



Deeply bound T_{bb} and near-threshold BB^* scattering with a relativistic heavy quark action

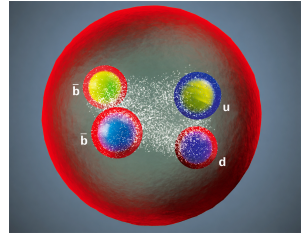
Andres Stump
Jeremy R. Green

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Humboldt-Universität zu Berlin

T_{bb} tetraquark and BB^* scattering

- Bottom analog of T_{cc} : $\bar{b}bud$, $I(J^P) = 0(1^+)$
→ appears in the BB^* system
 - Not (yet) seen in experiment
 - Broad consensus: **deeply bound**
[Leskovec et al. 2019; Alexandrou et al. 2024; Hoffmann and Meinel 2026]
 - Almost nothing known about BB^* scattering close to/above threshold
- **Goal:** Investigate T_{bb} binding energy *and* BB^* scattering



Why the BB^* system is hard

- Large $B^{(*)}$ masses
→ Dense finite-volume spectrum
- Large operator basis needed:
 - **Local** tetraquark operators:
Essential for T_{bb}
 - **Bilocal** two-meson operators:
For the BB^* scattering
- Small $B-B^*$ splitting (≈ 40 MeV)
 - Low B^*B^* threshold
 - Limits range of validity of single-channel Lüscher formalism

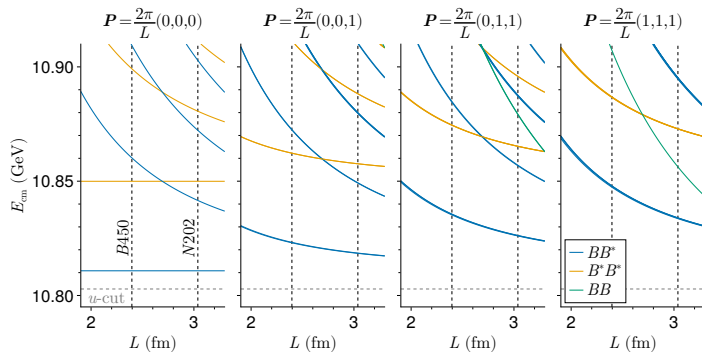


Figure: Noninteracting $B^{(*)}B^*$ c.m. levels

1. Lattice setup
2. B and B^* mesons
3. Local tetraquark operators in distillation
4. Results for the BB^* system (Exploratory!)
 - T_{bb} binding energy
 - Finite-volume BB^* spectrum
 - Single-channel s -wave Lüscher analysis

Lattice setup

CLS gauge ensembles (B450, N202):

- $N_f = 3$ Wilson clover fermions
- $SU(3)$ -flavour-symmetric point:
 - physical $\frac{1}{3}(m_u + m_d + m_s)$
 - $m_\pi \approx 420$ MeV

Ensemble	$N_s^3 \times N_t$	a [fm]	L [fm]	N_v
B450	$32^3 \times 64$	0.0749	2.4	32
N202	$48^3 \times 128$	0.0633	3.0	68

Bottom quark:

- Use RHQ action [Aoki, Kuramashi, and Tominaga 2003]:
(asymmetry between space and time)
- 5-parameter action
- Tuned nonperturbatively [Hudspith and Mohler 2022]:
 - 5 physical bottomonium masses
 - $c^2 = 1$

B and B^* mesons

Dispersion relation:

- Investigated speed-of-light parameter

$$E_B(\mathbf{p})^2 = m_B^2 + c_B^2 \mathbf{p}^2$$

→ Significantly above one on N202

- Possible reasons:

- Large excited-state contamination at $\mathbf{p} \neq 0$
(e.g. due to the large smearing radius)
- Mistuning of the RHQ action

- Variational analysis with covariant-derivative operators:

→ Reduces estimates of c_B^2 ($c_B^2 = 1.12(3)$)
→ Indicates large excited-state contamination

→ Need to handle excited states *and* correct for the dispersion relation for $B^{(*)}B^*$ states

Future plans: Repeat calculation with improved $B^{(*)}$ operator

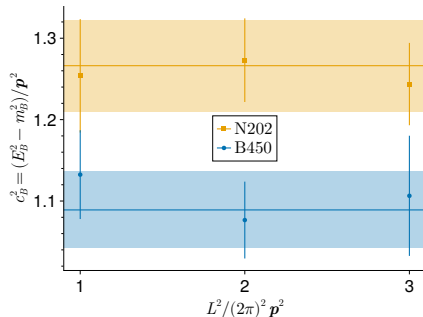


Figure: Speed-of-light parameter for B

Local tetraquark operators in distillation

Distillation + position-space sampling for local tetraquark operators

Laplacian Heaviside (LapH) smearing:

- Perambulator: $\tau(t', t) = V(t')^\dagger S(t', t) V(t)$
- $V(t)$ contains N_v lowest eigenvectors of $\Delta(t)$

Local tetraquark operators (e.g. diquark-antidiquark):

$$\mathcal{O}^T(t) \sim (\bar{b}b u d)(P, t)$$

- N_v^5 cost scaling in “traditional” distillation

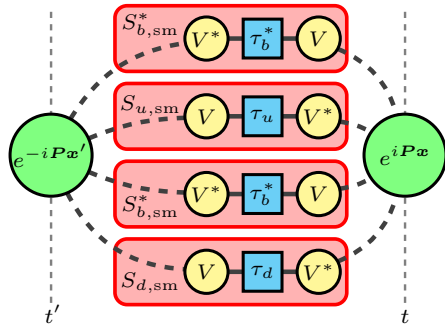
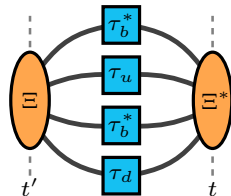
Position-space sampling method [AS and Green 2026]

- Explicitly compute smeared propagators (N_v^3 cost)

$$S_{f, \text{sm}}(t', t) = V(t') \cdot \tau_f(t', t) \cdot V(t)^\dagger$$

- Sum only over regular sparse grids with random offsets

→ Makes tetraquark operators affordable in distillation!



Results for the BB^* system (Exploratory!)

Large operator basis:

- Bilocal two-meson operators (BB^* and B^*B^*)
- Local tetraquark operators (local $B^{(*)}B^*$, BB_0^* , diquark-antidiquark)

$D = \frac{L}{2\pi} P$	(0, 0, 0)	(0, 0, 1)	(0, 1, 1)	(1, 1, 1)
Irrep	T_1^+	A_2	A_2	A_2
# bilocal operators	12	9	6	9
Total # operators	15	13	10	13

T_{bb} binding energy

- Well below the left-hand cut from one-pion exchange
→ cannot apply Lüscher
- Deeply bound ($e^{-m_\pi L}$ volume effects)
→ T_{bb} is the finite-volume ground state
- Difficulty: no clear plateau in the effective energy
→ fits at late, noisy times
 - Likely too large LapH smearing radius
 - N202: cross-check with Generalized Pencil of Function method

→ Binding energies are **lower bounds**

$$m_B + m_{B^*} - m_{T_{bb}} = \begin{cases} 53(8) \text{ MeV} & (\text{B450}), \\ 61(17) \text{ MeV} & (\text{N202}) \end{cases}$$

- In agreement with other RHQ results at similar m_π
[Hoffmann and Meinel 2026; Vujmilovic et al. 2026]

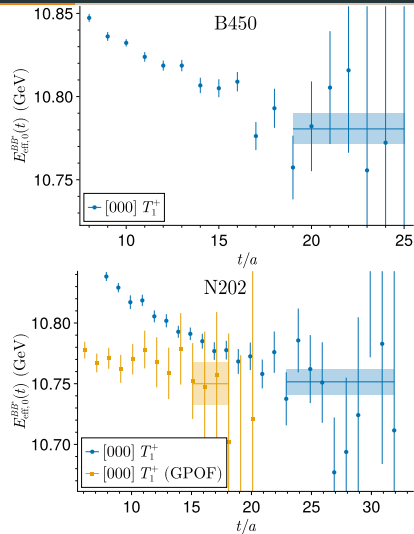


Figure: Ground state E_{eff} of BB^* system

Extracting excited $B^{(*)}B^*$ energies

Extract $\Delta E_n^{B^{(*)}B^*}$ to the nearest noninteracting level
 $\rightarrow$ strong (empirical) cancellation of excited-state effects

Two fitting strategies:

1. Fits to effective energy differences ("ratio" fits)

$$\Delta E_{\text{eff},n}^{B^{(*)}B^*}(t) = E_{\text{eff},n}^{B^{(*)}B^*}(t) - E_{\text{eff}}^{B^{(*)}(p_1)}(t) - E_{\text{eff}}^{B^*(p_2)}(t)$$

- Fit only where all E_{eff} have plateaued (if possible)

2. Combined fits to the three effective energies

Spread between the strategies $\rightarrow$ systematic error

Dispersion-relation correction:

$\rightarrow$ Use *continuum* noninteracting energies

$$E_n^{B^{(*)}B^*} = \Delta E_n^{B^{(*)}B^*} + \sqrt{m_{B^{(*)}}^2 + \mathbf{p}_1^2} + \sqrt{m_{B^*}^2 + \mathbf{p}_2^2}$$

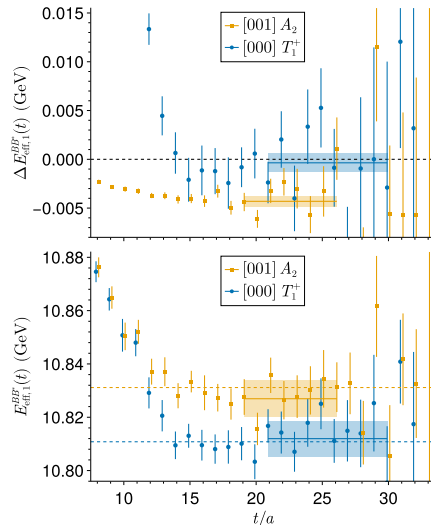


Figure: First excited state on N202:
 ΔE_{eff} (top) and E_{eff} (bottom)

Finite-volume $B^{(*)}B^*$ spectrum

- Ground state (T_{bb}) below u -cut
→ not usable for Lüscher analysis
- Use the **first excited states**
(well below B^*B^*)

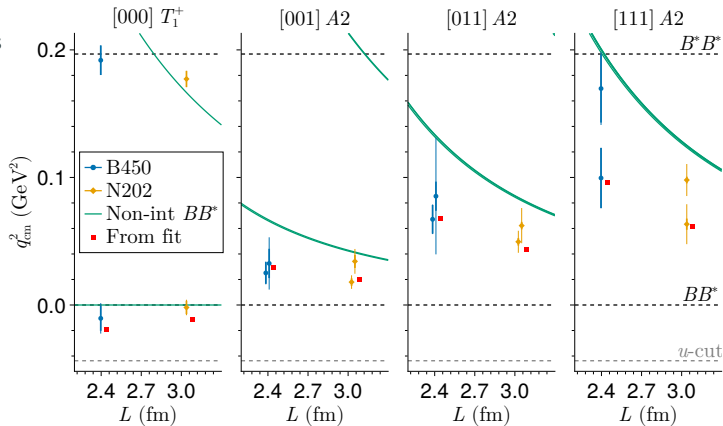


Figure: Excited $B^{(*)}B^*$ states

BB^* scattering near threshold

Single-channel s -wave Lüscher analysis

- Linear effective range expansion
- Combined fit to both ensembles
- Good fit quality
→ Consistent values for phase shift
- Crossing with the pole condition:
 - Strong indication of **shallow virtual bound state**
 - $2(2)_{\text{stat}}(1)_{\text{sys}}$ MeV below threshold
 - Distortion due to one-pion exchange?
- Heavy-quark spin symmetry:
 - A bound BB^* state has a near-degenerate B^*B^* partner
 - Relation to T_{cc} and T'_{cc} in $DD^*-D^*D^*$ system [Whyte, Wilson, and Thomas 2025]

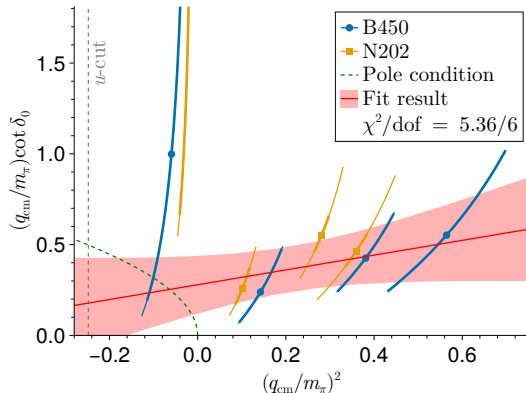


Figure: s -wave BB^* scattering phase shift

- Investigation of BB^* system with RHQ action:
 - $m_\pi \approx 420$ MeV
 - Large basis of local and bilocal operators in distillation
- Binding energy of the T_{bb} : ~ 55 MeV (lower bound)
→ Consistent with other RHQ results
- BB^* scattering near threshold:
 - s -wave Lüscher analysis (good fit quality)
 - Found a shallow **virtual bound state** 2 MeV below threshold
- Future plans:
 - Improve $B^{(*)}$ -meson and tetraquark operators → More trustworthy spectrum
 - Improve finite volume analysis (multiple channels, LHC)
 - Multiple lattice spacings/volumes

Thank you for your attention!

References

References

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References

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Backup slides

B and B^* effective energies

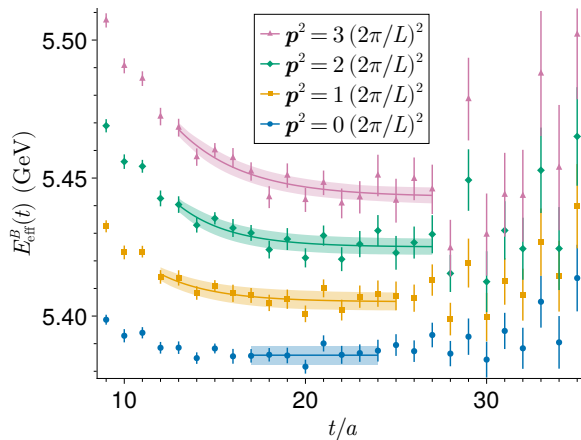


Figure: Effective energy E_{eff}^B of the B meson on N202 for the four lowest momenta.

T_{bb} dispersion relation

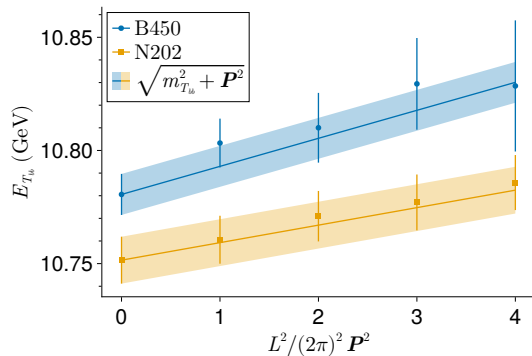


Figure: Ground-state energy vs. the integer total momentum squared. The curves show the continuum relation $E^2 = m_{T_{bb}}^2 + P^2$ evaluated at the respective lattice masses.

Comparison of the two fitting strategies

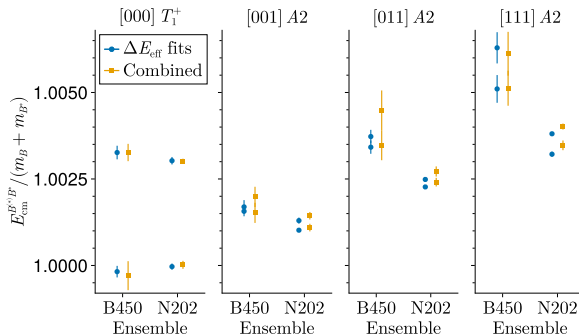


Figure: Excited finite-volume $B^{(*)}B^*$ energies from both fitting strategies and both ensembles, converted to the c.m. frame and normalized by the threshold energy $m_B + m_{B^*}$.

CLS gauge ensembles and RHQ parameters

Ensemble	$N_s^3 \times N_t$	β	a [fm]	$\kappa_u = \kappa_d = \kappa_s$	N_{cfg}	N_{src}	N_v	N_{sep}
B450	$32^3 \times 64$	3.46	0.0749	0.136890	1612	8	32	8
N202	$48^3 \times 128$	3.55	0.0633	0.137000	899	8	68	8

Ensemble	κ_b	ν	r_s	c_E	c_B
B450	0.054727	2.167	1.981	1.947	2.135
N202	0.068125	1.784	1.631	1.845	1.932

- $N_f = 3$ Wilson clover fermions
- $SU(3)$ -flavour-symmetric point with physical $\frac{1}{3}(m_u + m_d + m_s)$
- $m_\pi \approx 420$ MeV
- RHQ action for the b quark, tuned to 5 bottomonium masses and $c^2 = 1$