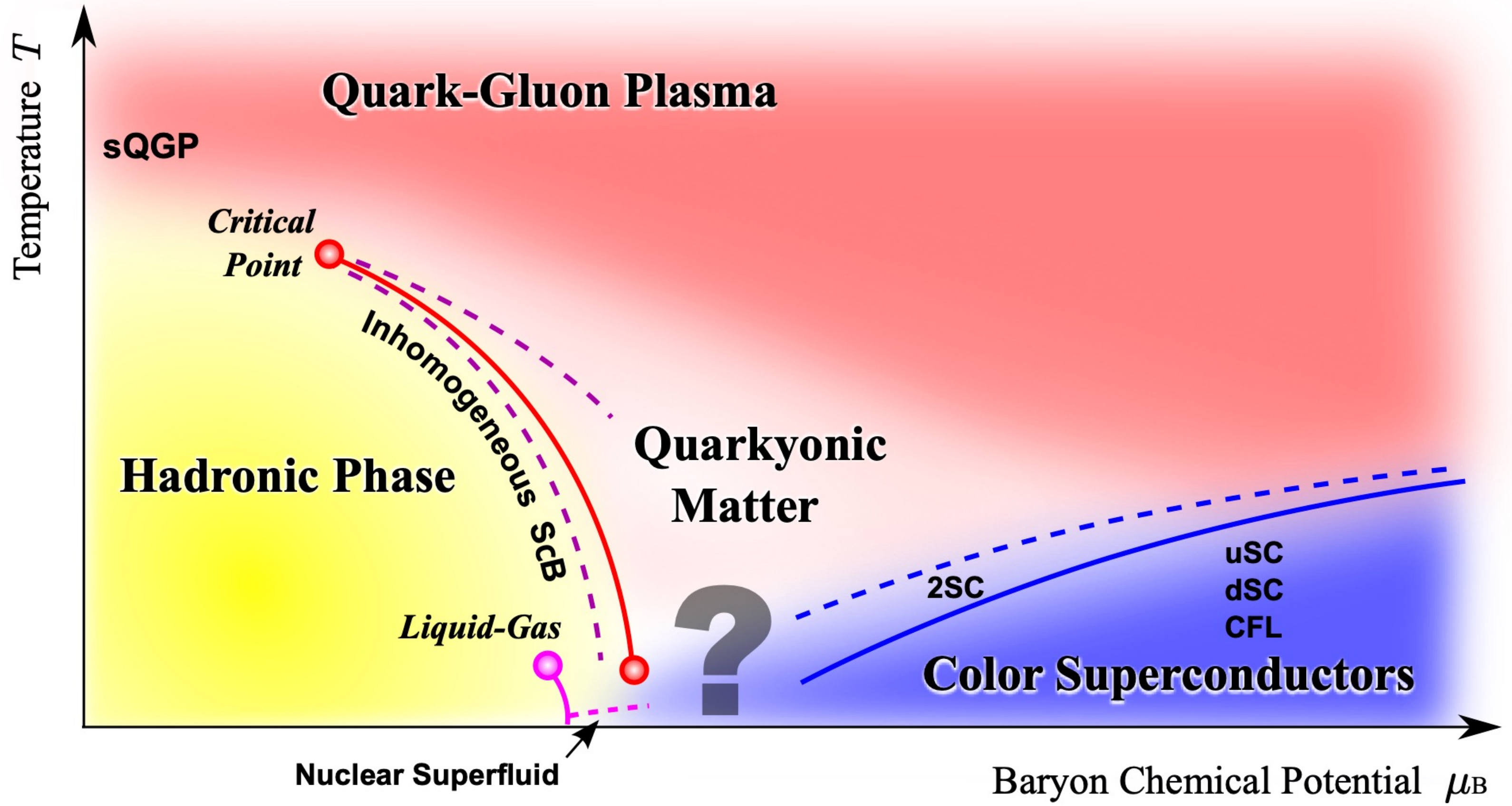
Higgs-Confinement Continuity in SU(2) Gauge-Higgs Model with Color-Flavor-Locking Potential

Yusuke Shimada (YITP, Kyoto University)

Based on on-going work with Arata Yamamoto (RIKEN)

Motivation: QCD Phase

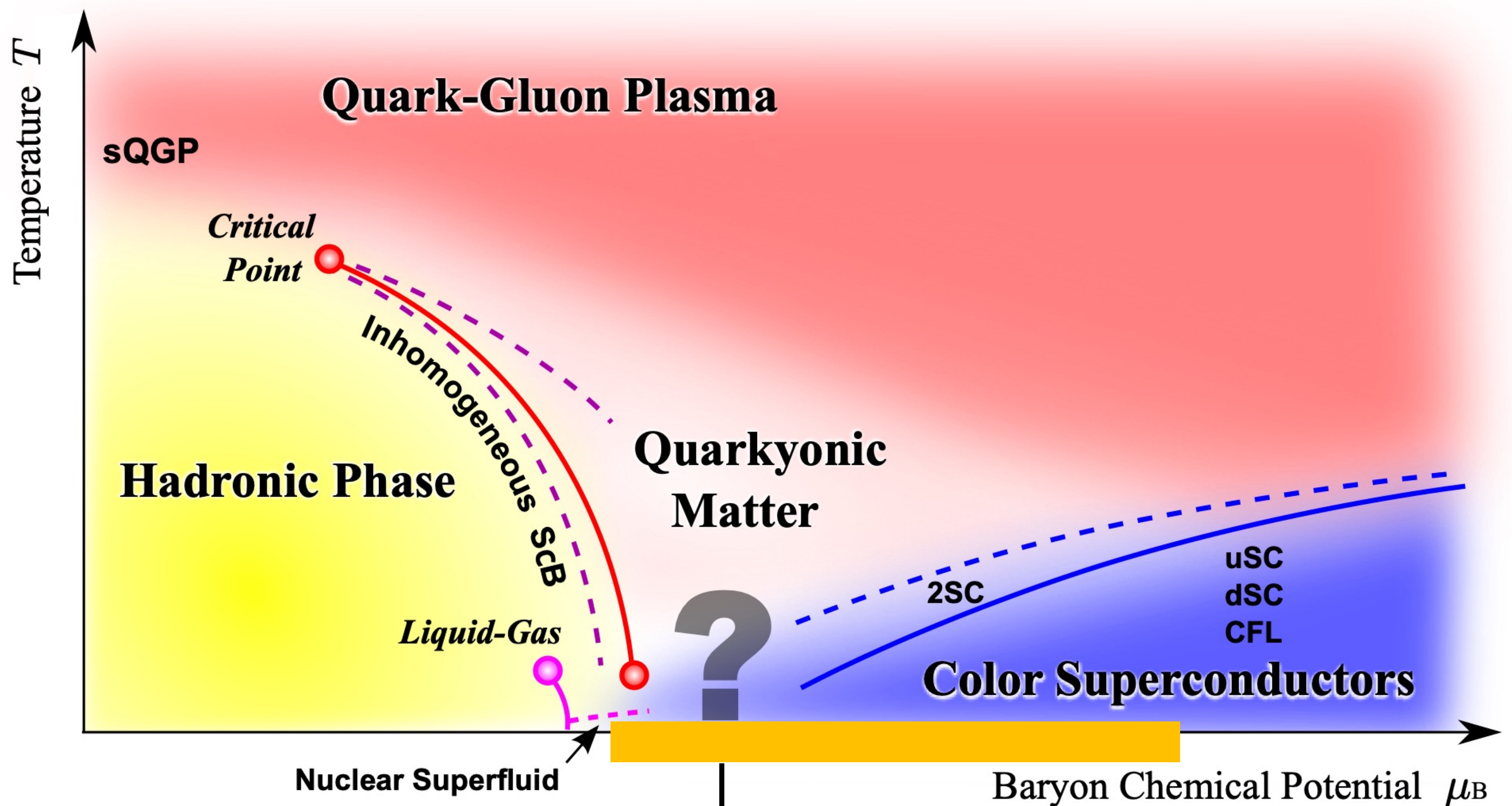
Phase transitions of QCD → Phase Diagram



Fukushima, Hatsuda [2011]

Motivation: QCD Phase in Middle Density

Phase transitions of QCD → Phase Diagram



Fukushima, Hatsuda [2011]

What occurs in middle density?

Continuity in Middle Density

Two regions in middle density : **Hadronic matter** vs **Quark matter**

Hadron Superfluid	**Color Superconductor**
• Chiral symmetry breaking	• Color-flavor locking (Higgs)
$SU(3)_L \times SU(3)_R \to SU(3)_V$	$SU(3)_C \times SU(3)_V \to SU(3)_{C+V}$
• Dibaryon BEC	• Diquark condensate
$\langle BB \rangle \neq 0 \to U(1)_B$ SSB	$\langle qq \rangle \neq 0 \to U(1)_B$ SSB

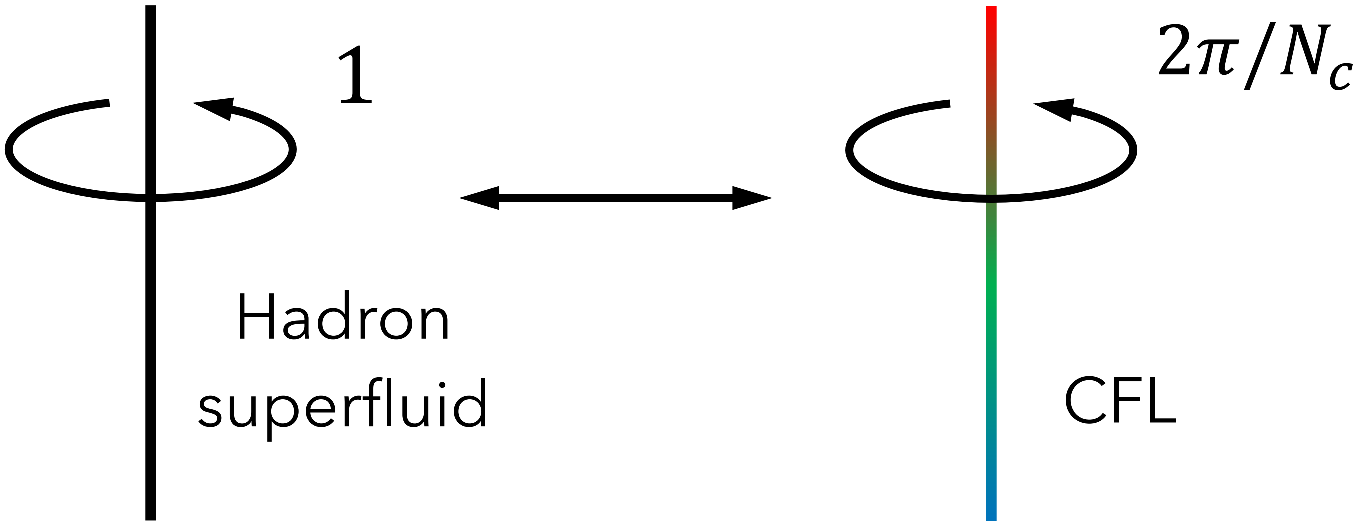
Low / High-density regions possess the same symmetry.

→ *Two regions can be connected w/o PT* (**quark-hadron continuity**)

Recent Studies on QH-Continuity

Vortex may detect phase transition.

Since quark matter is color-charged, the AB phase around a vortex distinguishes two regions. cf. Cherman et al. [2019, 2024]



Two AB phases distinguish regions.

Or, AB phase can be smoothly connected. (Y. Hayashi[2024])

How does the AB phase change between the two regimes?

QH-Continuity and HC-Continuity

Finite-density calculations suffer from the sign problem.

Hadronic matter (confined) vs. color Higgs regime

$\rightarrow$ We can use gauge-Higgs model instead!

QCD

Hadron Superfluid	Color Superconductor

=

Gauge-Higgs Model

Confined Phase	Higgs Phase

Our Calculations - Abstract

We consider 3d $SU(2)$ gauge-Higgs model ($N_f = 2$, fund. rep.)
with $U(1)$ symmetry broken. cf. Y. Hayashi[2024]
direct calculation for Higgs-conf. crossover using lattice!

- **Bulk**: Confined/Higgs regimes are connected by a crossover.
- **Vortex**: Non-trivial AB phase is observed in the Higgs regime. Its fate on the confined side is the open question. (on-going)

All results are preliminary.

Action: Gauge-Higgs Model with CFL

Our Fields

$U_\mu(x)$: $SU(2)$ gauge link

$\Phi(x)$: color/flavor charged Higgs field $\Phi(x) \to \underbrace{G}_{\text{color}}\, \Phi(x)\, \underbrace{F}_{\text{flavor}}$

Higgs condensate is color-flavor-locked by CFL potential

$$S_{CFL} = \lambda \sum_x \mathrm{tr}\left\{\Phi^\dagger(x)\Phi(x) - \frac{1}{2}\left[\mathrm{tr}\,\Phi^\dagger(x)\Phi(x)\right]1\right\}^2$$

or, simply $\Phi(x) \in U(2)$ taking $\lambda \to \infty$ (our choice).

Action: $U(1)$ Breaking

To break $U(1)$ symmetry of $\Phi(x)$ (or $\det\Phi(x)$), we introduce

$\phi_0(x)$: $U(1)$ Higgs field and following actions.

$$S_0 = -\kappa_0 \sum_{x,\mu} \mathrm{Re}[\phi_0^*(x)\phi_0(x+\hat{\mu})]\,, \quad S_\epsilon = -\epsilon \sum_{x} \mathrm{Re}[\phi_0^*(x)\det\Phi(x)]\,.$$

Large κ_0 breaks $U(1)_{\phi_0}$. S_ϵ locks $U(1)$ breaking of ϕ_0 and $\det\Phi$.

→ Large κ_0 and ϵ provide $\Phi(x)$ vortex. ($\kappa_0 = \epsilon = 10$ is taken.)

$\Phi(x) \sim \langle qq\rangle$, $\phi_0(x) \sim \langle BB\rangle$ # Fradkin and Shenker [1979] has no $U(1)$ breaking

Action: $SU(2)$ Gauge-Higgs Model

Finally, our model is $S = S_g + S_h + S_\epsilon + S_0$.

$$S_g = -\beta \sum_p \frac{1}{2} \mathrm{Re\,tr}\, U_p\,, \quad S_h = -\kappa \sum_{x,x' \in l} \mathrm{Re\,tr}\left[\Phi^\dagger(x) U_l(x) \Phi(x')\right]\,,$$

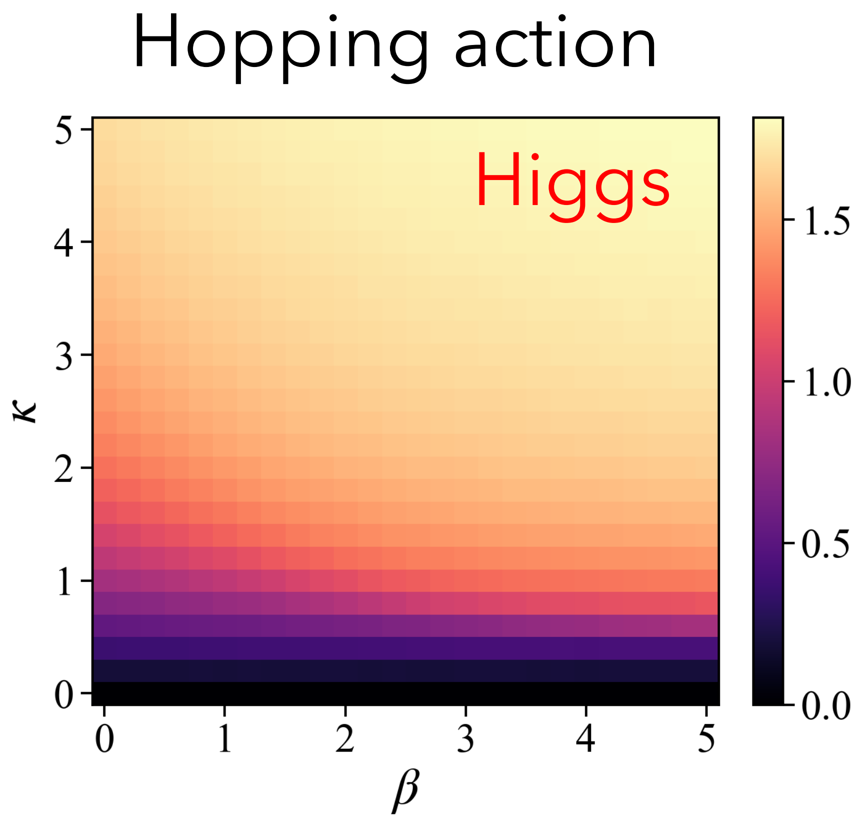
$$S_0 = -\kappa_0 \sum_{x,\mu} \mathrm{Re}[\phi_0^*(x)\phi_0(x+\hat{\mu})]\,, \quad S_\epsilon = -\epsilon \sum_x \mathrm{Re}[\phi_0^*(x) \det \Phi(x)]\,.$$

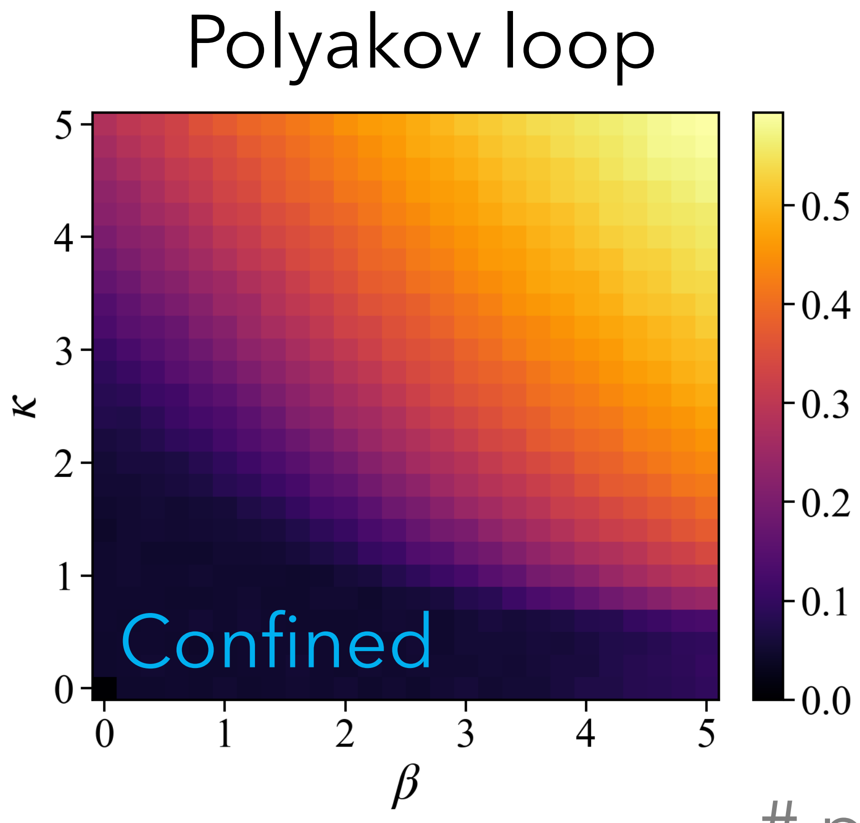
U_l : $SU(2)$ gauge link, Φ: $U(2)$ Higgs field, ϕ_0: $U(1)$ Higgs field.

$\kappa_0 = \epsilon = 10$ is taken and (β, κ) PD is considered in $3d$ using HMC.

Phase Diagram

$L = 8$ calculations provide the phase diagram in $(\beta, \kappa) \in [0,5]^2$.





preliminary

---

hopping action $= \left\langle \frac{1}{N_l} \sum_{x,x' \in l} \mathrm{Re}\,\mathrm{tr}\left[\Phi^\dagger(x) U_l(x) \Phi(x')\right] \right\rangle = \frac{1}{N_l} \frac{\partial \ln Z}{\partial \kappa}$.

Are There Phase Transitions?

Phase transitions ~ change of symmetries.

- $SU(2)_c$: NOT broken (gauge symmetry)
- $SU(2)_f$: NOT broken (CFL Higgs) cf. Bonati et al. [2021]
- $U(1)$: always broken ($\kappa_0 = 10$)

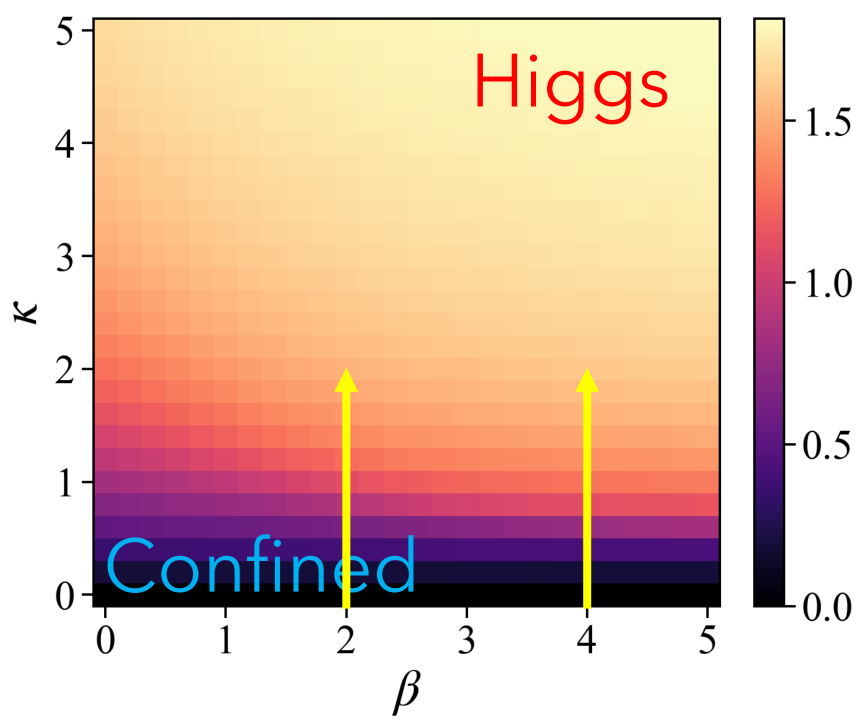
$\longrightarrow$ If any transition exists in the bulk, it must be 1st order.

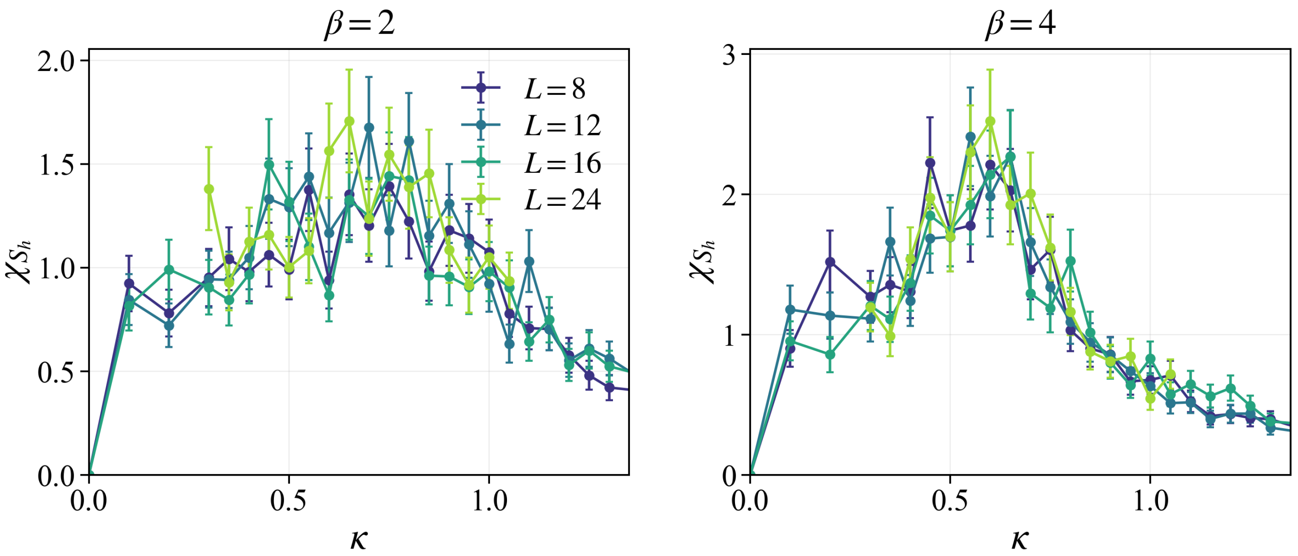
At $\beta \to \infty$, $SU(2)_c^{\mathrm{global}} \times SU(2)_f \to SU(2)_{c+f}$ 2nd order PT exists.

Is There a 1st Order Phase Transition?

To detect conf-Higgs 1st order PT, we measure susceptibility of hopping action along κ direction (yellow line).

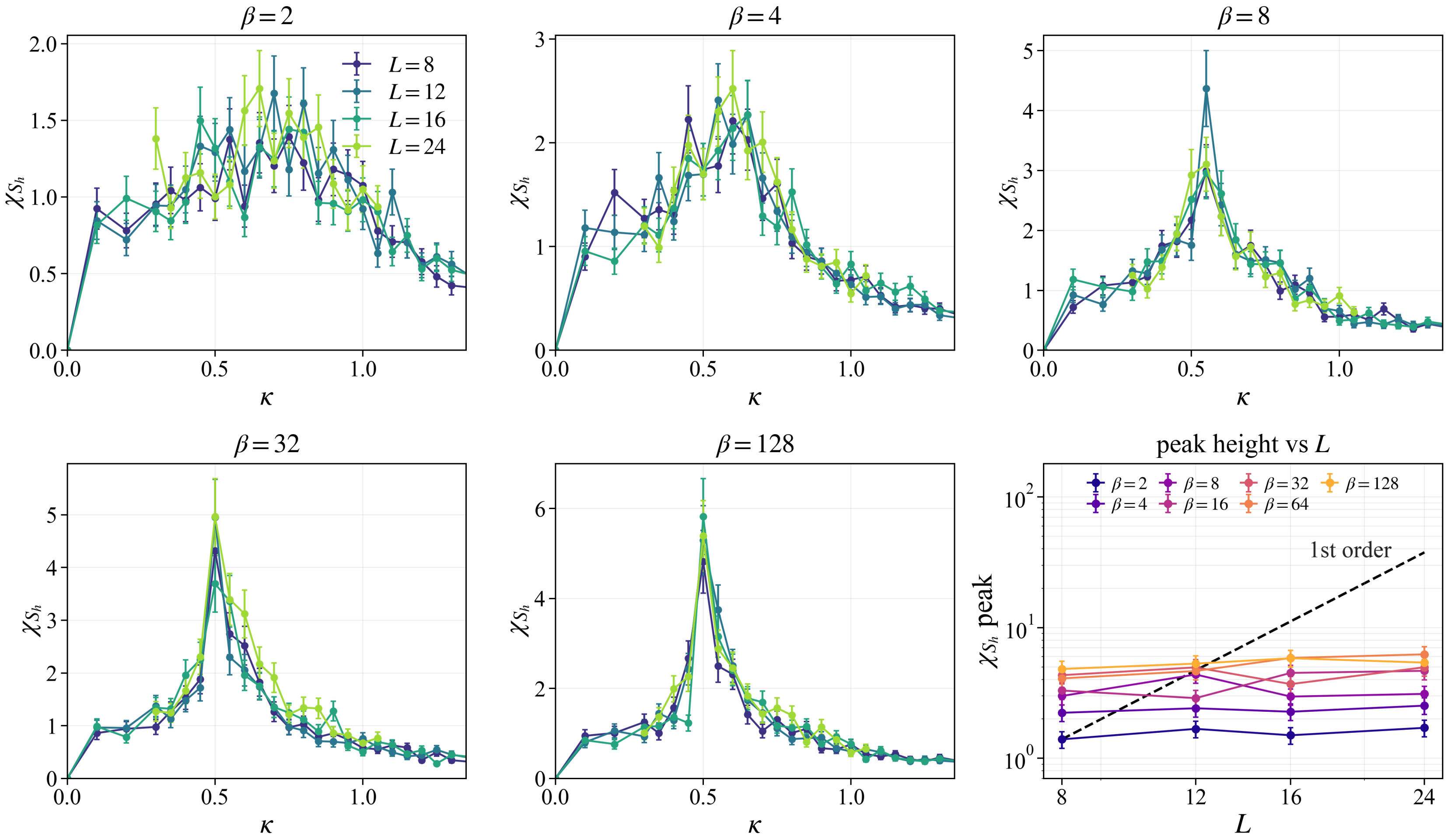
Hopping action





\# preliminary

Is There a 1st Order Phase Transition?



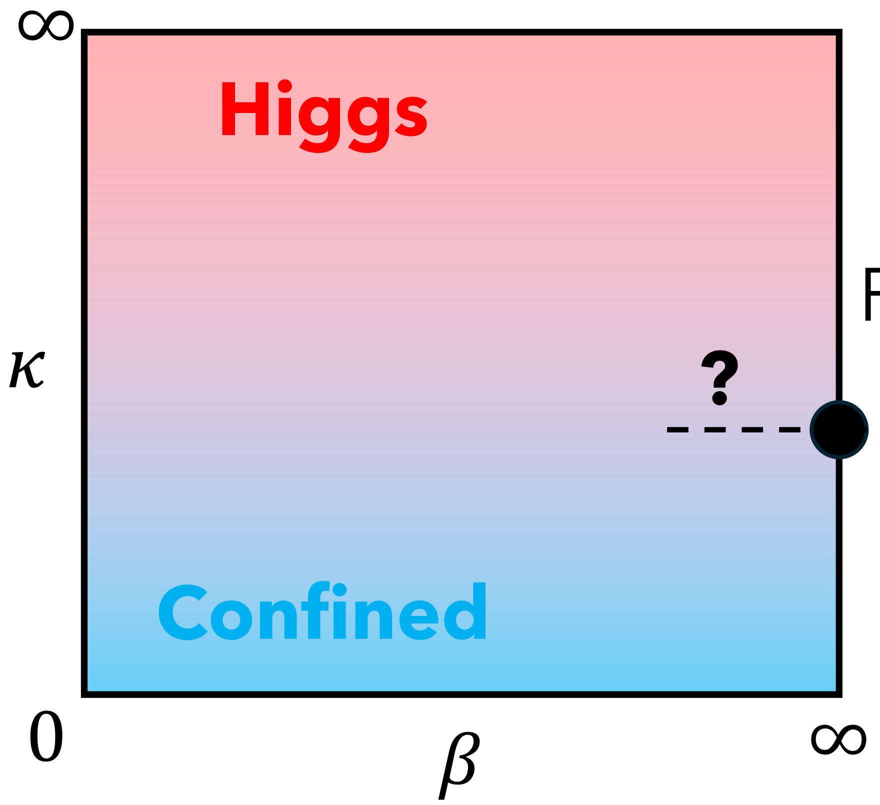
preliminary

The peak of the susceptibility does not grow with L. **No 1st order PT**.

Bulk Crossover of Gauge-Higgs Model

$SU(2)$ gauge-(multi-)Higgs model with $U(1)$ symmetry broken

$\rightarrow$ confinement-Higgs continuity is realized!



For $\beta \leq 128$, no 1st order PT

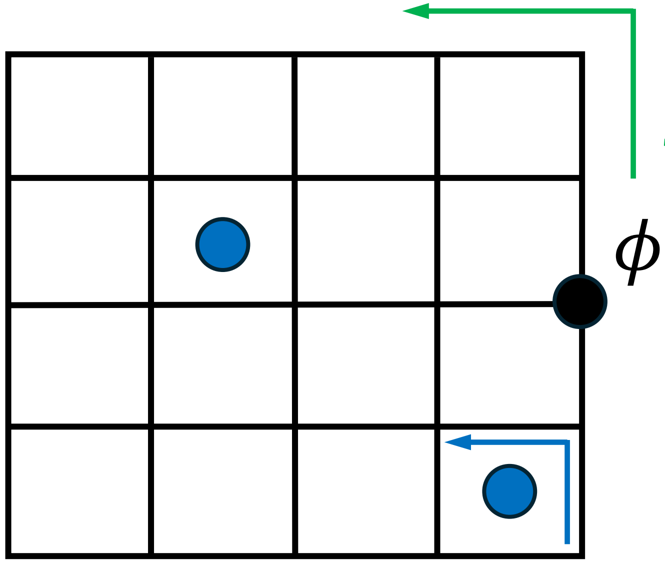
$\kappa_c \sim 0.483$

$O(4)$ class 2nd order PT

How about Vortices?

Recent interest: AB phase around vortex in conf/Higgs regime

In the xy-plane, we take OBC and fix the total winding of $\phi_0 = e^{i\theta}$.



$\oint d\theta(x) = 2\pi Q$

$\phi_0(x) = e^{i\theta(x)}$

$\oint d\theta(x) = 2\pi q_i$

Each vortex has winding q_i and total

$$\sum_i q_i = Q$$

is fixed. How it fragments into q_i is dynamics, not input.

In our model, $\arg\det\Phi(x)$ is locked to $\theta(x)$ by S_ε .
$\rightarrow$ the winding is imposed on $\Phi(x)$ as well.

Non-Abelian Vortices

For non-Abelian (color-charged) vortex,

$$\Phi \sim \begin{pmatrix} e^{i\theta} & \\ & 1 \end{pmatrix} = \underbrace{\begin{pmatrix} e^{\frac{i\theta}{2}} & \\ & e^{\frac{i\theta}{2}} \end{pmatrix}}_{U(1)} \underbrace{\begin{pmatrix} e^{\frac{i\theta}{2}} & \\ & e^{-\frac{i\theta}{2}} \end{pmatrix}}_{SU(2)}.$$

This is $q = 1$ vortex.

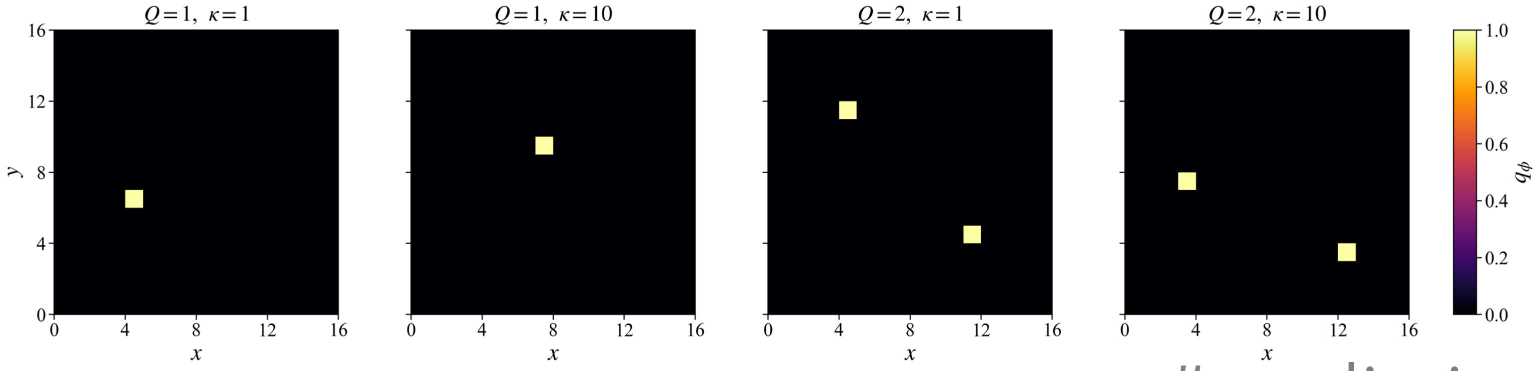
$SU(2)$ factor at $\theta = 2\pi$ is -1_2 : non-trivial Z_2 phase.

Abelian (color-neutral) vortex $\Phi \sim e^{i\theta} 1_2$ is $q = 2$, trivial charge.

By imposing $Q = 1, 2$, we obtain NA vortices.

$Q = 1 \rightarrow \text{one } q = 1$

$Q = 2 \rightarrow \text{two } q = 1$



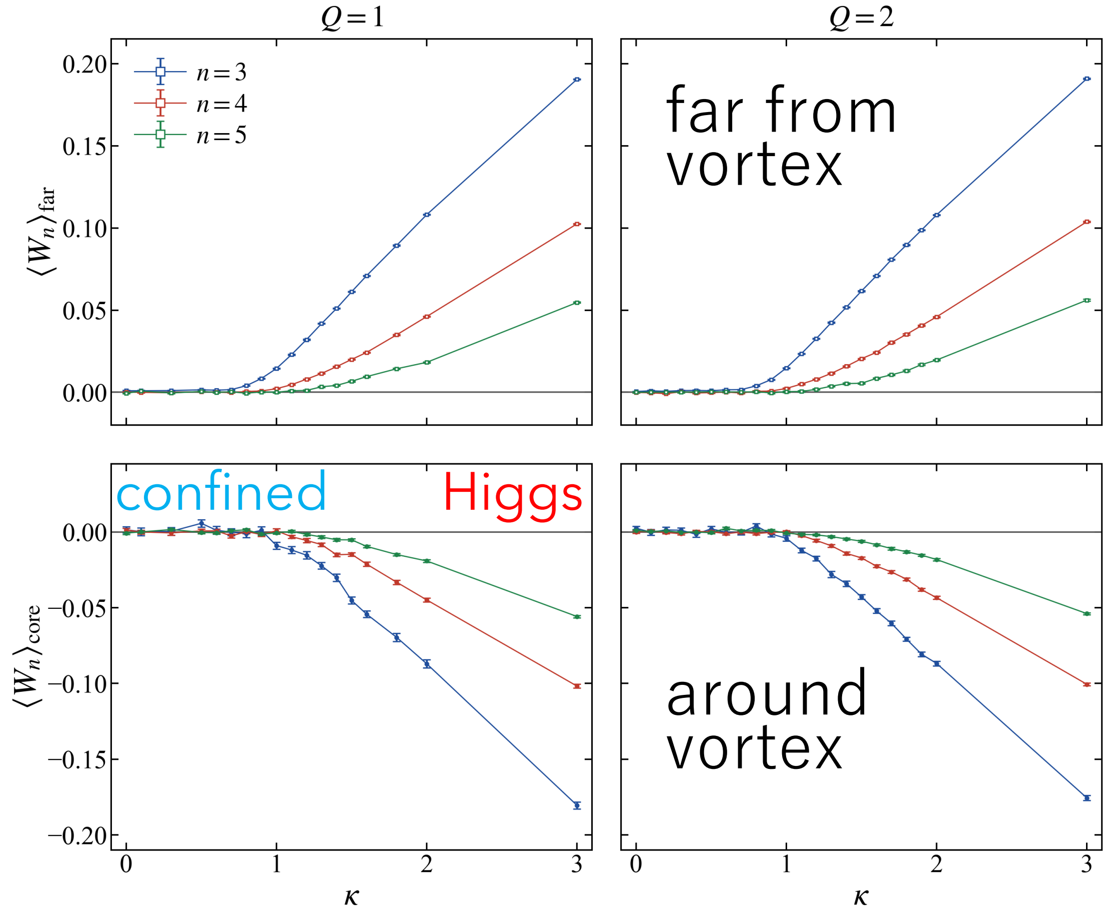
preliminary

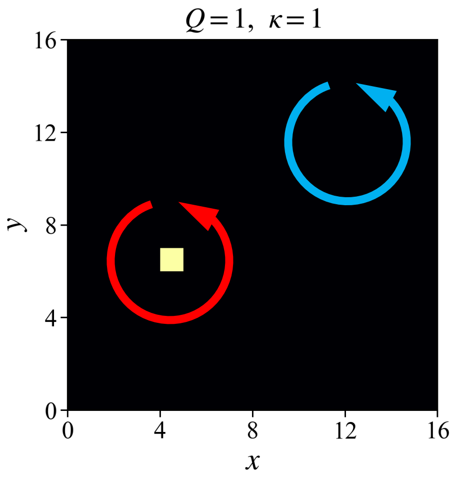
Wilson Loop around Vortices

Recent Question: Do the AB phase or the Wilson loop around vortices distinguish conf/Higgs regimes?

Wilson Loop around Vortices

Recent Question: Do the AB phase or the Wilson loop around vortices distinguish conf/Higgs regimes?





core/far Wilson loops are

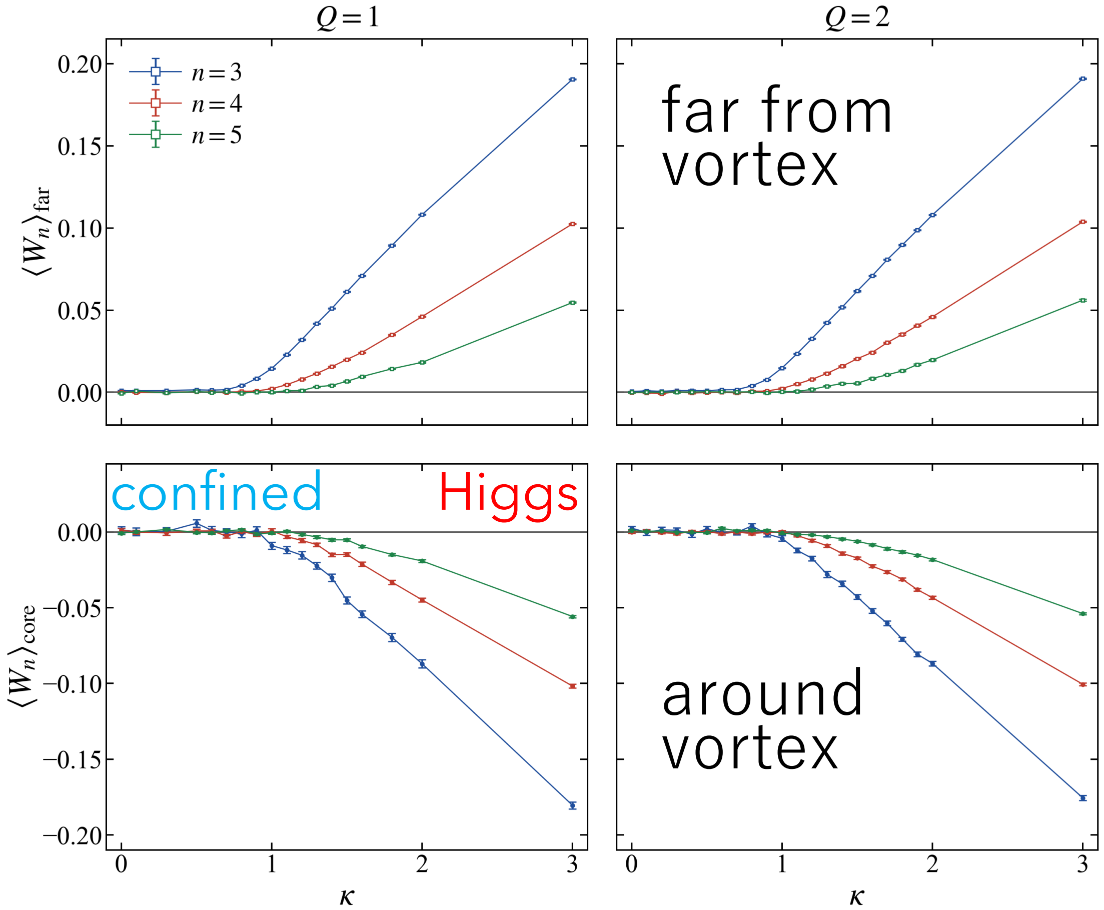
$$W_n = \prod U_l$$

for $n{\times}n$ loops.

preliminary

Wilson Loop around Vortices

Recent Question: Do the AB phase or the Wilson loop around vortices distinguish conf/Higgs regimes?



- Non-trivial AB phase is observed in Higgs regime.
- Toward the confined regime, Wilson loop decreases to zero smoothly – no jump.
- AB phase in confined regime is not accessible due to the severe area-law suppression.

preliminary

Summary

We studied Higgs-confinement continuity in $SU(2)$ gH model.

The model contains a two-flavor fundamental (CFL) Higgs, and U(1) breaking. Confined/Higgs ~ Hadron/CFL.

- **Bulk** : confined / Higgs regimes are connected by a crossover.

- **vortex** : NA vortex with a non-trivial AB phase is realized.

Detecting the change in AB phase requires alternative vortex-insertion or measurement techniques.

Changing κ_0, ϵ controls the vortex : small ϵ unlocks ϕ_0 and $\det \Phi$, and the CFL vortex splits into fractional vortices.

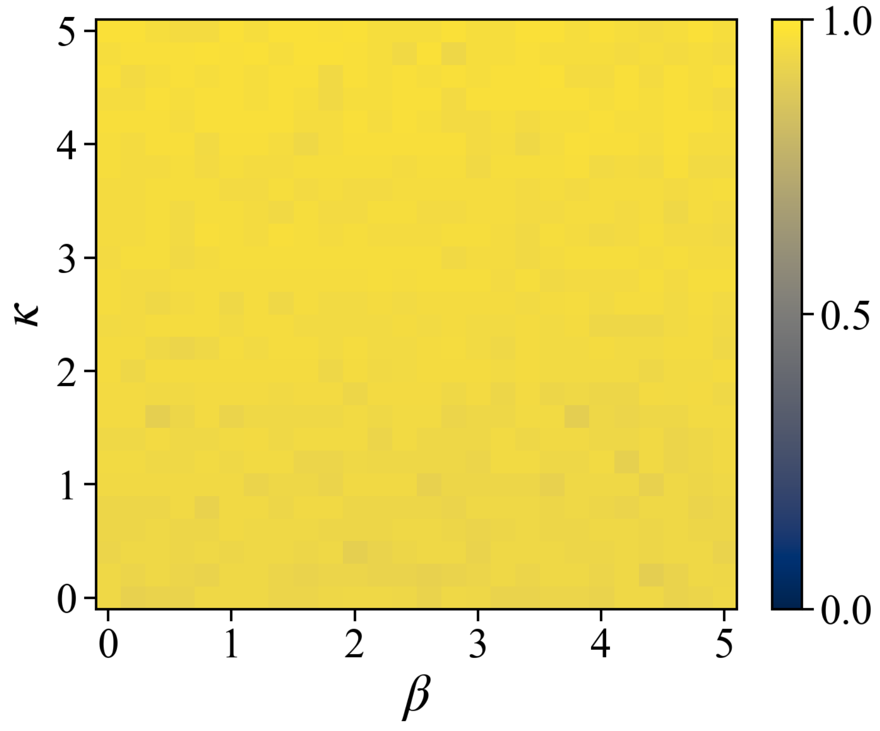
Thank you!

Appendix – U(1) Breaking Check

The phase of $\Phi(x)$ is calculated w/ our parameter $\kappa_0 = \epsilon = 10$.

In broken phase, $\sum_x \det\Phi(x)$ should have non-zero modulus.

$\rightarrow$ configuration average is also non-zero



Left: plot of $\langle |Y| \rangle$ at $L = 8$, $N_{\text{config}} = 100$,

$$Y := \frac{1}{V}\sum_x \det\Phi(x) \ .$$

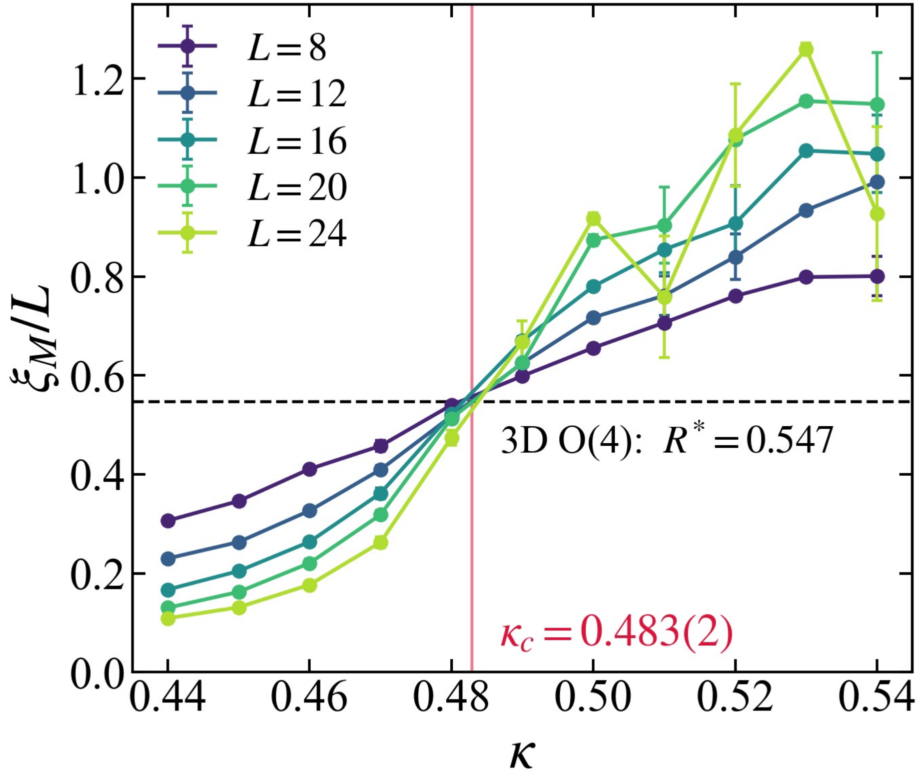
preliminary

Appendix – 2nd Order PT at $\beta = \infty$

At $\beta = \infty$, gauge field is frozen and global $SU(2)_c$ symmetry can be broken.

This is $SU(2)_c^{\text{global}} \times SU(2)_f \rightarrow SU(2)_{c+f}$ or $SO(4) \rightarrow SO(3)$.

With $\Phi(x) = e^{i\alpha} V(x), V(x) = v_0(x) + v(x) \cdot \sigma, M^a = L^{-3} \sum_x v^a(x)$.



ξ_M: correlation length of M^a:

$$G(k) = L^{-3} \sum_a \left| \sum_x v^a(x) e^{ik\cdot x} \right|^2$$

$$\xi_M = \frac{1}{2\sin(\pi/L)} \sqrt{\frac{\langle G(0) \rangle}{\langle G(k_{\min}) \rangle} - 1}$$

The crossing value $R^* = 0.547$.

This is 3D O(4).

preliminary