Diquark properties from heavy-light tetraquark operators

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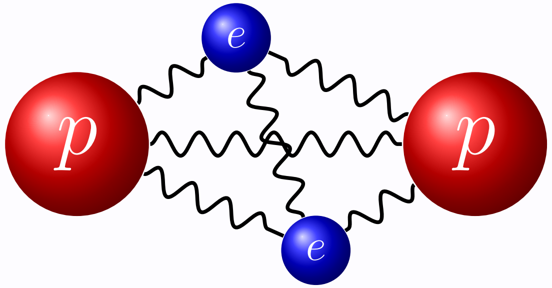
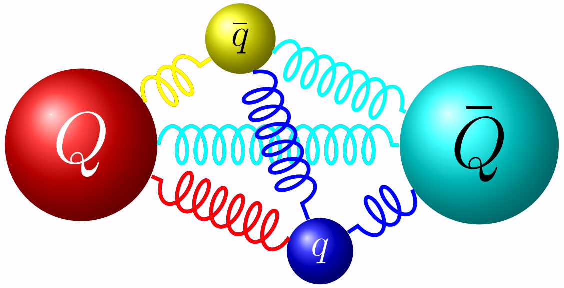
Overview

1. Heavy-Light Tetraquarks and Born-Oppenheimer EFT
2. Diquarks in Baryons and Tetraquarks
3. Relevant color and flavor Structures
4. N_f- and N_c-dependence of Diquarks
5. Towards Calculations on the Lattice

Heavy-Light Tetraquarks

- Plethora of experimentally seen tetraquarks with heavy Q and light q quarks:
  - $\bar{Q}Q\bar{q}q$ (hidden heavy flavor), e.g. Z_b, Z_c (aka $T_{b\bar{b}}, T_{c\bar{c}}$)
  - $QQ\bar{q}\bar{q}$ (open hidden flavor), e.g. T_{bb}, T_{cc}
- The presence of heavy b and c quarks provides a separation of scales $\Rightarrow$ opportunity for effective descriptions

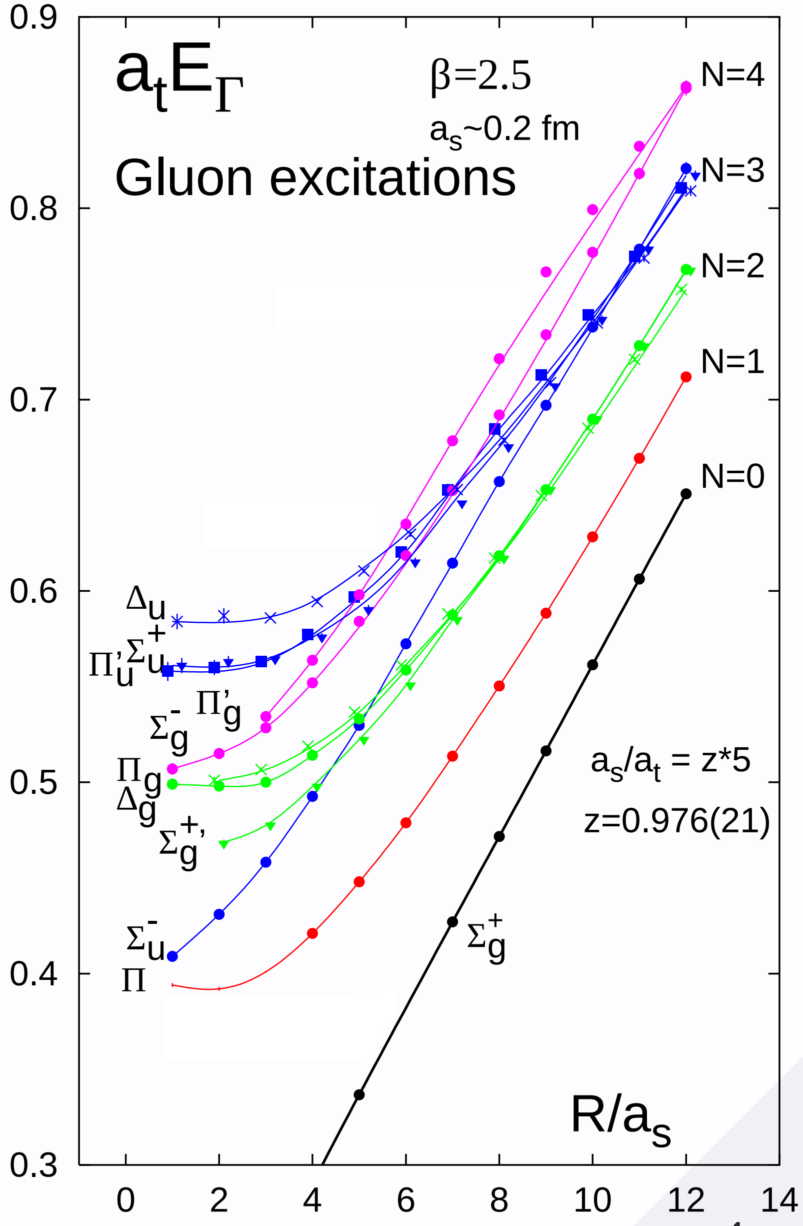




[image] Juge, Kuti, Morningstar, PRL90 (2003), hep-lat/0207004 (modified)[1]

Born-Oppenheimer Effective Field Theory[2] [3] (BOEFT)

- Separate light degrees of freedom *LDF* (glue, u, d) from heavy degrees of freedom.
- **At 1st order**: Heavy quarks are static & provide a potential at separation r
- **Lattice input**: static potentials $V(r)$ for Born-Oppenheimer quantum number of cylindrical symmetry $\Rightarrow$ solve Schrödinger equation



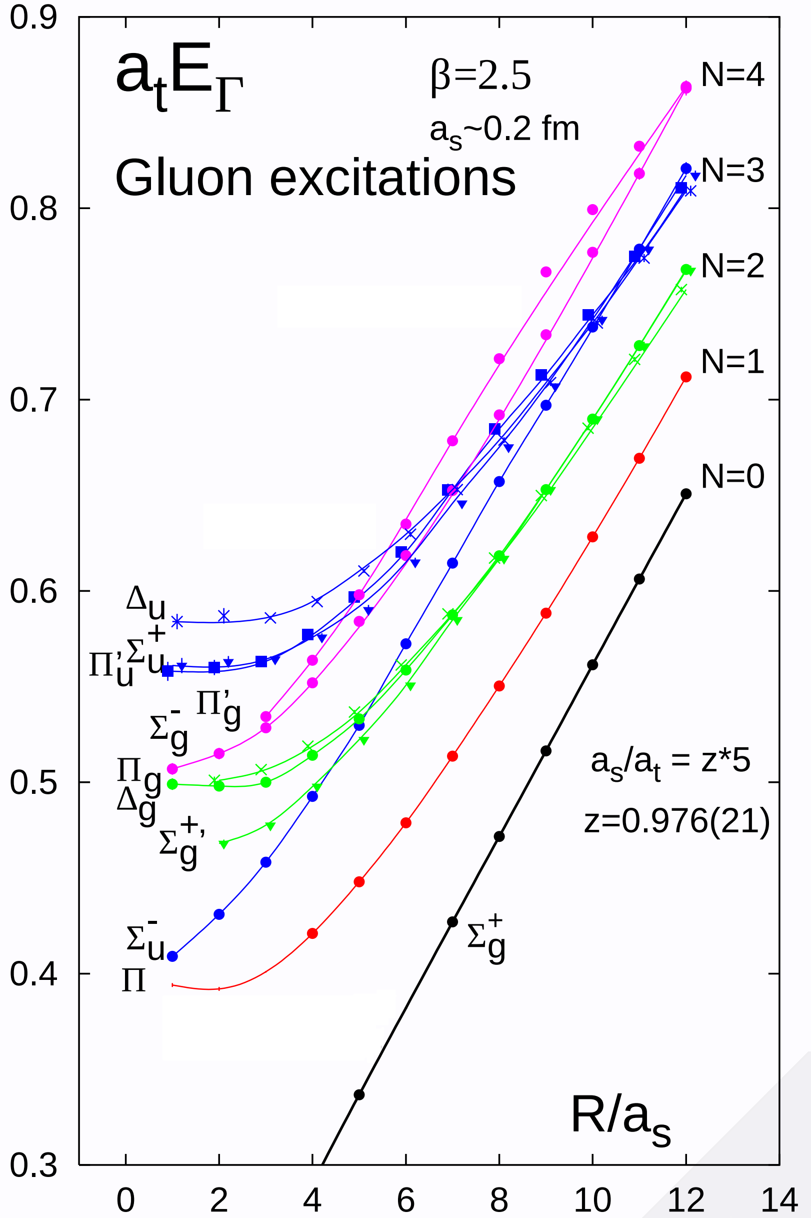
[1] Herr et.al. [2306.09902] Höllwieser et.al.[2501.15670]
[2] Foster et.al. [hep-lat/9811010] Sharma et.al. [2603.29581]
[3] Kelvin-Lee,Ishii [2606.19840], Francis et.al. [2106.09080] + many others

Born-Oppenheimer EFT (BOEFT)

- Interested in potentials $V(\mathbf{r})$
  - obtained from 2-point correlators
- constant in limit $\mathbf{r} \to 0$ from LDF

$$E(r) = V(r) + \Lambda + \mathcal{O}(r^2)$$

- energy Λ from light dof
  - gluonic $\Rightarrow$ gluelump [1]
  - colored $q\bar{q} \Rightarrow$ adjoint meson [2]
  - colored $qq \Rightarrow$ diquark [3]



Application: T_{cc} in BOEFT without experimental input

- Adjoint meson/diquark energies are input parameters in [1]
  - Adjoint meson $\Rightarrow \chi_{c1}(3872)$ (hidden flavor)
  - Diquark $\Rightarrow T_{cc}^{+}(3875)$ (open flavor)
- *treated as a free parameter in* [1] *and fixed by experiment!*

$$V_{\Sigma_g^+} = \begin{cases} \frac{\kappa_3}{r} + \Lambda_{0^+} + A_{\Sigma_g^+} r^2 & r < R_{\Sigma_g^+} \\ F_{\Sigma_g^+} e^{-r/d}/r^2 & r > R_{\Sigma_g^+} \end{cases},$$

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Goal: Provide the diquark mass differences from the lattice!
      Avoid Model dependence of V(r)!
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Light Degrees of Freedom: A Unified Approach

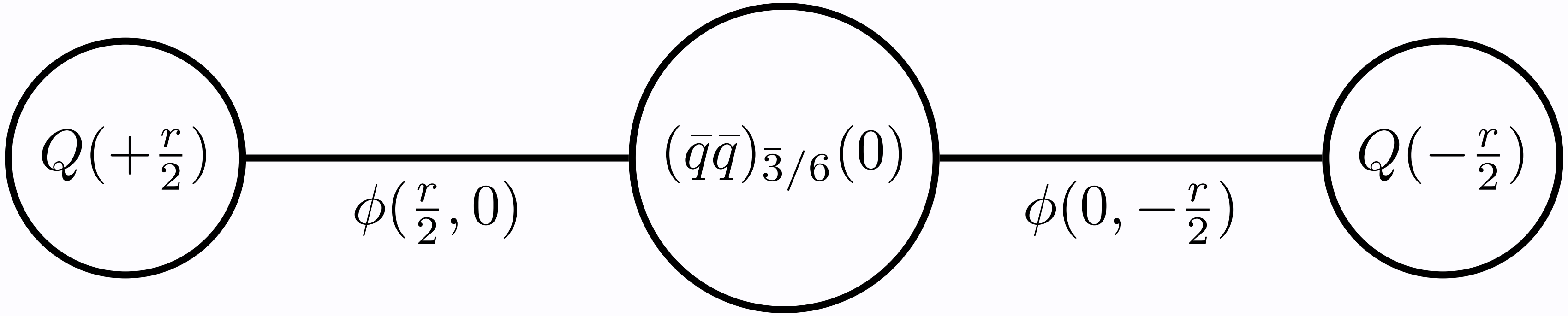
- Light degrees of freedom can carry spin, isospin, color
  - We want an approach which can include everything
- Schematic BOEFT tetraquark operator for QQ pair in color representation $\bar{R}$:

$$O_R = \underbrace{Q\left(t; +\frac{\mathbf{r}}{2}\right)}_{\text{static quark}} \underbrace{\phi_R\left(+\frac{\mathbf{r}}{2}, \mathbf{0}\right)}_{\text{Wilson line in R}} \underbrace{H^a_R(t, \mathbf{0})}_{\text{light dof}} \underbrace{T^a_R}_{\text{color}} \phi_R\left(\mathbf{0}, \mathbf{R} - \frac{\mathbf{r}}{2}\right) Q\left(t; -\frac{\mathbf{r}}{2}\right)$$

- Operator can incorporate all desired quantum numbers and is constructed to have vanishing overlap with quarkonium + hadron states in the EFT [1]

Diquarks from BOEFT operators

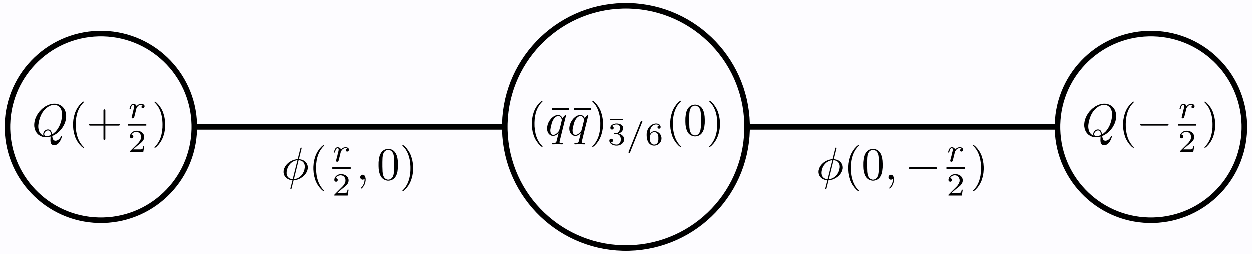
- Diquarks are **not physical** objects! They are not observable.
- We want to determine associated properties required for a BOEFT approach
- Start with BOEFT-operators $\Rightarrow$ matching notion of a diquark.
- Diquark energy arises in the zero-separation limit of these operators



see e.g. [1] Kelvin-Lee,Ishii [2606.19840], Francis et.al. [2106.09080]

Gauge Invariant Approach for Tetraquarks

- Extension of approach with static-light baryon operators [1]
- Consider BOEFT Operator with *static* quarks Q with spatial separation r, for light dof's $\bar{q}\bar{q}$:



- Two-point correlation functions of these operators are **gauge-invariant** on the lattice.
- Parameter Λ from exponential decay in Euclidean time

$$\lim_{|\mathbf{r}|\to 0} \langle O(t)O^\dagger(t')\rangle \to \exp\left(-(t-t')\Lambda\right),$$

Structure for Varying and N_c and Light N_f

- ensembles at $N_f = 2, 3$ available
- adjoint Meson in $N_c \times \bar{N}_c$, diquarks in $N_c \times N_c$

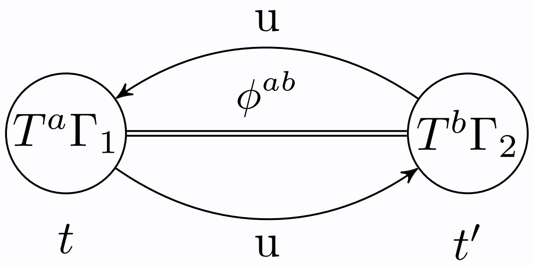
$$N_c \times \bar{N}_c = \underbrace{(N_c^2 - 1)}_{\text{adjoint}} + \underbrace{1}_{\text{singlet}} \qquad \mathrm{QCD}: \; 3 \times \bar{3} = 8 + 1$$

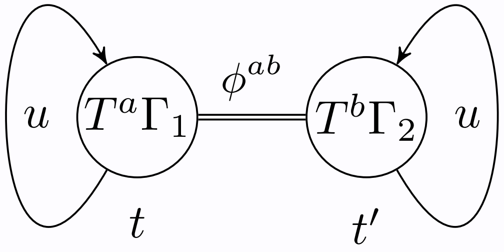
$$N_c \times N_c = \underbrace{(N_c^2 + N_c)/2}_{\text{symmetric}} + \underbrace{(N_c^2 - N_c)/2}_{\text{anti-symmetric}} \qquad \mathrm{QCD}: \; 3 \times 3 = \bar{3} + 6$$

- Same reasoning applies to **flavor**:
    - Flavour **symmetric, and antisymmetric diquarks**
    - Flavour **nonsinglet and singlet adjoint mesons**

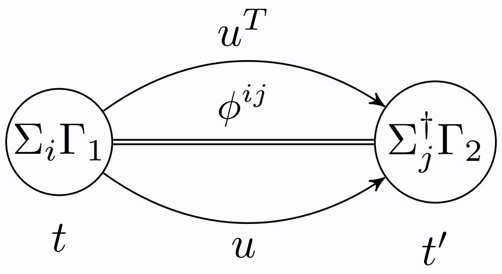
Diagrams to be Evaluated

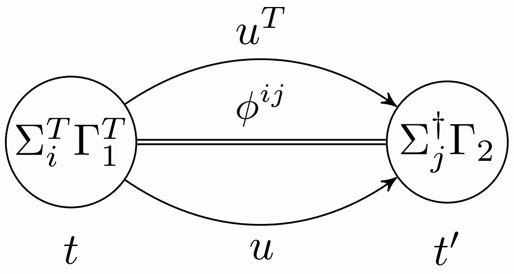
- Only singlet (adjoint) mesons have explicit N_f dependence

$C_8^{1_F}(t,t') =$ 

$- \quad N_f$ 

- Diquarks diagrams have the same form for all N_f and N_c

$C^{\mathrm{S|AS}}(t,t') =$ 

$\mp$ 

- Flavor symmetric and antisymmetric channel selected by $\pm$
- Some correlators vanish due to Pauli exclusion
- Temporal Wilson line comes from static quark propagation

Wilson Lines in Higher Representations

- can be expressed in fundamental link variables:

$$\phi^{ab}_{\mathrm{adj}}(t,t') = \mathbf{Tr}\left[U(t,t')T^a U(t,t')^\dagger T^b\right]$$
$$\phi^{\ell\ell'}_{\bar{3}}(t,t') = \mathbf{Tr}\left[U^T(t,t')\underline{T}^\ell U(t,t')\underline{T}^{\ell'\dagger}\right]$$
$$\phi^{\sigma\sigma'}_{6}(t,t') = \mathbf{Tr}\left[U^T(t,t')\underline{\Sigma}^\sigma U(t,t')\underline{\Sigma}^{\sigma'\dagger}\right]$$

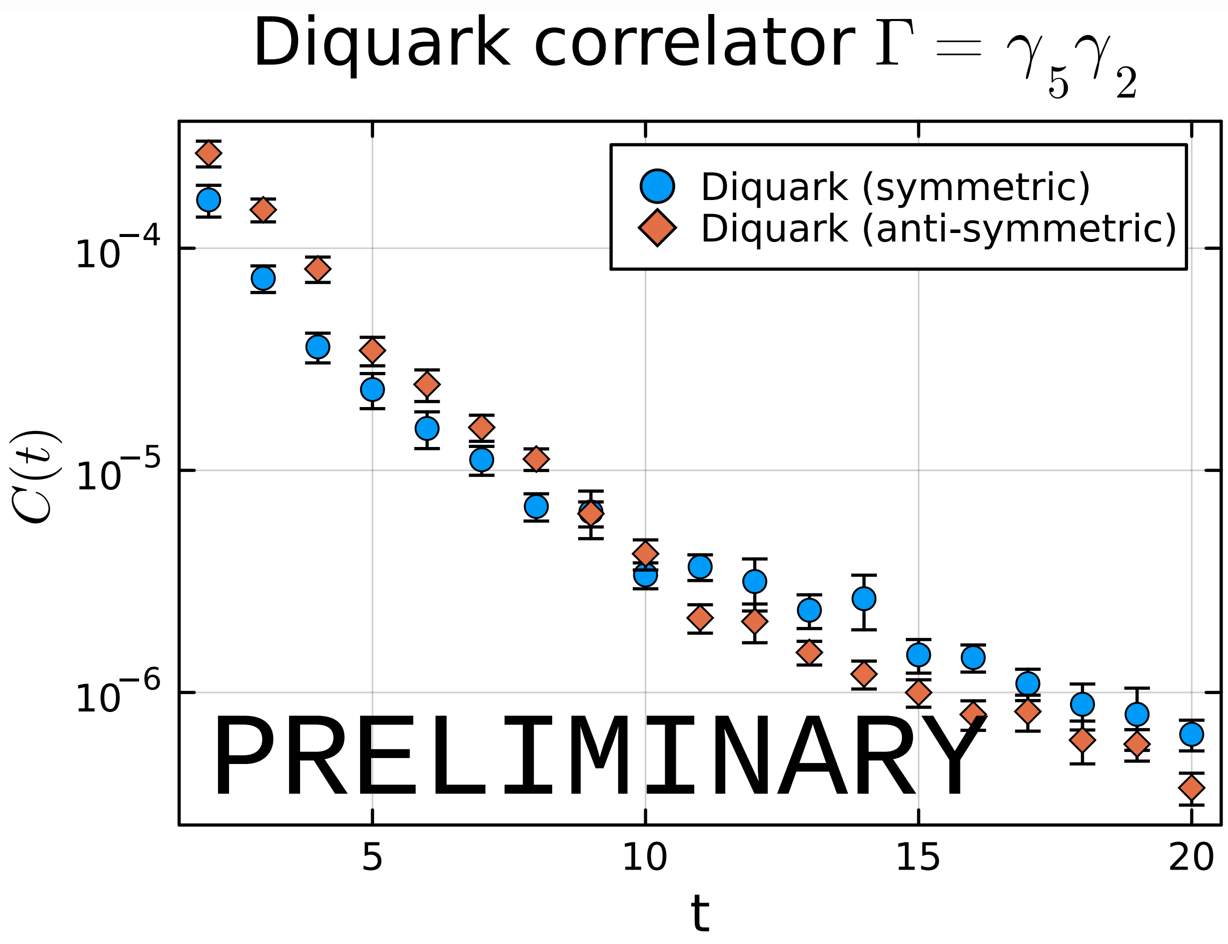
- Where T_a are in the adjoint representation, $\underline{T}^\ell$ are in the anti-symmetric, and $\underline{\Sigma}^\sigma$ are in the symmetric representation
- $U(t,t')$ are the usual fundamental link variables
- $\Rightarrow$ straightforward to implement on the lattice!

Towards Lattice Calculations

- Tests on one ensemble with $N_f = 3 + 1$, $m_\pi \approx 400$ MeV and open BC on a 96×32^3 lattice, in total 4000 configurations
- Measured with distillation with $N_\nu = 100$ for light flavours, HYP smearing for gauge fields
- Initial study of non-singlet adjoint mesons in $[1]$ performed on same ensemble

A First Look at the Signal

- On a small subset of configurations
- Signal for both diquarks seen
- Noticeably noisier than adjoint mesons with matching statistics



Summary

- Multi-quark operators allow gauge-invariant construction of diquarks in higher colour representations
- Derived the structure of Wick diagrams for all N_f and N_c
- Lattice determination is feasible and will be carried out

Next steps

- Use full statistics available and perform robust analysis
- Improve performance with generalized Fierz identities
- Consider all $J = 0, 1$ channels and include flavour-singlet adjoint mesons

Thank you