Toward a continuum limit determination of light meson charge radii with large-volume, physical point $N_f = 2 + 1$ lattice QCD

Kohei Sato
(Center for Computational Sciences, University of Tsukuba)

Hiromasa Watanabe (Keio University)
Takeshi Yamazaki (University of Tsukuba)
for PACS Collaboration

26 July – 1 Aug 2026
LATTICE 2026
University of Maryland, College Park

筑波大学
計算科学研究センター
Center for Computational Sciences

Program for Promoting Researches on the Supercomputer Fugaku

Large-scale lattice QCD simulation and development of AI technology

Target: π^+, K^+, K^0 charge radii

Toward a continuum limit determination of light meson charge radii with large-volume, physical point $N_f = 2 + 1$ lattice QCD

Data characteristics: Large volume and physical point

+

Analysis Method : without fit ansatz

sukuba)

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(mean-square) charge radius ··· a quantity that characterizes the structure of hadrons.
It represents the spread of the charge distribution.

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(input from lattice QCD)

form factor(output)

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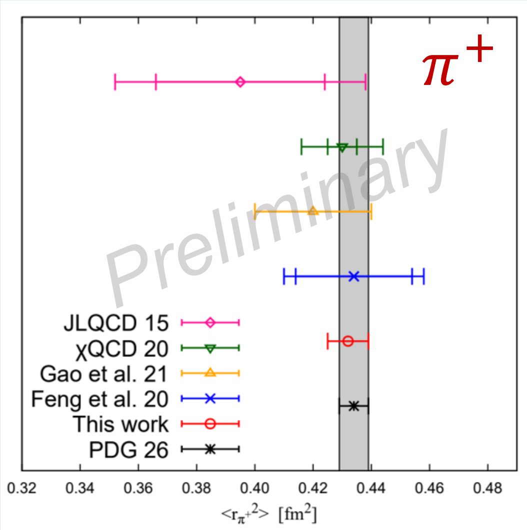
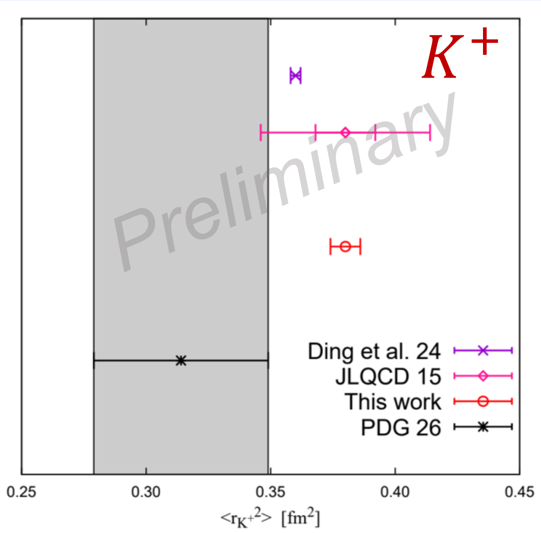
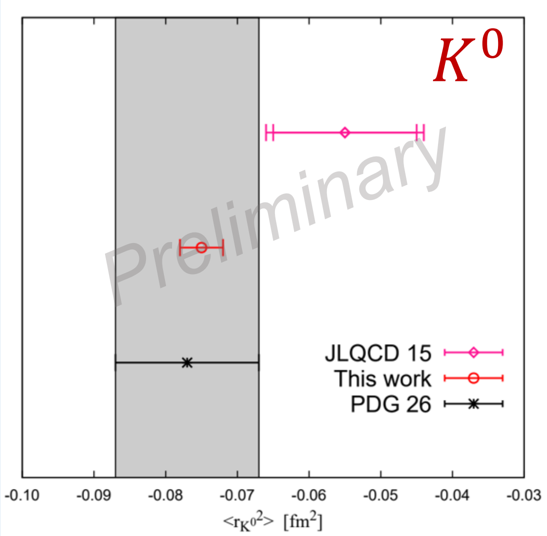
Why study the π and K meson charge radii ?

1. Establish reference values from lattice QCD calculations
2. Problems in dispersive analysis for the $e^+e^- \to \pi^+\pi^-$ experiment
3. AMBER experiment







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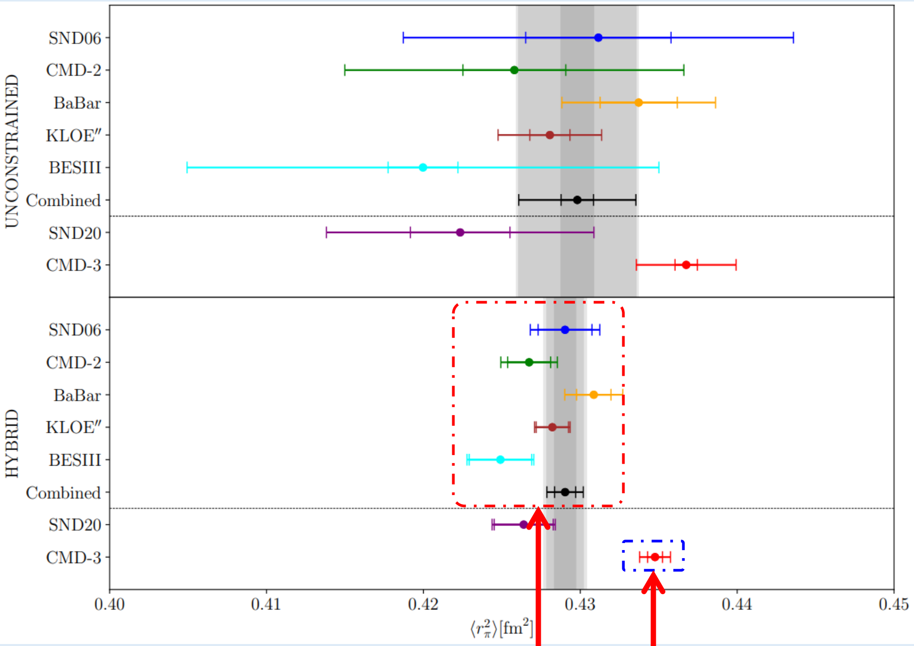
$$\sigma(e^+e^- \to \pi^+\pi^-)(s) = \frac{\pi\alpha^2}{3s}\sigma_\pi^3(s)\left|F_\pi^V(s)\right|^2$$

$$\langle r_\pi^2 \rangle = 6\frac{dF_\pi^V(s)}{ds}\bigg|_{s=0} = \frac{6}{\pi}\int_{4M_\pi^2}^{\infty} ds \frac{\mathrm{Im}\,F_\pi^V(s)}{s^2}$$

charge radius

R. Aliberti et al., Physics Reports 1143 (2025) 1-158



Inconsistencies between the experiments

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AMBER
Apparatus for Meson and Baryon Experimental Research

Bernhard Ketzer, " AMBER - First results and future plans", EINN2025

Phase-1: 2023 → 2031 (conventional beams)

- $\bar{p}$ production cross section measurements for DM searches (pbarX)
- Proton radius: high-energy $\mu - p$ elastic scattering (PRM)
- Pion and kaon quark PDFs: Drell-Yan processes (DY)

Phase-2: 2031 → 2041 (high-intensity kaon beam)

- K quark and gluon PDFs (DY, prompt photons)
- Strange meson spectroscopy
- Meson charge radii
- Meson-photon reactions in Primakoff kinematics

AMBER Collaboration, Jan Friedrich,"Hadron research with AMBER at CERN", Rev.Mex.Fis.Suppl. 3 (2022) 3, 0308009.

→ Precise measurement of the K meson charge radius is planned

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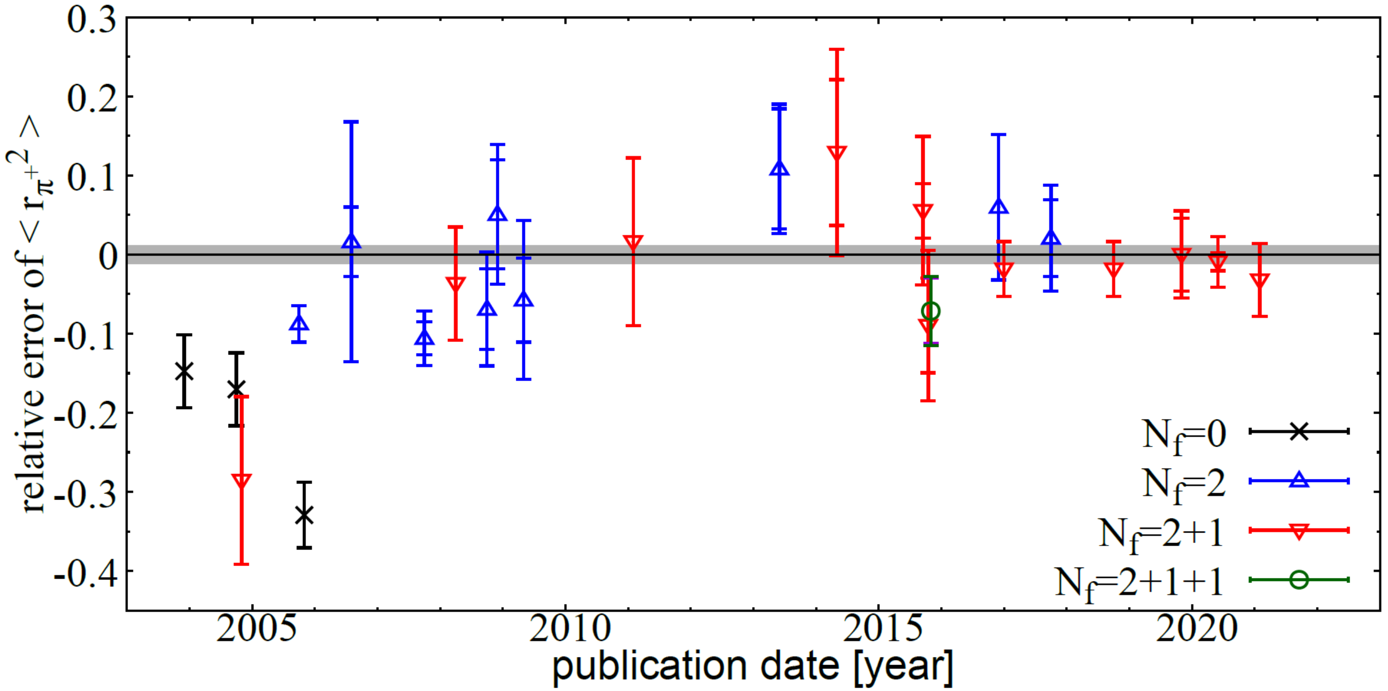

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  Recently : consistent
- ✓ Error : Lattice > Experimental
  There are 4 main systematic errors
  - ➢ Chiral extrapolation
  - ➢ Continuum extrapolation
  - ➢ Finite volume effect
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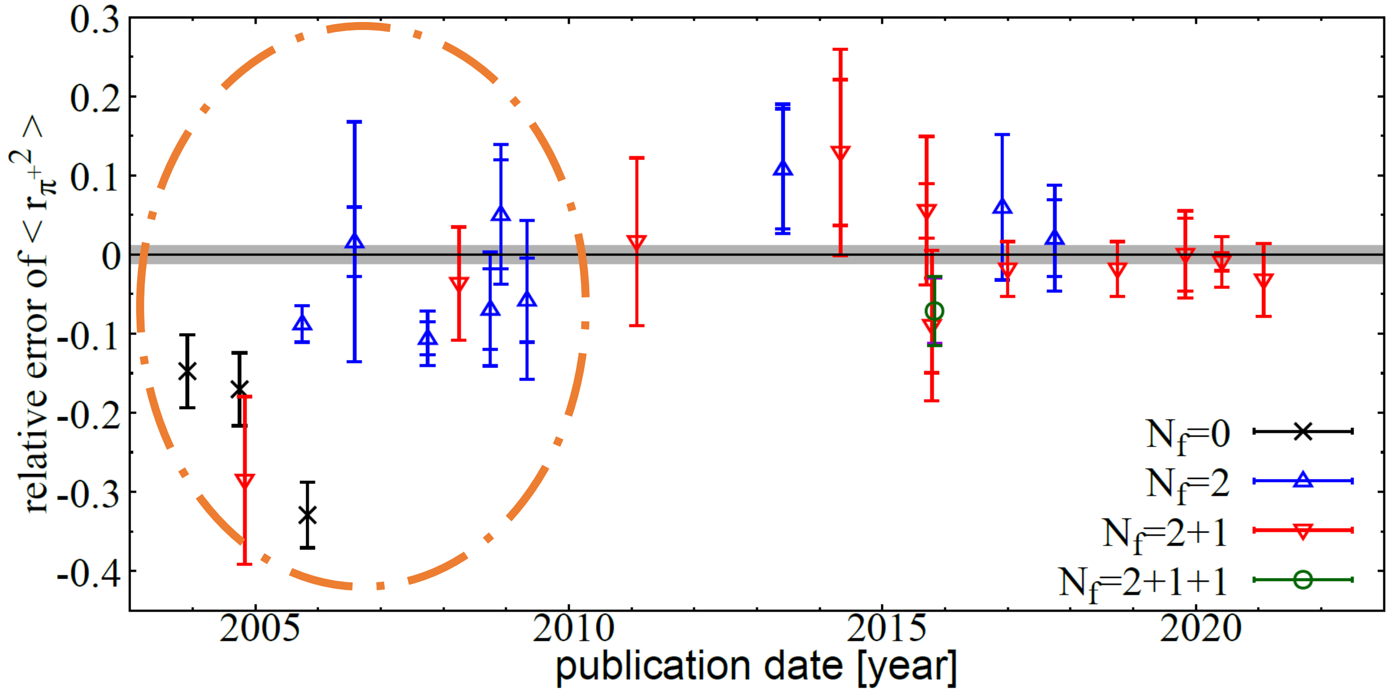

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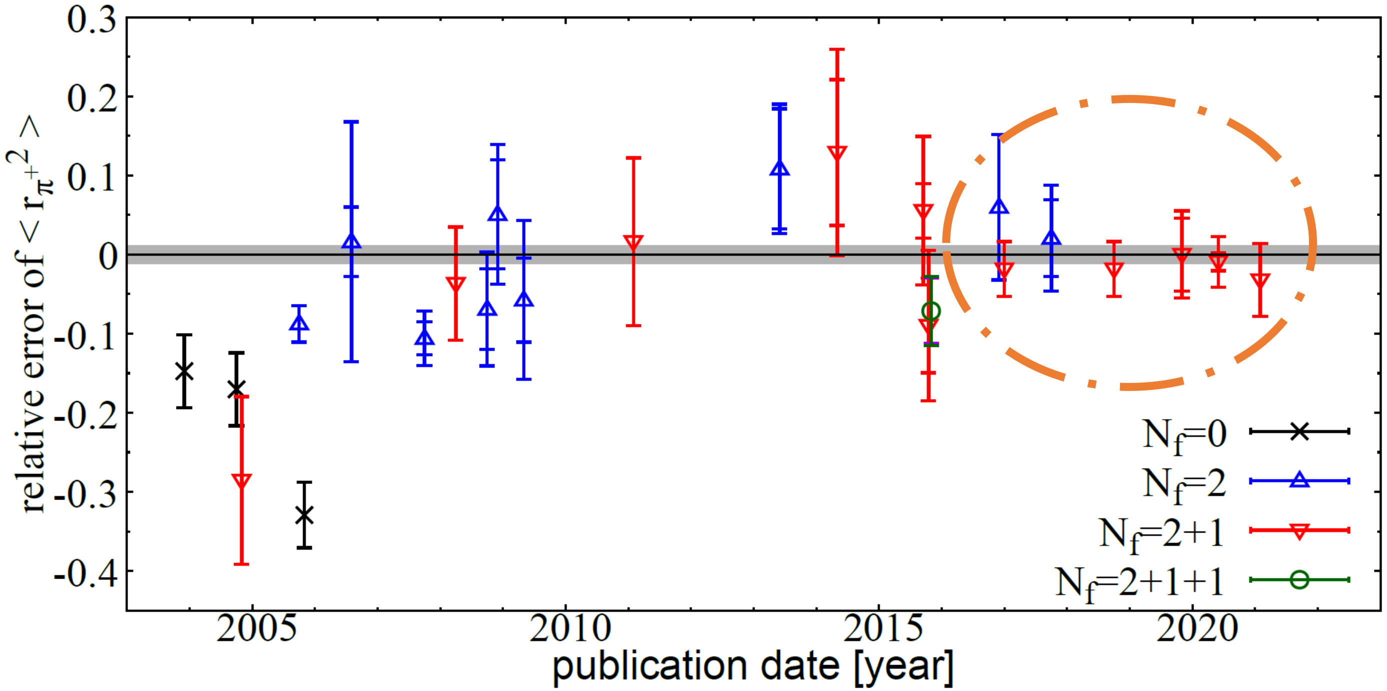

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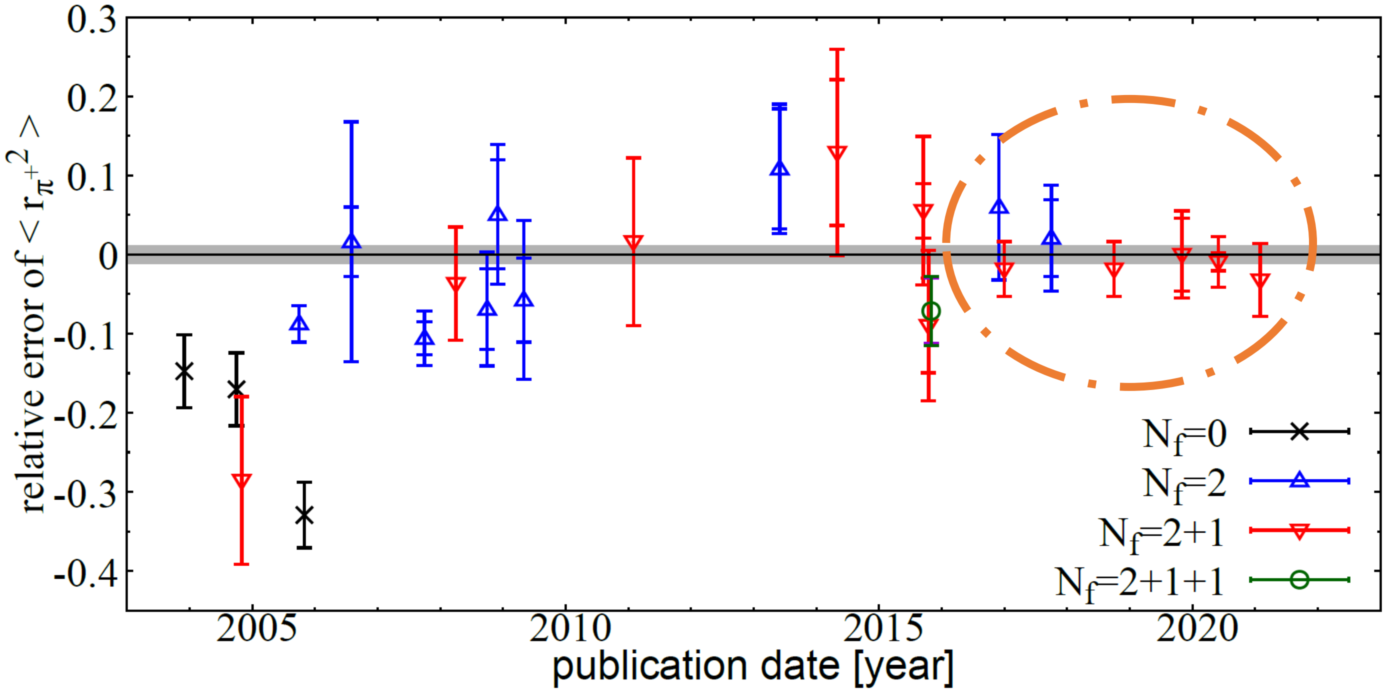

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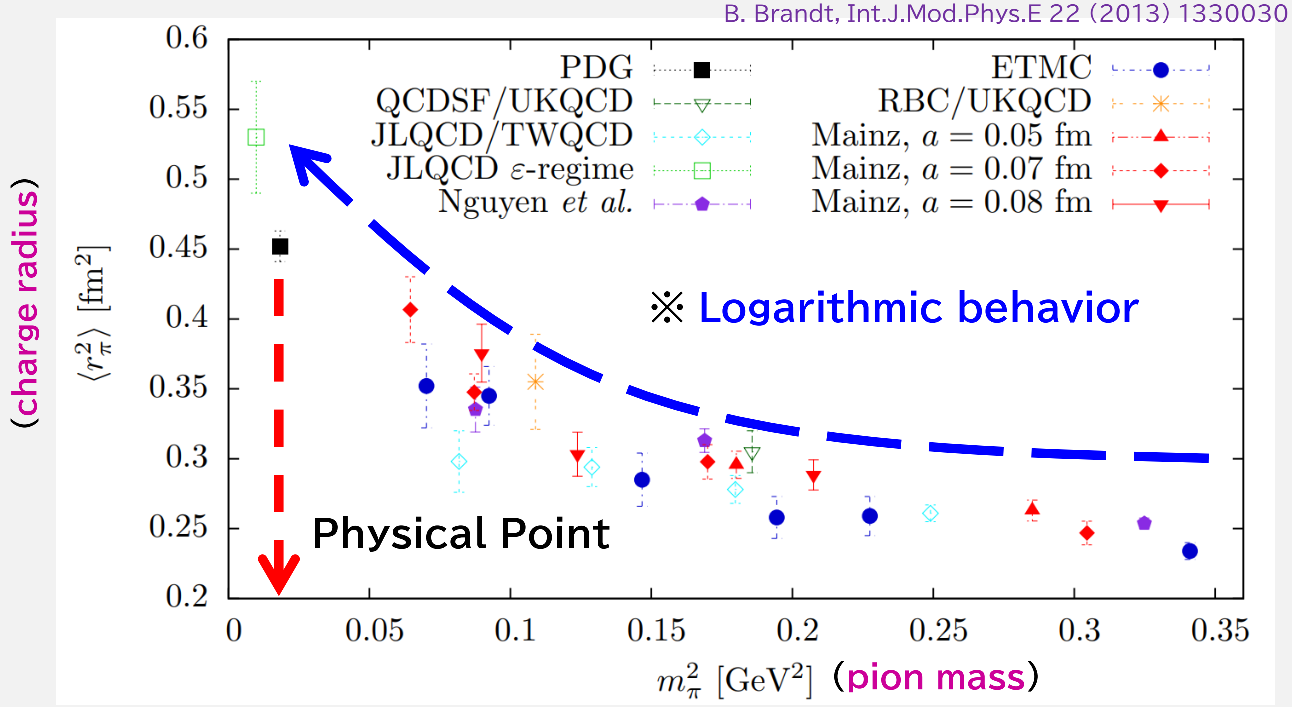
By changing theoretical parameters, we can compute physics in various worlds.

B. Brandt, Int.J.Mod.Phys.E 22 (2013) 1330030



- ✓ To reproduce the real world, we must use the parameters that correspond to our physical universe.
- ✓ Use chiral extrapolation to obtain values at the physical point.

relative error of $< r_{\pi^+}{}^2 >$
-0.4
2005 2010 2015 2020
publication date [year]

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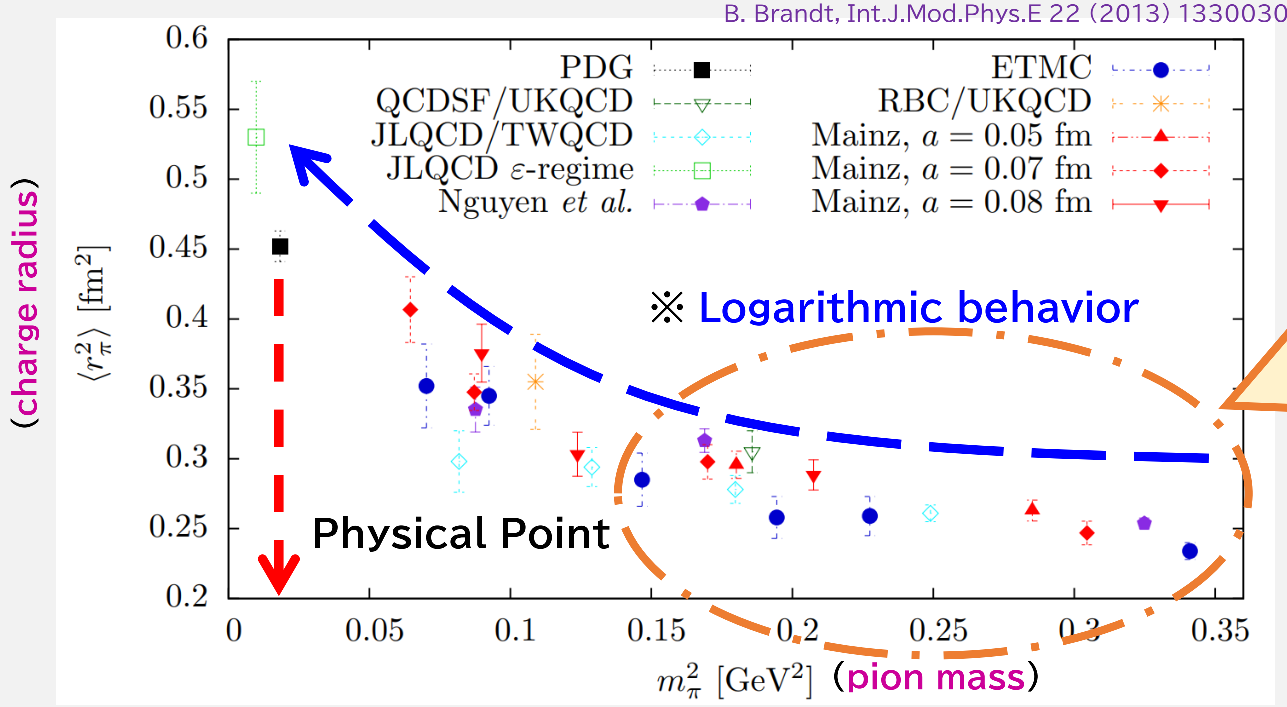

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Earlier lattice QCD calculations

Many simulations were performed at heavier pion masses

We needed to extrapolate these results to the physical point using chiral extrapolation

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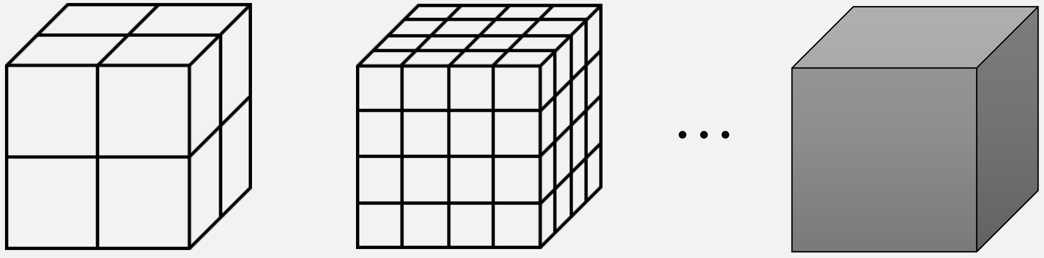
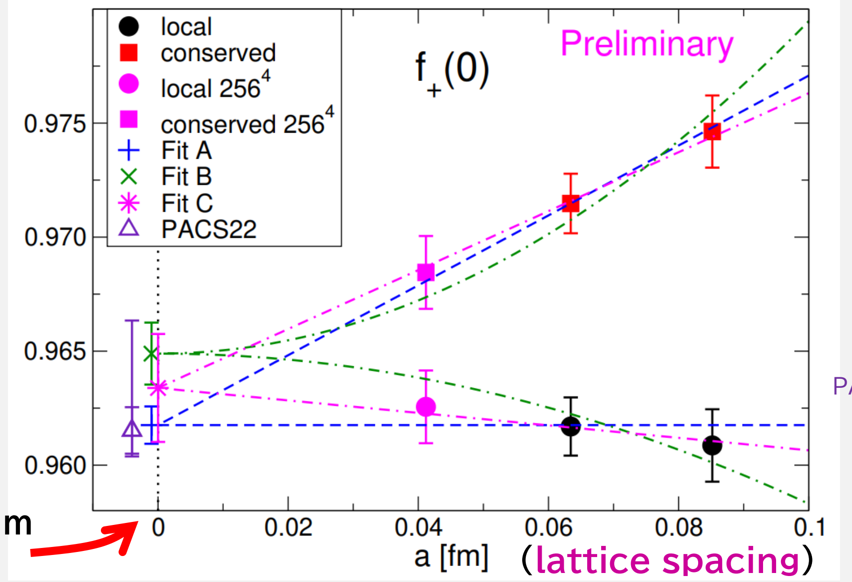
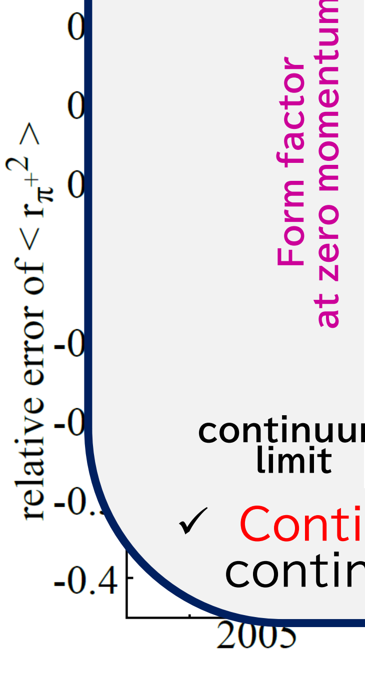
Lattice calculations discretize spacetime.





PACS, PoS LATTICE2023 (2024) 276

✓ Continuum extrapolation derives a value at continuum limit from discrete data.



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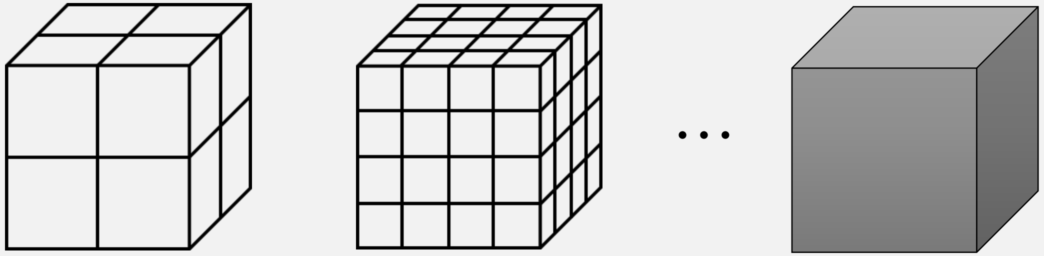
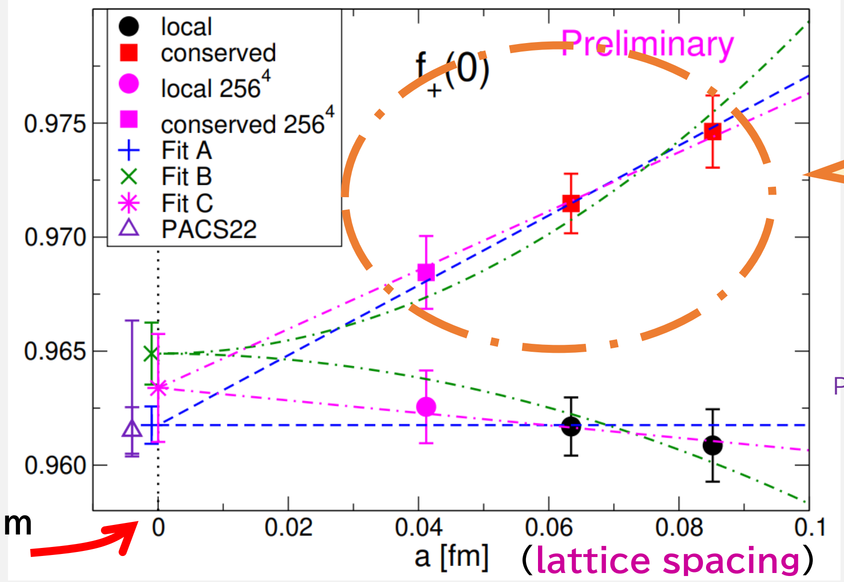


Coarse lattice

Fine lattice

Continuum
(real world)



PACS, PoS LATTICE2023 (2024) 276

✓ Continuum extrapolation derives a value at continuum limit from discrete data.

By the discretization of space-time

Values change depending on the lattice spacing

We need to calculate at multiple lattice spacings and then extrapolate

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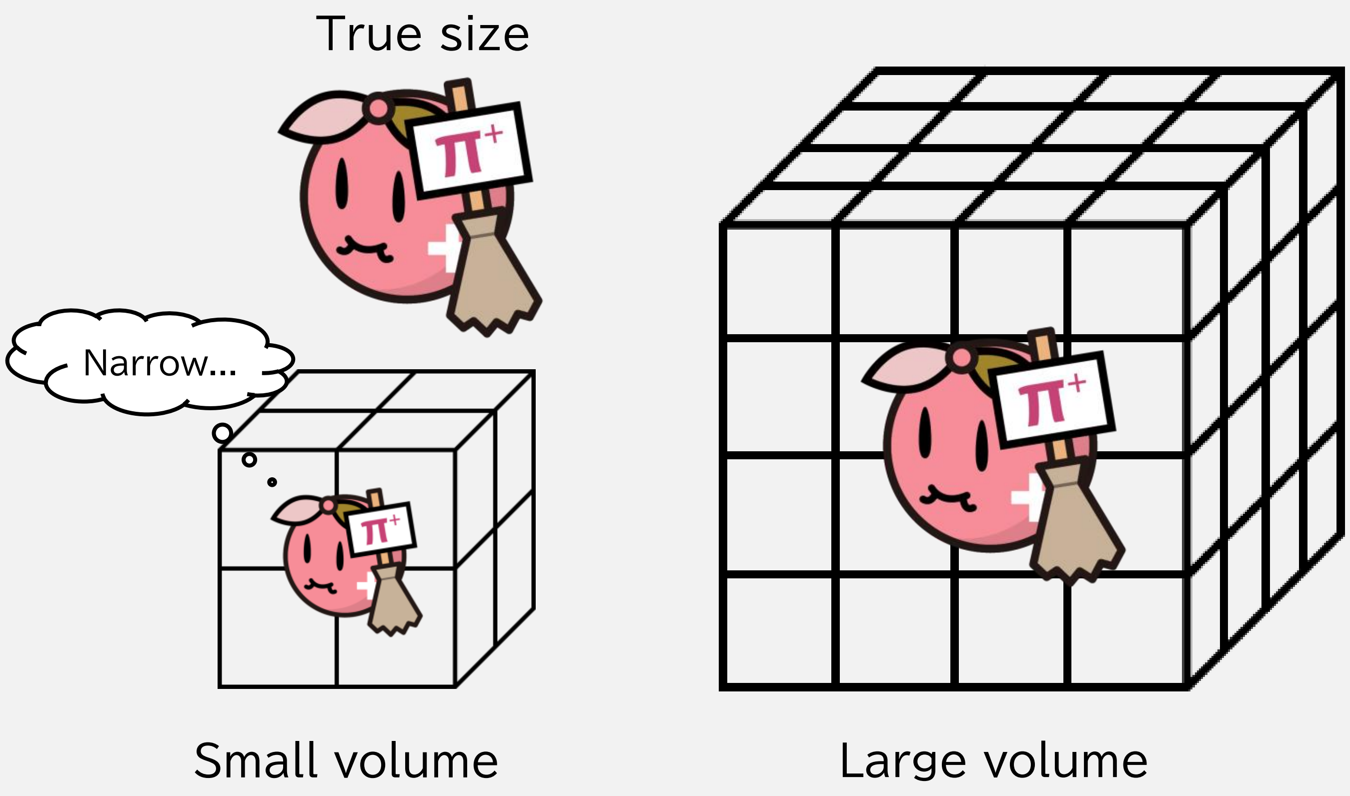

✓ Finite-volume effects from small volume.

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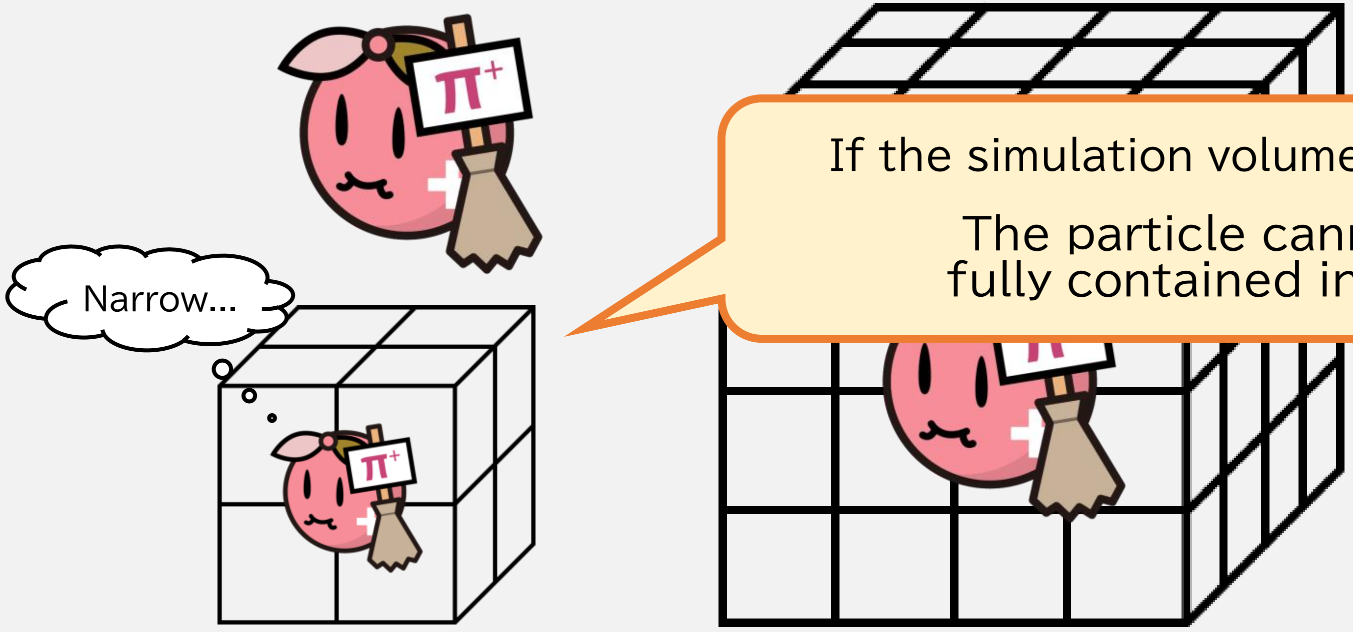
True size



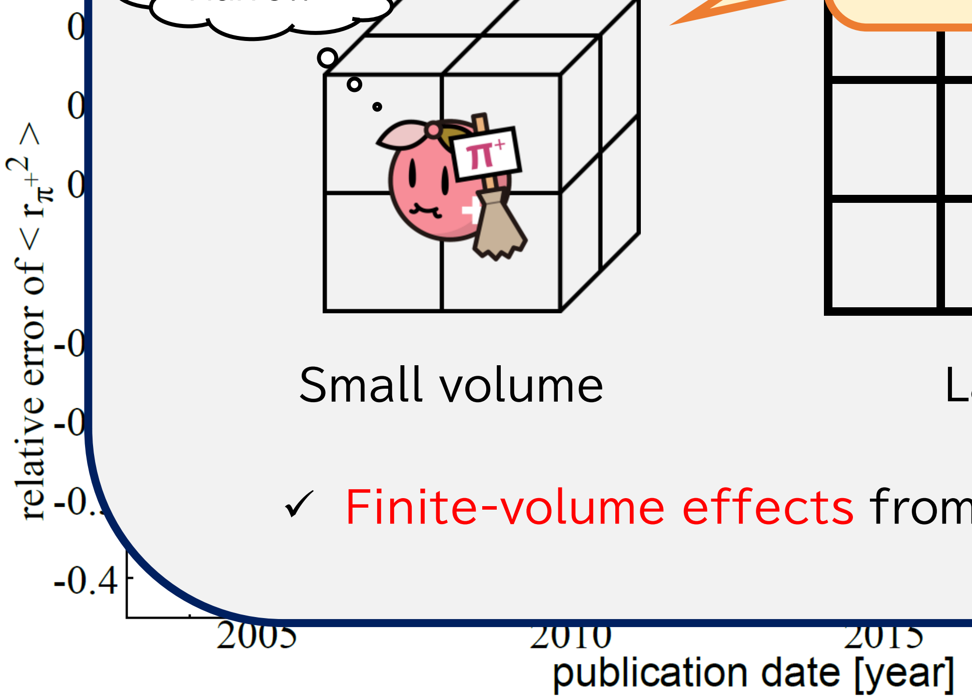
If the simulation volume is too small

The particle cannot be fully contained in space

Small volume

Large volume

✓ Finite-volume effects from small volume.



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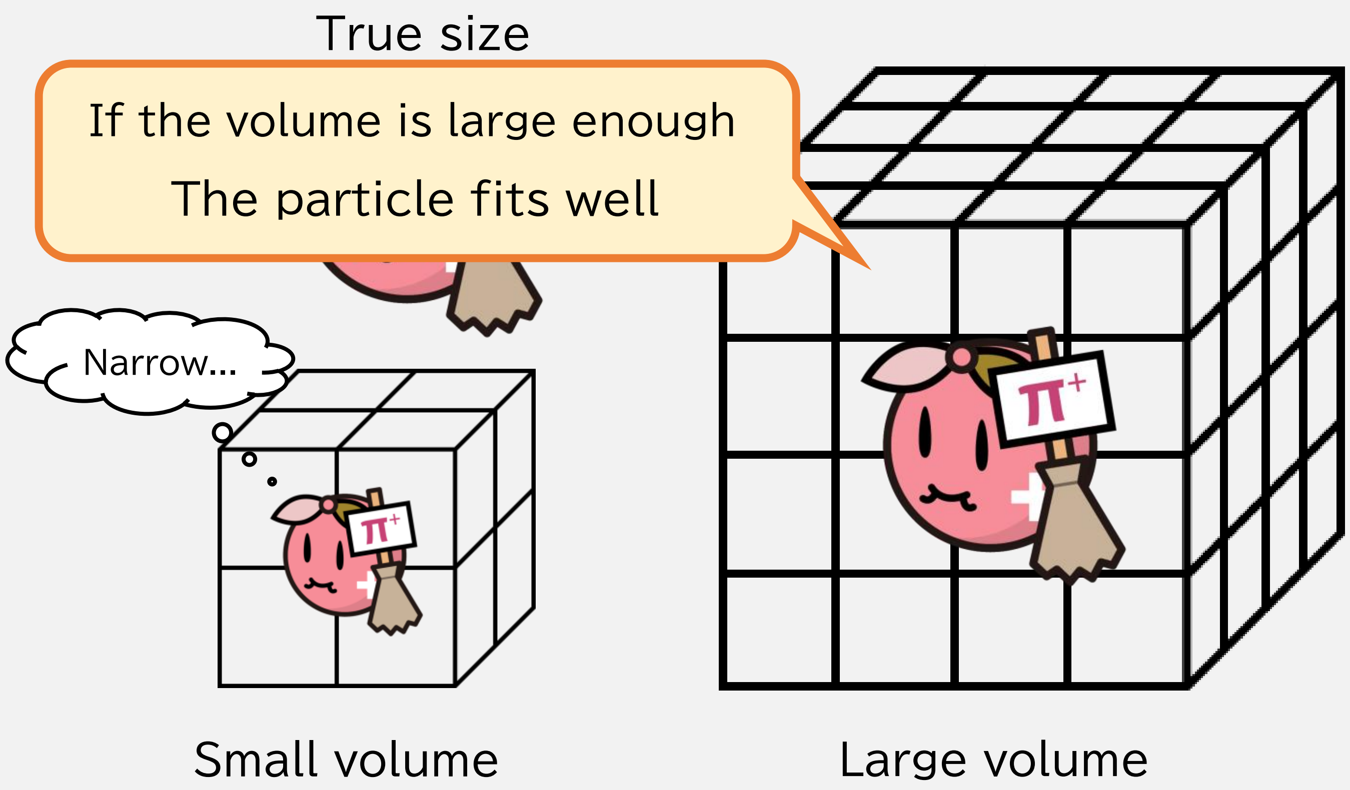

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PACS10 configurations

…Large-volume configurations at physical point
(3 lattice spacings).

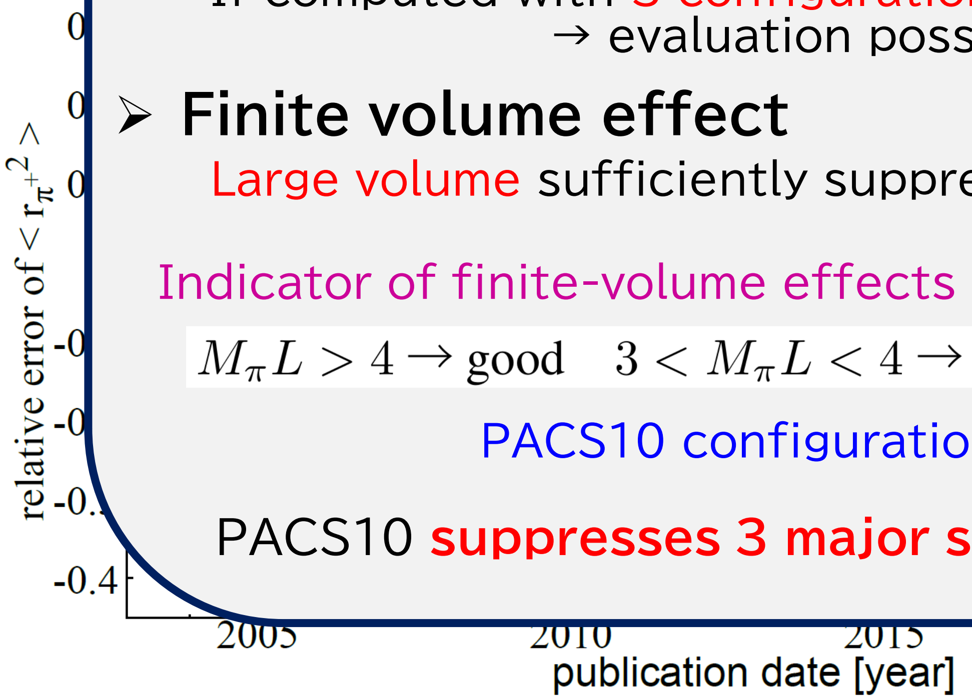
- **Chiral extrapolation**
  At physical point → almost unnecessary → good!!
  Hadron masses reproduced within 5% error
- **Continuum extrapolation**
  If computed with 3 configurations
  → evaluation possible → good!!
- **Finite volume effect**
  Large volume sufficiently suppresses effects → good!!

Indicator of finite-volume effects

$M_\pi L > 4 \rightarrow \text{good} \quad 3 < M_\pi L < 4 \rightarrow \text{soso} \quad M_\pi L < 3 \rightarrow \text{bad}$

PACS10 configurations ~ 7

PACS10 **suppresses 3 major systematic errors.**



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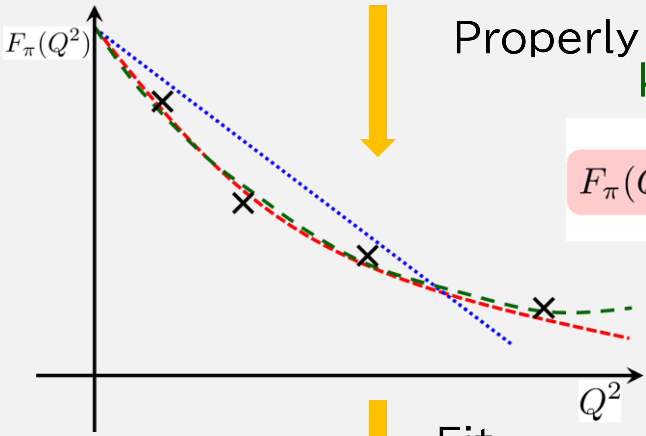
Systematic errors from traditional analysis methods

3-pt functions (lattice QCD)

Known function

$$\frac{\sum_{\vec{x}_{\rm sink},\vec{x}} \langle \pi^+(\vec{x}_{\rm sink}, t_{\rm sink}) V_4(\vec{x},t) \pi^+(0)^\dagger \rangle e^{-ipx_1}}{\sum_{\vec{x}_{\rm sink},\vec{x}} \langle \pi^+(\vec{x}_{\rm sink}, t_{\rm sink}) V_4(\vec{x},t) \pi^+(0)^\dagger \rangle} = \frac{E_\pi(p) + m_\pi}{2E_\pi(p)} e^{-(E_\pi(p)-m_\pi)t} F_\pi(Q^2)$$

Form factors (desired quantity)



Properly combine lattice data and known functions.

$$F_\pi(Q^2) = \frac{\text{Input from lattice calculation}}{\text{Known function}}$$

Fit

$$\langle r_\pi^2 \rangle = -6 \frac{\mathrm{d}}{\mathrm{d}Q^2} F_\pi(Q^2) \Big|_{Q^2=0}$$

Fit ansatz

$$F_\pi(Q^2) = \begin{cases} \dfrac{1}{1+Q^2/M_{\rm pole}^2} \\ 1 + \sum_k f_k (Q^2)^k \\ \sum_k a_k z^k \quad \left(z := \dfrac{\sqrt{4m_\pi^2 + Q^2} - \sqrt{4m_\pi^2}}{\sqrt{4m_\pi^2 + Q^2} + \sqrt{4m_\pi^2}} \right) \\ 1 + \dfrac{1}{f_\pi^2} [2l_6^r(\mu) Q^2 + 4H(m_\pi^2, Q^2, \mu^2)] \end{cases}$$

✓ The fit ansatz error arises from the choice of the fitting function and the fitting range.

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2005, 2010, 2015, 2020

publication date [year]

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model-independent method

✓ Key idea: **Calculate spatial moment**

U. Aglietti et al., Phys.Lett.B324,85(1994); UKQCD, Nucl.Phys.B444,401(1995) ; C. Bouchard et al., PoS LATTICE2016,170(2016); PACS, Phys.Rev.D104, 074514(2021)

➢ Differentiation of Fourier transform

$$\left.\frac{\mathrm{d}\tilde{C}(p)}{\mathrm{d}p^2}\right|_{p^2=0} = \left.\frac{\mathrm{d}}{\mathrm{d}p^2}\sum_x C(x)e^{-ipx}\right|_{p^2=0} = -\frac{1}{2}\sum_x x^2 C(x)$$

($N_{space} \to \infty$, a :finite ; $L = N_{space} a \to \infty$)

infinite volume limit

In a finite volume, higher-order contamination appears.

$$\sum_x x^2 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

$$\sum_x x^4 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

Xu Feng et al., Phys.Rev.D101,051502(R)(2020)

add these higher-order moments to reduce the contamination

$$R(t) = \alpha_1 (x^2\text{moment}) + \alpha_2 (x^4\text{moment}) = \left.\frac{\mathrm{d}}{\mathrm{d}Q^2}F_\pi(Q^2)\right|_{Q^2=0} + (f_3 + f_4 + \cdots)$$

α_1, α_2: parameters which set to cancel out the contamination

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3pt function
(input from lattice QCD)

form factor(output)

(mean-square) charge radius … a quantity that characterizes the structure of hadrons.
It represents the spread of the charge distribution.

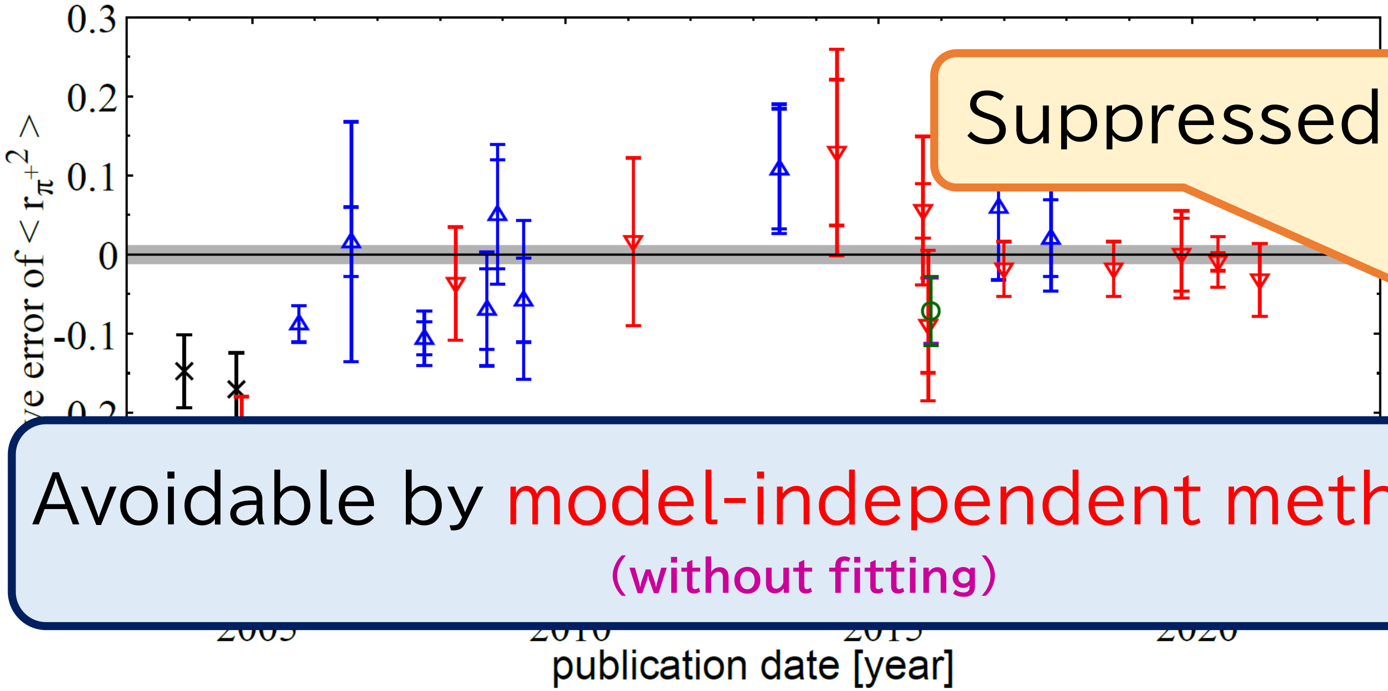
$$\langle r_\pi^2 \rangle = -6 \frac{\mathrm{d}}{\mathrm{d}Q^2} \underbrace{F_\pi(Q^2)}\Big|_{Q^2=0}$$

electromagnetic form factor

$$\boxed{\langle \pi^+(p_f) | V_\mu | \pi^+(p_i) \rangle} = (p_f + p_i)_\mu F_\pi(Q^2)$$

electromagnetic current: $V_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f$

Momentum transfer : $Q^2 = -(p_f - p_i)^2 \geq 0$



< Initially : large difference

Suppressed with PACS10 configurations

or : Lattice > Experimental

ere are 4 main systematic errors

- Chiral extrapolation
- Continuum extrapolation
- Finite volume effect
- Fit ansatz

Avoidable by model-independent method
(without fitting)

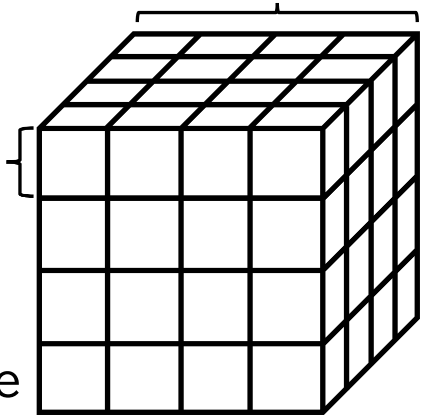
Simulation parameters

✓ Gauge configuration (PACS, PRD 99, 014504 (2019); PACS, PRD 106, 094505 (2022); PACS, PRD 109, 094505 (2024); PoS LATTICE2024 (2025) 318)

PACS10 configuration

$N_f = 2+1$ six-stout-smeared non-perturbative $O(a)$-improved Wilson action+ Iwasaki gauge action

β	$L^3 \times T$	L [fm]	a [fm]	a^{-1} [GeV]	m_π [MeV]	m_K [MeV]	N_{conf}	N_{meas}
2.20	256^4	10.5	0.041	4.8	142	514	20	$\mathcal{O}(500)$
2.00	160^4	10.1	0.063	3.1	138	506	20	$\mathcal{O}(4500)$~$\mathcal{O}(14000)$
1.82	128^4	10.9	0.085	2.3	136	499	20	$\mathcal{O}(4500)$~$\mathcal{O}(14000)$



Lattice size $128^4 \sim 256^4$

- Chiral extrapolation
  → Physical point
- Continuum extrapolation
  → 3 lattice spacings
- Finite volume effects
  → Large volume

✓ Measurement parameter

- 20 config. , 500~14000 meas. Per config.
- Periodic + anti-periodic correlation functions in time direction
- Forward + Backward correlation functions in time direction
- $|t_{sink} - t_{source}| = 3\text{~}4$ fm

→ ↑ Statistics
Reduce the wrapping-around effect
Systematic errors can be evaluated

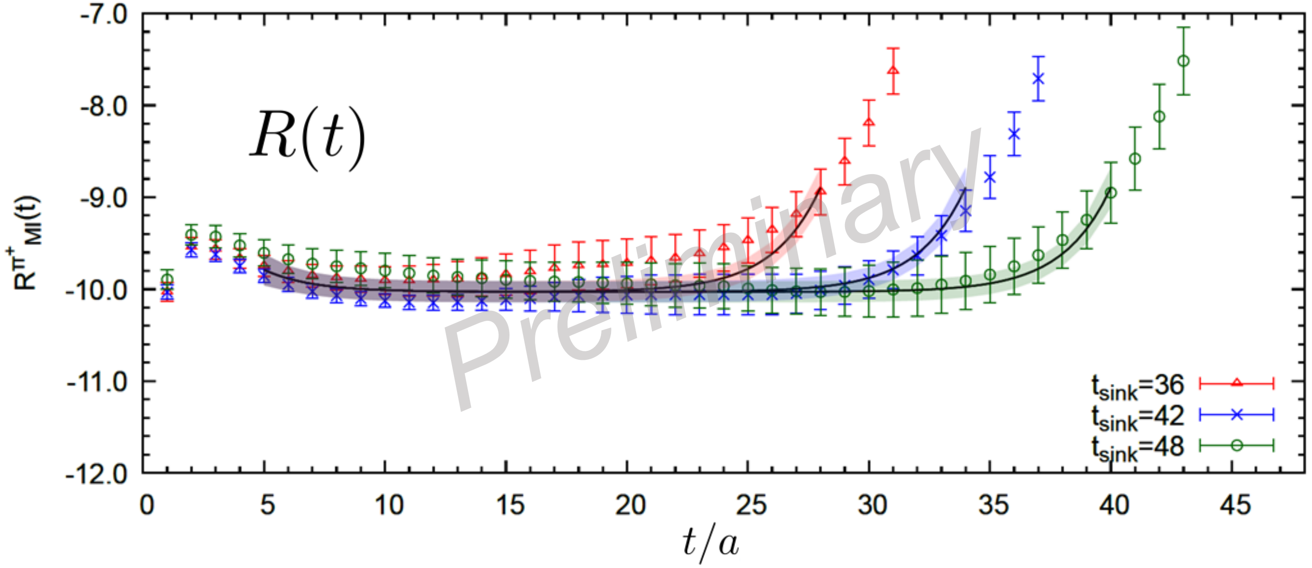
Result

✓ Model-independent method
(without fit ansatz)

$$\sum_x x^2 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

$$\sum_x x^4 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

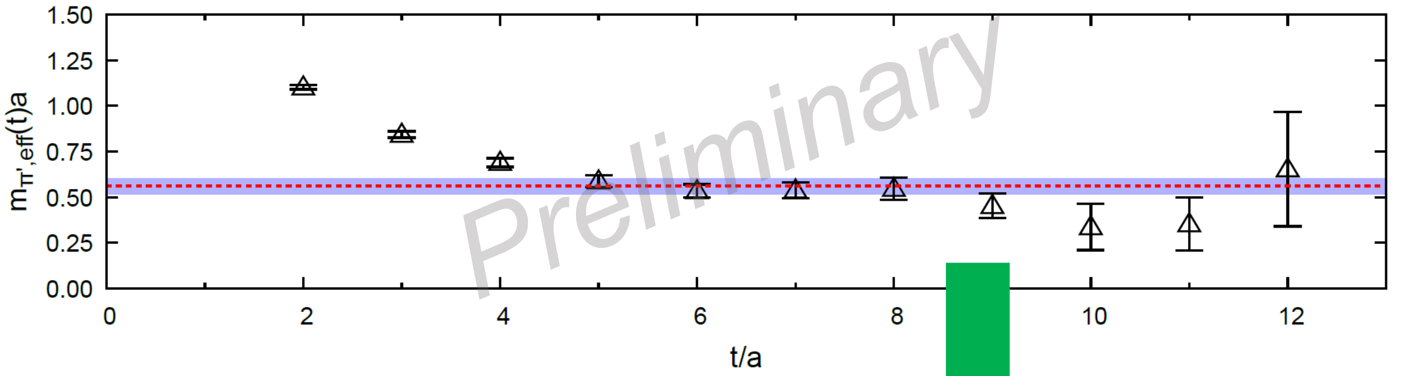
$$R(t) = \alpha_1 \,(x^2\text{moment}) + \alpha_2\,(x^4\text{moment})$$
$$= \frac{\mathrm{d}}{\mathrm{d}Q^2} F_\pi(Q^2)\Big|_{Q^2=0} + (f_3 + f_4 + \cdots)$$



Result

- ✓ Extraction including the first excited state

$$f(t;t_{\rm sink}) = \left.\frac{\mathrm{d}}{\mathrm{d}Q^2}F_{\pi^+}(Q^2)\right|_{Q^2=0} + A\,e^{-(m_{X'}-m_X)t} + B\,e^{-(m_{X'}-m_X)(t_{\rm sink}-t)}$$

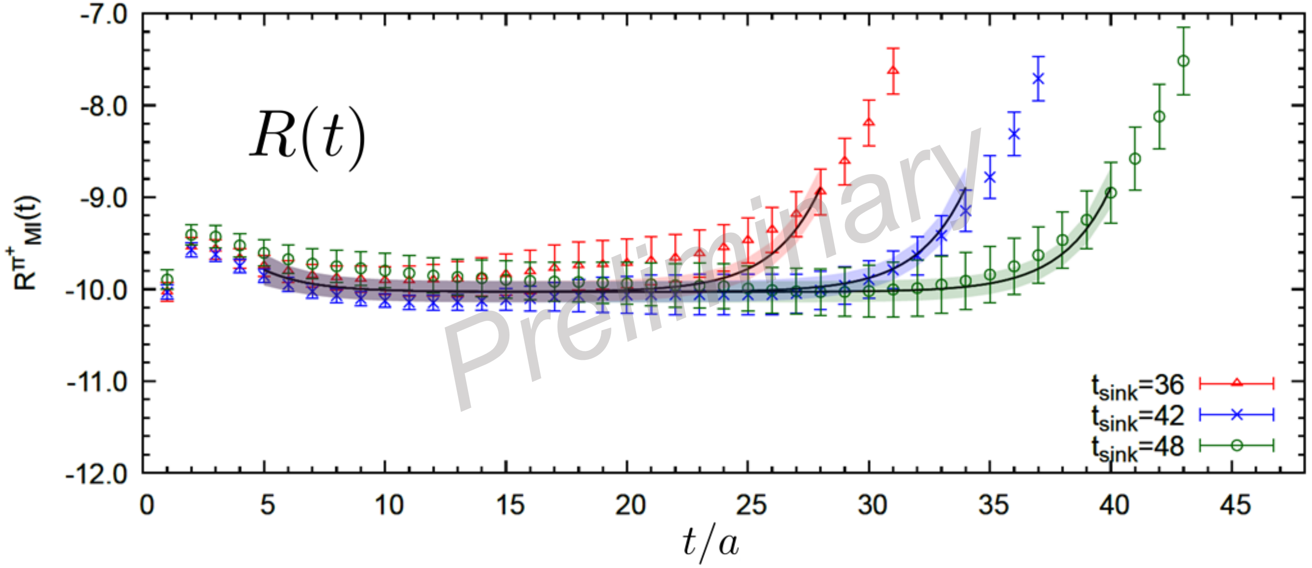


- ✓ Model-independent method (without fit ansatz)

$$\sum_x x^2 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

$$R(t) = \alpha_1\,(x^2\text{moment}) + \alpha_2\,(x^4\text{mo}\ldots\text{men}\ldots)$$

$$= \left.\frac{\mathrm{d}}{\mathrm{d}Q^2}F_\pi(Q^2)\right|_{Q^2=0} + (f_3 + f_4 + \cdots)$$



- ✓ From the plateau, we can see that the PACS10 configurations reproduce the experimental mass of the first excited state.

- ✓ We use experimental value as the mass of the first excited state

Result

✓ Model-independent method (without fit ansatz)

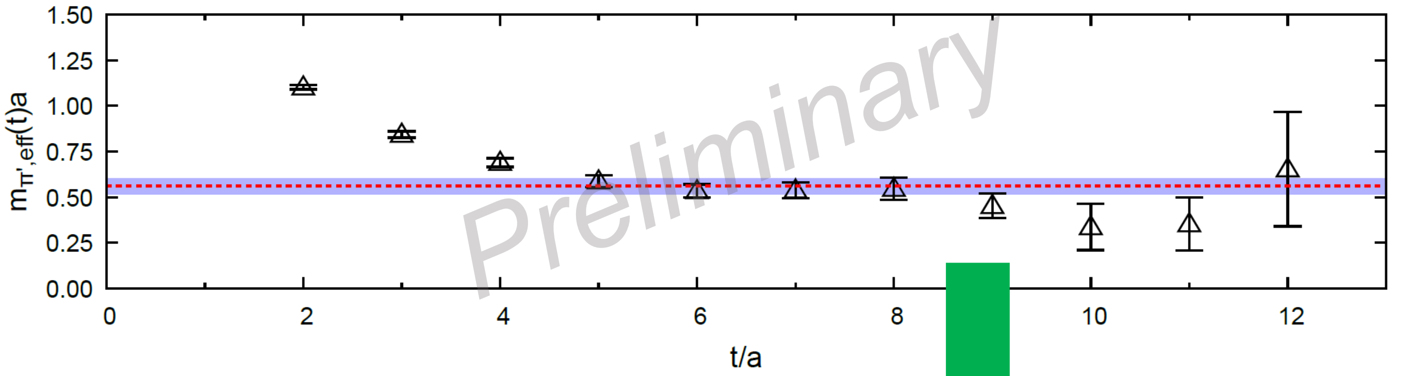
$$\sum_x x^2 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

$$R(t) = \alpha_1 \, (x^2\text{moment}) + \alpha_2 \, (x^4\text{m}$$

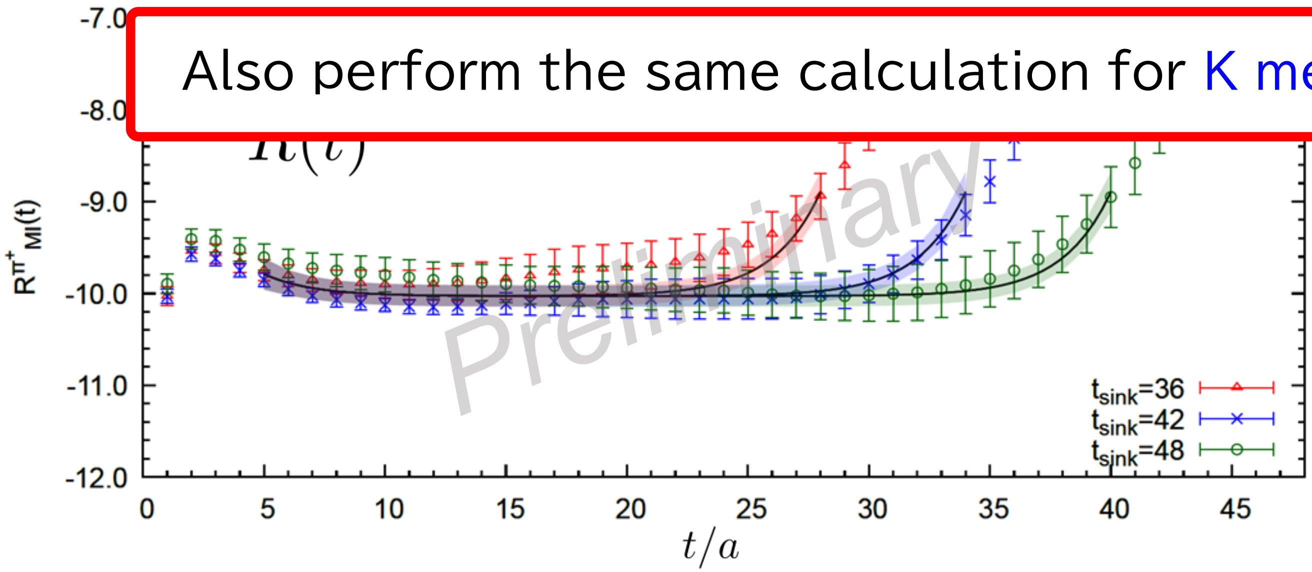
$$= \frac{\mathrm{d}}{\mathrm{d}Q^2} F_\pi(Q^2)\Big|_{Q^2=0} + (f_3 + f_4 + \cdots)$$

✓ Extraction including the first excited state

$$f(t; t_{\rm sink}) = \frac{\mathrm{d}}{\mathrm{d}Q^2} F_{\pi^+}(Q^2)\Big|_{Q^2=0} + A\, e^{-(m_{X'}-m_X)t} + B\, e^{-(m_{X'}-m_X)(t_{\rm sink}-t)}$$



Also perform the same calculation for K mesons and L160/L256 configuration.

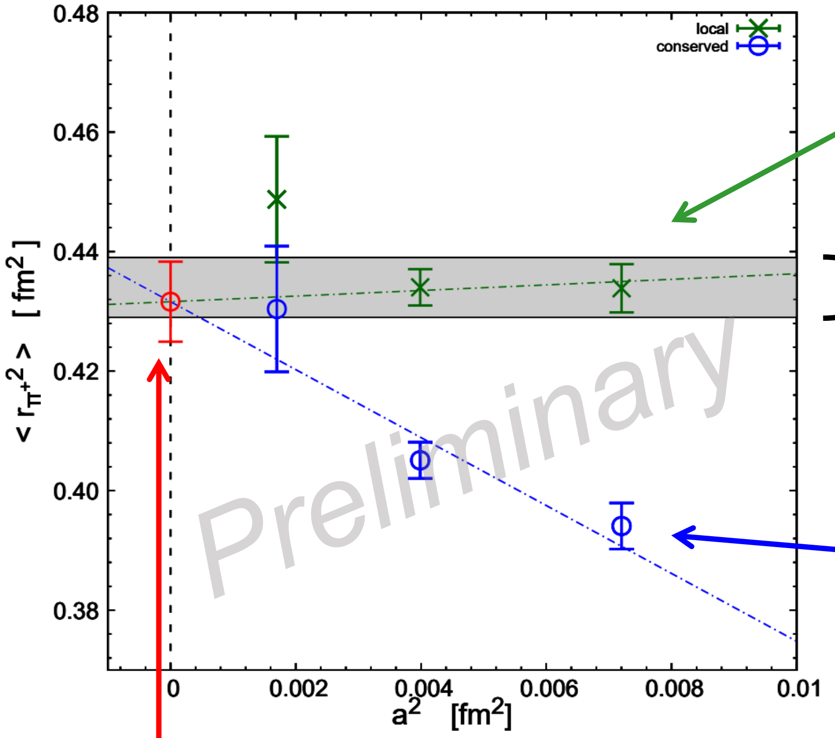


state.

✓ We use experimental value as the mass of the first excited state

Preliminary results

✓ π^+ charge radius



local vector current

$$V_4^{\mathrm{loc}}(x) = \bar{\psi}(x)\gamma_4\psi(x)$$

$$\langle \pi^+(p_f)|V_\mu|\pi^+(p_i)\rangle = (p_f + p_i)_\mu F_\pi(Q^2)$$

electromagnetic current: $V_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f$

Momentum transfer : $Q^2 = -(p_f - p_i)^2 \geq 0$

} PDG ($e + \pi \rightarrow e + \pi$ scattering)

point splitting conserved vector current

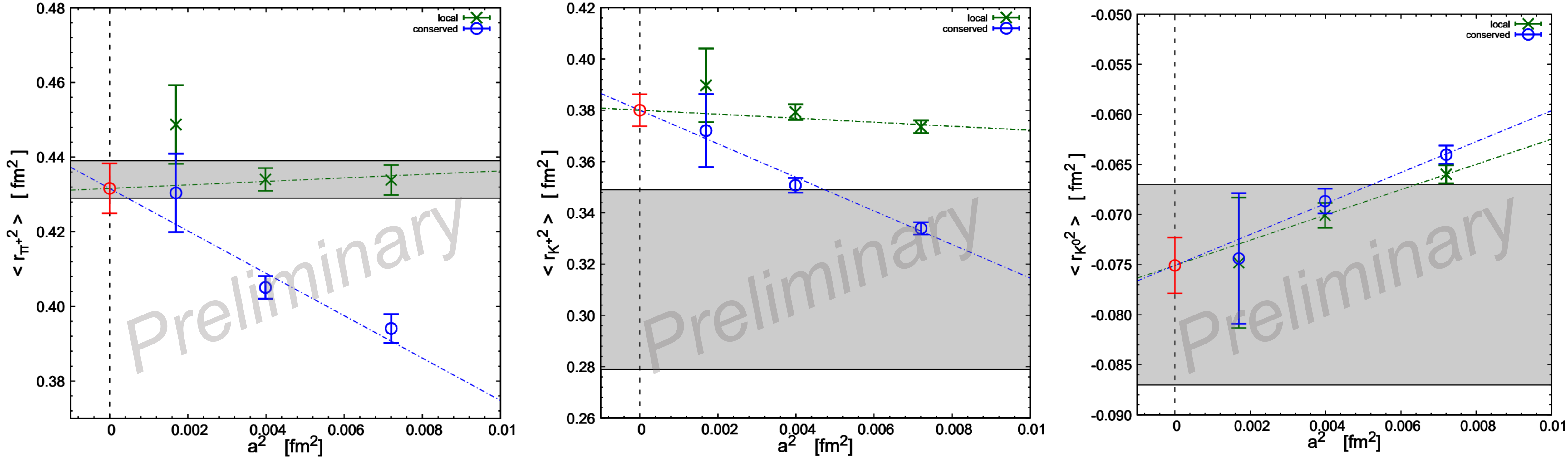
$$V_4^{\mathrm{cons}}(x) = \frac{1}{2}\left[\bar{\psi}(x+\hat{\mu})(1+\gamma_4)U_\mu^\dagger(x)\psi(x) - \bar{\psi}(x)(1-\gamma_4)U_\mu(x)\psi(x+\hat{\mu})\right]$$

Different a-dependencies in each current

We fit the data so that they match in the continuum limit.

Preliminary results

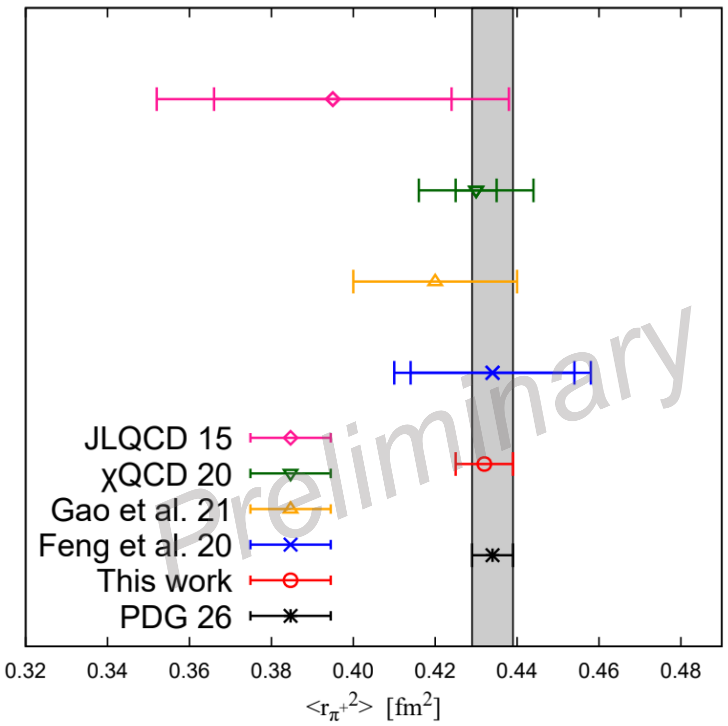
✓ π^+ charge radius ✓ K^+ charge radius ✓ K^0 charge radius



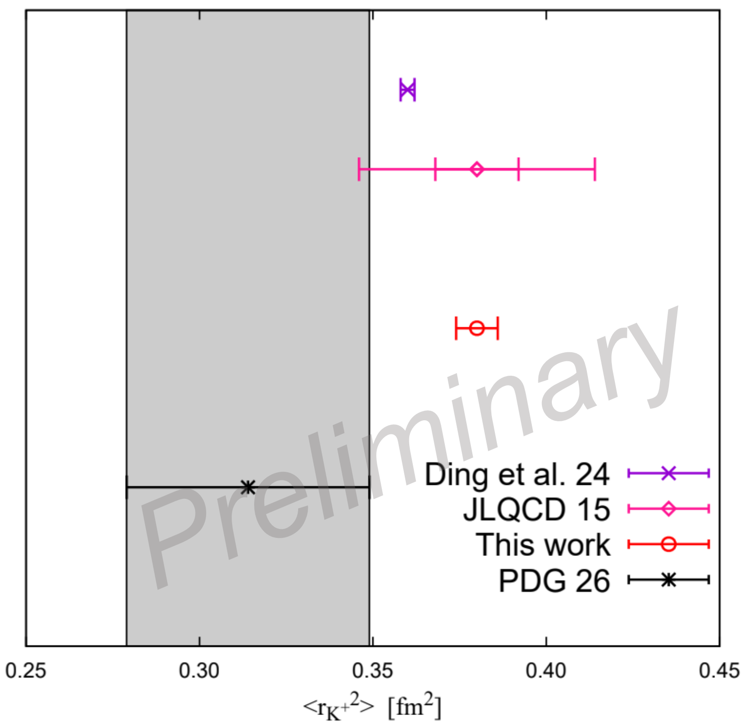
- ✓ As a preliminary extrapolation, we perform a linear fit in a^2.
- ✓ For the continuum limit values, our results agree with PDG within $\sim 2\sigma$.
- ✓ π^+ achieves the same level of experimental error, while K^+ and K^0 achieve precision exceeding the experiment (PDG).

Preliminary results

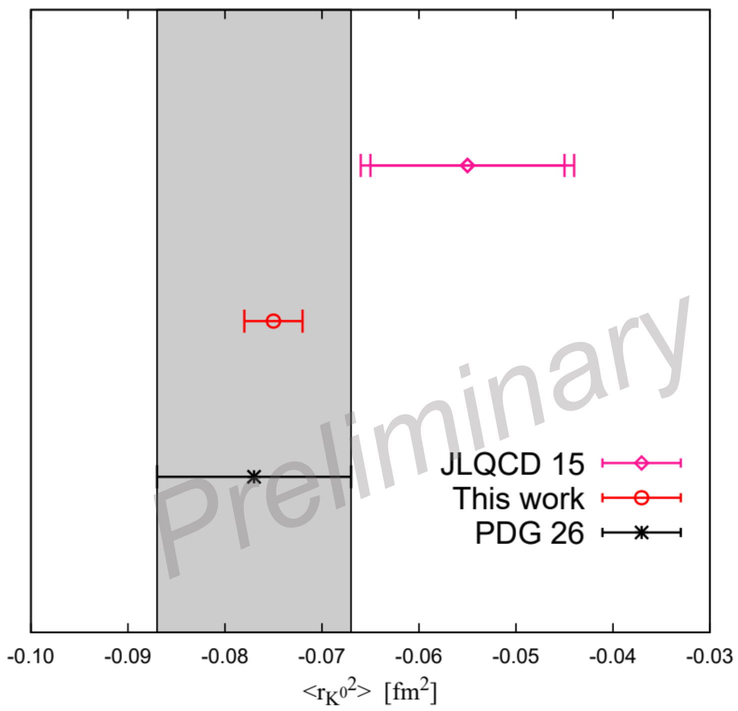
✓ π^+ charge radius



✓ K^+ charge radius



✓ K^0 charge radius



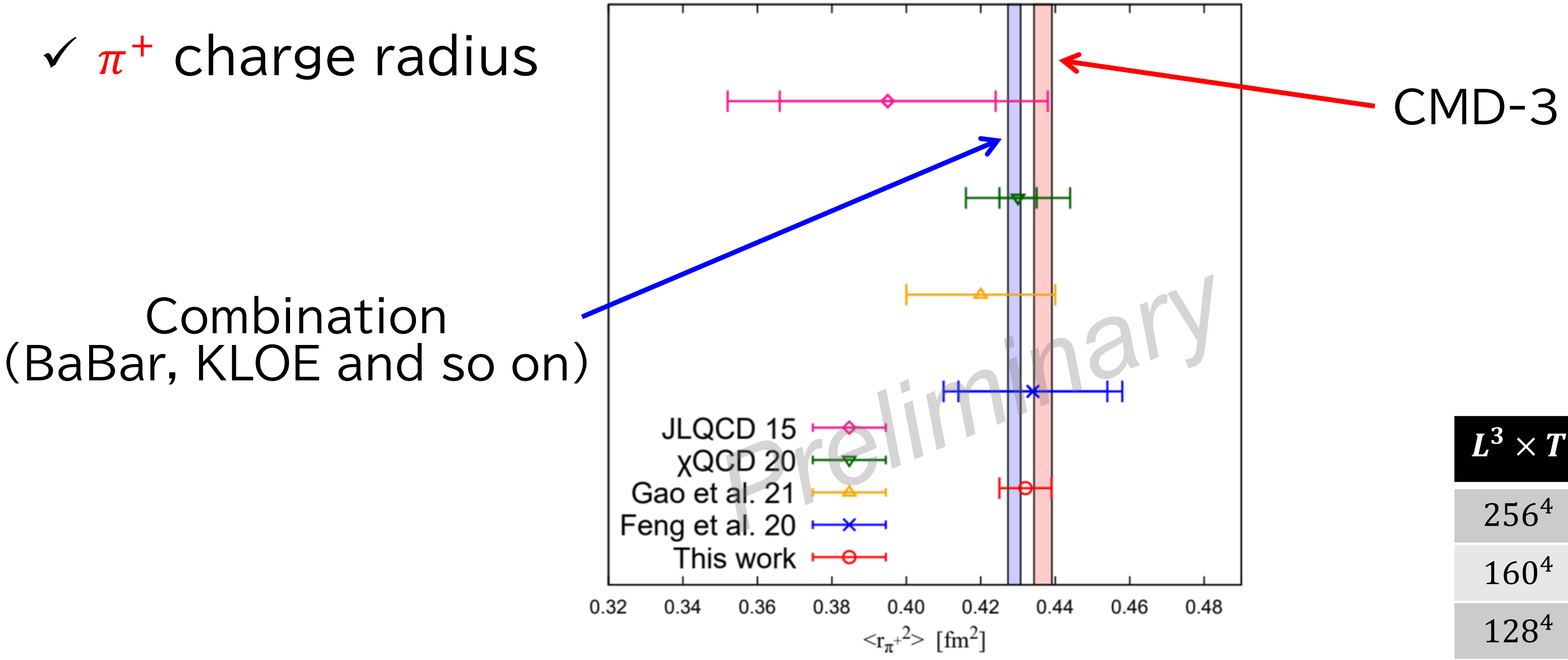
Although preliminary, our results are consistent with the experimental value(PDG) and the previous lattice calculations.

Some of our results exceed their precision.

Preliminary results

Comparison with the dispersive analysis for the $e^+e^- \to \pi^+\pi^-$ experiment

✓ π^+ charge radius



$L^3 \times T$	a [fm]	N_{meas}
256^4	0.041	$\mathcal{O}(500)$
160^4	0.063	$\mathcal{O}(4500)$~$\mathcal{O}(14000)$
128^4	0.085	$\mathcal{O}(4500)$~$\mathcal{O}(14000)$

At this point, we cannot draw any conclusions about the difference between these two values based on our values.

The statistics from the measurements on the finest lattice are not yet sufficient.

Summary

- ✓ Calculated charge radii of π^+, K^+, and K^0 on PACS10 configurations.
- ✓ Used the model-independent method.
- ✓ Although preliminary, our results are consistent with the experimental value(PDG) and the previous lattice calculations.
- ✓ Some of our results exceed their precision.

Future works :

- ✓ Increase statistics for PACS/L256
- ✓ Evaluate unaccounted-for systematic errors(source-sink separation, continuum extrapolation and so on).