The Neutron Electric Dipole Moment from Lattice QCD using a Background Electric Field

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Fangcheng He
New Mexico State University/LBNL

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Collaborators: Tom Blum, Taku Izubuchi, Luchang Jin, Hiroshi Ohki and Sergey Syritsyn

Outline

- The calculations of EDM on lattice using background field method

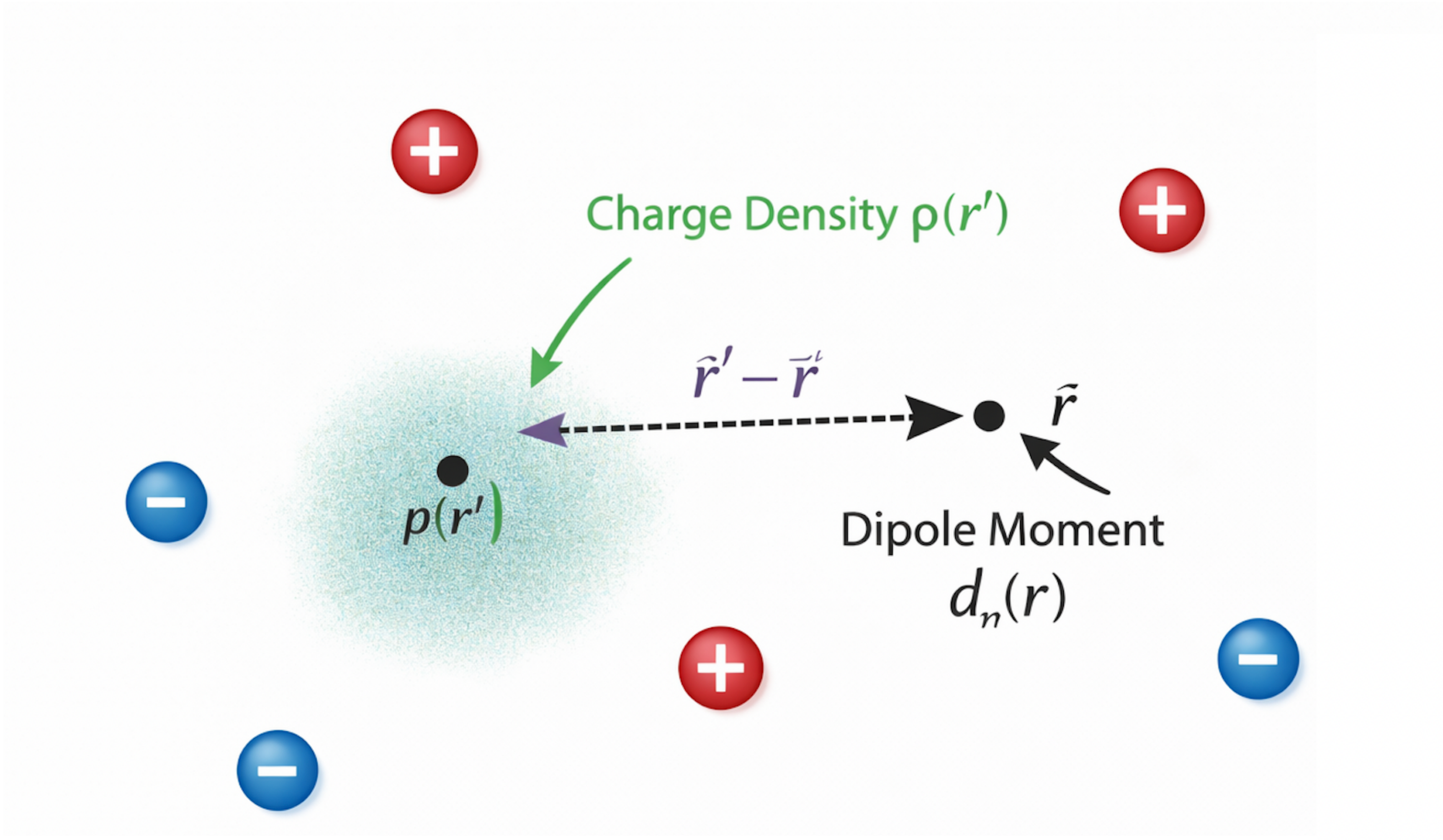
- Numerical results

- Summary and outlook

Neutron electric dipole moment

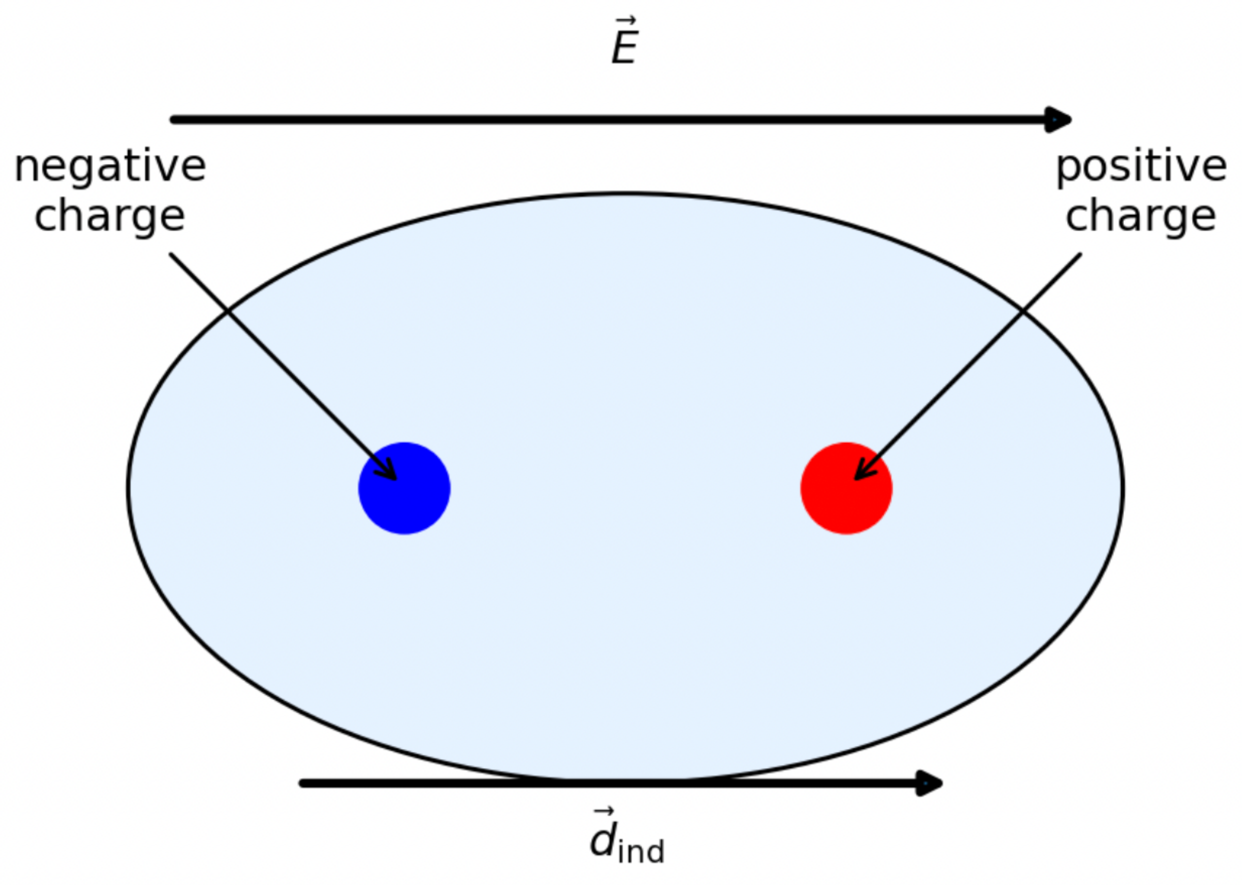
- **The electric dipole moment is a measure of the separation of positive and negative charges within a system:**

$$\overrightarrow{d}_{tot} = \alpha_E \overrightarrow{E} + d_n \overrightarrow{S}$$



$$\overrightarrow{d}(\vec{r}) = \int_V \rho(\overrightarrow{r'})(\overrightarrow{r'} - \vec{r}) d^3\overrightarrow{r'}$$

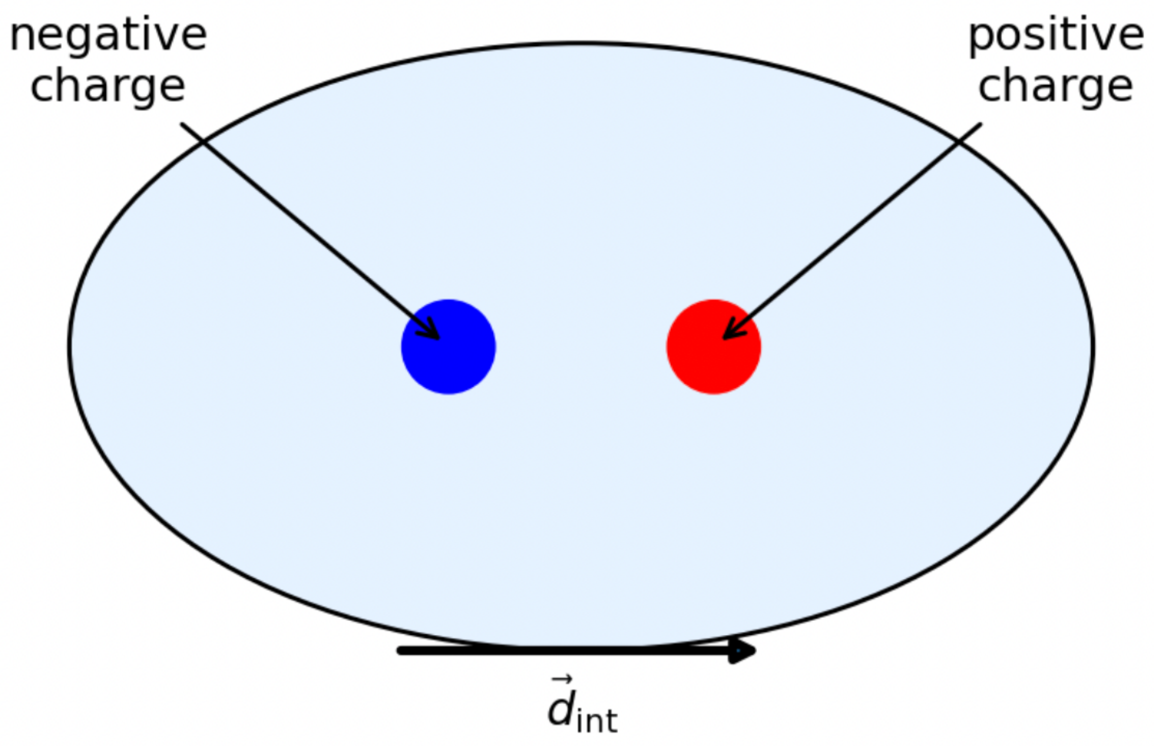
Induced neutron EDM



$\vec{d}_{\text{ind}} = \alpha_E \vec{E}$

spin-independent (CP-even)

Intrinsic neutron EDM



$\vec{d}_{\text{int}} = d_n \vec{S}$

spin-dependent (CP-odd)

The intrinsic nEDM is an important probe of CP violation!

Lattice calculation of the theta EDM

Topological charge

$$Q_{top} = \sum_t \sum_{\vec{y}} \frac{G^a_{\mu\nu}(t,\vec{y})\widetilde{G}^a_{\mu\nu}(t,\vec{y})}{32\pi^2}$$

- Form factor method

J. Dragos, et al. PRC 103 (2021); C. Alexandrou, et al. PRD 103(5) (2021)
T. Bhattacharya, et al. PRD 103(11) (2021);J. Liang, et al. PRD*108* (2023)

CP violation 3pt correlation function

$$\langle N J^\mu \bar{N}\rangle_{\cancel{CP}} = \langle N J^\mu \bar{N}\rangle + i\theta\langle N J^\mu \bar{N} Q_{top}\rangle + \mathcal{O}(\theta^2)$$

CP violation matrix element

$$\langle p',\sigma'|J^\mu|p,\sigma\rangle_{\cancel{CP}} = \bar{u}_{p',\sigma'}\left[F_1(Q^2)\gamma^\mu + \left(F_2(Q^2) + iF_3(Q^2)\gamma_5\right)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]u_{p,\sigma}\,,$$

CP even form factors **Electric dipole form factor**

Q^2 **extrapolation**

$$d_n = \frac{F_3(Q^2 \to 0)}{2m_n}$$

- Background electric field method

S. Aoki, et.al PRL 65 (1990);E. Shintani, et.al PRD 75 (2007); PRD 78 (2008);T. Izubuchi, et.al *PoS* LATTICE2007; M. Abramczyk, et. al .PRD 96 (2017)

Energy shift proportional to EDM

$$\delta E \propto d_n \vec{S}\cdot\vec{\mathcal{E}}$$

Determines nEDM directly at zero momentum.

$$R(t,t_{src};E) = \lim_{t\to\text{large}} \frac{\langle N_\uparrow(t)\overline{N}_\uparrow(t_{src})\rangle_{\theta,E}}{\langle N_\downarrow(t)\overline{N}_\downarrow(t_{src})\rangle_{\theta,E}} \propto e^{+d_N(\theta)Et}\,.$$

Challenges for lattice nEDM calculations:

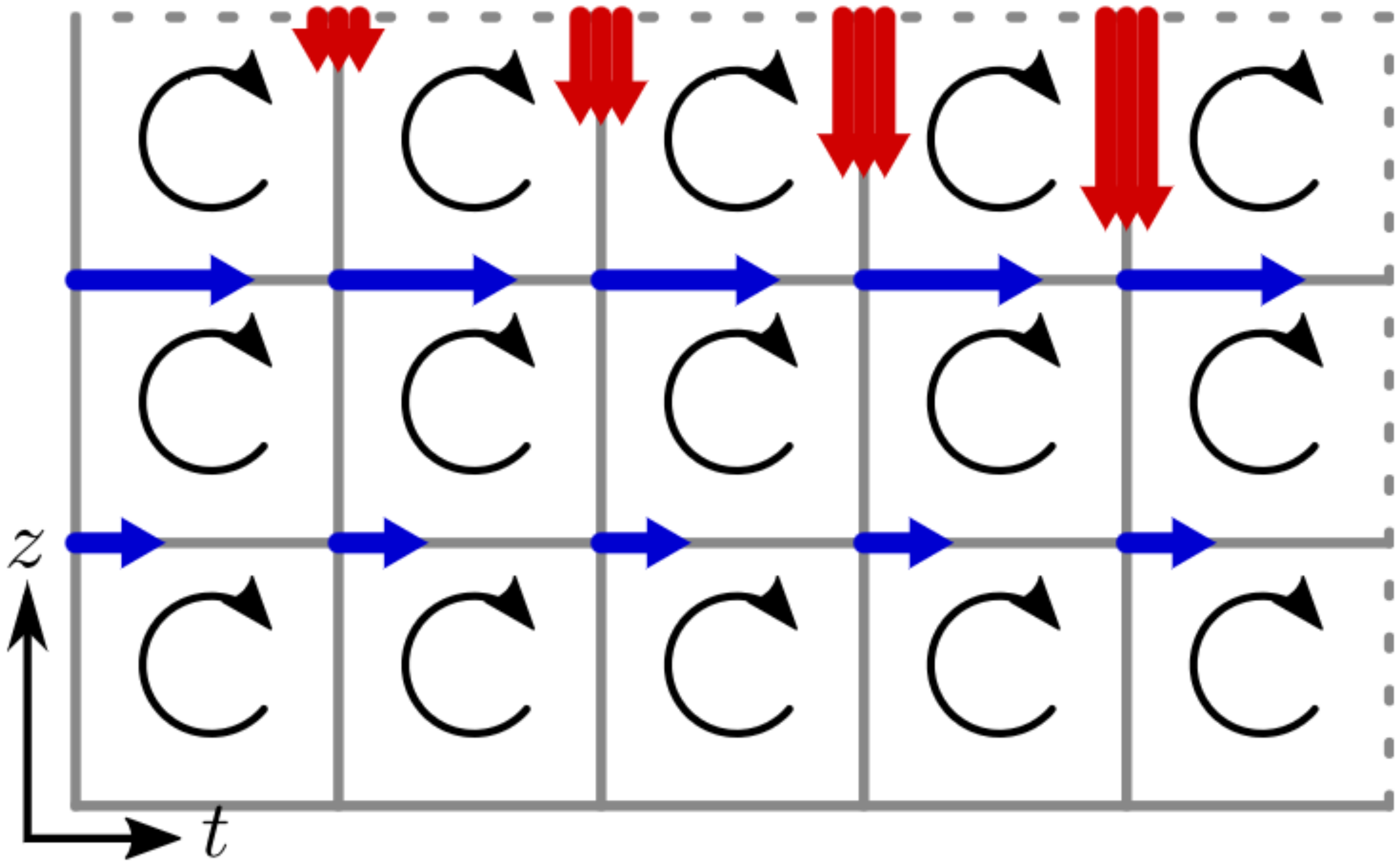
1. Large noise from Q_{top}

2. Excited-state contamination caused by parity mixing.

Background electric field method

- The constant background field on a periodic lattice

W. Detmold, B. C. Tiburzi, A. Walker-Loud, PRD (2010)



Effective Dirac operator

$$\not{D}_{QCD} \to \not{D}_{QCD} + iq\not{A}$$

The setup of U(1) gauge links

$$U_\mu \to e^{iqA_\mu} U_\mu$$

$$A_t(z,t) = \mathcal{E}_z z$$

$\mathcal{E}_z$ **: Electric-field strength**

$$A_z(L_z - 1, t) = -\mathcal{E}_z t \times L_z$$

Quantization condition $\quad e^{iq\mathcal{E}_z L_z L_t} = 1 \qquad qL_t L_z \mathcal{E}_z = 2\pi n$

Deformed nucleon ground state

- The Euclidean electric dipole operator induced by the electric field

Two point correlator $C_{2pt,\mathcal{E}} = \langle O(t_f) O^\dagger(0) e^{-S_{QCD} - i \int d^4x \mathcal{A}_\mu J^\mu_{EM}} \rangle$

Vector potential $\mathcal{A}^\mu = (0, 0, 0, z\mathcal{E}_z).$

Electric dipole interaction
(Parity odd, CP even, anti-Hermitian)

$$\Delta_H = i\mathcal{E}_z \mathcal{D}_z = i\mathcal{E}_z \int d^3r \, z \, \rho_{EM}(\vec{r})$$

- Deformed nucleon ground state to first order in the electric field

Right eigenstate

$$|\mathcal{N}_0^R\rangle = |N_0\rangle + i\mathcal{E}_z \sum_i \frac{\langle N_i^* | \mathcal{D}_z | N_0 \rangle}{m_{N_0} - m_{N_i^*}} |N_i^*\rangle + O(\mathcal{E}_z^2) \,,$$

$|N_i\rangle$: **positive-parity nucleon state**
$|N_i^*\rangle$: **negative-parity nucleon state**

Left eigenstate

$$\langle \mathcal{N}_0^L | = \langle N_0 | + i\mathcal{E}_z \sum_i \frac{\langle N_0 | \mathcal{D}_z | N_i^* \rangle}{m_{N_0} - m_{N_i^*}} \langle N_i^* | + O(\mathcal{E}_z^2) \,,$$

A positive parity projector by itself does not isolate the deformed nucleon ground state!

Feynman-Hellmann theorem

- The nEDM from Feynman-Hellman theorem

1. Energy shift

$$E_N = m_N + i\bar{\theta} d_n^{\theta} \, (\vec{\Sigma} \cdot \vec{\mathcal{E}})$$

2. Feynman-Hellman theorem

$$\left.\frac{\partial E_N^{\uparrow}}{\partial \bar{\theta}}\right|_{\bar{\theta}=0} = i \langle \mathcal{N}_0^{L,\uparrow} | q_{top}(0) | \mathcal{N}_0^{R,\uparrow} \rangle \Big|_{\mathcal{E}_z} ,$$

3. EDM from time-local qtop

$$d_n^{\theta} = \frac{1}{\mathcal{E}_z} \langle \mathcal{N}_0^{L,\uparrow} | q_{top}(0) | \mathcal{N}_0^{R,\uparrow} \rangle_{\mathcal{E}_z}$$

Time local qtop $q_{top}(0) = \sum_{\vec{y}} \frac{G^a_{\mu\nu}(0,\vec{y}) \widetilde{G}^a_{\mu\nu}(0,\vec{y})}{32\pi^2}$

- The matrix element of nEDM

Deformed nucleon ground states

$$|\mathcal{N}_0^R\rangle = |N_0\rangle + i\mathcal{E}_z \sum_i \frac{\langle N_i^* | \mathcal{D}_z | N_0 \rangle}{m_{N_0} - m_{N_i^*}} |N_i^*\rangle + O(\mathcal{E}_z^2) ,$$

$$\langle \mathcal{N}_0^L| = \langle N_0| + i\mathcal{E}_z \sum_i \frac{\langle N_0 | \mathcal{D}_z | N_i^* \rangle}{m_{N_0} - m_{N_i^*}} \langle N_i^*| + O(\mathcal{E}_z^2) ,$$

Spectral interpretation

$$d_n^{\theta} = \frac{1}{\mathcal{E}_z} \langle \mathcal{N}_0^{L,\uparrow} | q_{top}(0) | \mathcal{N}_0^{R,\uparrow} \rangle_{\mathcal{E}_z}$$

$$= 2 \sum_i \frac{\mathrm{Im}\left[\langle N_0 | q_{top} | N_i^* \rangle \langle N_i^* | \mathcal{D}_z | N_0 \rangle\right]}{m_{N_i^*} - m_{N_0}}$$

V. Baluni. PRD 19 (1979)

The nEDM arises from the interplay between CP-even electric dipole operator and the CP-odd topological charge operator.

Non-Hermitian GEVP (variational method)

- Extraction of ground state nEDM using non-Hermitian GEVP

1. Choose a parity basis

$$\psi_N^{\uparrow} = \frac{1}{4}(1+\gamma_4)(1+\Sigma_z)O$$

$$\psi_{N*}^{\uparrow} = \frac{1}{4}(1-\gamma_4)(1+\Sigma_z)O$$

2. Build the 2x2 correlator

$$C_{2pt,\mathcal{E}}^{\psi\psi^\dagger,\uparrow}(t_f) = \begin{pmatrix} \langle \psi_N^{\uparrow}(t_f)\psi_N^{\uparrow,\dagger}(0)\rangle_{\mathcal{E}_z} & \langle \psi_N^{\uparrow}(t_f)\psi_{N*}^{\uparrow,\dagger}(0)\rangle_{\mathcal{E}_z} \\ \langle \psi_{N*}^{\uparrow}(t_f)\psi_N^{\uparrow,\dagger}(0)\rangle_{\mathcal{E}_z} & \langle \psi_{N*}^{\uparrow}(t_f)\psi_{N*}^{\uparrow,\dagger}(0)\rangle_{\mathcal{E}_z} \end{pmatrix}$$

3. Non-Hermitian GEVP $v_L^\dagger C_{2pt,\mathcal{E}}^{\psi\psi^\dagger,\uparrow}(t_f) v_R = e^{-m_0(t_f-t_0)} v_L^\dagger C_{2pt,\mathcal{E}}^{\psi\psi^\dagger,\uparrow}(t_0) v_R$

4. Construct the optimal operator

$$O_{\mathcal{N}_0}^{L,\uparrow} = v_{L,0}^*\psi_N^{\uparrow} + v_{L,1}^*\psi_{N*}^{\uparrow}$$

$$O_{\mathcal{N}_0}^{\dagger,R,\uparrow} = v_{R,0}\psi_N^{\uparrow,\dagger} + v_{R,1}\psi_{N*}^{\uparrow,\dagger}$$

5. Calculate the nEDM

$$d_n/\bar{\theta} \approx \frac{1}{\mathcal{E}_z}\frac{\langle O_{\mathcal{N}_0}^{L,\uparrow}(t_f)\, q_{top}(\tau)\, O_{\mathcal{N}_0}^{\dagger R,\uparrow}(0)\rangle_{\mathcal{E}_z}}{\langle O_{\mathcal{N}_0}^{L,\uparrow}(t_f)\, O_{\mathcal{N}_0}^{\dagger R,\uparrow}(0)\rangle_{\mathcal{E}_z}} \rightarrow \frac{1}{\mathcal{E}_z}\langle \mathcal{N}_0^{L,\uparrow}|q_{top}(0)|\mathcal{N}_0^{R,\uparrow}\rangle_{\mathcal{E}_z}$$

Challenges for lattice nEDM calculations:

1. Large noise from $Q_{top} = \sum_{\tau}\sum_{\vec{y}} \frac{G^a_{\mu\nu}(\tau,\vec{y})\widetilde{G}^a_{\mu\nu}(\tau,\vec{y})}{32\pi^2}$

2. Excited-state contamination caused by parity mixing.

Our method:

1. Use time-local, spatially summed qtop $q_{top}(\tau) = \sum_{\vec{y}} \frac{G^a_{\mu\nu}(\tau,\vec{y})\widetilde{G}^a_{\mu\nu}(\tau,\vec{y})}{32\pi^2}$

2. Solve a non-Hermitian GEVP to isolate the deformed ground state

Outline

- The calculations of EDM on lattice using background field method

- **Numerical results**

Lattice size	Lattice spacing	Pion mass
$24^3 \times 64$	0.1105fm	340MeV
$24^3 \times 64$	0.1105fm	420MeV
$24^3 \times 64$	0.1105fm	576MeV

- Summary and outlook

nEDM from different nucleon interpolating operators

- Chiral covariant and non-covariant nucleon interpolating operators

$U(1)_A$ **rotation** $q_i \rightarrow e^{i\theta_A\gamma_5} q_i,$ Y. Ema, T. Gao, M. Pospelov, A. Ritz, PRD (2024)

Chiral non-covariant operators

$$O_1 = \epsilon^{abc}(d^{aT}C\gamma_5 u^b)d^c\,,$$

$$O_2 = -\epsilon^{abc}(d^{aT}Cu^b)\gamma_5 d^c\,,$$

Chiral covariant operators

$$O_m = O_1 - O_2$$

$$O_m \rightarrow e^{3i\theta_A\gamma_5}O_m$$

Chiral covariant operators

$$O_p = O_1 + O_2$$

$$O_p \rightarrow e^{-i\theta_A\gamma_5}O_p$$

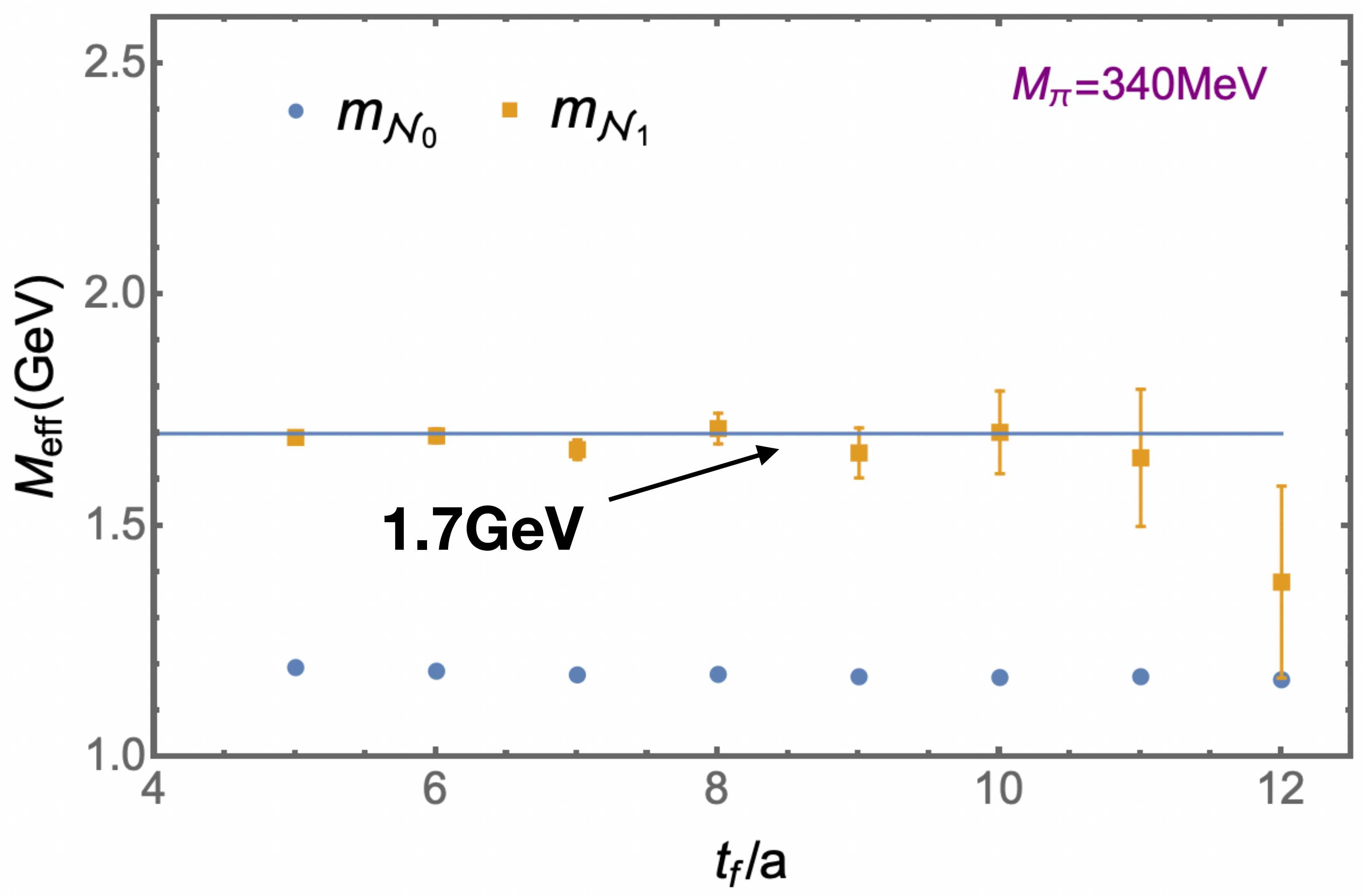
1. The result from the operator O_p has larger excited state contamination

2. The results from the non-covariant operator O_1 and the covariant operator O_m are consistent

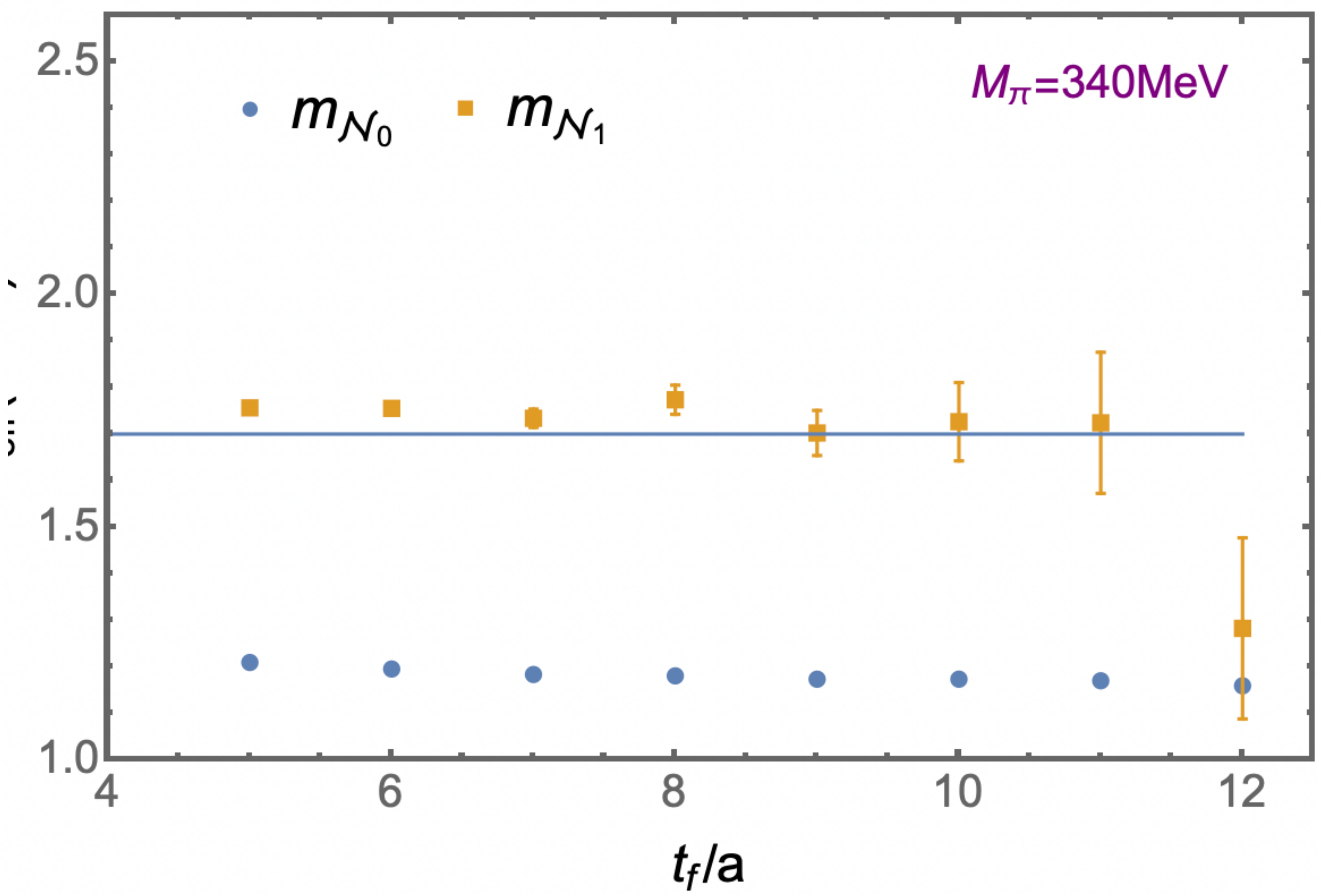
Energy spectrum

- Energy levels extracted using different interpolating operators

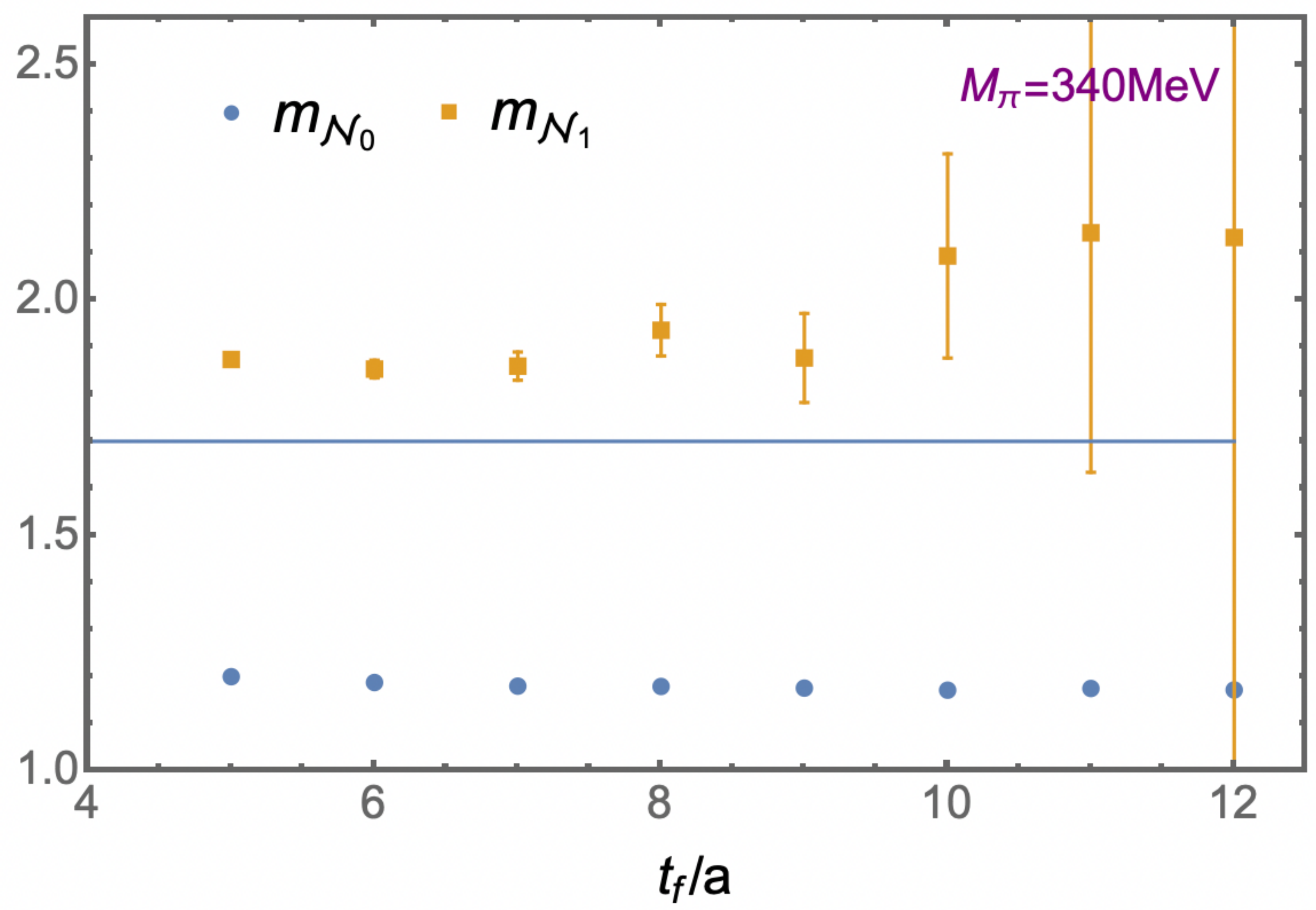
O_1



O_m



O_p



What does the missing energy level mean?

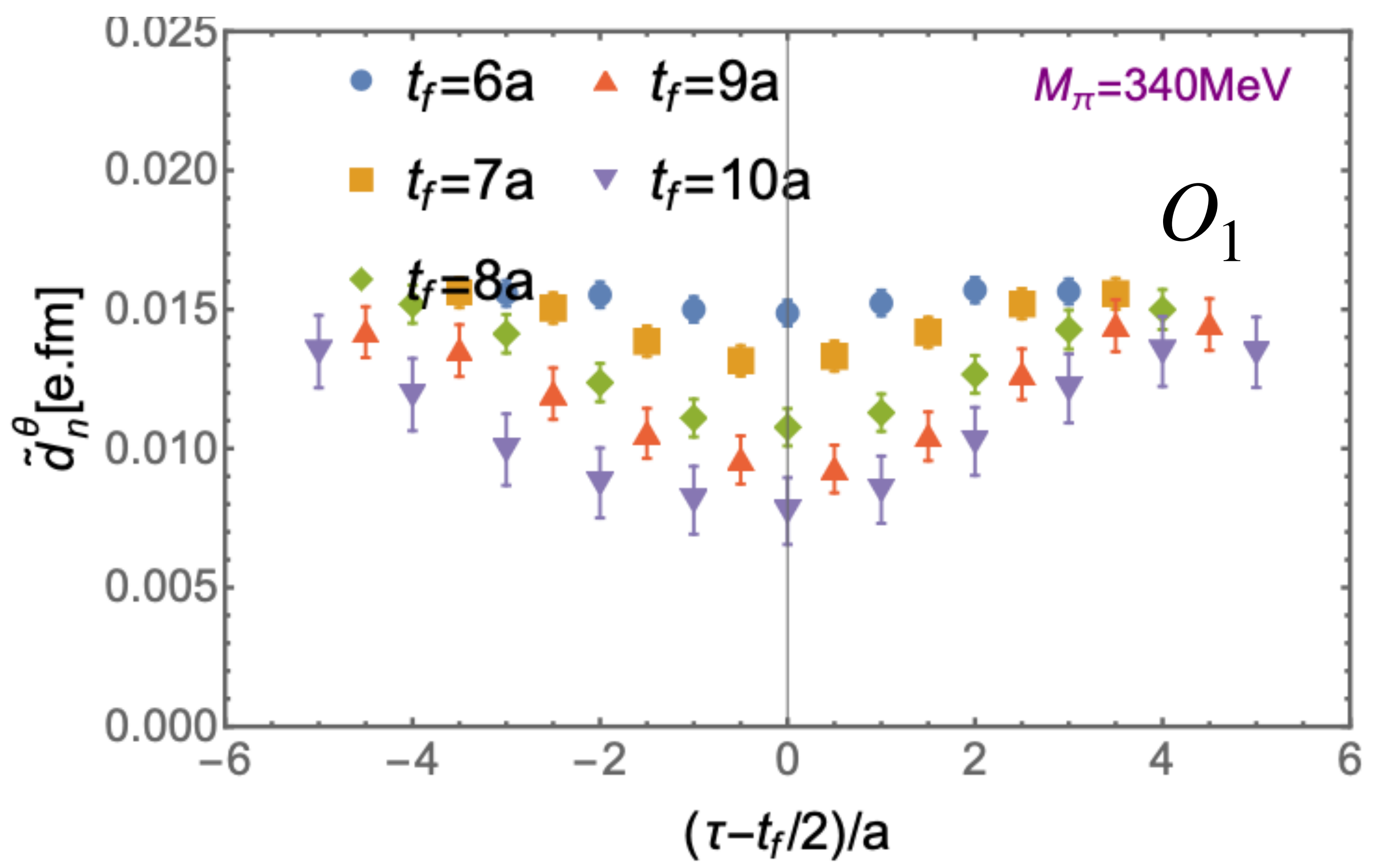
The excited state with a mass around 1.7GeV physically exists; O_p has insufficient overlap to isolate it as a separate GEVP eigenstate. Therefore, GEVP method can not remove its contamination to the ground state matrix element.

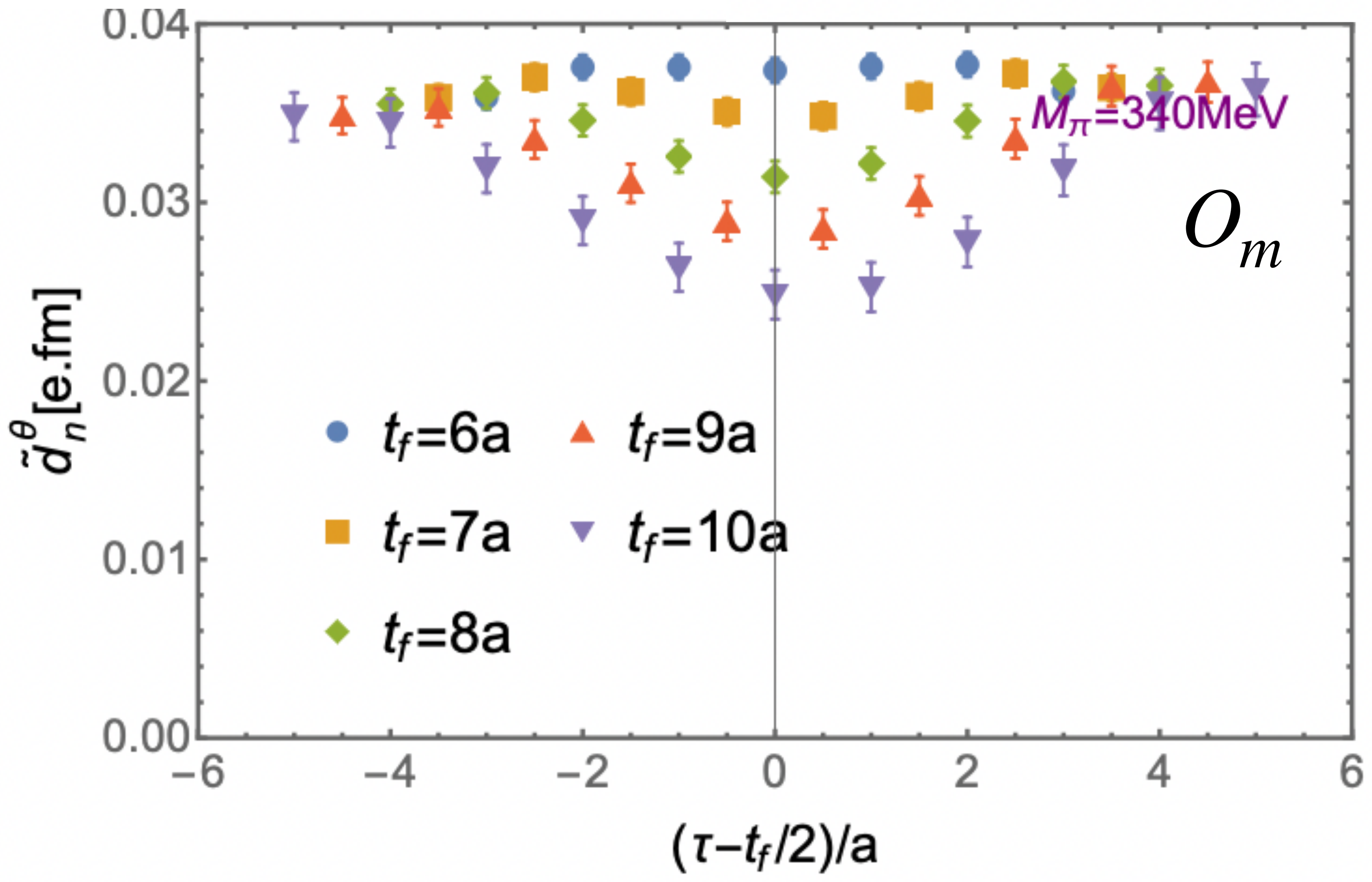
Comparison between parity and variational operators

- The results with positive-parity operator $\tilde{d}_n^{\theta}(t_f,\tau) = \dfrac{1}{\mathcal{E}_z}\dfrac{\langle \psi_N^{\uparrow}(t_f)\, q_{top}(\tau)\, \psi_N^{\uparrow\dagger}(0)\rangle_{\mathcal{E}_z}}{\langle \psi_N^{\uparrow}(t_f)\psi_N^{\uparrow\dagger}(0)\rangle_{\mathcal{E}_z}}$

Positive-parity component

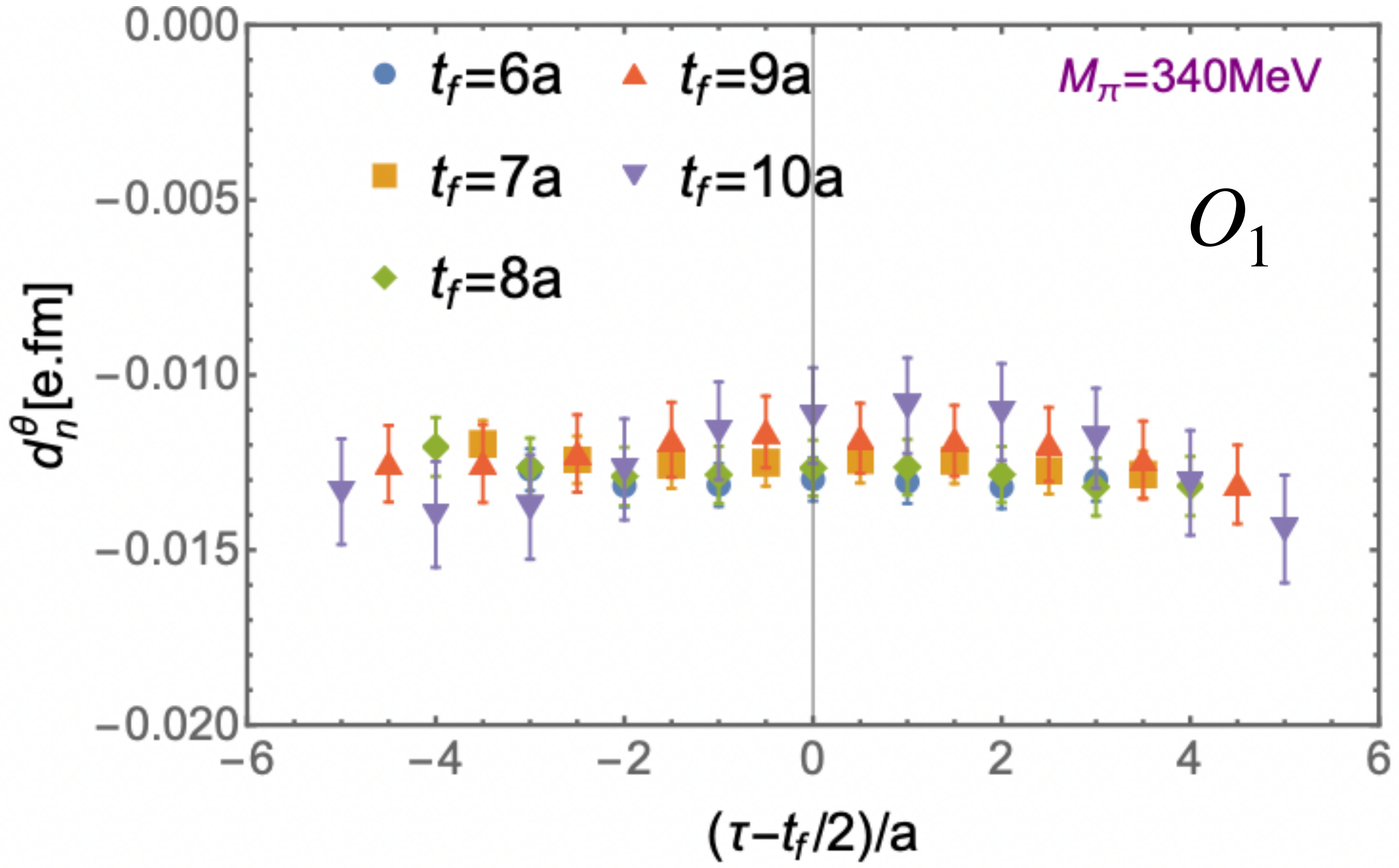
$$\psi_N^{\uparrow} = \frac{1}{4}(1+\gamma_4)(1+\Sigma_z)O$$

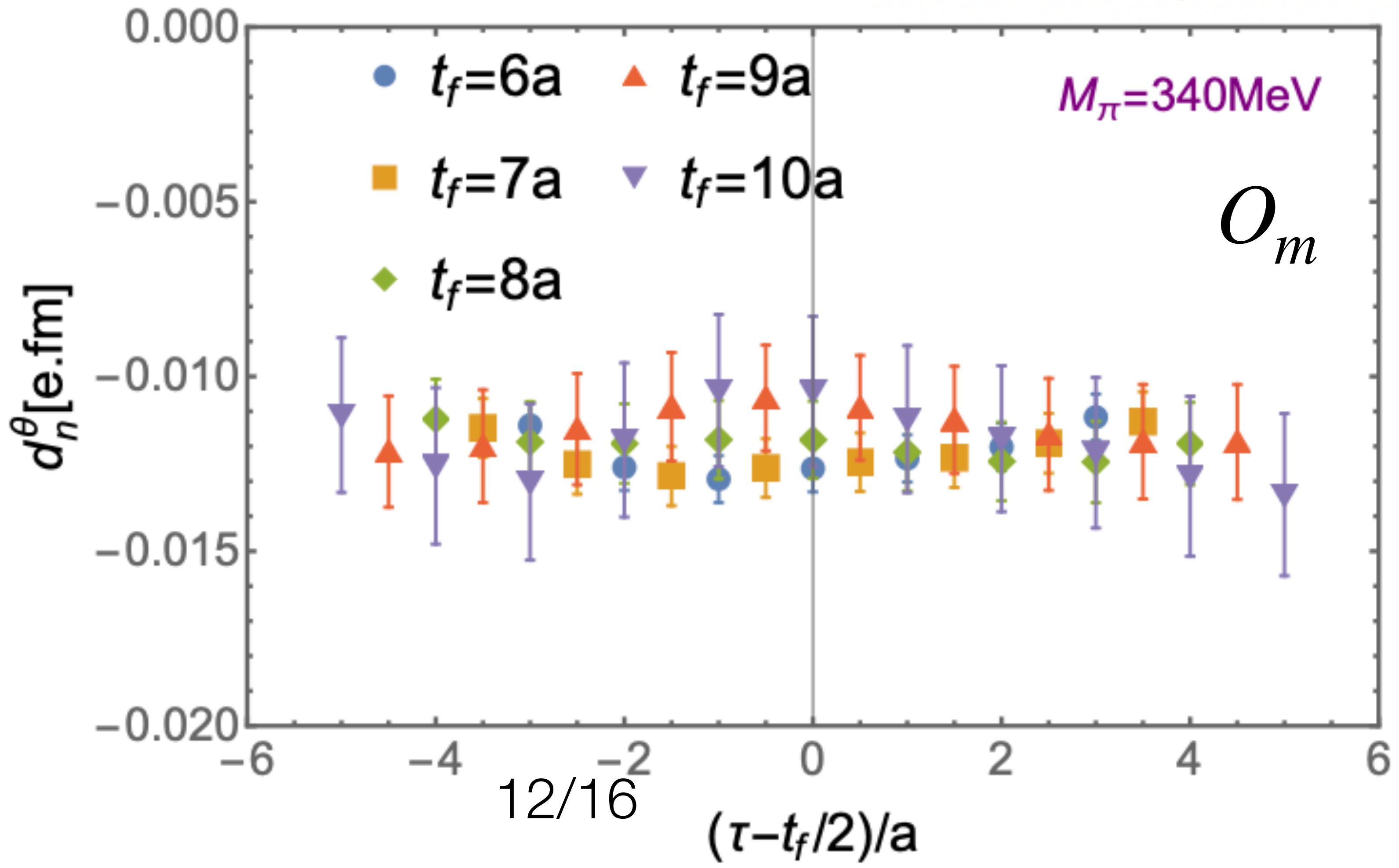




1. **Strong excited state contamination**
2. **Large operator discrepancy between O_1 and O_m**
3. **Even the sign is unreliable**

- The results with variational operators $d_n^{\theta} = \dfrac{1}{\mathcal{E}_z}\dfrac{\langle O_{\mathcal{N}_0}^{L,\uparrow}(t_f)\, q_{top}(\tau)\, O_{\mathcal{N}_0}^{\dagger R,\uparrow}(0)\rangle_{\mathcal{E}_z}}{\langle O_{\mathcal{N}_0}^{L,\uparrow}(t_f)\, O_{\mathcal{N}_0}^{\dagger R,\uparrow}(0)\rangle_{\mathcal{E}_z}}$



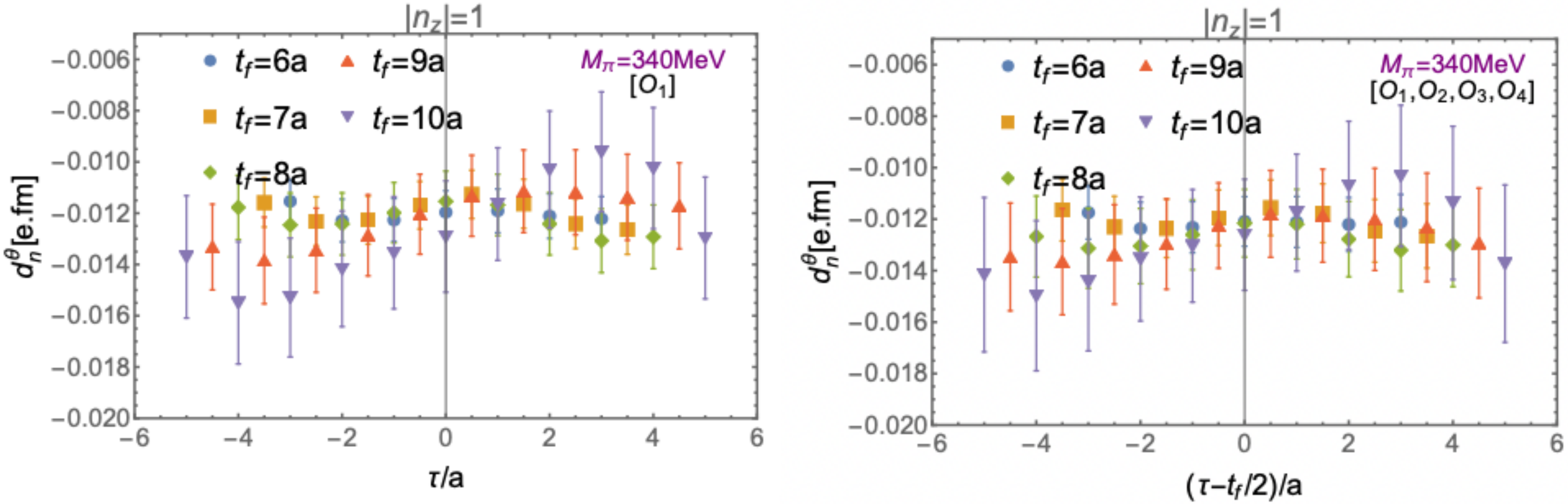


1. **Weak excited state contamination**
2. **The results of different interpolators are consistent**

nEDM from multi-operator GEVP

- Suppression of excited state contamination using a multi-operator GEVP

$$O_1 = \epsilon^{abc}(d^{aT}C\gamma_5 u^b)d^c\,, \qquad O_2 = -\epsilon^{abc}(d^{aT}Cu^b)\gamma_5 d^c\,,$$
$$O_3 = \epsilon^{abc}(d^{aT}C\gamma_4\gamma_5 u^b)d^c\,, \qquad O_4 = \epsilon^{abc}(d^{aT}C\gamma_4 u^b)\gamma_5 d^c,$$



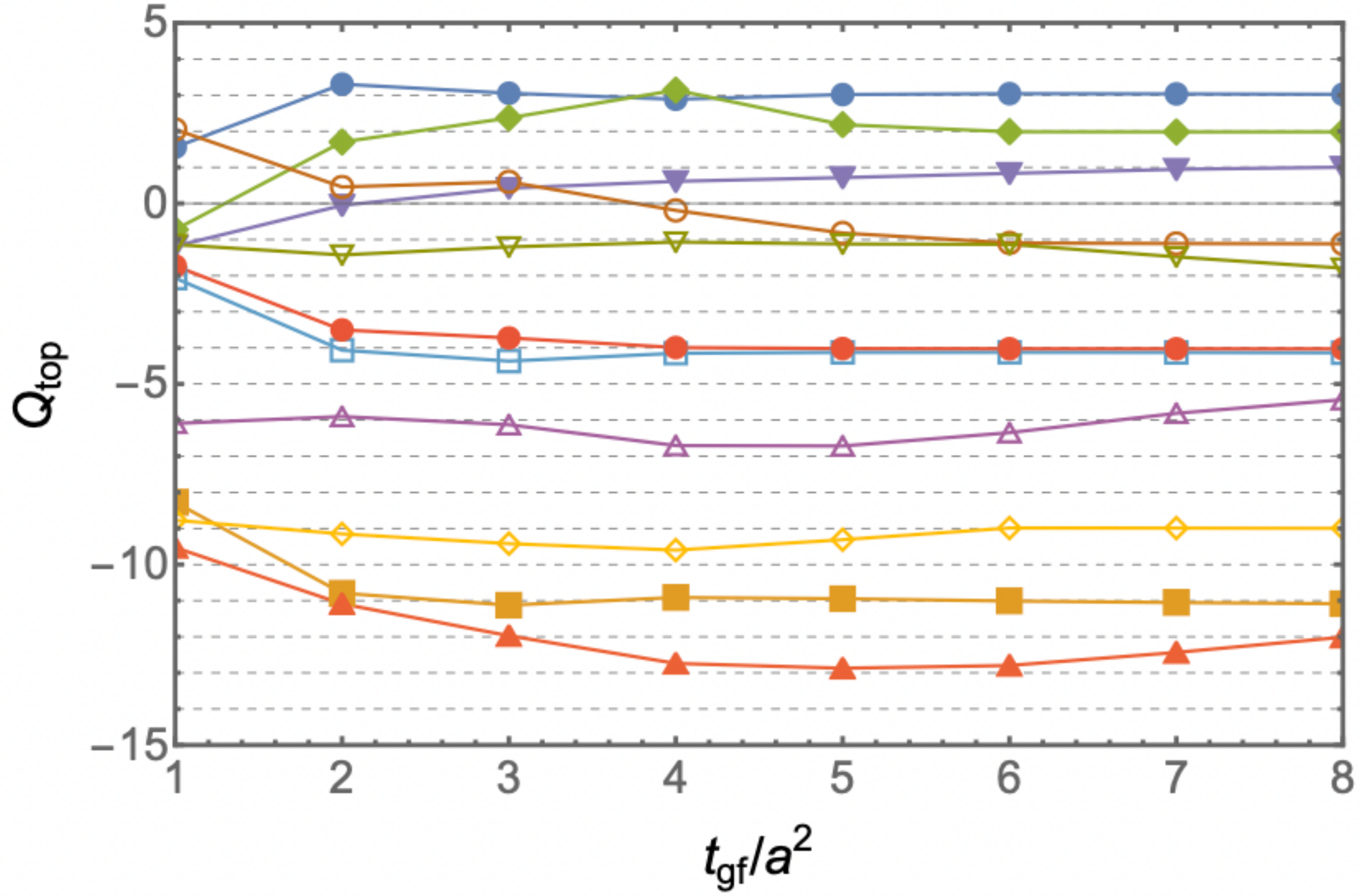
Gradient flow

Gradient flow $\frac{d}{dt_{\rm GF}} B_\mu(t_{\rm GF}) = D_\mu G_{\mu\nu}(t_{\rm GF})\,, \quad B_\mu(0) = A_\mu$ **smoothing radius** $r = (8t_{gf})^{1/2}$

[M.Luscher, JHEP08:071; 1006.4518]

1 Global topology stabilizes

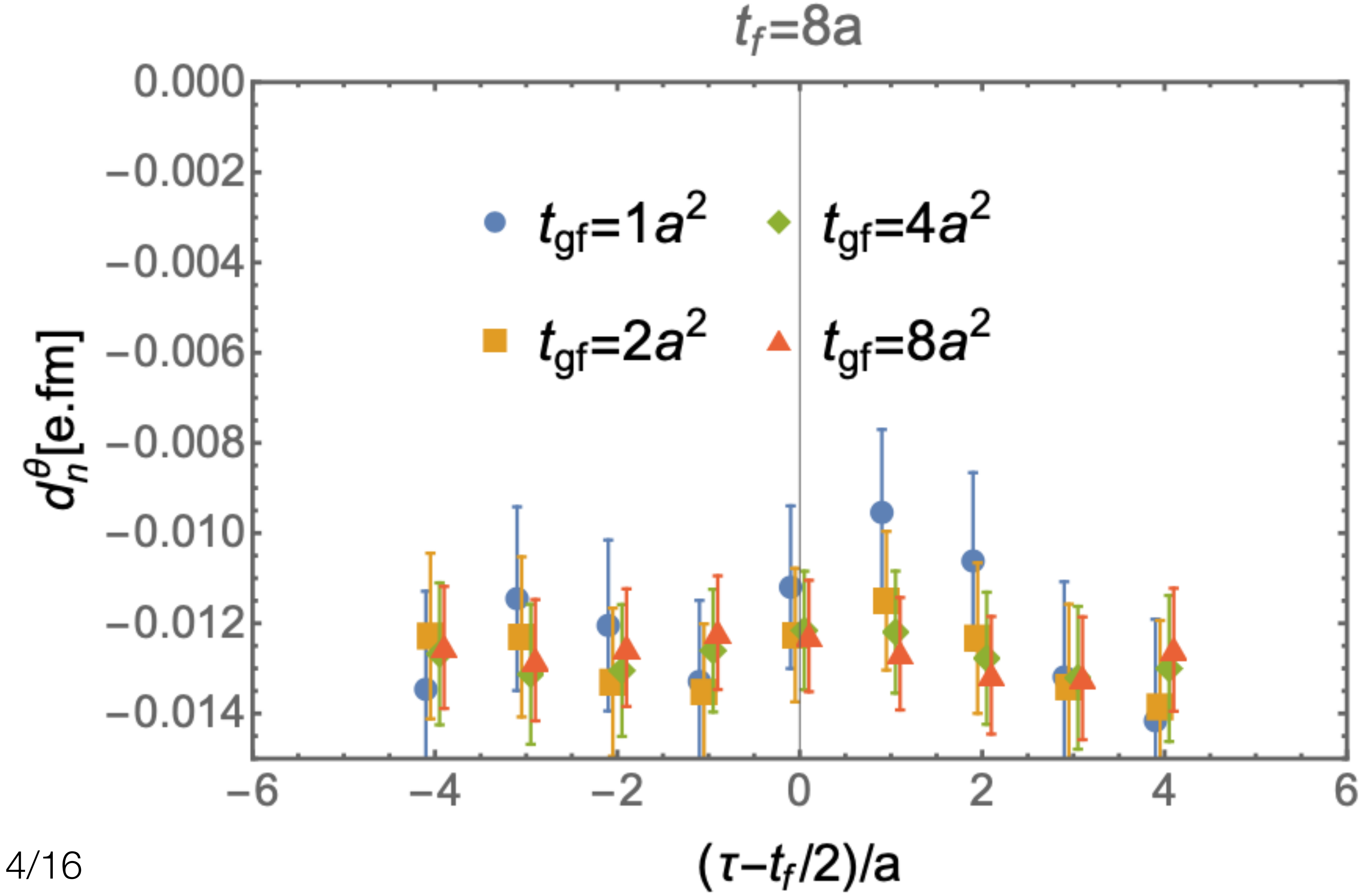
The fluctuations are suppressed, and each configuration approaches a stable near-integer Qtop plateau.



2. The extracted nEDM remains unchanged

nEDM with time-local q_{top}

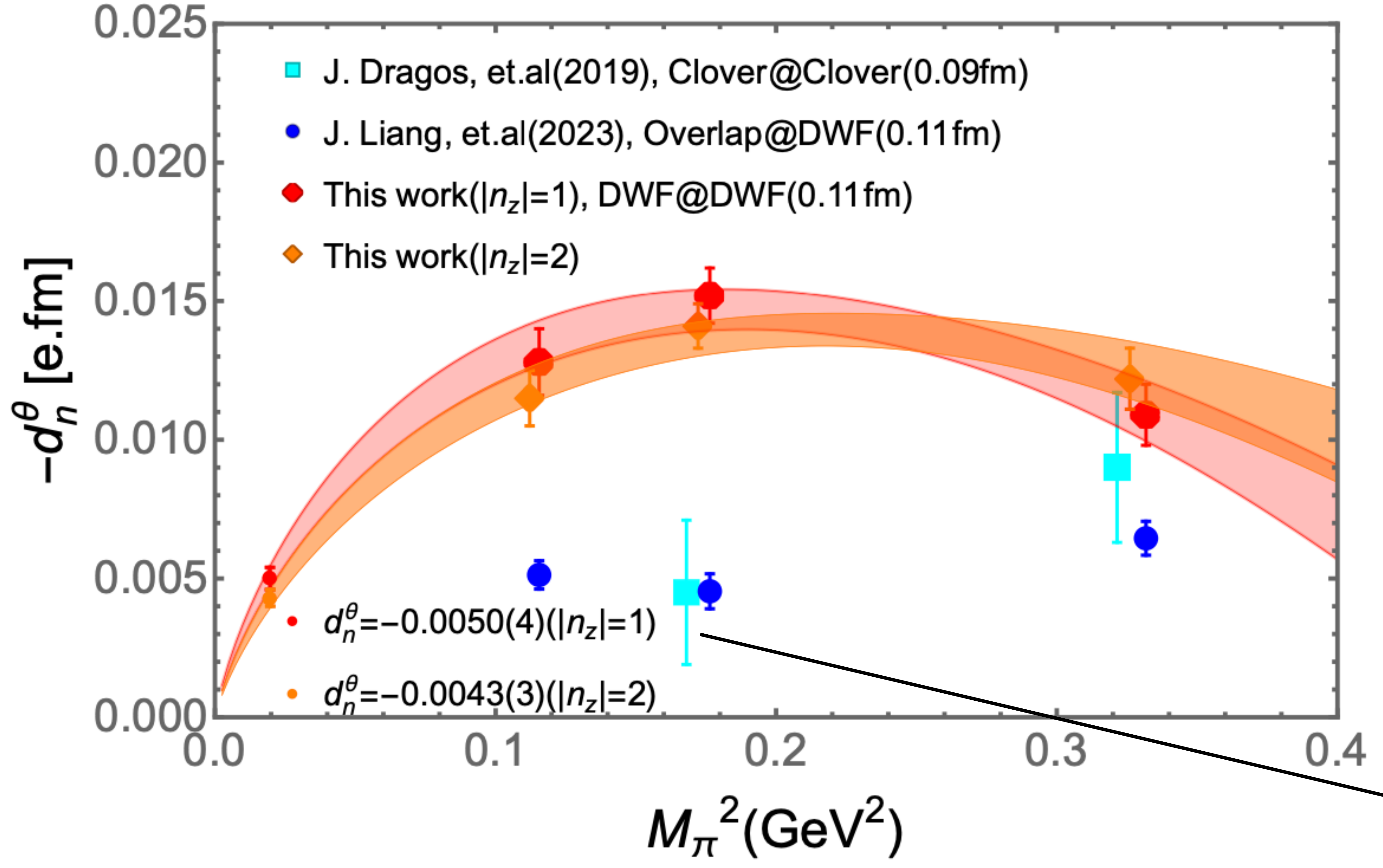
$$d_n^\theta(t_f,\tau) = \frac{1}{\mathcal{E}_z}\frac{\langle O^{L,\uparrow}_{\mathcal{N}_0}(t_f)\, q_{top}(\tau;t_{gf})\, O^{\dagger R,\uparrow}_{\mathcal{N}_0}(0)\rangle_{\mathcal{E}_z}}{\langle O^{L,\uparrow}_{\mathcal{N}_0}(t_f)\, O^{\dagger R,\uparrow}_{\mathcal{N}_0}(0)\rangle_{\mathcal{E}_z}}$$



Summary of nEDM results

- Chiral extrapolation and summary of the nEDM result at the physical point

Chiral extrapolation



$$d_n^\theta = c_0 m_\pi^2 + c_1 m_\pi^2 log\left(\frac{m_\pi^2}{m_{N,phy}^2}\right)$$

nEDM results at the physical point

		$[10^{-3}\, e\cdot \mathrm{fm}]$
Dragos et al(2019)	[36]	$d_n/\theta = -1.5(7)$
Alexandrou et al(2020)	[37]	$\|d_n/\bar\theta\| = 0.9(2.4)$
Bhattacharya et al (2021)	[38]	$d_n/\bar\theta = -28(18)(54)$
Liang et al (2023)	[39]	$d_n/\bar\theta = -1.48(0.14)(0.31)$
This work		$d_n/\bar\theta = -5.0(0.4)(0.8)$

Form factor method

$$\langle p',\sigma'|J^\mu|p,\sigma\rangle_{\cancel{CP}} = \bar u_{p',\sigma'}\left[F_1(Q^2)\gamma^\mu + \left(F_2(Q^2)+iF_3(Q^2)\gamma_5\right)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]u_{p,\sigma},$$

Electric dipole moment $\quad d_n = \dfrac{F_3(Q^2\to 0)}{2m_n}$

Summary and outlook

- To obtain a statistically significant nEDM, we propose to use a local definition of the topological charge on a single time-slice, motivated by the Feynman–Hellmannn theorem.
- We use GEVP method to isolate the deformed ground nucleon state in the presence of electric field, suppressing the excited state contamination and gradient flow dependence.
- The nEDM calculated using different interpolating operators are consistent with each other.
- Conventional systematic errors like discretisation, finite-volume effect and chiral extrapolation effects will be addressed in the future work.

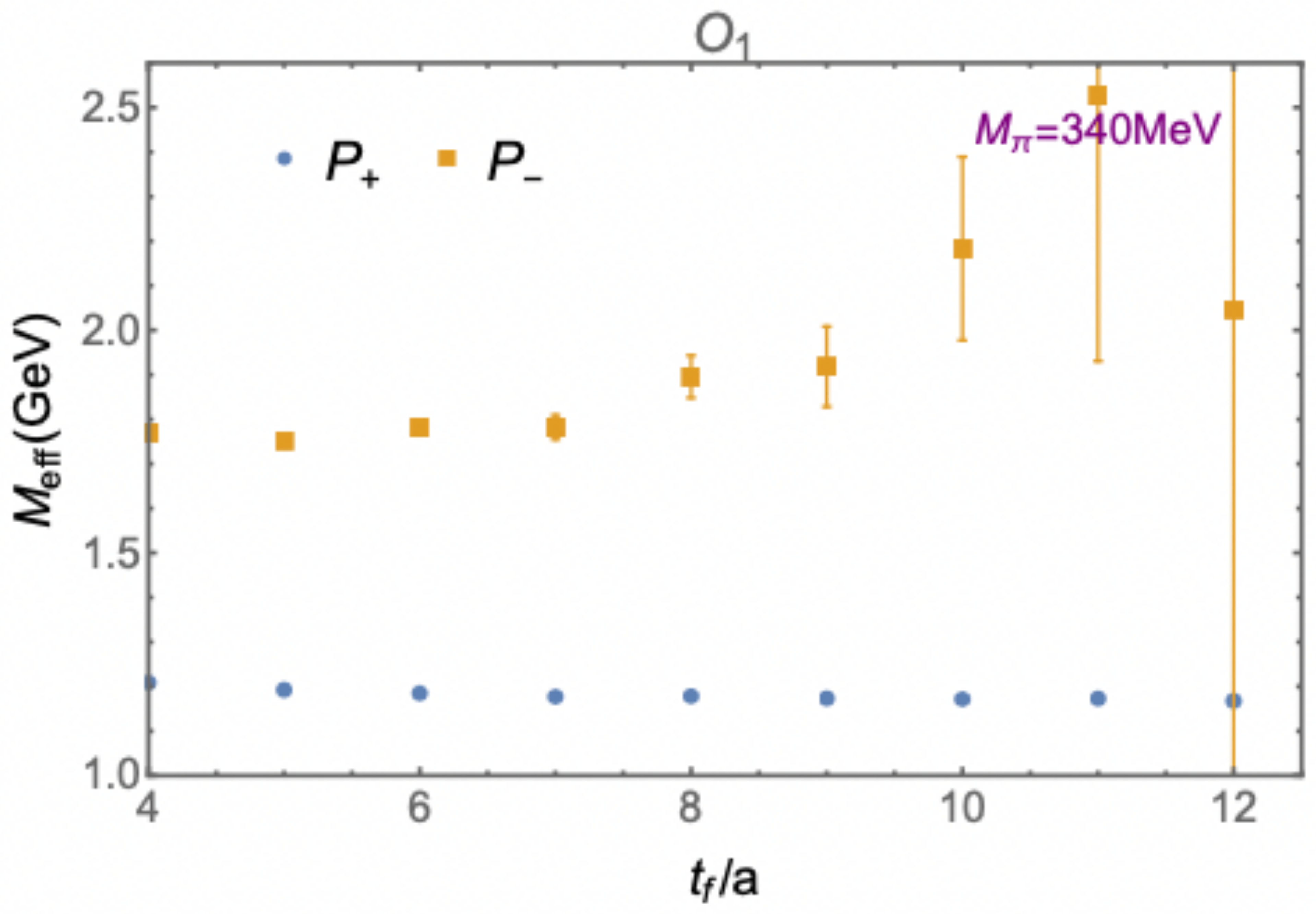
Thank you for your attention!

Backup slides

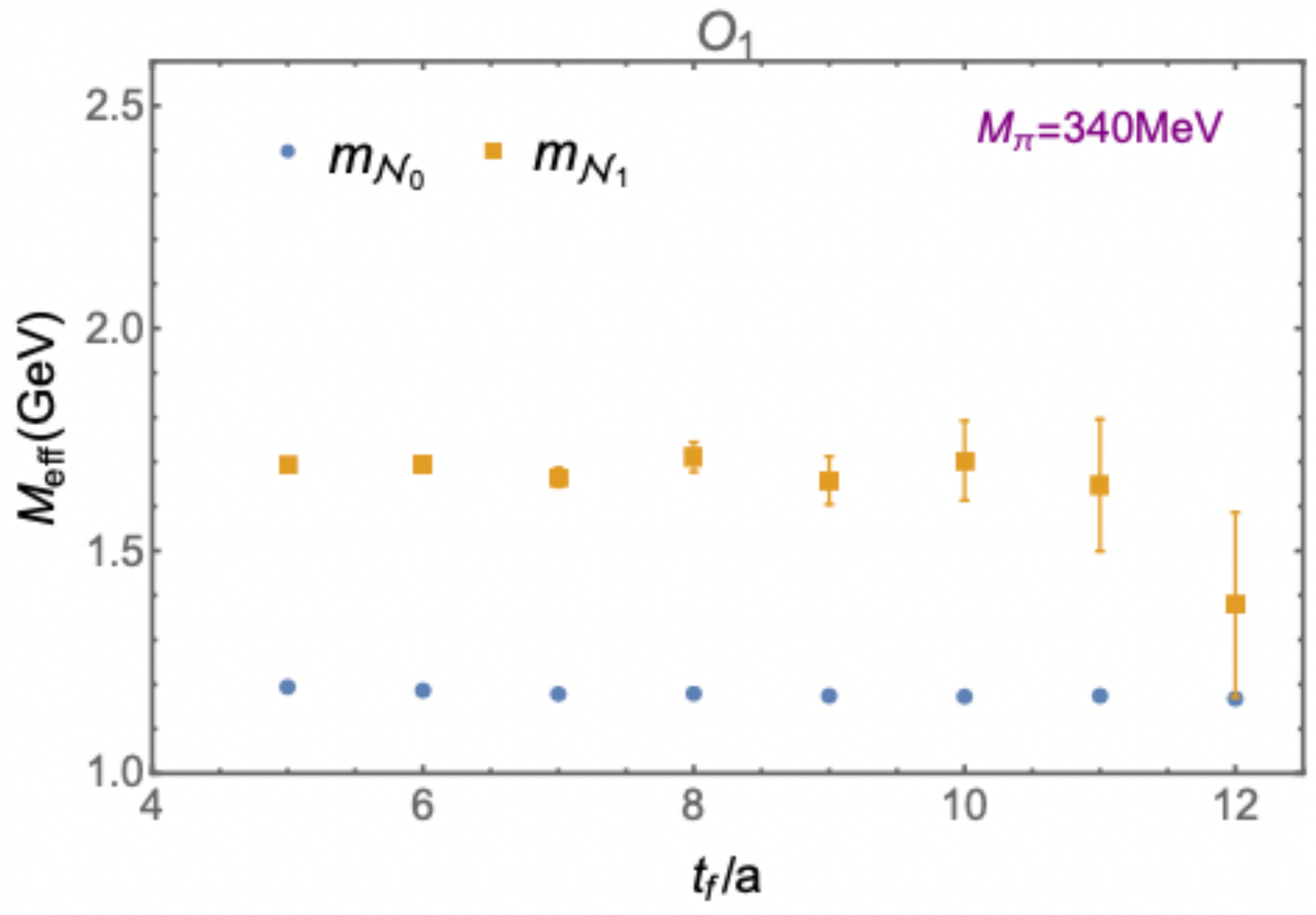
Effective mass

- Effective masses of parity and variational operators

Effective masses of parity operators



Effective masses of variational operators



All mode average and low mode average

T. Blum, T. Izubuchi and E. Shintani, PRD 88 (2013)

2pt with exact CG	1
2pt with sloppy CG	64
low mode all to all 2pt	Volume

AMA $$\langle(\mathcal{O}\ldots)_{\rm AMA}\rangle_{N_{\rm sl}} = \langle(\mathcal{O}\ldots)_{\rm sl}\rangle_{N_{\rm sl}} + \langle((\mathcal{O}\ldots)_{\rm ex} - (\mathcal{O}\ldots)_{\rm sl})\rangle_{N_{\rm ex}}\ ,$$

AMA + LMA $$\langle(\mathcal{O}\ldots)_{\rm LMA}\rangle = \langle(\mathcal{O}\ldots)_{\rm low}\rangle_{N_{\rm V}} + \langle((\mathcal{O}\ldots)_{\rm AMA} - (\mathcal{O}\ldots)_{\rm low})\rangle_{N_{\rm sl}}$$