Inclusive P-wave quarkonium decay widths from pNRQCD

Julian Mayer-Steudte[⋆1]

Nora Brambilla[1], Viljami Leino[2], Panayiotis Panayiotou[1], Andrea Shindler[3], Antonio Vairo[1], Xiang-Peng Wang[4]

[1]Technical University of Munich and [2]University of Southern Denmark and [3]RWTH Aachen and [4]Central China Normal University

Based on

N. Brambilla, V. Leino, JMS, P. Panayiotou, A. Shindler, A. Vairo, X.-P. Wang, arXiv:2607.13024

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July 29 2026

Motivation

Motivation: Inclusive hadronic decay widths

- **Inclusive decay widths** of quarkonium can be determined from experiments
- Theoretical computations from **first principles** are a long-standing challenge
- **NRQCD** factorizes:
  - Short-distance coefficients (SDCs): perturbation theory
  - Long-distance matrix elements (LDME): non-perturbative lattice computations
  - LDME depends on the **external state**
- Here: **strongly coupled pNRQCD** for P-wave decay:
  LDME factorizes into a state-dependent factor and a **universal gluonic correlator** $\mathcal{E}_3$
- Similar universal objects are required for **S-wave decay** and **production** rates
- P-wave decay serves as a **use case** scenario for developing **new techniques**

Intro

Intro: pNRQCD factorization

- Quarkonia are **non-relativistic** systems characterized by the **hierarchy of scales**:

  $m_Q \gg m_Q v \gg m_Q v^2, \Lambda_{\mathrm{QCD}}$

  $v \ll c$: relative velocity, m_Q: mass of heavy quark flavor Q
- **Strongly coupled pNRQCD**: $m_Q v^2 \ll \Lambda_{\mathrm{QCD}}$
  LDME factorizes into a state-dependent factor and a **universal gluonic quantity**
- Factorization equation, expanded in v at leading order:
  (Bodwin et.al. PRD51,1125(1995)), (Brambilla et.al. PRL 88,012003(2002))

$$\Gamma\left(\chi_{QJ} \to \mathrm{LH}\right) = \frac{1}{m_Q^2}\left|R'_{\chi_Q}(0)\right|^2 \left(\frac{3N_c}{2\pi m_Q^2} 2\mathrm{Im} f_1\left({}^3P_J\right) + 2\mathrm{Im} f_8\left({}^3S_1\right)\frac{2T_F}{9N_c m_Q^2}\frac{3N_c}{2\pi}\mathcal{E}_3\right)$$

Intro: pNRQCD factorization

$$\Gamma\left(\chi_{QJ}\to \mathrm{LH}\right)=\frac{1}{m_Q^2}\boxed{\left|R'_{\chi_Q}(0)\right|^2}\left(\frac{3N_c}{2\pi m_Q^2}\boxed{2\mathrm{Im}f_1\left({}^3P_J\right)}+\frac{2T_F}{9N_c m_Q^2}\frac{3N_c}{2\pi}\boxed{2\mathrm{Im}f_8\left({}^3S_1\right)}\mathcal{E}_3\right)$$

- $\boxed{\left|R'_{\chi_Q}(0)\right|^2}$: Squared derivative of the radial P-wave quarkonium wavefunction at the origin
- $\boxed{\text{SDC}}$ known up to $\mathcal{O}(\alpha_s^3)$: $2\mathrm{Im}f_8\left({}^3S_1\right)$, $2\mathrm{Im}f_1\left({}^3P_J\right)(\Lambda)\sim -n_f\frac{4C_F}{9N_c}\alpha_s^3\log\left(\frac{\Lambda}{2m_Q}\right)$

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- LDME with a universal correlator: $\mathcal{E}_3$
  (Brambilla et.al.:PRL88,012003(2002); PRD67,034018(2003); JHEP04,095(2020))
- LDME cancels factorization scale Λ

Takeaway: SDCs depend on the external states while the LDME is a universal quantity

Intro: $\mathcal{E}_3$

$$\mathcal{E}_3(\Lambda) = \frac{T_F}{N_c}\int_0^\infty \mathrm{d}t\, t^3\, \mathcal{E}(t)$$

$$\mathcal{E}(t) \equiv \langle 0| gE^{i,a}(t)\Phi^{ab}(t,0) gE^{i,b}(0)|0\rangle$$

$$E^{i,a} = \partial_0 A_i^a - \partial_i A_0^a - g f^{abc} A_0^b A_i^c$$

$$\Phi^{ab}(0,t) = \left[P\, e^{-ig\int_0^t dt A_0}\right]^{ab}$$

- $\mathcal{E}(t)$ is known at NNLO $\overline{\mathrm{MS}}$ at small t
- large t is non-perturbative
- $\mathcal{E}_3$ has logarithmic UV divergence from short-time region
- Divergence regulated by Λ and cancels IR logarithm of the singlet SDC

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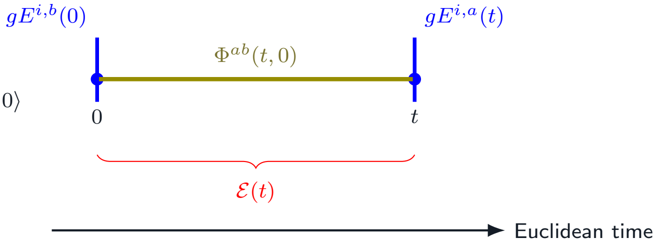

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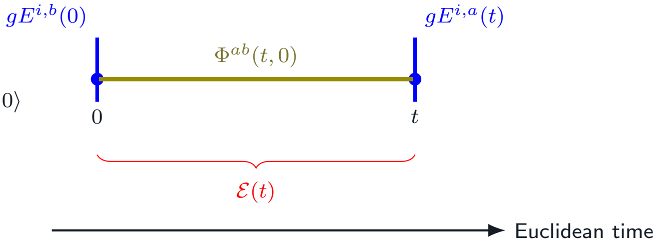

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We aim for a non-perturbative estimation of $\mathcal{E}_3$

Intro: Strategy for $\mathcal{E}_3$

- Split the integral into perturbative and non-perturbative part:

$$\mathcal{E}_3(\Lambda) = \mathcal{E}_3^{\text{pt}}(\Lambda, t_*) + \mathcal{E}_3^{\text{np}}(t_*)$$

$$\mathcal{E}_3^{\text{pt}}(\Lambda, t_*) = \frac{T_F}{N_c} \int_0^{t_*} \mathrm{d}t\, t^3\, \mathcal{E}^{\overline{\text{MS}}}(t)$$

$$\mathcal{E}_3^{\text{np}}(t_*) = \frac{T_F}{N_c} \int_{t_*}^{\infty} \mathrm{d}t\ t^3 G_{EE}^{\overline{\text{MS}}}(t)$$

- Computing the correlator on the lattice, matched to $\overline{\text{MS}}$: $G_{EE}^{\overline{\text{MS}}}(t)$
- Employing **gradient flow** to:
  - improve **signal-to-noise ratio**
  - **renormalize** the E-fields
  - **regulate the power divergence** of the Wilson line
- Here: First time computation of $\mathcal{E}_3$ in **quenched theory**

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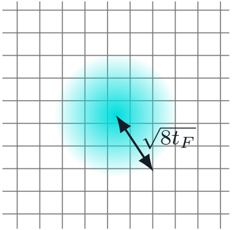
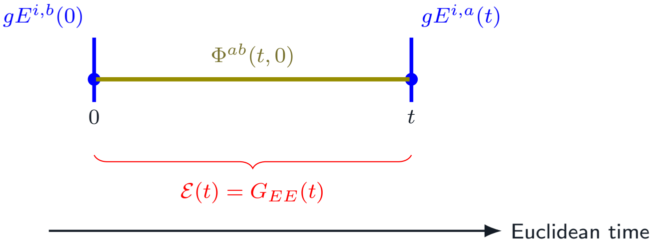


Intro: Power divergence

- On the lattice:

$$G_{EE}(t) = e^{-\delta m t} G_{EE}^{\text{sub}}$$
$$\delta m \propto \frac{1}{a}$$
$$\delta m \propto \frac{1}{\sqrt{8\tau_F}}$$



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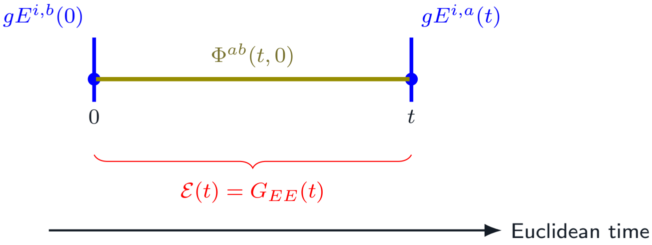
- At finite temperatures:

$$L_8 = e^{-\delta m/T} L_8^{\mathrm{sub}}$$



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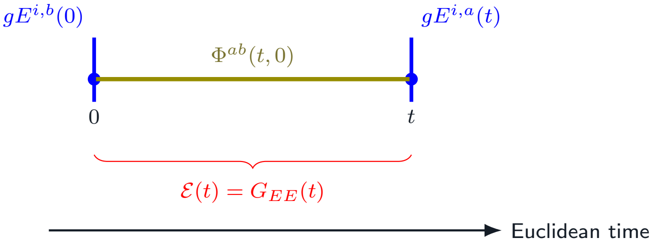

Takeaway: Find δm non-perturbatively from Polyakov loops at finite T

Computation of $\mathcal{E}(t)/\mathcal{E}_3^{\mathrm{np}}$

Warm up: Tackling the power divergence

- Polyakov loops at finite T and finite flow time:

$$\log L_8 = -\frac{\delta m}{T} + finite = -\frac{d(a)}{\sqrt{8\tau_F}T} + finite$$

- Parameter d parametrizes also for G_{EE}:

$$G_{EE}(t, \tau_F) = e^{-\frac{d}{\sqrt{8\tau_F}}t} G_{EE}^{\mathrm{sub}}(t)$$

- Measure flowed L_8 for different temperatures:
  - Spatial volume: large enough: 26^3, 30^3, 40^3
  - Temporal extent: $n_t = 4, 6, \ldots$ such that $T > T_C$

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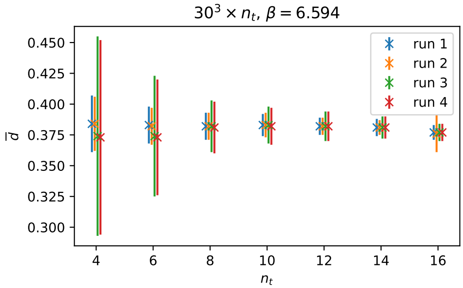
    - Fitting d for each ensemble with a Bayesian fit
    - Finding d for each lattice spacing a



Warm up: Tackling the power divergence

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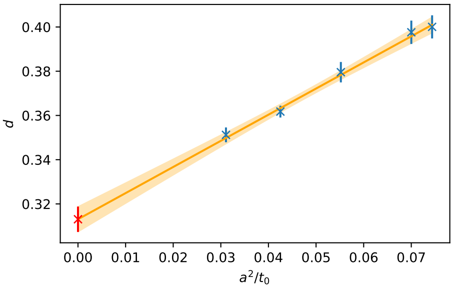
  - Fitting d for each ensemble with a Bayesian fit
  - Finding d for each lattice spacing a
  - Taking continuum limit of d

$$d = 0.315(6)$$



$\mathcal{E}(t)$: Computation on the lattice

- With gradient flow, the full matching reads:

$$G_{EE}^{\overline{\mathrm{MS}}}(t) = \lim_{\tau_F \to 0} e^{\left(\frac{d}{\sqrt{8\tau_F}} + m_0^{\overline{\mathrm{MS}}}\right)t} Z_{EE}^{\overline{\mathrm{MS}}} G_{EE}(\tau_F, t)$$

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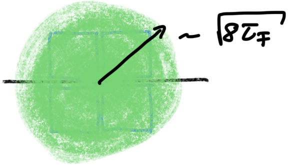
- τ_F: auxiliary gradient flow scale



$\sqrt{8\tau_F}$ smears out the lattice regulator

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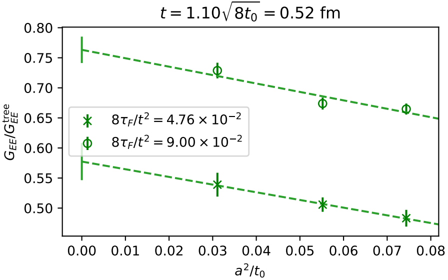
- $G_{EE}(\tau_F, t)$: Continuum limit of the flowed correlator



Continuum limit of $\frac{G_{EE}}{G_{EE}^{\mathrm{tree}}}$ (tree-level improvement)

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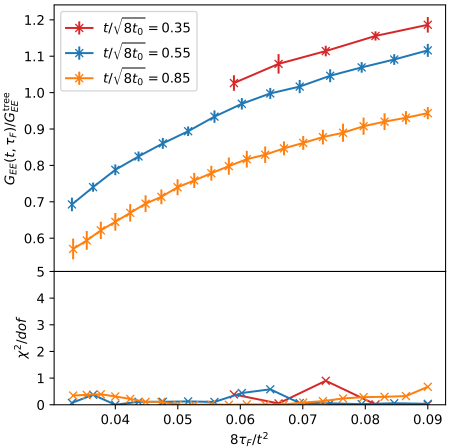
- $G_{EE}(\tau_F, t)$: Continuum limit of the flowed correlator
- $Z_{EE}^{\overline{\mathrm{MS}}}$: matching from gradient flow scheme to $\overline{\mathrm{MS}}$ scheme, $Z_{EE}^{\overline{\mathrm{MS}}} = 1 + \mathcal{O}(\alpha_s^2)$ (Brambilla and Wang, JHEP06,210,2024)



flowed correlator w/o subtraction of the divergence

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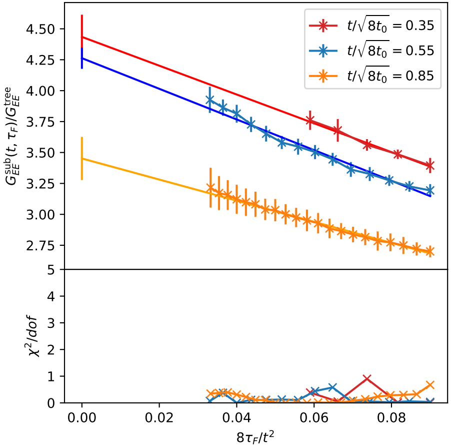
- d: parametrized subtraction of the power divergence, $d = 0.315(6)$



Zero-flow-time limit of the subtracted, flowed correlator

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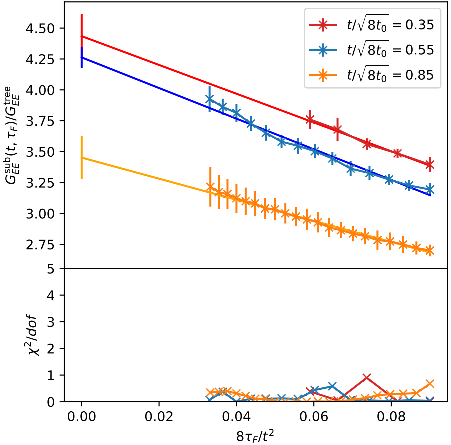

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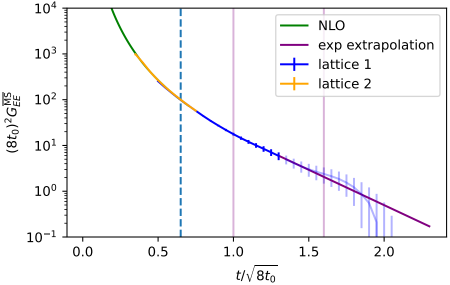
- d: parametrized subtraction of the power divergence, $d = 0.315(6)$
- $\lim_{\tau_F \to 0}$: zero flow time limit
- $m_0^{\overline{\mathrm{MS}}}$: finite and scheme dependent matching term
  Matching at $\alpha_s^{(n_f=0)}(1.17\,\mathrm{GeV}) = 0.26$ at NLO:
  $m_0^{\overline{\mathrm{MS}}}\sqrt{8t_0} = 0.476(12)$



matching the finite term at $\mathcal{O}(\alpha_s^2)$

Computation of $\mathcal{E}_3$

- Recall:

$$\mathcal{E}_3(\Lambda) = \mathcal{E}_3^{\rm pt}(\Lambda, t_*) + \mathcal{E}_3^{\rm np}(t_*)$$

- Numbers:
  - $t_* \approx 0.19\,\text{fm}$
  - $\Lambda = 2e^{-\gamma_E}/t_* \approx 1.17\,\text{GeV}$
  - $\mathcal{E}_3(1.17\,\text{GeV}) = 2.3 \pm 0.60^{\rm stat} \pm 0.15^{\rm syst} \pm 0.11^{\rm h.o.}$
- Run $\mathcal{E}_3$ to $m_c = M_{\chi_c}/2 = 1.76\,\text{GeV}$ and $m_b = M_{\chi_b}/2 = 4.95\,\text{GeV}$:

$$\mathcal{E}_3\left(\Lambda\right) = \mathcal{E}_3\left(\Lambda'\right) + \frac{24C_F}{\beta_0}\log\left(\frac{\alpha_s(\Lambda')}{\alpha_s(\Lambda)}\right)$$

- Final results for $\mathcal{E}_3$:

$$\mathcal{E}_3(M_{\chi_c}/2) = 3.14 \pm 0.60^{\rm stat.} \pm 0.15^{\rm syst.} \pm 0.11^{\rm h.o.}$$
$$\mathcal{E}_3\left(M_{\chi_b}/2\right) = 4.80 \pm 0.60^{\rm stat.} \pm 0.15^{\rm syst.} \pm 0.11^{\rm h.o.}$$

Computation of $\mathcal{E}_3$

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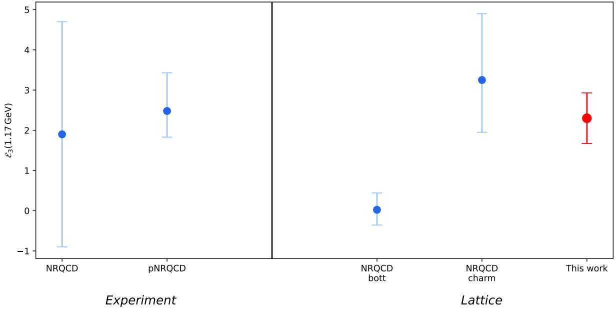

NRQCD: (Bodwin, Sinclair, Kim, PRL77,2376(1996))
pNRQCD: (Brambilla et.al. JHEP04,095(2020))

Final results for $\Gamma(\chi_{QJ} \to \mathrm{LH})$:

χ_{QJ}	$\Gamma\left(\chi_{QJ} \to \mathrm{LH}\right)_{\mathrm{exp}}$	$\Gamma\left(\chi_{QJ} \to \mathrm{LH}\right)$	trunc.	SDC	$\mathcal{E}_3$
$\chi_{c0}(1P)$	12.2 ± 0.5	13.13 ± 4.33	3.94	1.78	0.18
$\chi_{c1}(1P)$	0.552 ± 0.041	0.68 ± 0.27	0.20	0.03	0.18
$\chi_{c2}(1P)$	1.60 ± 0.06	1.64 ± 0.58	0.49	0.25	0.18
$\chi_{b0}(1P)$	-	1.07 ± 0.13	0.11	0.07	0.02
$\chi_{b1}(1P)$	-	0.13 ± 0.02	0.01	0.01	0.02
$\chi_{b2}(1P)$	-	0.24 ± 0.03	0.02	0.01	0.02
$\chi_{b0}(2P)$	-	1.37 ± 0.17	0.14	0.09	0.02
$\chi_{b1}(2P)$	-	0.16 ± 0.03	0.02	0.01	0.02
$\chi_{b2}(2P)$	-	0.31 ± 0.04	0.03	0.01	0.02
$\chi_{b0}(3P)$	-	1.58 ± 0.20	0.16	0.11	0.03
$\chi_{b1}(3P)$	-	0.19 ± 0.04	0.02	0.01	0.03
$\chi_{b2}(3P)$	-	0.35 ± 0.05	0.04	0.01	0.03

Error columns:

- trunc.: truncation in velocity expansion
- SDC: higher order perturbative expansion
- $\mathcal{E}_3$: errors from $\mathcal{E}_3$

Summary & Outlook

- Summary
  - Gradient flow serves as an excellent intermediate scale/regulator
  - Developed a new method to subtract the power divergence and match to $\overline{\mathrm{MS}}$
  - Theoretical prediction for $\Gamma\left(\chi_{cJ} \to \mathrm{LH}\right)$ in the regime of experimental measurements
- Outlook
  - Extending to unquenched lattice computation
  - Extending to S-wave decays with similar objects
  - Extending the Gradient flow method to production in pNRQCD

$$\mathcal{E}_{00} \propto \int_0^\infty dt \int_0^\infty dt' \langle \Phi_l^{\dagger ab}(0)\Phi_0^{\dagger ad}(0;t) gE^d(t) gE^e(t')\Phi_0^{ec}(0;t')\Phi_l^{bc}(0)\rangle$$

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- Outlook
  - Extending to unquenched lattice computation
  - Extending to S-wave decays with similar objects
  - Extending the Gradient flow method to production in pNRQCD

$$\mathcal{E}_{00} \propto \int_0^\infty dt \int_0^\infty dt' \langle \Phi_l^{\dagger ab}(0)\Phi_0^{\dagger ad}(0;t) gE^d(t) gE^e(t')\Phi_0^{ec}(0;t')\Phi_l^{bc}(0)\rangle$$

Thank you for sitting through my talk.

Sample frame title

This is some text in a sample frame. Don't waste your time and stay focused on the talks.

Takeaway: Knock Knock!! Who's there!?