Syracuse University

SOCS

Spins on Curved Surfaces

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Main objective

- **Study:**
  - discrete quantum spin systems on curved geometries
- **Investigate:**
  - Thermodynamic behavior
  - Scaling laws
  - Information scrambling
  - Out of Time Ordered Correlators (OTOCs)
- **Using:**
  - Density Matrix Renormalization Group (DMRG) – Matrix Product State (MPS)

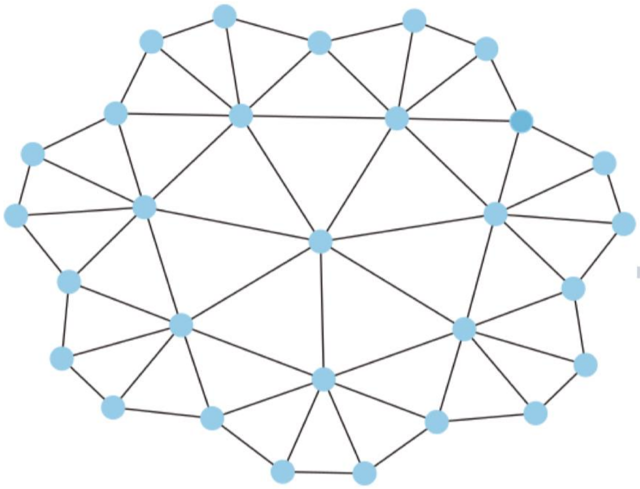
What Path?

- **<u>Model:</u>**
  - 1d and 2d Quantum Ising model on AdS space.
  - Samlodia et.al. **[*Eur.Phys.J.C* 86 (2026), *Phys.Rev.D* 113 (2026)]**
  - Goksu et.al. **[PhysRevD.109.054513 (2024)]**

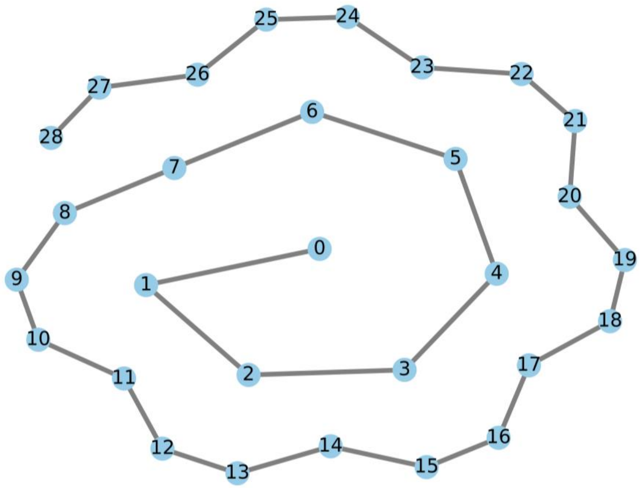
- **<u>What about curvature:</u>**
  - Site dependent couplings or,
  - Triangulated fully connected graphs

- **<u>MPS for 2-dimensions:</u>**
  - Start at the center node and traverse a path in spiral
  - Connect the other neighbors later

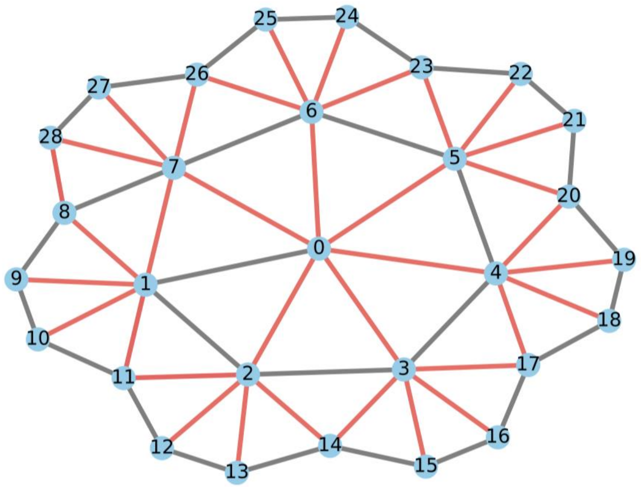
Ising model in AdS_{2+1}



Adjacency Graph of a Hyperbolic Tessellation



MPS description of the Hyperbolic Tessellation



Introduce long-range Interactions

{3,7} Tessellation

Ising model in AdS_{2+1} - Setup

$$H = -J_{zz} \sum_{\langle j,k \rangle} Z_j Z_k - \sum_j J_x X_j - m \sum_j Z_j$$

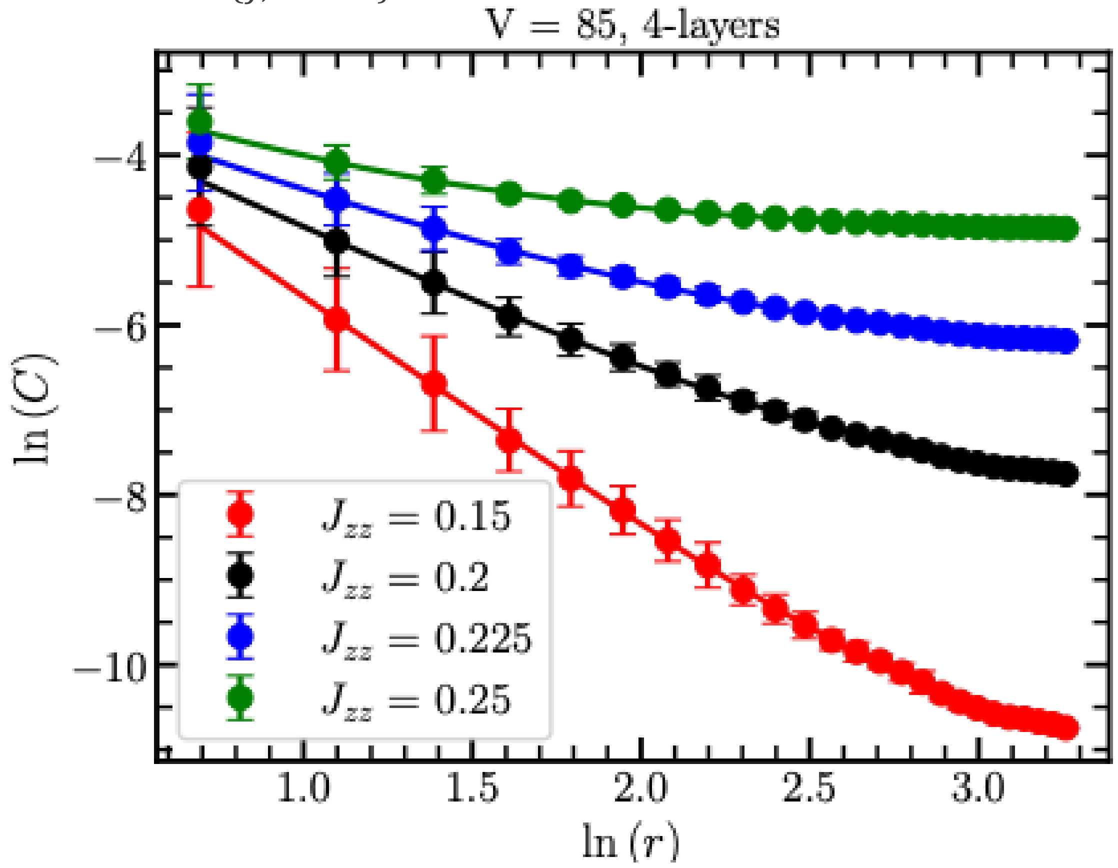
m = 0.001, fixed
J_x **= 1.0, fixed**

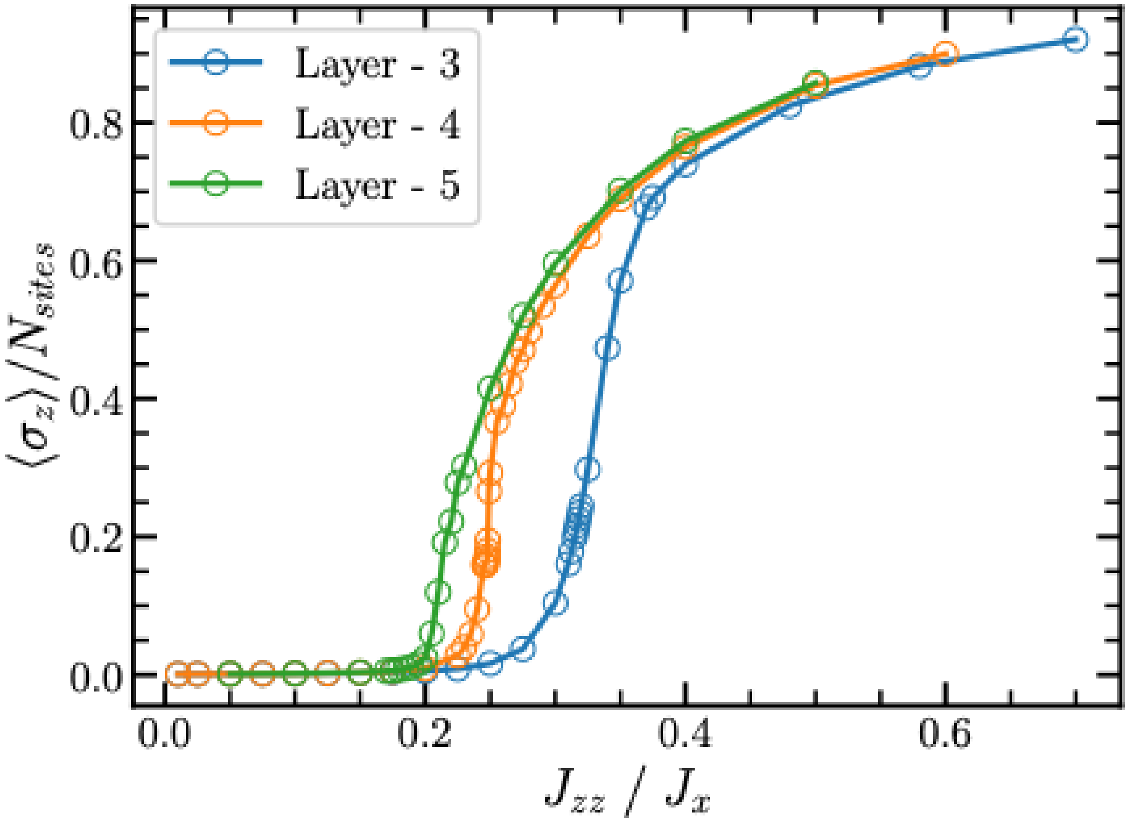
Number of layers	N_{sites}	N_b
3	29	21
4	85	54
5	232	147

TABLE I: Lattice volume for (3,7) hyperbolic tessellation

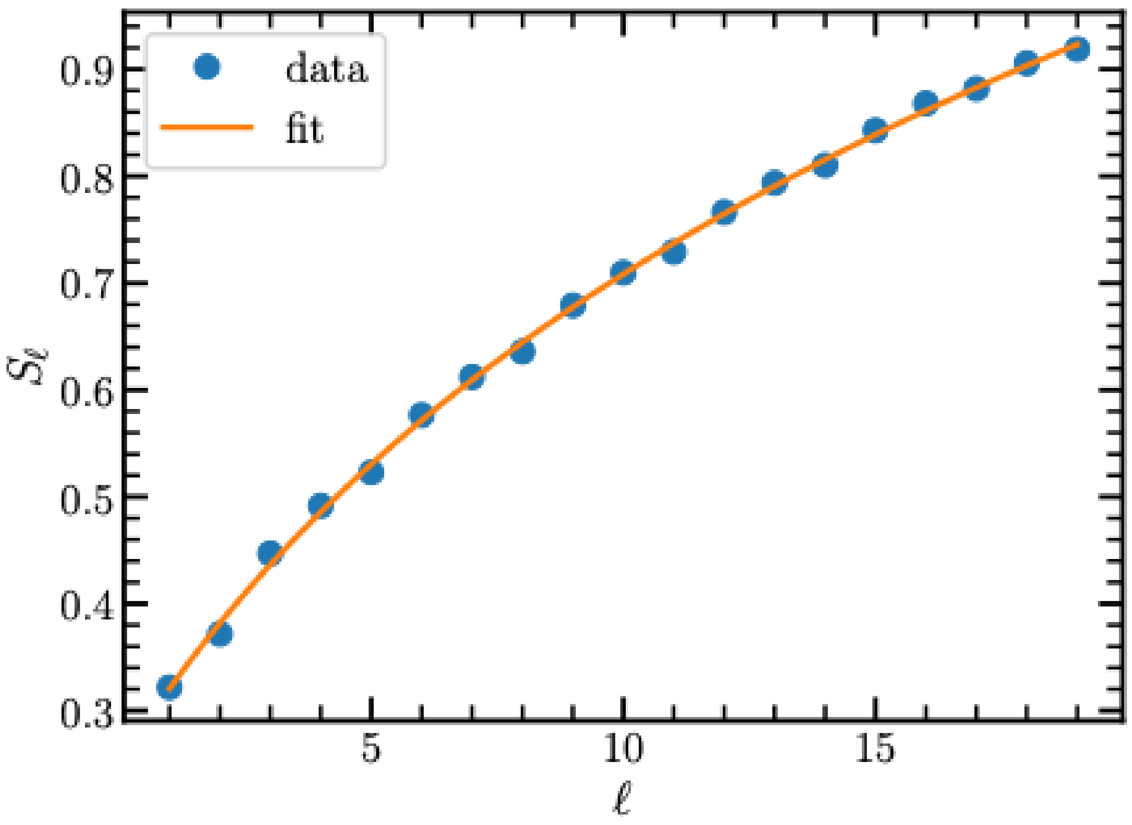
Ising model in AdS_{2+1} - Magnetization & Correlators

$$C(r) := \frac{1}{N_b} \sum_{\{j,k \in \delta D\}} \delta(r - |j - k|)(\langle Z_j Z_k \rangle - \langle Z_j \rangle \langle Z_k \rangle)$$

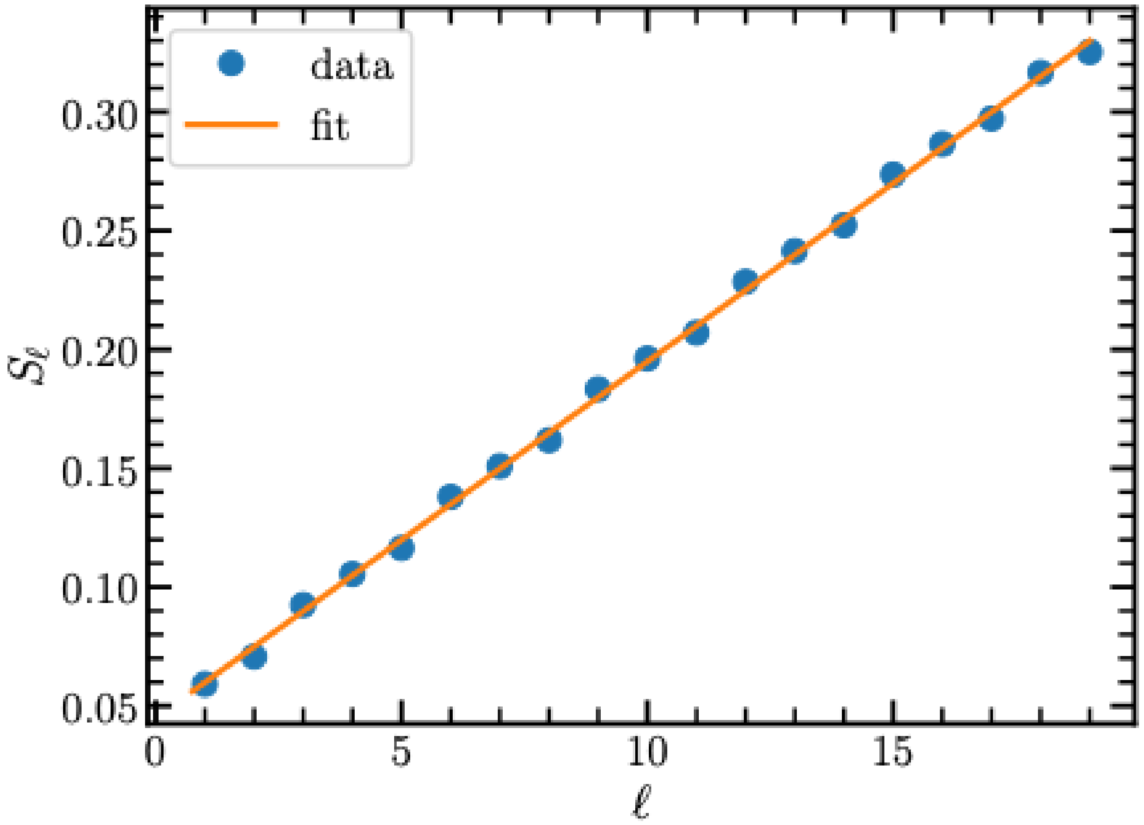




Ising model in AdS_{2+1} - Boundary Entanglement Entropy



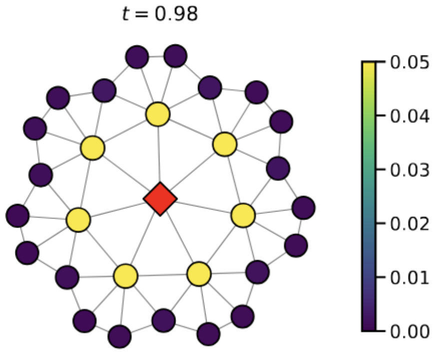
Critical Point – Logarithmic
3-layers



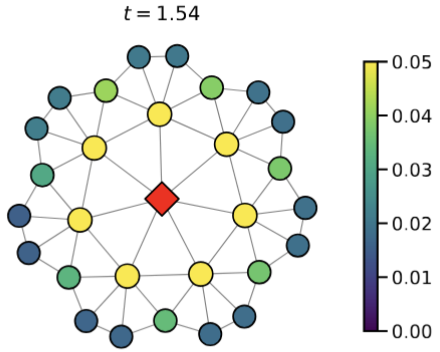
Away from Criticality – Linear
3-layers

$$O_{ij}(t) := \langle \left| \left[Z_i(t), Z_j(0) \right] \right|^2 \rangle = 2(1 - Re[\langle Z_i^\dagger(t) Z_j(0)^\dagger Z_i(t) Z_j(0) \rangle])$$

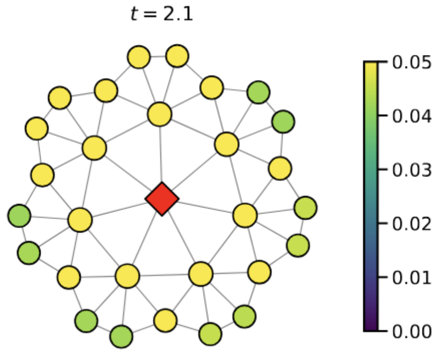
Ising model in AdS_{2+1} - OTOCs



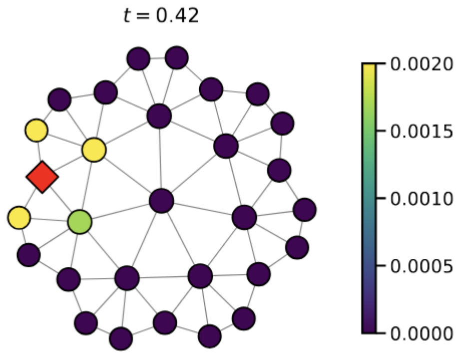
(a) $t = 0.98$, source at center node



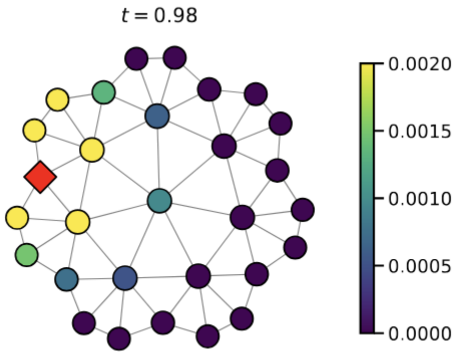
(b) $t = 1.54$, source at center node



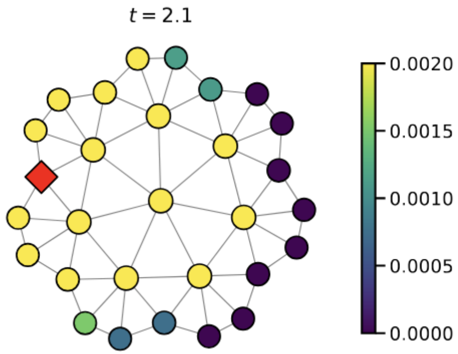
(c) $t = 2.1$, source at center node



(d) $t = 0.42$, source at first boundary node



(e) $t = 0.98$, source at first boundary node



(f) $t = 2.1$, source at first boundary node

Ising model in AdS_{1+1} - Setup

$$H = -J\sum_i \frac{(\eta_i+\eta_{i+1})}{2}\sigma_i^z\sigma_{i+1}^z + h\sum_i \eta_i\sigma_i^x + m\sum_i \eta_i\sigma_i^z$$

$$\eta_i = \cosh(\rho_i), \qquad \rho_i = -\ell_{\max} + i\,\frac{2\ell_{\max}}{N-1}$$

Controls curvature

Critical Point at J/h = 1

$$ds^2 = \cosh^2(\rho)dt^2 + d\rho^2$$

Ising model in AdS_{1+1} - Krylov State Complexity

$$C_K(t) = \sum_n n|\phi_n(t)|^2$$

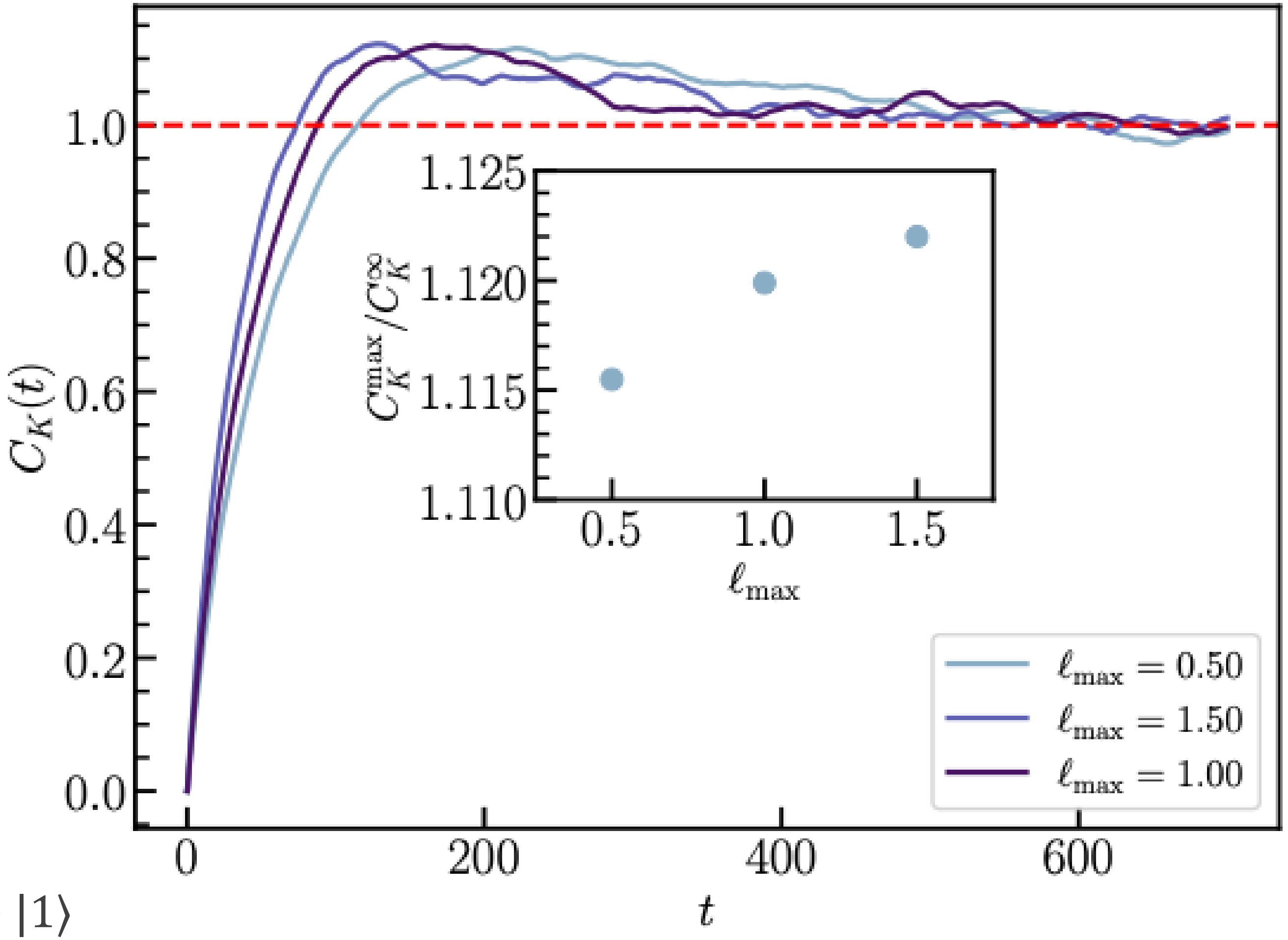
Krylov Basis Functions

n = 0,1,2…,K-1

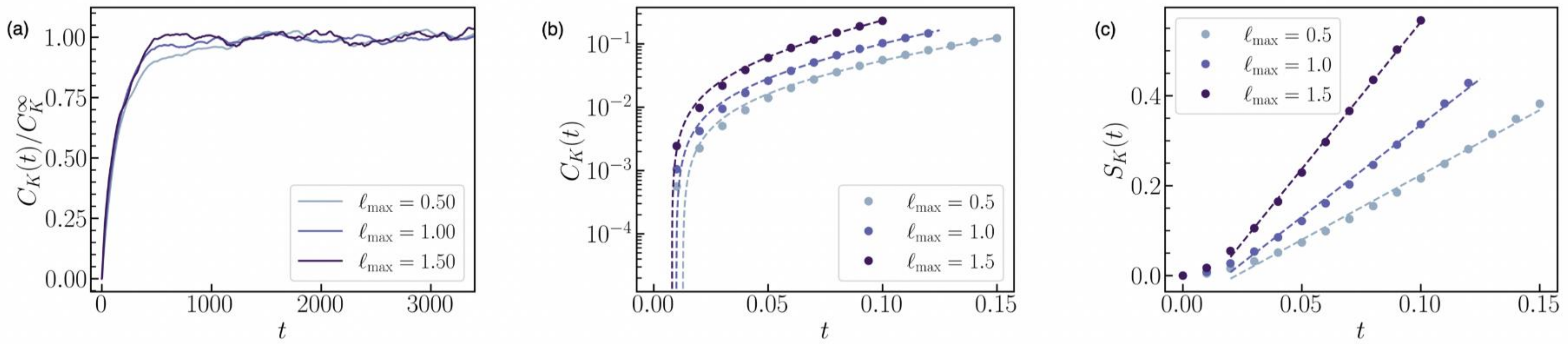
N = 11, J/h = 1.0, m = 0.25

Initial state

$$|\Psi_0\rangle = |+^y\rangle \otimes |1\rangle \otimes |+^y\rangle \otimes |1\rangle \ldots \otimes |+^y\rangle \otimes |1\rangle$$



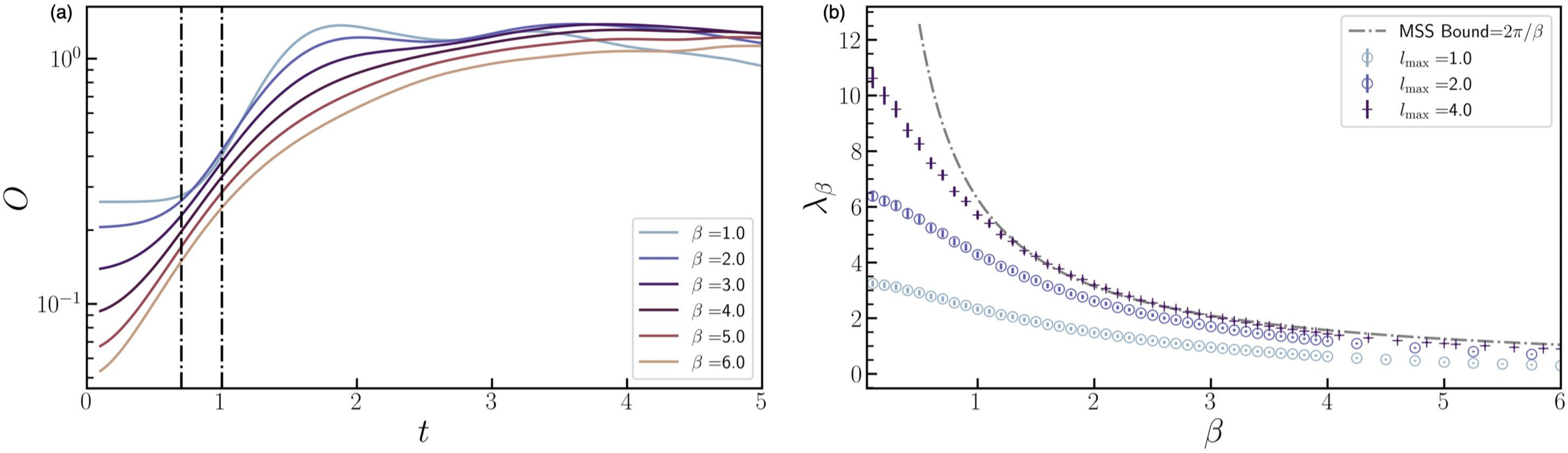
Ising model in AdS_{1+1} - Krylov Operator Complexity



$$S_K(t) = -\sum_n |\varphi_n(t)|^2 ln(|\varphi_n(t)|\verb|^|2)$$

1. Exponential Fit to Complexity
2. Linear Fit to Entropy

Ising model in AdS_{1+1} - Lyapunov Exponent from OTOCs



Fit exponentials to OTOCs – Lyapunov Exponent

N = 13, J/h = 1.0, m = 0.05

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Thank You

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