)		
$L^3 \cdot T$	128^4	160^4	256^4	128^4	168^4	256^4
L [fm]	10.9	10.2	10.5	10.7	10.5	∼10.5
a [fm]	0.08	0.06	0.04	0.08	0.06	∼0.04
m_π [GeV]	0.135	0.138	0.141	0.137	0.137	∼0.137
m_K [GeV]	0.497	0.505	0.514	0.498	0.497	∼0.497
N_{conf}	20	20	20	40	40	∼20
N_{meas}	< 2560		1280	5120		∼2800

Generation of PACS10$_c$/L256 is under way.

PACS10/L256, PACS10$_c$/L128,L168,L256 are preliminary.

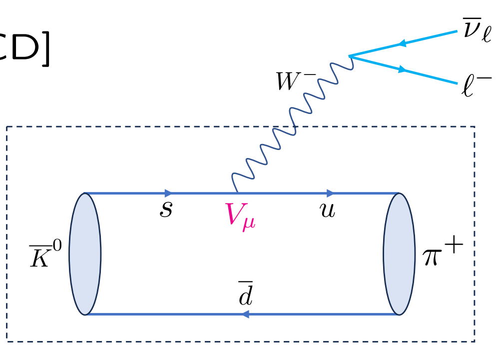
Lat26 : Nucleon FF by Nagatsuka(talk) and Tsuji(poster) ; Meson radius by Sato(talk)

- 2-, 3-point functions w/ $Z(2) \otimes Z(2)$ random source ['08 RBC-UKQCD]

$$C_{\pi,K}(t,p) = \langle 0 | O_{\pi,K}(t,\mathbf{p}) O^\dagger_{\pi,K}(0,\mathbf{p}) | 0 \rangle$$

$$C_{V_\mu}(t,t_{\text{sep}},p) = \langle 0 | O_K(t_{\text{sep}},\mathbf{0}) V_\mu(t,\mathbf{p}) O^\dagger_\pi(0,\mathbf{p}) | 0 \rangle$$

V_μ: Local vector current renormalized by Z_V
Conserved vector current



- finite $p^2 = (2\pi/L)^2 n$, $n = 0$–6, for q^2 interpolation
- two t_{sep} for investigation of excited state contributions

Calculation method w/ local vector current

Ratio $(0 \ll t \ll t_{\mathrm{sep}})$

Details in PACS:PRD101,9,094504(2020)

$$\frac{Z_\pi Z_K Z_V C_{V_\mu}(t, t_{\mathrm{sep}}, p)}{C_\pi(t,p) C_K(t_{\mathrm{sep}} - t, 0)} = M_\mu(p) + \underbrace{A(p)e^{-\Delta_\pi(p)t} + B(p)e^{-\Delta_K(t_{\mathrm{sep}}-t)}}_{\text{1st excited state, } \pi', K', \text{ contributions}} + \cdots$$

$$\Delta_\pi(p) = E_{\pi'}(p) - E_\pi(p), \Delta_K = M_{K'} - M_K$$

Extract $M_\mu(p) = \langle \pi(p) | V_\mu | K(0) \rangle$ w/ fit including 1st excited states

3-point function* $(0 \ll t \ll t_{\mathrm{sep}})$

$$C_{V_\mu}(t, t_{\mathrm{sep}}, p) = \langle 0 | O_K(t_{\mathrm{sep}}, \mathbf{0}) V_\mu(t, \mathbf{p}) O_\pi^\dagger(0, \mathbf{p}) | 0 \rangle$$

$$= \frac{Z_\pi Z_K}{Z_V} \frac{M_\mu(p)}{4 E_\pi M_K} e^{-E_\pi t} e^{-M_K(t_{\mathrm{sep}}-t)} + \cdots$$

$p^2 \equiv |\mathbf{p}|^2 = \left(\frac{2\pi}{L}\right)^2 n, \; n = 0\text{–}6$

with periodic boundary

$\cdots$ corresponds to excited state contributions

2-point function* $X = \pi, K$ $(0 \ll t)$

$$C_X(t, p) = \langle 0 | O_X(t, \mathbf{p}) O_X^\dagger(0, \mathbf{p}) | 0 \rangle = \frac{Z_X^2}{2E_X}\left(e^{-E_X t} + e^{-E_X(2T-t)}\right) + \cdots$$

*Averaging ones with periodic, anti-periodic temporal boundary conditions reducing wrapping around effect in 3pt, and doubling periodicity in 2pt

$Z_V = 1/\sqrt{F_\pi^{\mathrm{bare}}(0) F_K^{\mathrm{bare}}(0)}$ determined w/ electromagnetic form factor $F_{\pi,K}(0) = 1$

Conserved current case: $V_\mu \to \widetilde{V}_\mu$ and $Z_V = 1$

Calculation method

Details in PACS:PRD101,9,094504(2020)

$M_\mu(p) = \langle \pi(p) | V_\mu | K(0) \rangle$ extracted from ratio

$K_{\ell 3}$ form factors $f_+(q^2), f_0(q^2)$

$$\langle \pi(p) | V_\mu | K(0) \rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$p_K = (M_K, \mathbf{0}), p_\pi = (E_\pi, \vec{p})$
$q^2 = -(M_K - E_\pi)^2 + p^2$

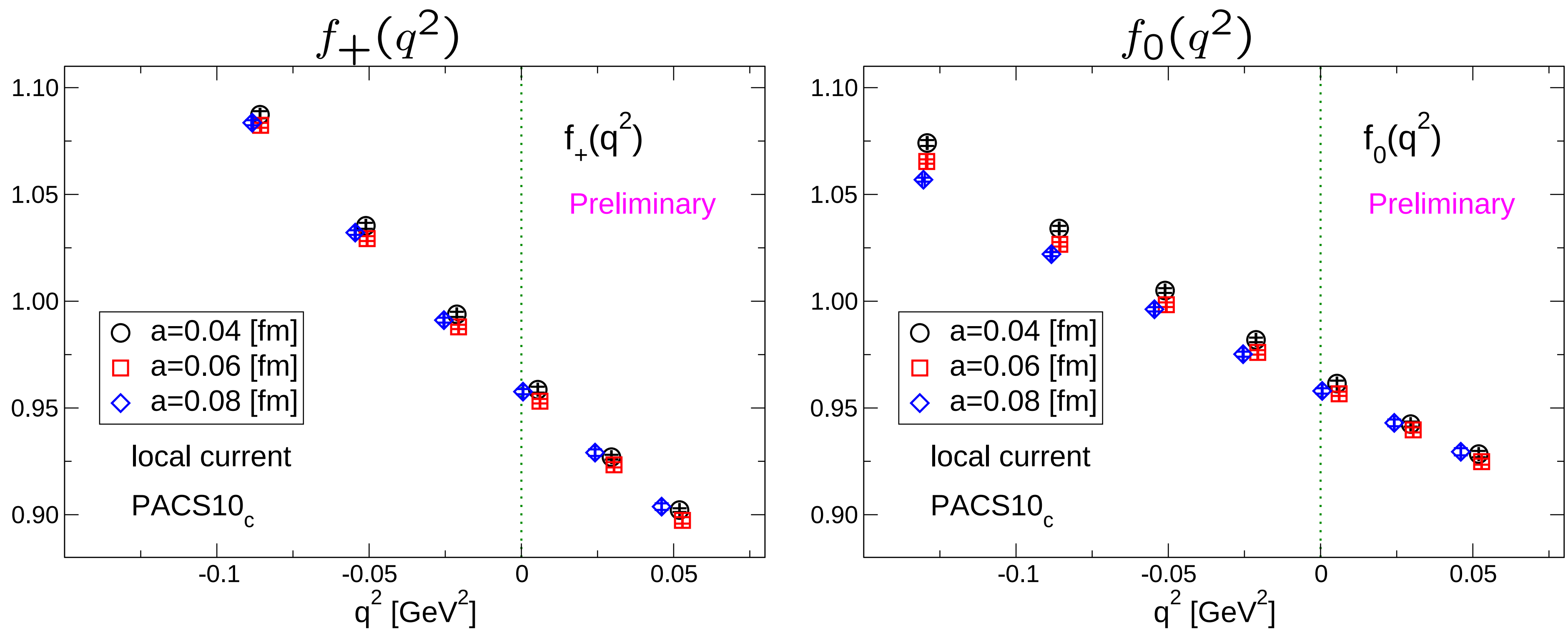
Determination of $|V_{us}|$

$|V_{us}|$ through $|V_{us}| f_+(0) = 0.21654(41)$ [Moulson:PoS(CKM2016)033(2017)]

$M_4(p), M_i(p) \rightarrow f_+(q^2), f_0(q^2)$ at each q^2 except for $p = 0$, where only $f_0(q^2)$

$\rightarrow q^2$ interpolation for $f_+(q^2), f_0(q^2) \rightarrow f_+(0)\,(= f_0(0))$

$f_+(q^2)$ and $f_0(q^2)$ with PACS10$_c$ (L128,L168,L256 local current)



Clear signal at three lattice spacings

Several data in tiny q^2 region thanks to huge volume

Small a dependence in $q^2 \sim 0$ region

q^2 interpolation with PACS10$_c$

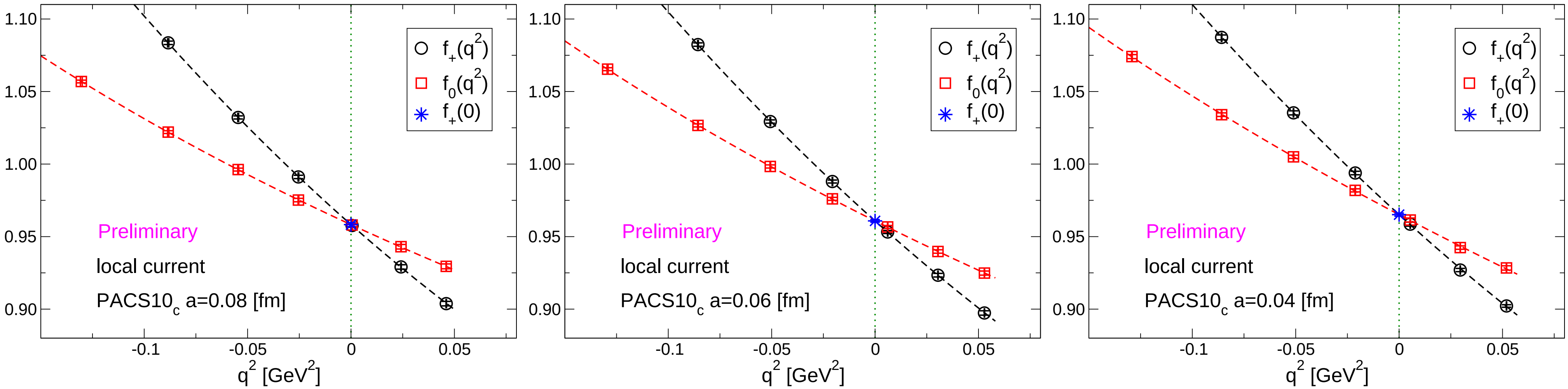
Fit based on NLO SU(3) ChPT with $f_+(0) = f_0(0)$ [PACS:PRD106,9,094501(2022)]

$$f_+(q^2) = 1 - \frac{4}{F_0^2} L_9(\mu) q^2 + K_+(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^+ q^4$$
$$f_0(q^2) = 1 - \frac{8}{F_0^2} L_5(\mu) q^2 + K_0(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^0 q^4$$

K_+, K_0: known functions ['85 Gasser, Leutwyler] w/ $\mu = 0.77$ GeV, $F_0 = 0.11205$ GeV

free parameters: $L_5(\mu), L_9(\mu), c_2^+, c_2^0, c_0$

F_0 estimated from FLAG $F^{\mathrm{SU(2)}}/F_0$ w/ $F^{\mathrm{SU(2)}} = 0.129$ GeV



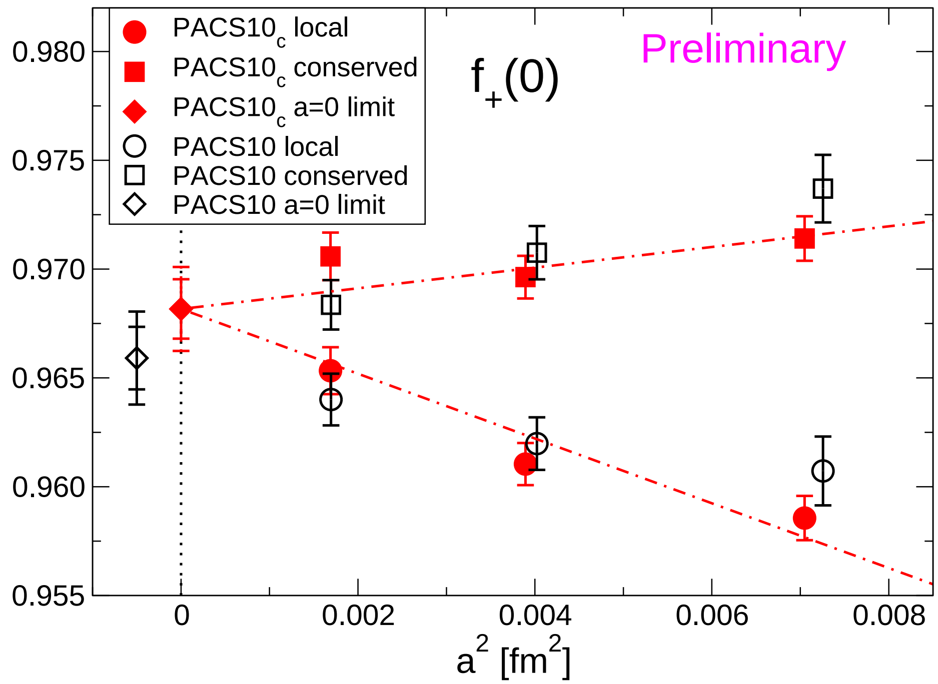
Simultaneous fit for (f_+, f_0) works well.

Tiny extrapolation to physical M_{π^-} and M_{K^0} using same formulas

a[fm]	M_π[MeV]	M_K[MeV]
0.08	137	498
0.06	137	497
~0.04	~137	~497

$m_{\pi^-} = 139.57061$ MeV, $m_{K^0} = 497.611$ MeV

a dependence of $f_+(0)$ with PACS10$_c$



$N_f = 2+1$ results from [PoS(LATTICE2024)227(2024)]

$N_f = 2+1+1$ results comparable with $N_f = 2+1$ results

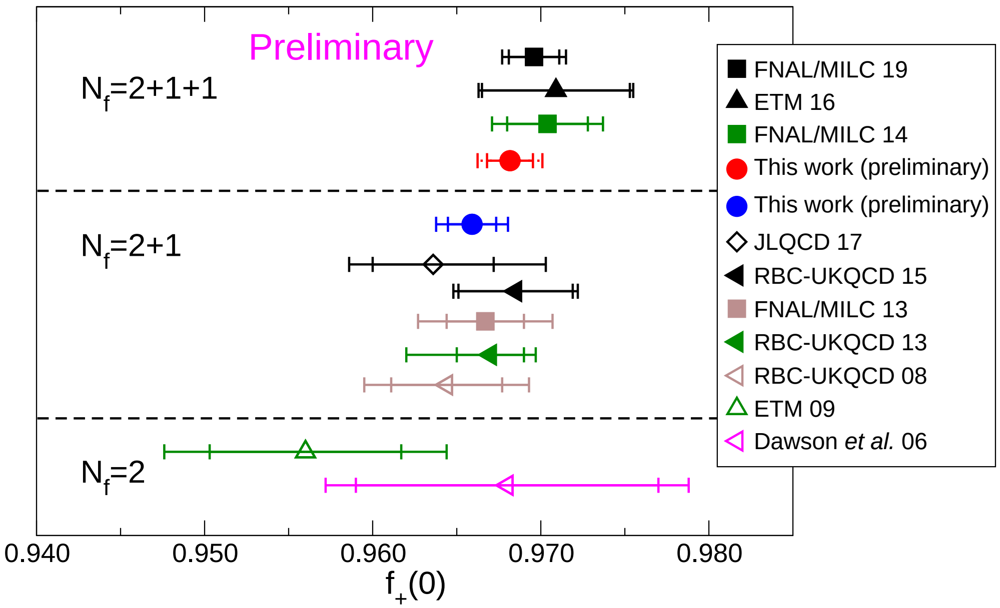
dynamical charm effect might be smaller than statistical error

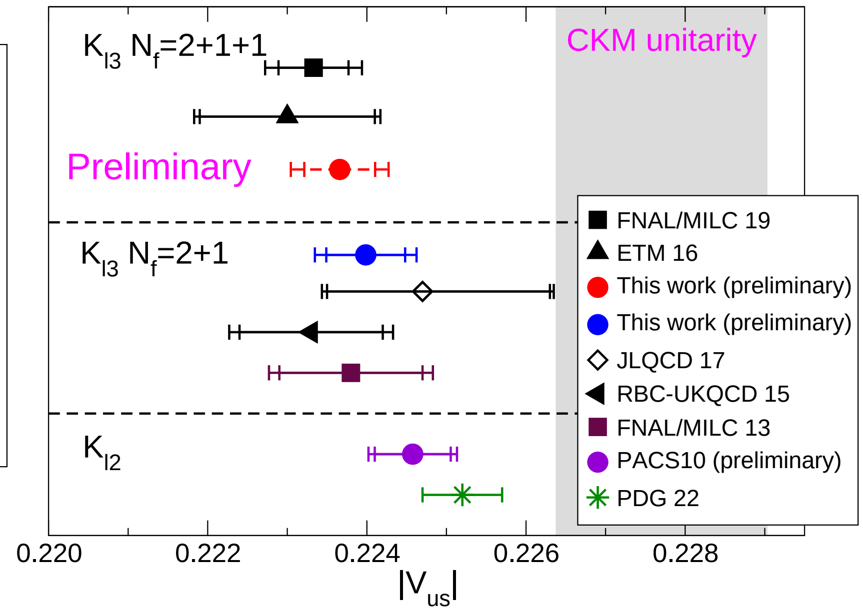
Calculation on $N_f = 2+1+1$ 256^4 lattice are very preliminary.

Systematic error of $N_f = 2+1+1$ result in $a = 0$ limit is roughly estimated.

$f_+(0)$ and $|V_{us}|$

our $N_f = 2+1$ result in PoS(LATTICE2024)227(2024)





inner, outer = statistical, total(stat.+sys.)   inner, outer = lattice, total(lat.+exp.)

Standard model (SM) prediction using $|V_{us}| \approx \sqrt{1-|V_{ud}|^2}$, grey band: ['20 Hardy, Towner]

Rough estimate of systematic error in our $N_f = 2+1+1$ result

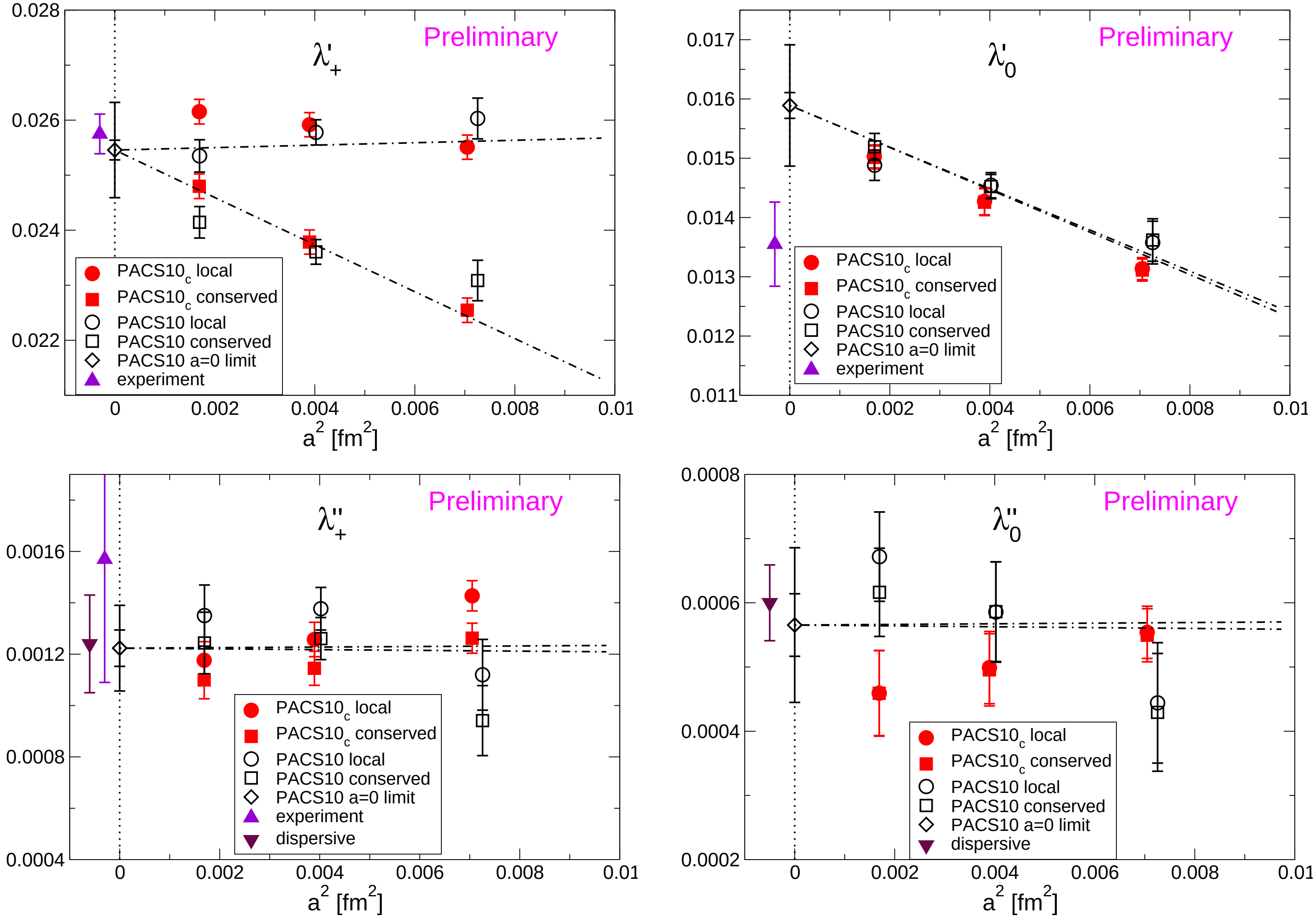
$f_+(0)$: Reasonably agree with previous lattice calculations $\lesssim 1\sigma$

$|V_{us}|$ from $K_{\ell 3}$ using $|V_{us}|f_+(0) = 0.21654(41)$ ['17 Moulson]

$|V_{us}|$ from $K_{\ell 2}$ using $\dfrac{|V_{us}|}{|V_{ud}|}\dfrac{f_K}{f_\pi} = 0.27683(35)$ ['19 Di Carlo *et al.*]

$2 \sim 3\sigma$ difference from CKM unitarity (grey band)

Slope and curvature for $f_{+,0}(q^2)$ $\lambda'_{+,0} = \frac{M_{\pi^-}^2}{f_+(0)} \frac{df_{+,0}(q^2)}{d(-q^2)}, \; \lambda''_{+,0} = \frac{M_{\pi^-}^4}{f_+(0)} \frac{d^2 f_{+,0}(q^2)}{d(-q^2)^2}$



PACS10: Systematic error mainly from fit form dependence ($q^2 \to 0, a \to 0$)

PACS10$_c$: no $a \to 0$ extrapolation yet

Summary

PACS10$_c$ configurations in $N_f = 2 + 1 + 1$ QCD

$V \gtrsim (10\text{fm})^4$ in physical point at three lattice spacings

Preliminary results of $f_+(q^2)$ from PACS10$_c$/L128,L168,L256

Stable $q^2 \to 0$ interpolation with data near $q^2 = 0$

Similar to PACS10 result

$f_+(0)$ and $|V_{us}|$: consistent with previous lattice results

$2 \sim 3\sigma$ difference of $|V_{us}|$ from CKM unitarity

Future work

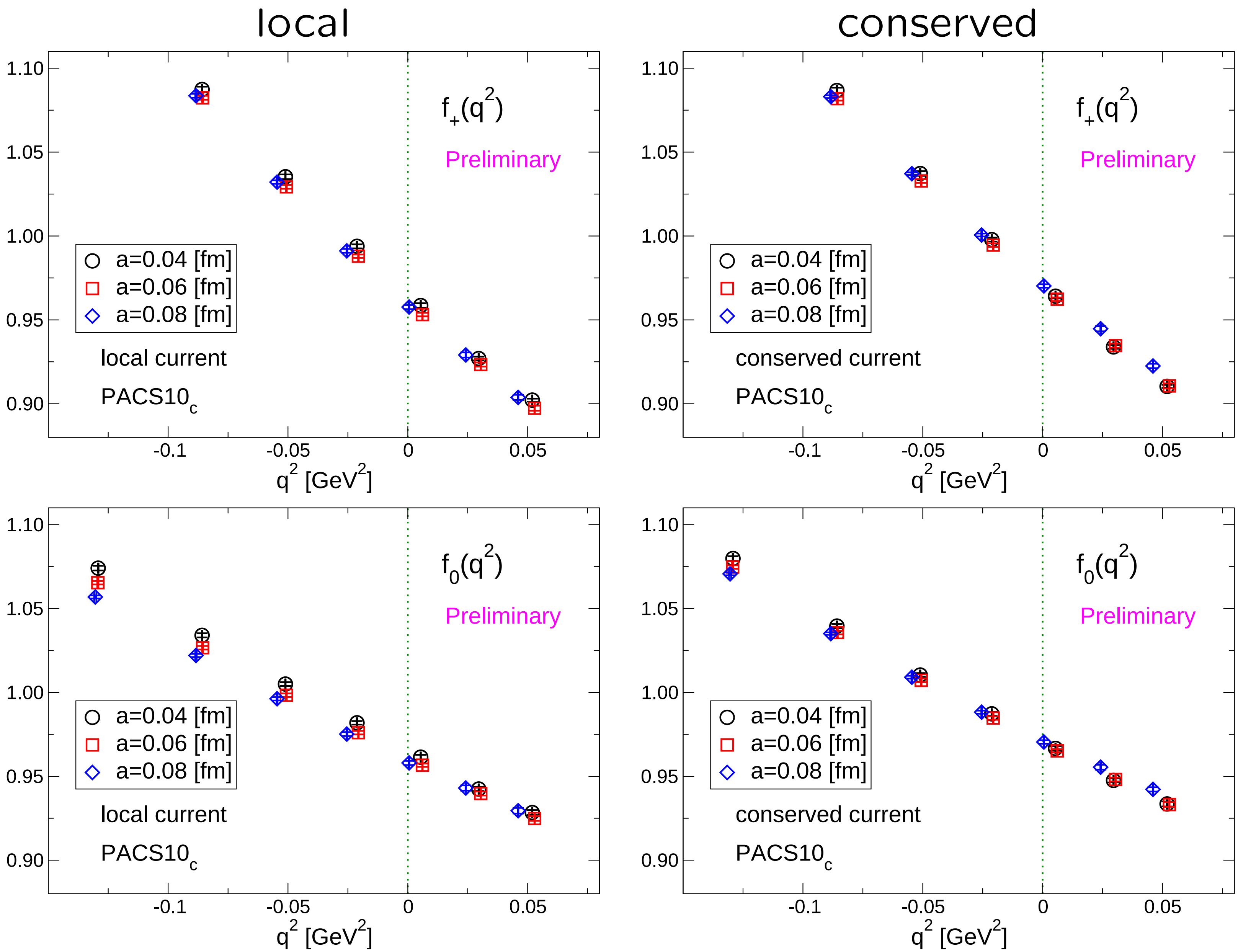
Continue calculation with PACS10$_c$/L256 Reliable $a \to 0$ extrapolation

$|V_{us}|$ w/ phase space integral with PACS10$_c$

Estimate systematic uncertainties *e.g.*, isospin breaking

Back up

$f_+(q^2)$ and $f_0(q^2)$ with PACS10$_c$ (local and conserved currents)



Phase space integral I_K^ℓ (PACS10 result)

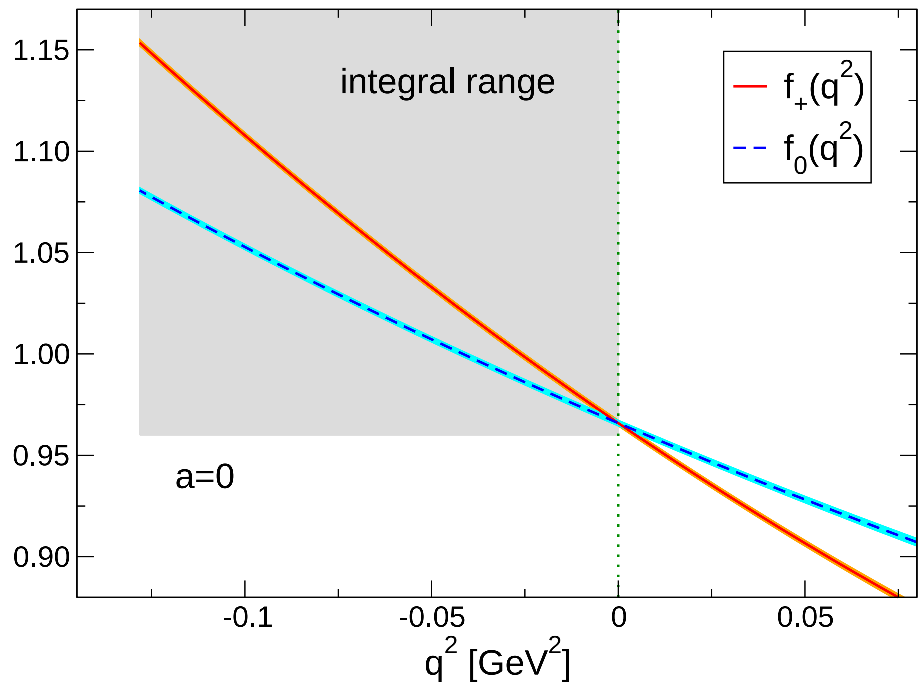
$\Gamma_{K_{\ell 3}} = C_{K_{\ell 3}} (|V_{us}| f_+(0))^2 I_K^\ell$ $\Gamma_{K_{\ell 3}}$: decay width, $C_{K_{\ell 3}}$: known factor, $\ell = e, \mu$

$|V_{us}| f_+(0) = 0.21654(41)$ ['17 Moulson]

$\leftarrow$ I_K^ℓ from dispersive representation of experimental $\overline{F}_{+,0}(t)$

$$I_K^\ell = \int_{m_\ell^2}^{(M_K - M_\pi)^2} dt \left(J_+(t) \overline{F}_+^2(t) + J_0(t) \overline{F}_0^2(t) \right), \quad \overline{F}_{+,0}(t) = \frac{f_{+,0}(-t)}{f_+(0)}$$

$J_{+,0}(t)$: known function ['84 Leutwyler, Roos]



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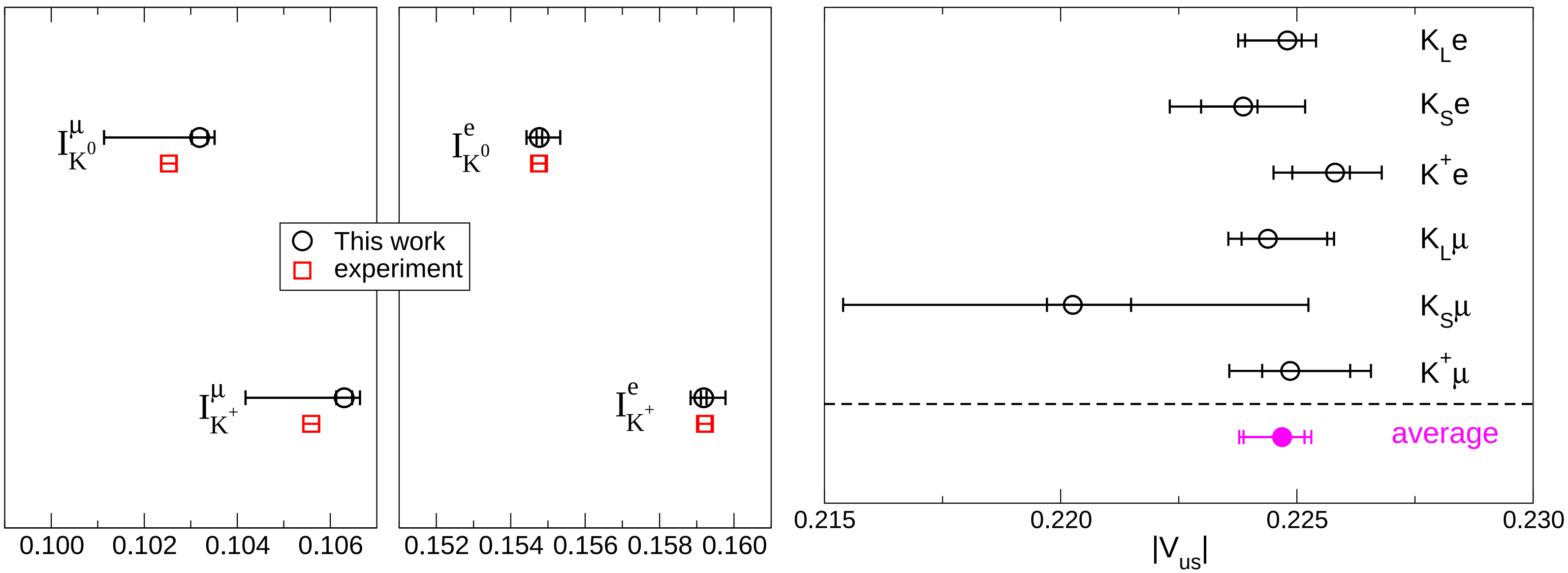

inner: stat. error; outer: (stat.+sys.) error

Reasonably agree with experimental values ['10 Antonelli *et al.*]

Large uncertainty from fit form of $a \to 0$