

Toward a continuum limit determination of light meson charge radii with large-volume, physical point $N_f = 2 + 1$ lattice QCD

Kohei Sato

(Center for Computational Sciences, University of Tsukuba)

Hiromasa Watanabe (Keio University)

Takeshi Yamazaki (University of Tsukuba)

for PACS Collaboration

26 July – 1 Aug 2026

LATTICE 2026

University of Maryland, College Park

Target: π^+ , K^+ , K^0 charge radii

Toward a continuum limit determination of **light meson charge radii**
with **large-volume, physical point** $N_f = 2 + 1$ lattice QCD

Data characteristics: **Large volume** and **physical point** (Tsukuba)
+
Analysis Method : **without fit ansatz**

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Introduction

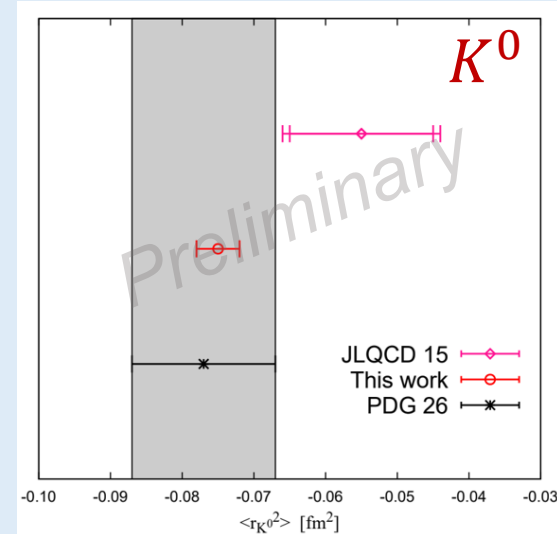
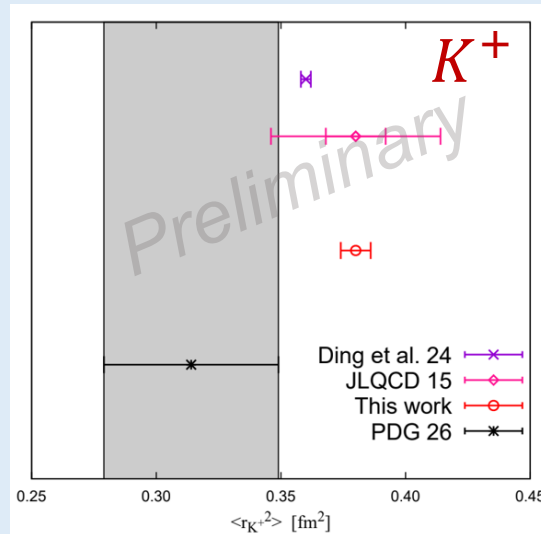
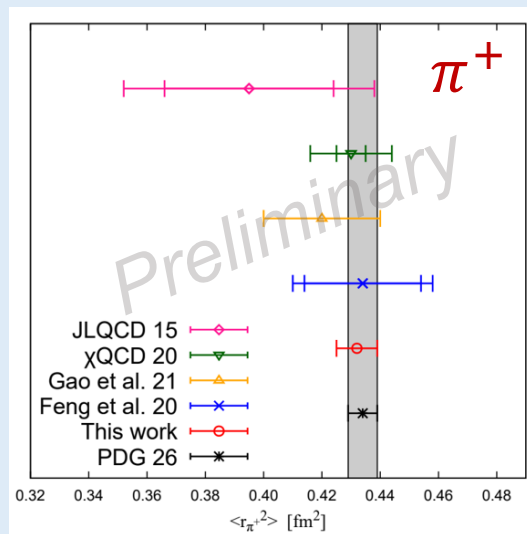
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It represents the spread of the charge distribution.

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Why study the π and K meson charge radii ?

1. Establish reference values from lattice QCD calculations
2. Problems in dispersive analysis for the $e^+e^- \rightarrow \pi^+\pi^-$ experiment
3. AMBER experiment



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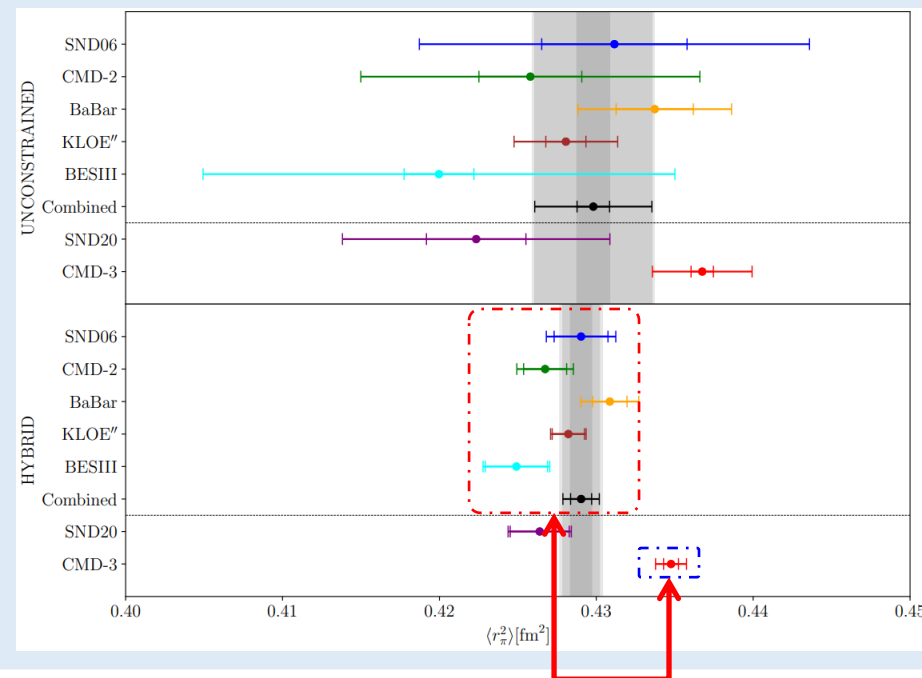
1. Establish reference values from lattice QCD calculations
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R. Aliberti et al., Physics Reports 1143 (2025) 1-158

$$\sigma(e^+e^- \rightarrow \pi^+\pi^-)(s) = \frac{\pi\alpha^2}{3s} \sigma_\pi^3(s) |F_\pi^V(s)|^2$$

$$\langle r_\pi^2 \rangle = 6 \frac{dF_\pi^V(s)}{ds} \Big|_{s=0} = \frac{6}{\pi} \int_{4M_\pi^2}^{\infty} ds \frac{\text{Im} F_\pi^V(s)}{s^2}$$

charge radius



Inconsistencies between the experiments

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3. AMBER experiment

Bernhard Ketzer, "AMBER - First results and future plans", EINN2025



Phase-1: 2023 → 2031 (conventional beams)

- $\bar{p}$ production cross section measurements for DM searches (pbarX)
- Proton radius: high-energy $\mu - p$ elastic scattering (PRM)
- Pion and kaon quark PDFs: Drell-Yan processes (DY)

Phase-2: 2031 → 2041 (high-intensity kaon beam)

- K quark and gluon PDFs (DY, prompt photons)
- Strange meson spectroscopy
- Meson charge radii
- Meson-photon reactions in Primakoff kinematics

AMBER Collaboration, Jan Friedrich, "Hadron research with AMBER at CERN", Rev.Mex.Fis.Suppl. 3 (2022) 3, 0308009.

→ Precise measurement of the K meson charge radius is planned

Introduction

3pt function
(input from lattice QCD)

form factor(output)

(mean-square) charge radius

... a quantity that characterizes the structure of hadrons. It represents the spread of the charge distribution.

$$\langle r_\pi^2 \rangle = -6 \frac{d}{dQ^2} \underbrace{F_\pi(Q^2)}_{\text{electromagnetic form factor}} \Big|_{Q^2=0}$$

electromagnetic form factor

$$\left[\begin{array}{ll} \langle \pi^+(p_f) | V_\mu | \pi^+(p_i) \rangle = (p_f + p_i)_\mu F_\pi(Q^2) \\ \text{electromagnetic} & \text{Momentum} \\ \text{current:} & \text{transfer :} \\ V_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f & Q^2 = -(p_f - p_i)^2 \geq 0 \end{array} \right]$$

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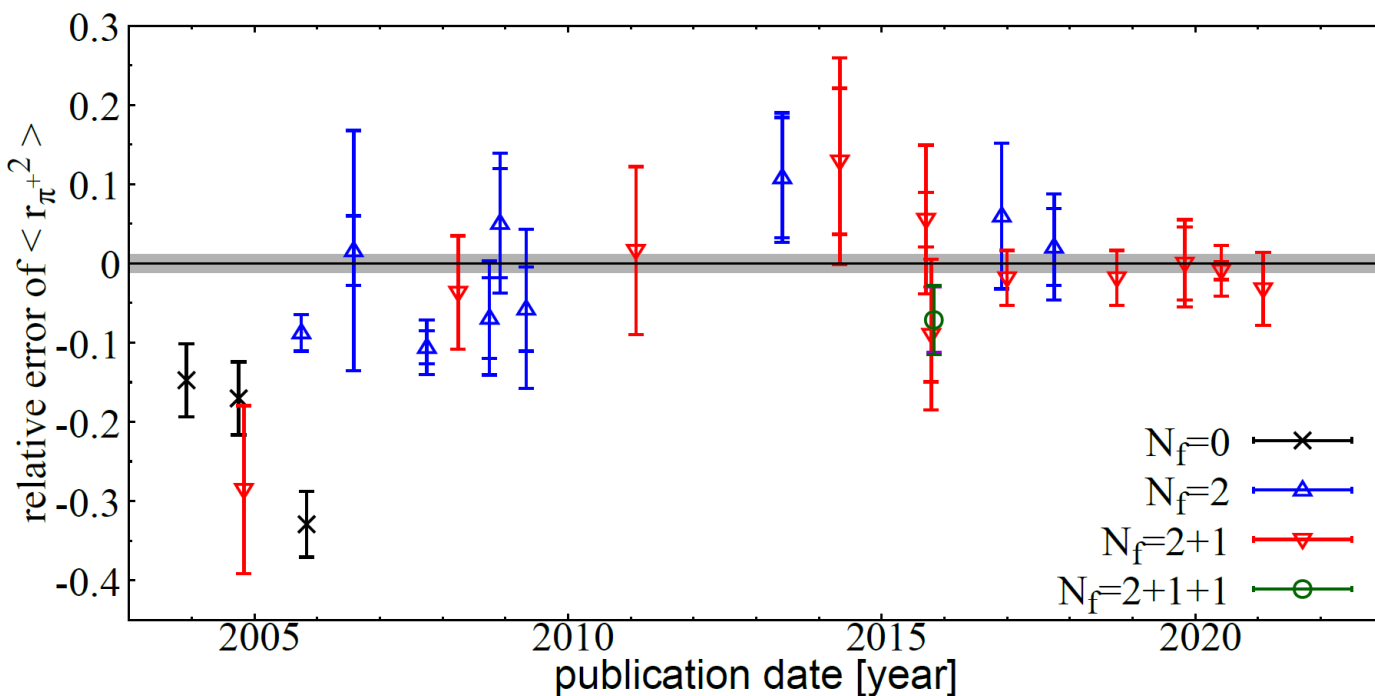
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- ✓ Initially : large difference
Recently : consistent
- ✓ Error : Lattice > Experimental
- There are 4 main systematic errors
 - Chiral extrapolation
 - Continuum extrapolation
 - Finite volume effect
 - Fit ansatz

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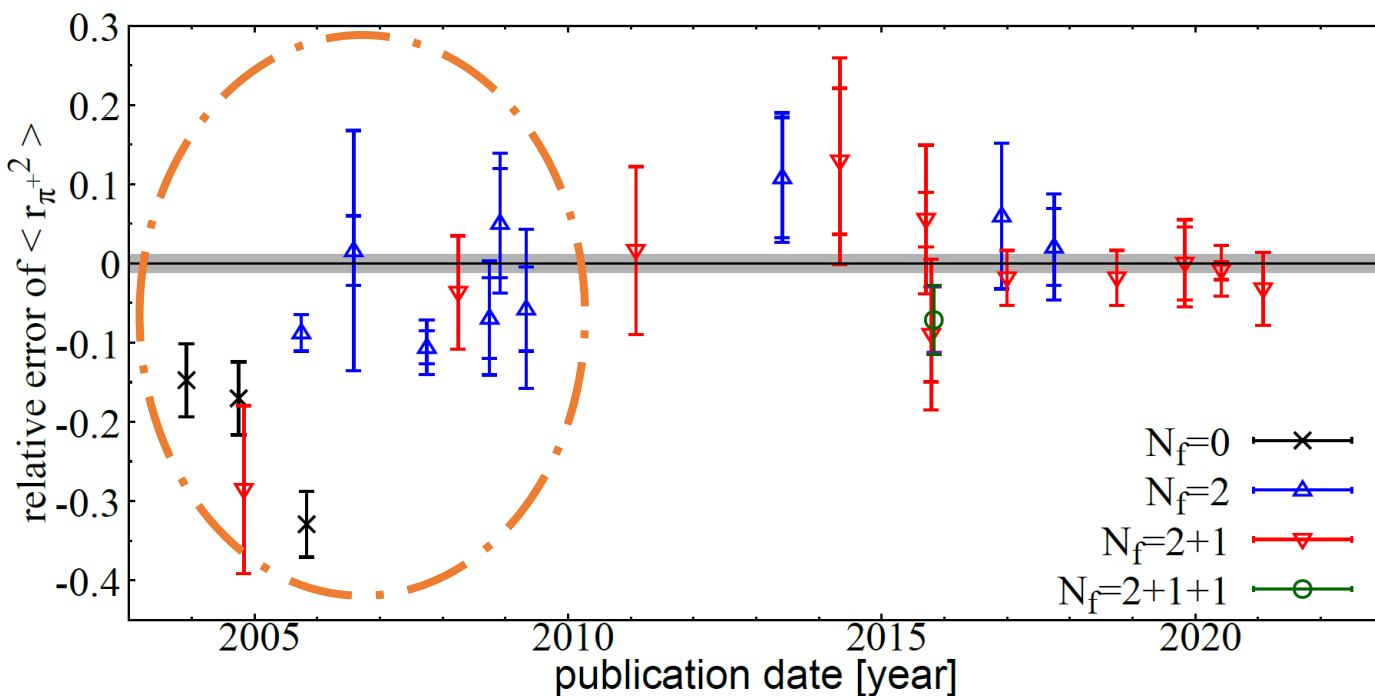
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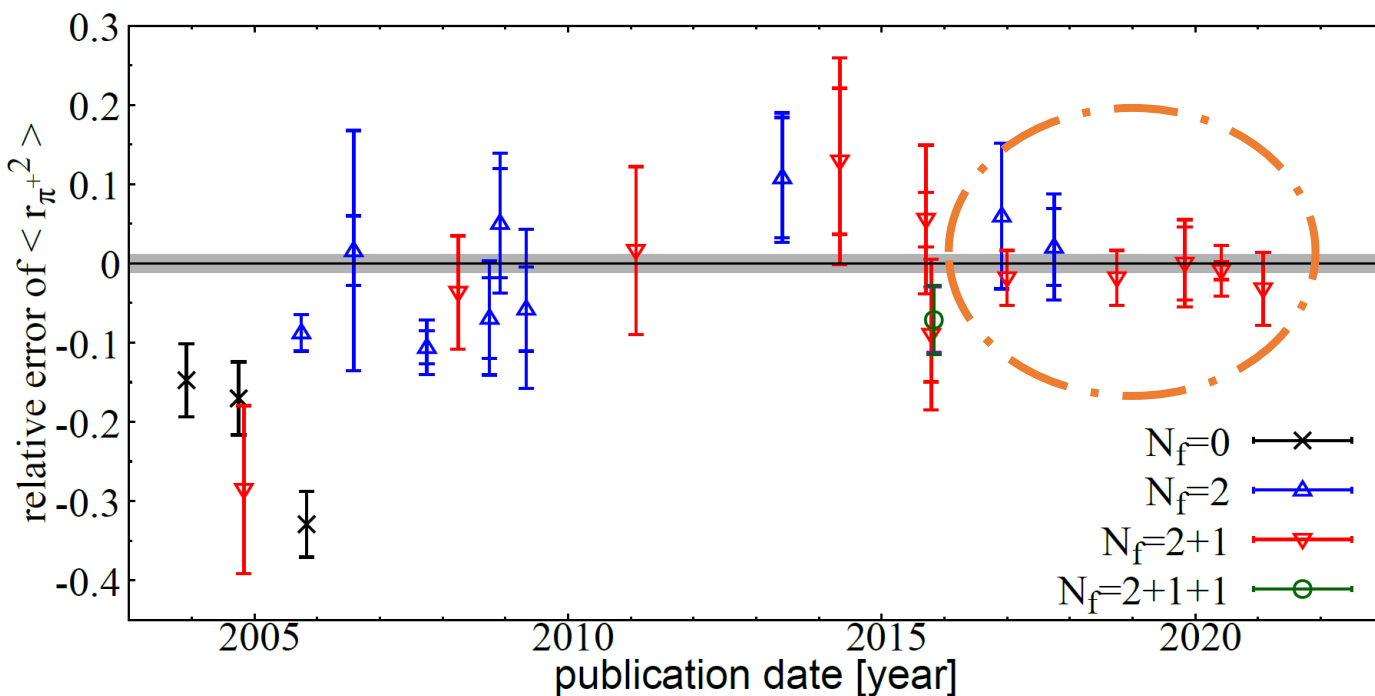
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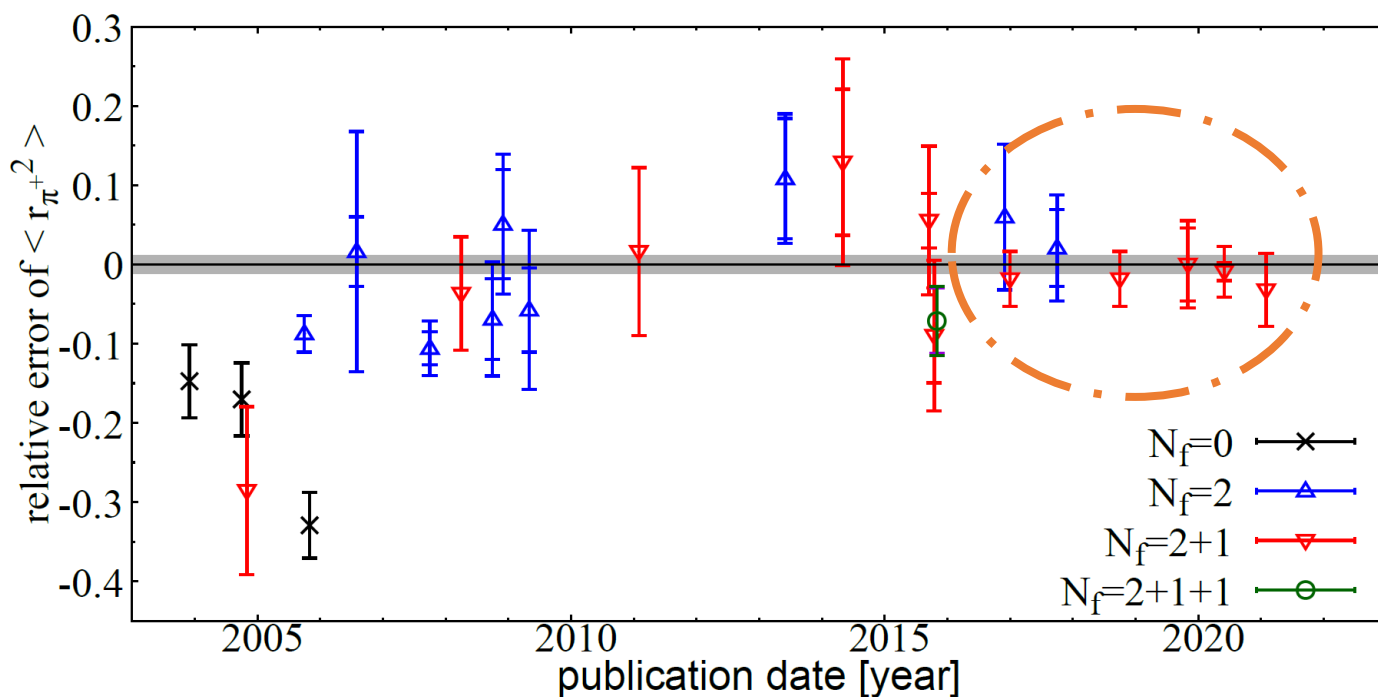
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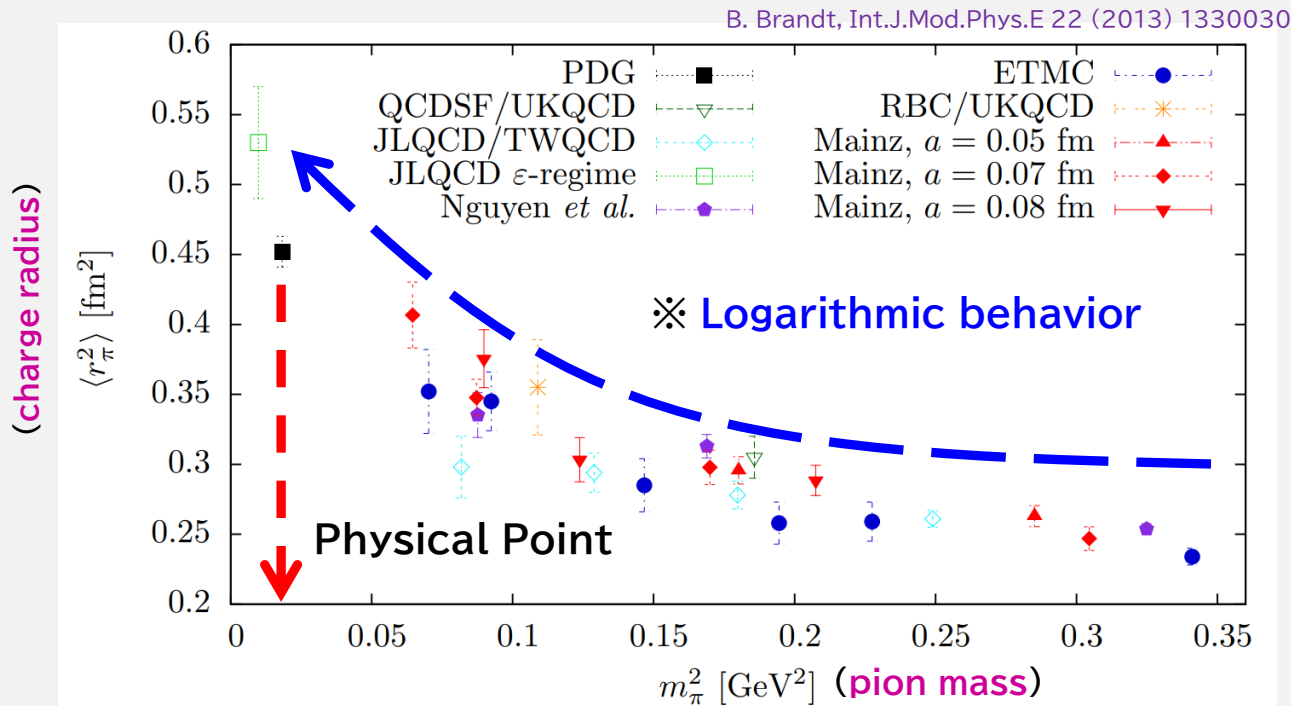


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(input from lattice QCD) form factor(output)

(m) By **changing theoretical parameters**, we can compute **physics in various worlds**.



- ✓ To reproduce the real world, we must use the **parameters that correspond to our physical universe**.
- ✓ Use **chiral extrapolation** to obtain values at the physical point.

characterizes the structure of hadrons.
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Electric
current:
 $\bar{\psi}_f \gamma_\mu \psi_f$

Momentum
transfer :
 $Q^2 = -(p_f - p_i)^2 \geq 0$

Initially : large difference

Recently : consistent

For : **Lattice > Experimental**

There are 4 main systematic errors

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form factor(output)

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characterizes the structure of hadrons.

Earlier lattice QCD calculations

Many simulations were performed at **heavier pion masses**



We needed to extrapolate these results to the physical point using **chiral extrapolation**

recently : consistent

or : **Lattice > Experimental**

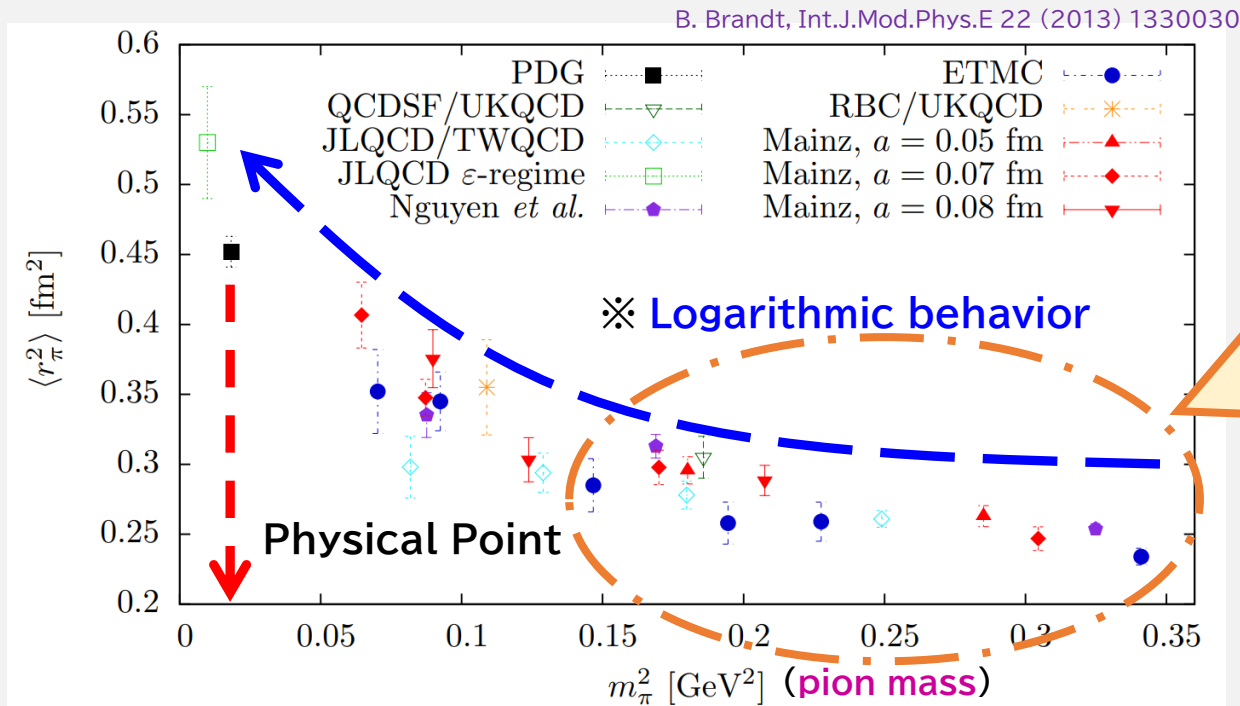
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➤ **Chiral extrapolation**

➤ Continuum extrapolation

➤ Finite volume effect

➤ Fit ansatz



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2005

2010

2015

2020

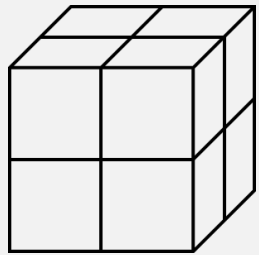
publication date [year]

Introduction

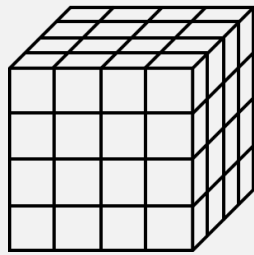
3pt function
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form factor(output)

Lattice calculations discretize spacetime.

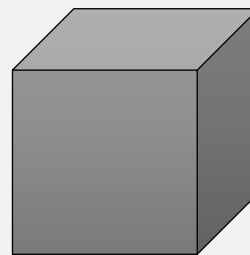


Coarse lattice



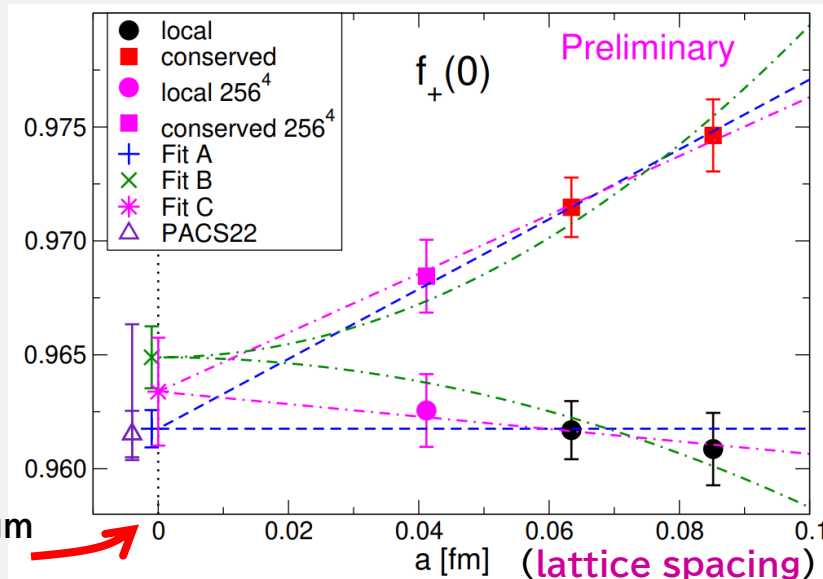
Fine lattice

...



Continuum
(real world)

Form factor
at zero momentum



✓ Continuum extrapolation derives real values from discrete data.

characterizes the structure of hadrons.
the spread of the charge distribution.

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etic
current:
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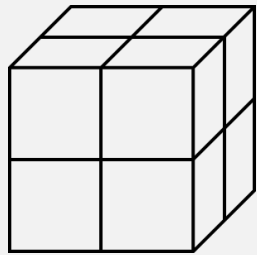
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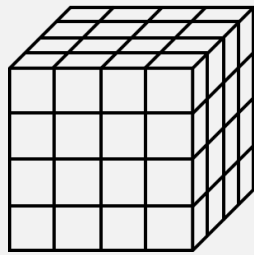
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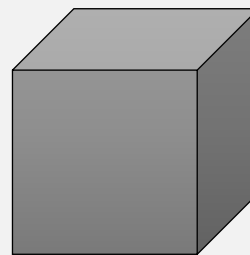


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Continuum
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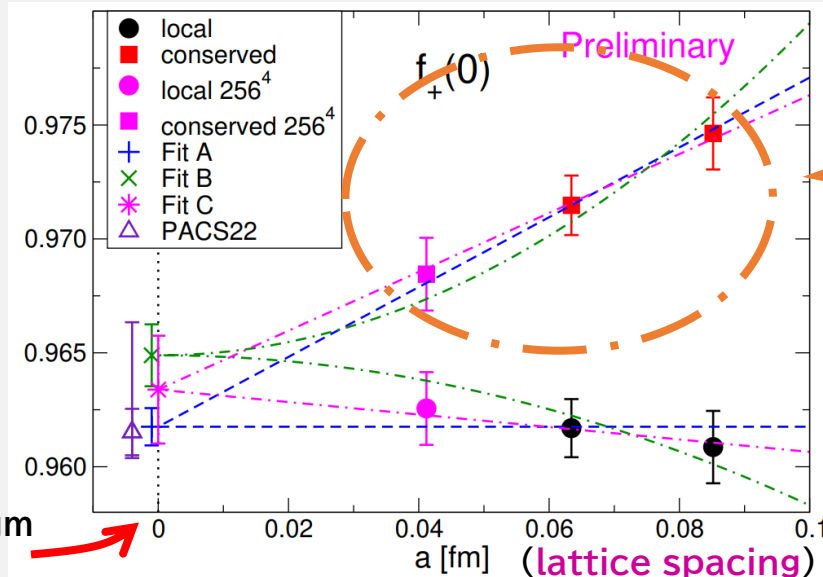
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By the discretization of space-time
Values change depending on the
lattice spacing

We need to calculate at **multiple lattice spacings** and then **extrapolate**

Form factor
at zero momentum



PACS, PoS LATTICE2023
(2024) 276

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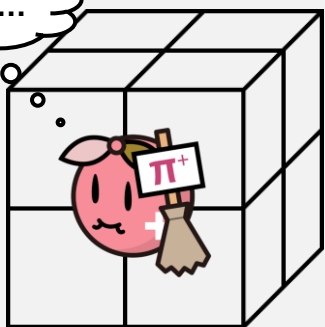
(m)

Lattice calculations use **finite spacetime volume**.

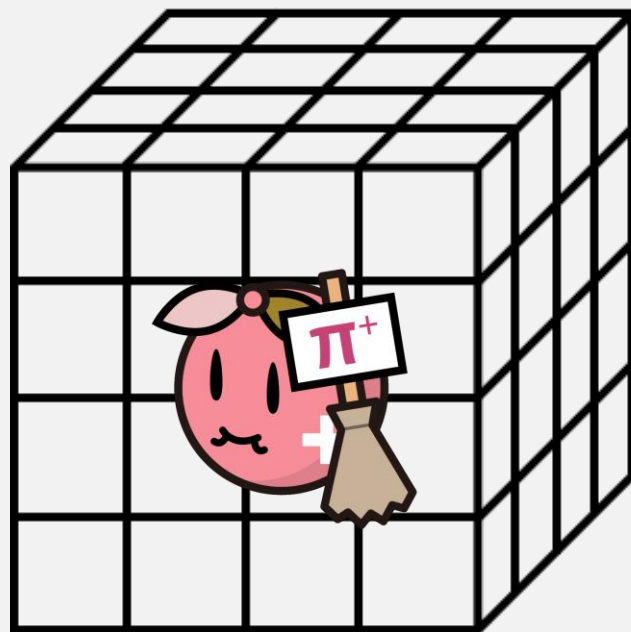
True size



Narrow...



Small volume



Large volume

✓ **Finite-volume effects** from small volume.

characterizes the structure of hadrons.
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etic
current:
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Momentum
transfer :

$$Q^2 = -(p_f - p_i)^2 \geq 0$$

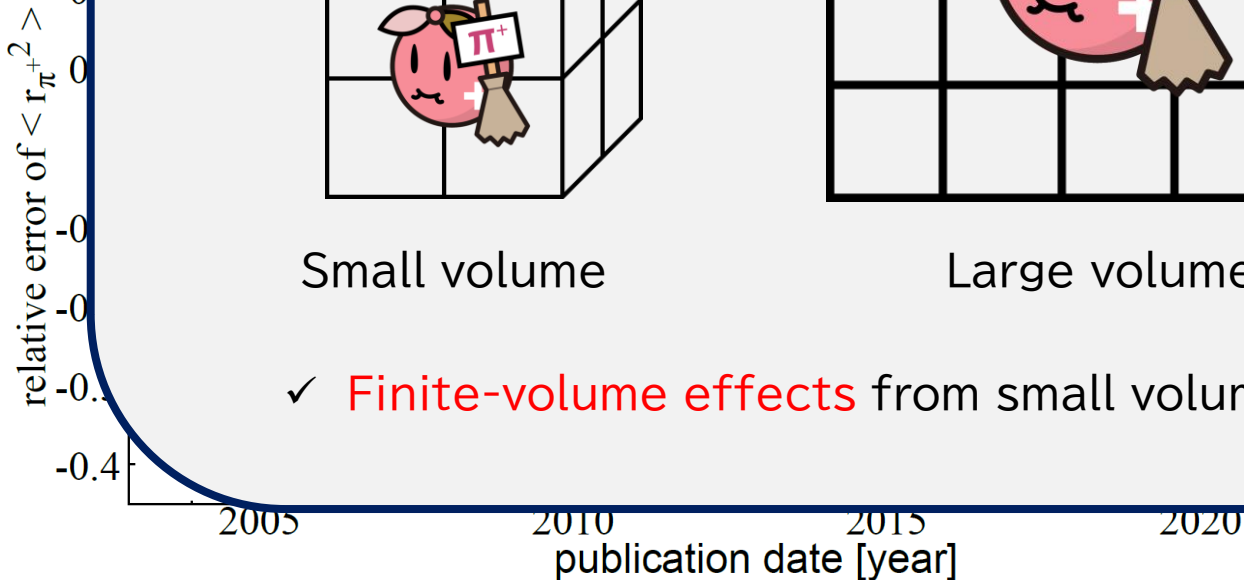
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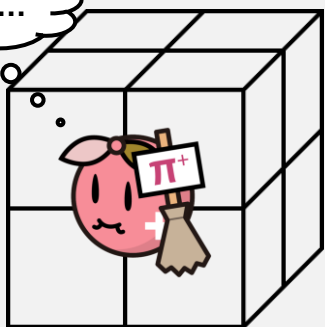
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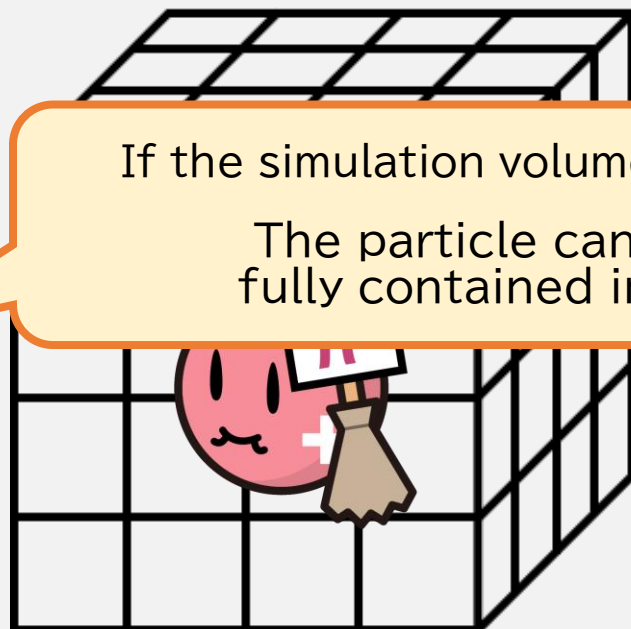


Narrow...



Small volume

If the simulation volume is too small
The particle cannot be
fully contained in space



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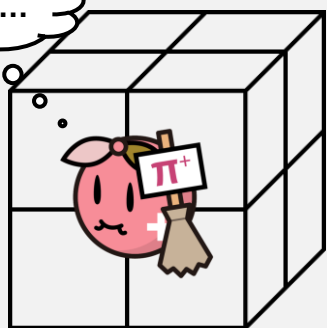
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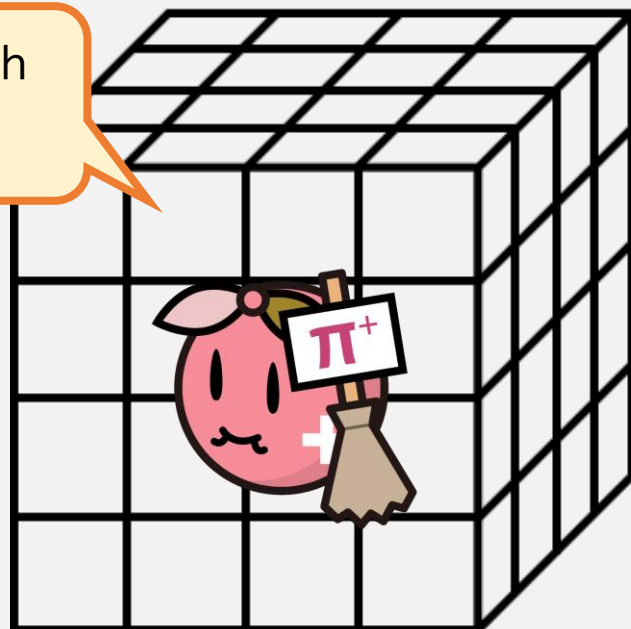
True size

If the volume is large enough
The particle fits well

Narrow...



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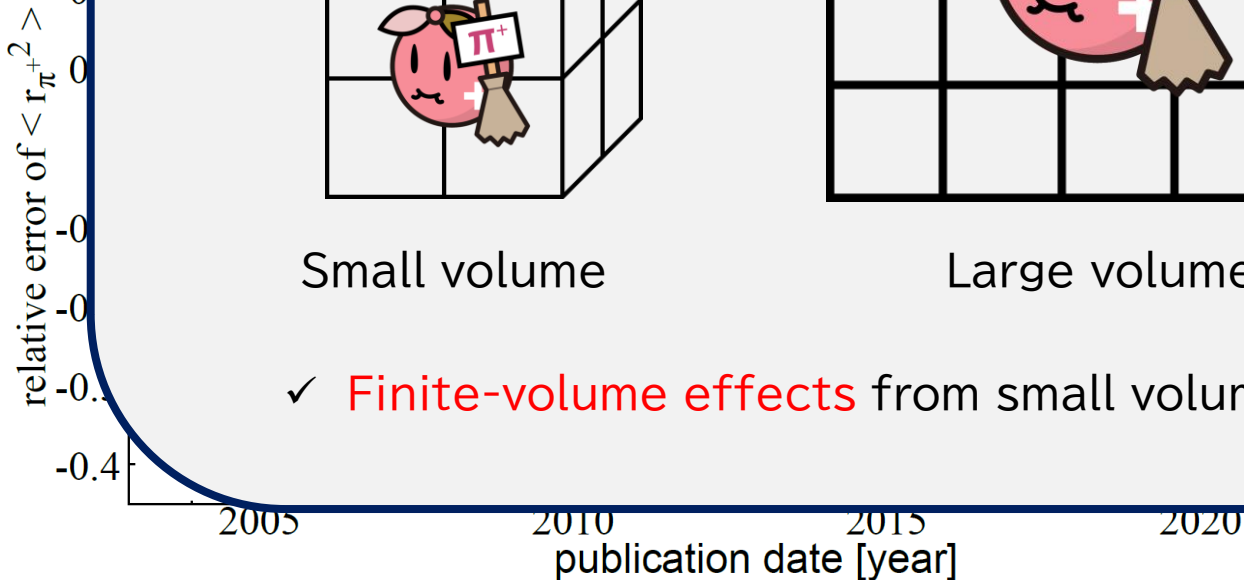
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PACS10 configurations

...Large-volume configurations at physical point
(3 lattice spacings).

➤ Chiral extrapolation

At physical point → almost unnecessary → good!!

Hadron masses reproduced within 5% error

➤ Continuum extrapolation

If computed with 3 configurations
→ evaluation possible → good!!

➤ Finite volume effect

Large volume sufficiently suppresses effects → good!!

Indicator of finite-volume effects

$M_\pi L > 4 \rightarrow \text{good}$ $3 < M_\pi L < 4 \rightarrow \text{soso}$ $M_\pi L < 3 \rightarrow \text{bad}$

PACS10 configurations ~ 7

PACS10 suppresses 3 major systematic errors.

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(m) Systematic errors from traditional analysis methods

3-pt functions (lattice QCD)

Known function

$$\frac{\sum_{\vec{x}_{\text{sink}}, \vec{x}} \langle \pi^+(\vec{x}_{\text{sink}}, t_{\text{sink}}) V_4(\vec{x}, t) \pi^+(0)^\dagger \rangle e^{-ipx_1}}{\sum_{\vec{x}_{\text{sink}}, \vec{x}} \langle \pi^+(\vec{x}_{\text{sink}}, t_{\text{sink}}) V_4(\vec{x}, t) \pi^+(0)^\dagger \rangle} = \frac{E_\pi(p) + m_\pi}{2E_\pi(p)} e^{-(E_\pi(p) - m_\pi)t} F_\pi(Q^2)$$

Form factors (desired quantity)

Properly combine lattice data and known functions.

$$F_\pi(Q^2) = \frac{\text{Input from lattice calculation}}{\text{Known function}}$$

$$F_\pi(Q^2) = \begin{cases} \frac{1}{1 + Q^2/M_{\text{pole}}^2} \\ 1 + \sum_k f_k(Q^2)^k \\ \sum_k a_k z^k \left(z := \frac{\sqrt{4m_\pi^2 + Q^2} - \sqrt{4m_\pi^2}}{\sqrt{4m_\pi^2 + Q^2} + \sqrt{4m_\pi^2}} \right) \\ 1 + \frac{1}{f_\pi^2} [2l_6^r(\mu)Q^2 + 4H(m_\pi^2, Q^2, \mu^2)] \end{cases} \quad \text{Fit ansatz}$$

$$\langle r_\pi^2 \rangle = -6 \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0}$$

✓ The fit ansatz error arises from the choice of the fitting function and the fitting range.

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model-independent method

✓ Key idea: Calculate spatial moment

U. Aglietti et al., Phys.Lett.B324,85(1994); UKQCD, Nucl.Phys.B444,401(1995);
C. Bouchard et al., PoS LATTICE2016,170(2016); PACS, Phys.Rev.D104, 074514(2021)

➤ Differentiation of Fourier transform

$$\left. \frac{d\tilde{C}(p)}{dp^2} \right|_{p^2=0} = \frac{d}{dp^2} \sum_x C(x) e^{-ipx} \Big|_{p^2=0} = -\frac{1}{2} \sum_x x^2 C(x)$$

($N_{space} \rightarrow \infty$, a :finite ; $L = N_{space}a \rightarrow \infty$)

infinite volume limit

We can suppress the contamination at large volume.

$$\sum_x x^2 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

$$\sum_x x^4 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

Xu Feng et al.,
Phys.Rev.D101,051502(R)(2020)

add these higher-order moments
to reduce the contamination

$$R(t) = \alpha_1 (x^2 \text{moment}) + \alpha_2 (x^4 \text{moment})$$

$$= \frac{d}{dQ^2} F_\pi(Q^2) \Big|_{Q^2=0} + (f_3 + f_4 + \dots)$$

α_1, α_2 : parameters which set to cancel out the contamination

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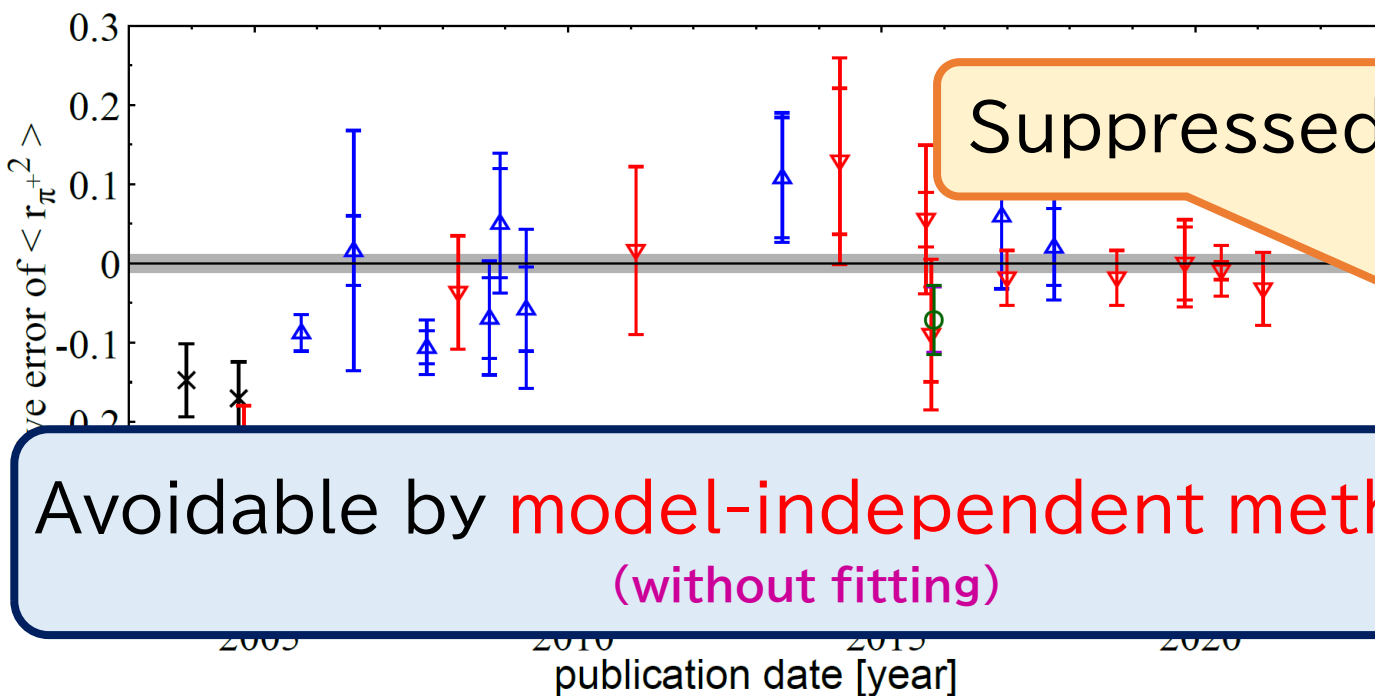
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$$\langle r_\pi^2 \rangle = -6 \frac{d}{dQ^2} \underbrace{F_\pi(Q^2)}_{\text{electromagnetic form factor}} \Big|_{Q^2=0}$$

electromagnetic form factor

$$\left[\begin{array}{ll} \langle \pi^+(p_f) | V_\mu | \pi^+(p_i) \rangle = (p_f + p_i)_\mu F_\pi(Q^2) \\ \text{electromagnetic current:} & \text{Momentum transfer:} \\ V_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f & Q^2 = -(p_f - p_i)^2 \geq 0 \end{array} \right]$$



Suppressed with PACS10 configurations

or : Lattice > Experimental

there are 4 main systematic errors

- Chiral extrapolation
- Continuum extrapolation
- Finite volume effect
- Fit ansatz

Outline

- ✓ Introduction
 - Charge radius
 - Main systematic errors
- ✓ Application to PACS10 configurations
 - Analysis of π^+ , K^+ , and K^0 charge radii using model-independent method
- ✓ Summary

Simulation parameters

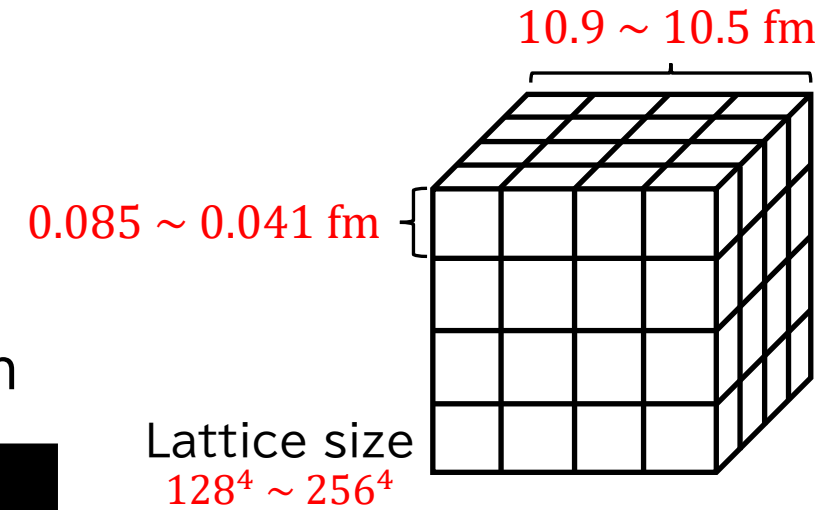
✓ Gauge configuration (

PACS, PRD 99, 014504 (2019);
PACS, PRD 106, 094505 (2022);
PACS, PRD 109, 094505 (2024);
PoS LATTICE2024 (2025) 318

PACS10 configuration

$N_f = 2 + 1$ six-stout-smeared non-perturbative
 $O(a)$ -improved Wilson action + Iwasaki gauge action

β	$L^3 \times T$	L [fm]	a [fm]	a^{-1} [GeV]	m_π [MeV]	m_K [MeV]	N_{conf}	N_{meas}
2.20	256^4	10.5	0.041	4.8	142	514	20	$\mathcal{O}(500)$
2.00	160^4	10.1	0.063	3.1	138	506	20	$\mathcal{O}(4500) \sim \mathcal{O}(14000)$
1.82	128^4	10.9	0.085	2.3	136	499	20	$\mathcal{O}(4500) \sim \mathcal{O}(14000)$



✓ Measurement parameter

- 20 config. , 500~14000 meas. Per config.
- **Periodic** + **anti-periodic** correlation functions in time direction
- **Forward** + **Backward** correlation functions in time direction
- $|t_{\text{sink}} - t_{\text{source}}| = 3 \sim 4 \text{ fm}$

- Chiral extrapolation
→ **Physical point**
- Continuum extrapolation
→ **3 lattice spacings**
- Finite volume effects
→ **Large volume**

↑ Statistics

→ Reduce the wrapping-around effect
Systematic errors can be evaluated

Result

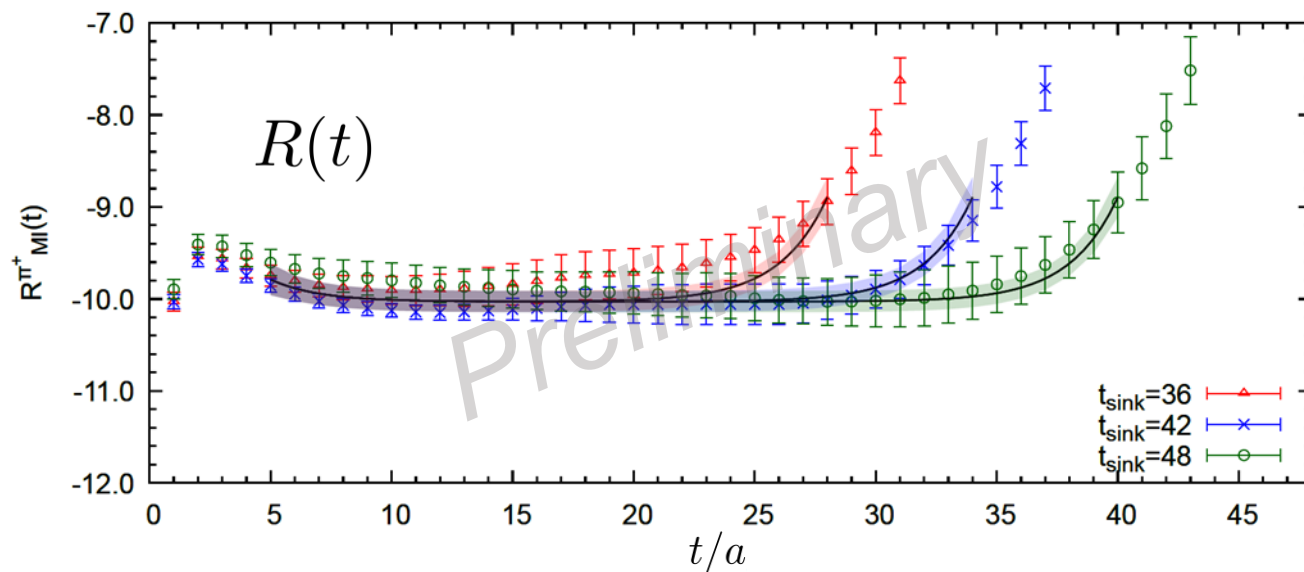
- ✓ Model-independent method
(without fit ansatz)

$$\sum_x x^2 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

$$\sum_x x^4 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

$$R(t) = \alpha_1 (x^2 \text{moment}) + \alpha_2 (x^4 \text{moment})$$

$$= \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + (f_3 + f_4 + \dots)$$



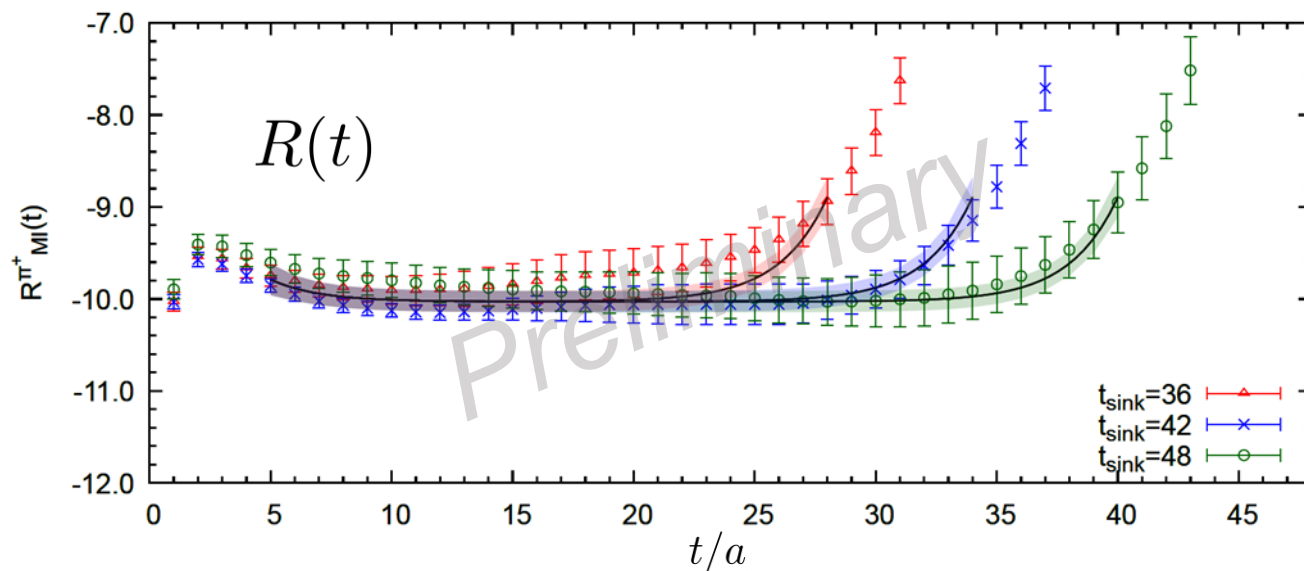
Result

- ✓ **Model-independent method (without fit ansatz)**

$$\sum_x x^2 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

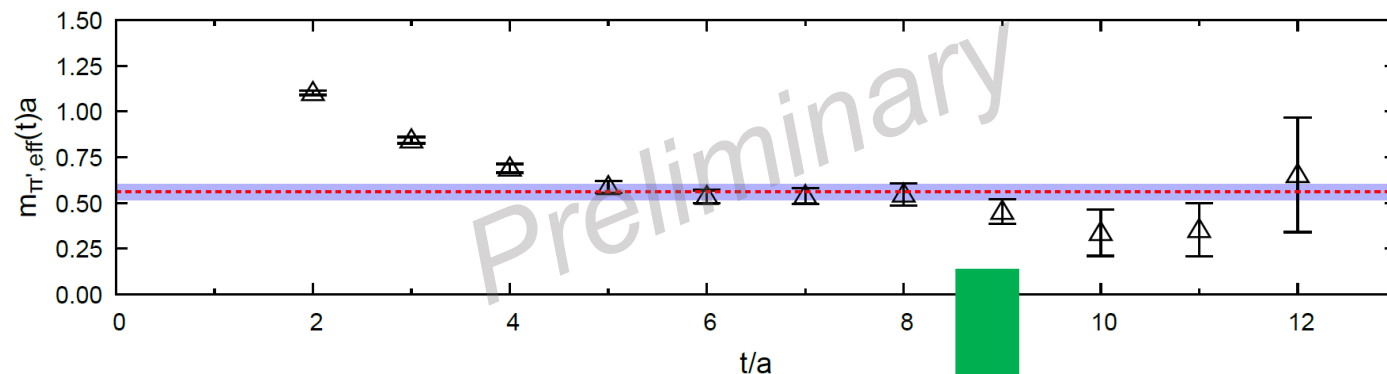
$$R(t) = \alpha_1 (x^2 \text{moment}) + \alpha_2 (x^4 \text{moment}) + \dots$$

$$= \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + (f_3 + f_4 + \dots)$$



- ✓ Extraction including the **first excited state**

$$f(t; t_{\text{sink}}) = \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + A e^{-(m_{X'} - m_X)t} + B e^{-(m_{X'} - m_X)(t_{\text{sink}} - t)}$$



- ✓ From the plateau, we can see that the PACS10 configurations **reproduce the experimental mass of the first excited state**.

- ✓ We use experimental value as the mass of the first excited state

Result

- ✓ Model-independent method (without fit ansatz)

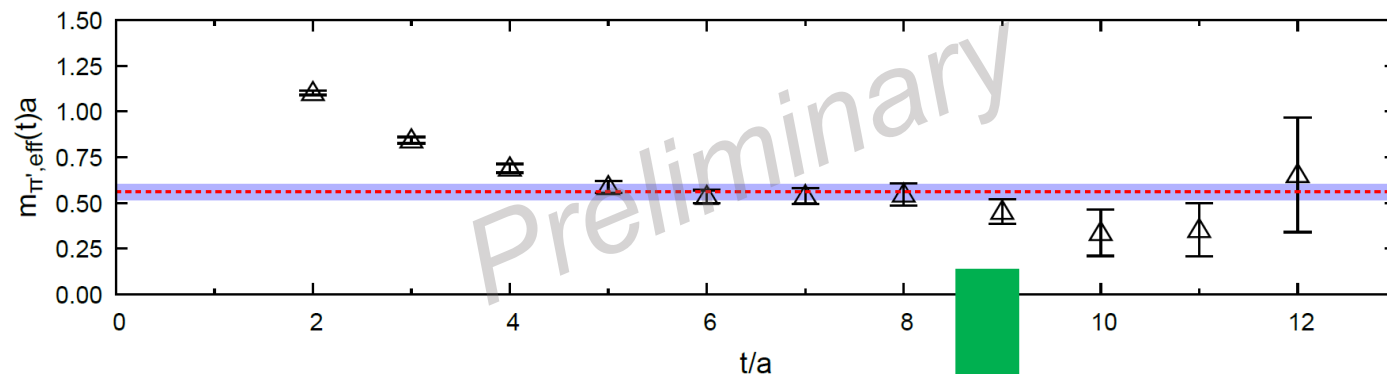
$$\sum_x x^2 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

$$R(t) = \alpha_1 (x^2 \text{moment}) + \alpha_2 (x^4 \text{moment})$$

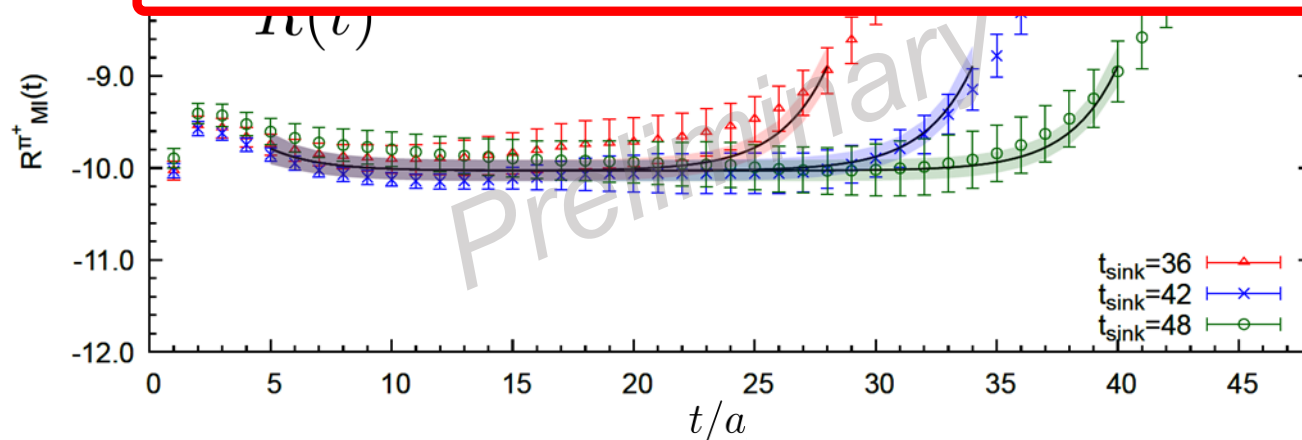
$$= \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + (f_3 + f_4 + \dots)$$

- ✓ Extraction including the first excited state

$$f(t; t_{\text{sink}}) = \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + A e^{-(m_{X'} - m_X)t} + B e^{-(m_{X'} - m_X)(t_{\text{sink}} - t)}$$



Also perform the same calculation for K mesons and L160/L256 configuration.

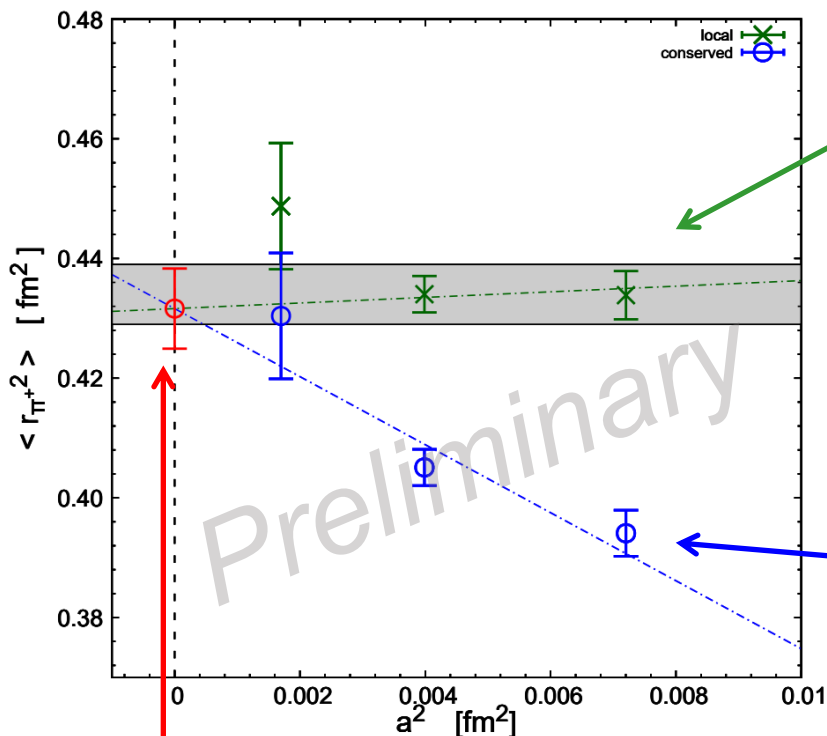


state.

- ✓ We use experimental value as the mass of the first excited state

Preliminary results

✓ π^+ charge radius



local vector current

$$V_4^{\text{loc}}(x) = \bar{\psi}(x)\gamma_4\psi(x)$$

} PDG ($e + \pi \rightarrow e + \pi$ scattering)

point splitting conserved vector current

$$V_4^{\text{cons}}(x) = \frac{1}{2} [\bar{\psi}(x + \hat{\mu})(1 + \gamma_4)U_\mu^\dagger(x)\psi(x) - \bar{\psi}(x)(1 - \gamma_4)U_\mu(x)\psi(x + \hat{\mu})]$$

$$\left[\begin{array}{ll} \langle \pi^+(p_f) | V_\mu | \pi^+(p_i) \rangle = (p_f + p_i)_\mu F_\pi(Q^2) & \\ \text{electromagnetic current:} & \text{Momentum transfer:} \\ V_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f & Q^2 = -(p_f - p_i)^2 \geq 0 \end{array} \right]$$

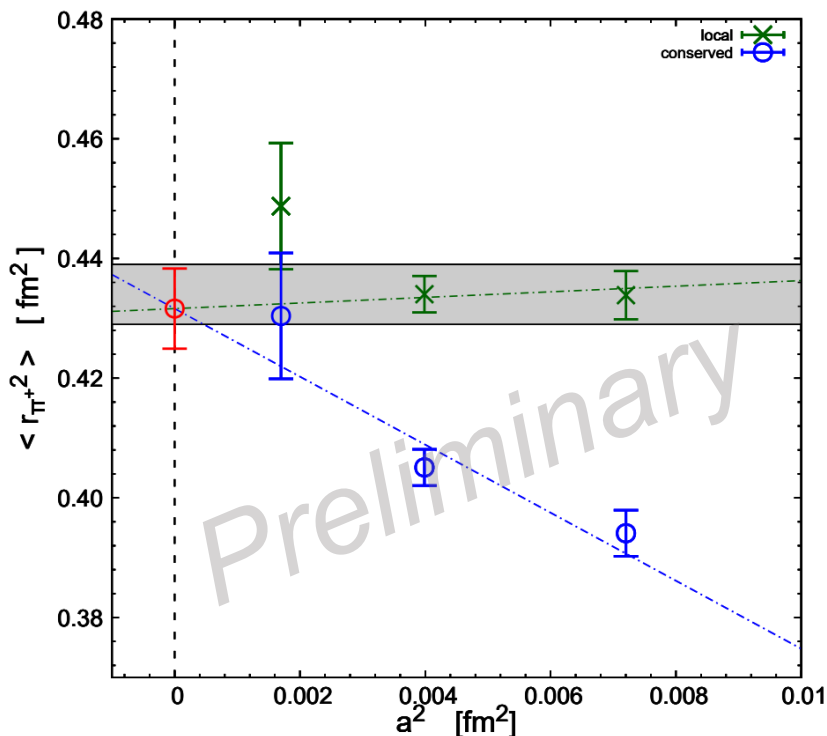


Different a -dependencies in each current

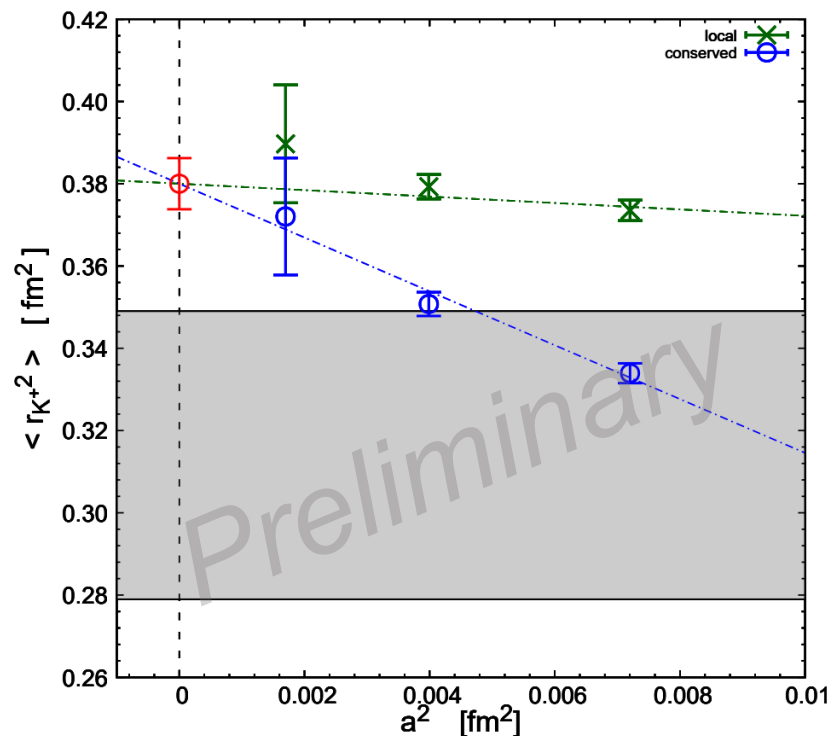
We fit the data so that they **match in the continuum limit.**

Preliminary results

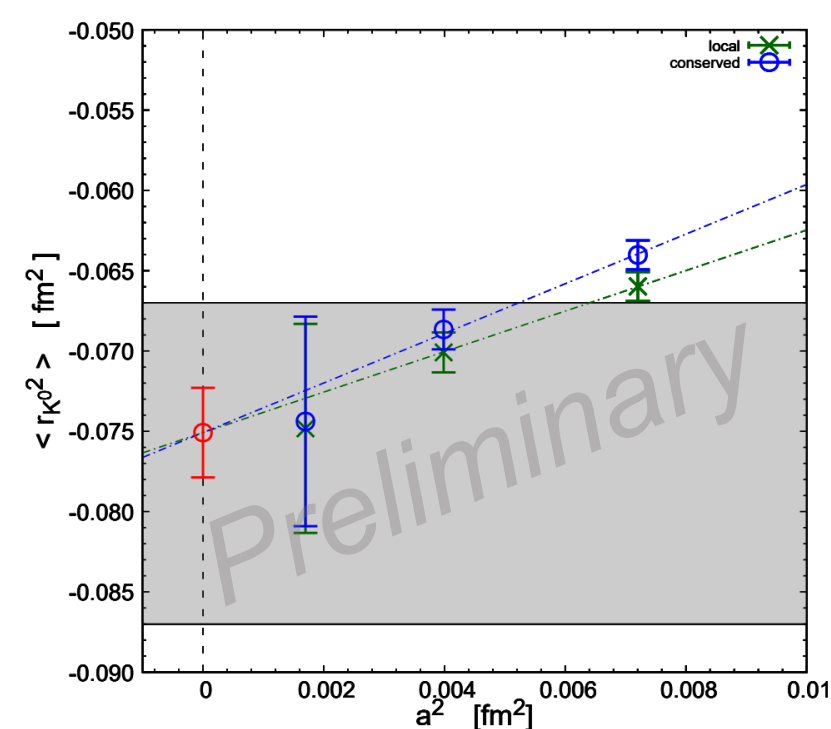
✓ π^+ charge radius



✓ K^+ charge radius



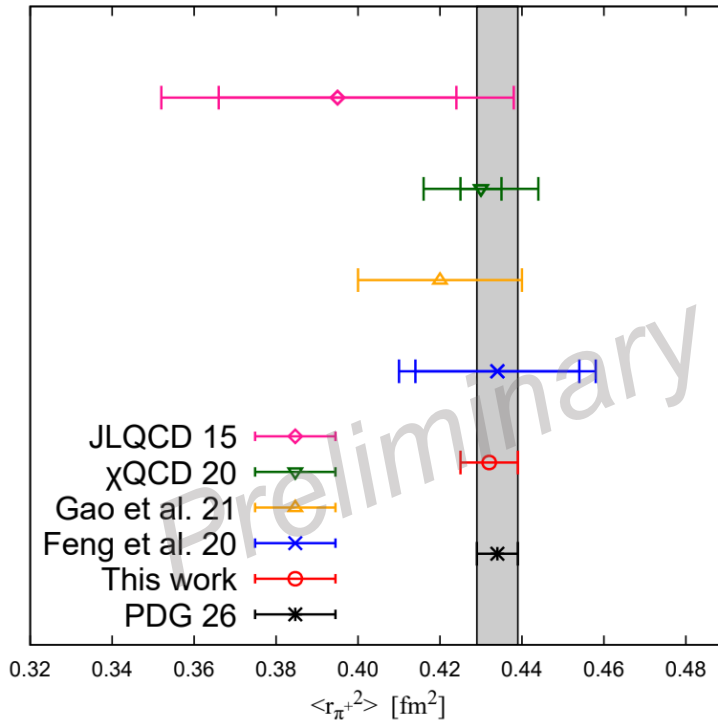
✓ K^0 charge radius



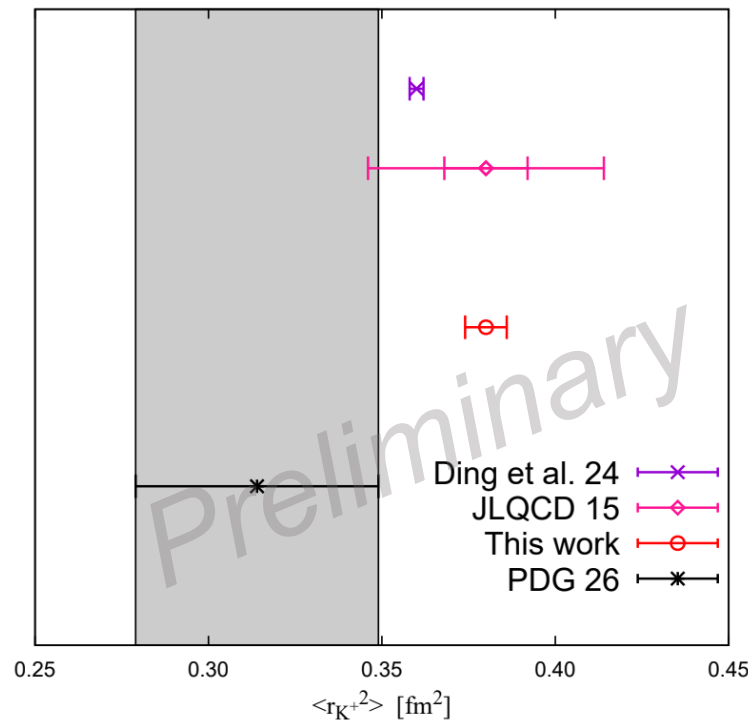
- ✓ As a preliminary extrapolation, we perform a **linear fit in a^2** .
- ✓ For the continuum limit values, results **agree with PDG** within $\sim 2\sigma$.
- ✓ π^+ achieves the **same level of experimental error**, while K^+ and K^0 achieve precision **exceeding the experiment (PDG)**.

Preliminary results

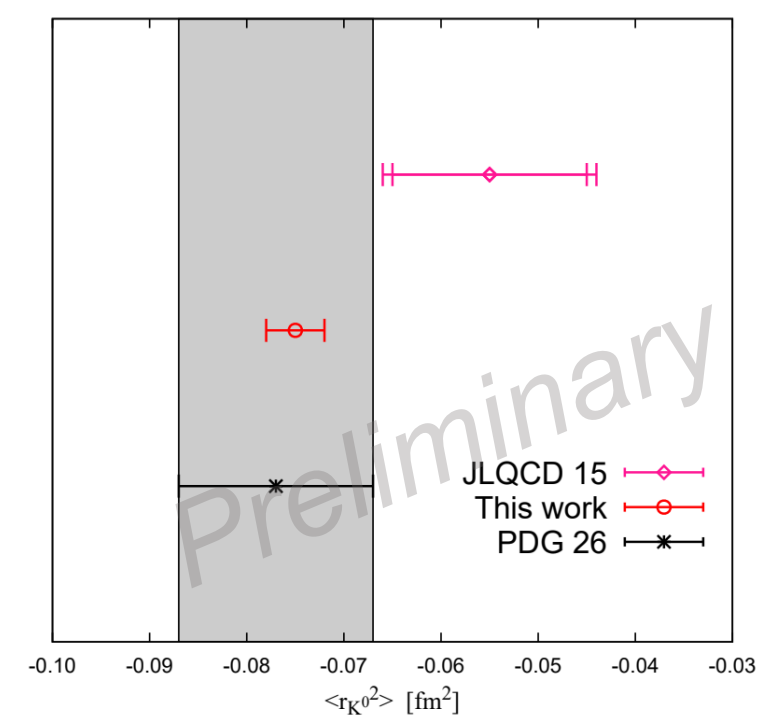
✓ π^+ charge radius



✓ K^+ charge radius



✓ K^0 charge radius



Although preliminary, the results are consistent with the experimental value(PDG) and the results of the previous lattice calculations.

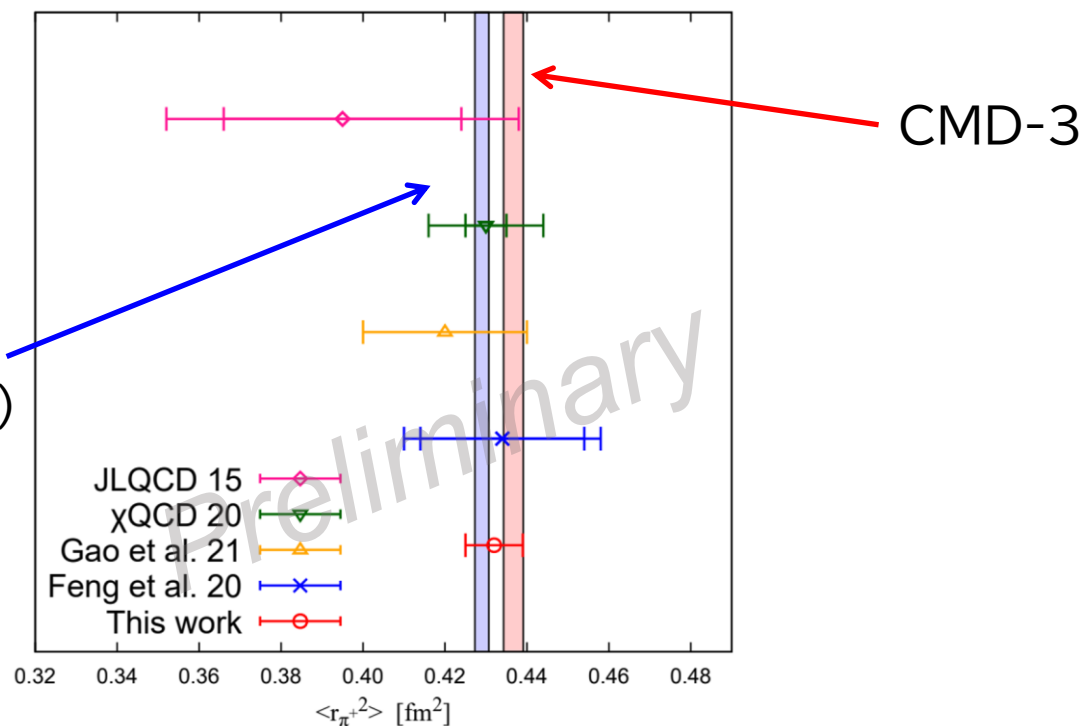
Some of our results exceed their precision.

Preliminary results

Comparison with the dispersive analysis for the $e^+e^- \rightarrow \pi^+\pi^-$ experiment

✓ π^+ charge radius

Combination
(BaBar, KLOE and so on)



At this point, we cannot draw any conclusions about the difference between these two values based on our value.

➡ The statistics from the measurements on the finest lattice are not yet sufficient.

Summary

- ✓ Calculated charge radii of π^+ , K^+ , and K^0 on PACS10 configurations.
- ✓ Used the model-independent method.
- ✓ Although preliminary, the results are consistent with the experimental value(PDG) and the results of the previous lattice calculations.
- ✓ Some of our results exceed their precision.

Future works :

- ✓ Increase statistics for PACS/L256
- ✓ Evaluate unaccounted-for systematic errors(source-sink separation, continuum extrapolation and so on).